Do static atoms outside a Schwarzschild black hole spontaneously excite?

Hongwei Yu and Wenting Zhou

Department of Physics and Institute of Physics, Hunan Normal University, Changsha, Hunan 410081, China

(Received 28 June 2007; published 28 August 2007)

The spontaneous excitation of a two-level atom held static outside a four dimensional Schwarzschild black hole and in interaction with a massless scalar field in the Boulware, Unruh, and Hartle-Hawking vacuums is investigated, and the contributions of the vacuum fluctuations and radiation reaction to the rate of change of the mean atomic energy are calculated separately. We find that, for the Boulware vacuum, the spontaneous excitation does not occur and the ground-state atoms are stable, while the spontaneous emission rate for excited atoms in the Boulware vacuum, which is well behaved at the event horizon, is not the same as that in the usual Minkowski vacuum. However, for both the Unruh vacuum and the Hartle-Hawking vacuum, our results show that the atom would spontaneously excite, as if there were an outgoing thermal flux of radiation or as if it were in a thermal bath of radiation at a proper temperature which reduces to the Hawking temperature in the spatial asymptotic region, depending on whether the scalar field is in the Unruh or Hartle-Hawking vacuum.

DOI: 10.1103/PhysRevD.76.044023 PACS numbers: 04.70.Dy, 04.62.+v, 42.50.Ct, 97.60.Lf

I. INTRODUCTION

Spontaneous emission is one of the most important features of atoms, and so far mechanisms such as vacuum fluctuations [1,2], radiation reaction [3], or a combination of them [4] have been put forward to explain why spontaneous emission occurs. The ambiguity in physical interpretation arises because of the freedom in the choice of ordering of commuting operators of the atom and field in a Heisenberg picture approach to the problem. The controversy was resolved when Dalibard, Dupont-Roc, and Cohen-Tannoudji (DDC) [5,6] proposed a formalism which distinctively separates the contributions of vacuum fluctuations and radiation reaction by demanding a symmetric operator ordering of atom and field variables. The DDC formalism has recently been generalized to study the spontaneous excitation of uniformly accelerated atoms in interaction with vacuum fluctuations of scalar and electromagnetic fields in a flat space-time [7–11], and these studies show that, when an atom is accelerated, the delicate balance between vacuum fluctuations and radiation reaction that ensures the ground-state atom's stability in vacuum is altered, making possible the transitions to excited states for ground-state atoms even in vacuum.

Inspired by an equivalence principle-type argument, i.e., the same accelerated atoms are seen by comoving observers as static ones in a uniform "gravitational field," one may wonder what happens if an atom is held static in a curved space-time, such as that of a black hole, for example. Do static atoms spontaneously excite outside a black hole, and if they do, will the excitation rate be what one expects assuming the existence of Hawking radiation from black holes? Answers to these questions may reveal a relationship between the Hawking radiation and the spontaneous excitation of atoms outside a black hole, and thus provide an alternative derivation of Hawking radiation. When we move to study the spontaneous excitation of static atoms interacting with vacuum fluctuations of quantum fields in a curved space-time, a delicate issue then arises as to how the vacuum state of the quantum fields is determined. Normally, a vacuum state is associated with nonoccupation of positive frequency modes. However, the positive frequency of field modes is defined with respect to the time coordinate. Therefore, to define positive frequency, one has to first specify a definition of time. In a spherically symmetric black hole background, one definition is the Schwarzschild time, t, and it is a natural definition of time in the exterior region. The vacuum state, defined by requiring normal modes to be positive frequency with respect to the Killing vector $\partial/\partial t$, is called the Boulware vacuum. Other possibilities that have been proposed are the Unruh vacuum [12] and the Hartle-Hawking vacuum [13]. The Unruh vacuum is defined by taking modes that are incoming from $\mathcal{J}^-$ to be positive frequency with respect to $\partial/\partial t$, while those that emanate from the past horizon are taken to be positive frequency with respect to the Kruskal coordinate $\bar{u}$, the canonical affine parameter on the past horizon. The Hartle-Hawking vacuum, on the other hand, is defined by taking the incoming modes to be positive frequency with respect to $\bar{v}$, the canonical affine parameter on the future horizon, and outgoing modes to be positive frequency with respect to $\bar{u}$. The calculations of the values of physical observables, such as the expectation values of the energy-momentum tensor and the response rate of an Unruh detector in these vacuum states, have yielded the following physical understanding:

(i) The Boulware vacuum corresponds to our familiar concept of a vacuum state at large radii, but is problematic in the sense that the expectation value of the energy-momentum tensor, evaluated in a free-falling frame, diverges at the horizon.

(ii) The Unruh vacuum is the vacuum state that best approximates the state that we would obtain following the gravitational collapse of a massive body, since in the spatial asymptotic region, it corresponds to an outgoing flux of blackbody radiation at the Hawking temperature.

(iii) The Hartle-Hawking state, however, does not correspond to our usual notion of a vacuum, as it has thermal radiation incoming to the black hole from infinity and describes a black hole in equilibrium with a sea of thermal radiation.

In the current paper, we would like to apply the DDC formalism to study the spontaneous excitation of a static two-level atom outside a four dimensional Schwarzschild black hole in interaction with massless quantum scalar fields in all the above three vacuum states, aiming to answer the question of whether a static atom outside a black hole spontaneously excites. We also hope to gain more insight into the physical meaning of the vacuum states proposed so far in the black hole space-time, as well as to reveal a relationship between the Hawking radiation and spontaneous excitation of atoms. Let us note that, recently, we have already studied the spontaneous excitation of a static two-level atom interacting with massless scalar fields in both the Unruh vacuum and the Hartle-Hawking vacuum outside a $1 + 1$ dimensional Schwarzschild black hole and found that the atom spontaneously excites as if there is thermal radiation at the Hawking temperature emanating from the black hole [14].

II. GENERAL FORMALISM

Let us consider a two-level atom in interaction with a quantum real massless scalar field outside a Schwarzschild black hole. The metric of the space-time can be written in terms of the Schwarzschild coordinates as

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where M is the mass of the black hole. Without loss of generality, we assume a pointlike two-level atom on a stationary space-time trajectory $x(\tau)$, where τ denotes the proper time on the trajectory. The stationarity of the trajectory guarantees the existence of stationary atomic states, $|+\rangle$ and $|-\rangle$, with energies $\pm\frac{1}{2}\omega_0$ and a level spacing ω_0. The atom's Hamiltonian which controls the time evolution with respect to τ is given, in Dicke's notation [15], by

$$H_A(\tau) = \omega_0 R_3(\tau), \tag{2}$$

where $R_3 = \frac{1}{2}|+\rangle\langle+| - \frac{1}{2}|-\rangle\langle-|$ is the pseudospin operator commonly used in the description of two-level atoms [15]. The free Hamiltonian of the quantum scalar field that governs its time evolution with respect to τ is

$$H_F(\tau) = \int d^3k\omega_{\vec{k}} a^\dagger_{\vec{k}} a_{\vec{k}} \frac{dt}{d\tau}. \tag{3}$$

Here $a^\dagger_{\vec{k}}$, $a_{\vec{k}}$ are the creation and annihilation operators with momentum $\vec{k}$. The interaction between the atom and the quantum field is assumed to be described by a Hamiltonian [7],

$$H_I(\tau) = \mu R_2(\tau)\phi(x(\tau)), \tag{4}$$

where μ is a coupling constant which we assume to be small, $R_2 = \frac{1}{2}i(R_- - R_+)$, and $R_+ = |+\rangle\langle-|$, $R_- = |-\rangle\langle+|$. The coupling is effective only on the trajectory $x(\tau)$ of the atom.

We can now write down the Heisenberg equations of motion for the atom and field observables. The field is always assumed to be in its vacuum state $|0\rangle$. We will separately discuss the two physical mechanisms that contribute to the rate of change of atomic observables: the contribution of vacuum fluctuations and that of radiation reaction. For this purpose, we can split the solution of field ϕ of the Heisenberg equations into two parts: a free or vacuum part ϕ^f, which is present even in the absence of coupling, and a source part ϕ^s, which represents the field generated by the interaction between the atom and the field. Following DDC [5,6], we choose a symmetric ordering between atom and field variables and consider the effects of ϕ^f and ϕ^s separately in the Heisenberg equations of an arbitrary atomic observable G. Then, we obtain the individual contributions of vacuum fluctuations and radiation reaction to the rate of change of G. Since we are interested in the spontaneous excitation of the atom, we will concentrate on the mean atomic excitation energy $\langle H_A(\tau)\rangle$. The contributions of vacuum fluctuations (vf) and radiation reaction (rr) to the rate of change of $\langle H_A\rangle$ can be written as (cf. Refs. [5–7])

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm vf} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' C^F(x(\tau), x(\tau')) \frac{d}{d\tau}\chi^A(\tau, \tau'), \tag{5}$$

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm rr} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' \chi^F(x(\tau), x(\tau')) \frac{d}{d\tau}C^A(\tau, \tau'), \tag{6}$$

with $|\rangle = |a, 0\rangle$ representing the atom in the state $|a\rangle$ and the field in the vacuum state $|0\rangle$. Here the statistical functions of the atom, $C^A(\tau, \tau')$ and $\chi^A(\tau, \tau')$, are defined as

$$C^A(\tau, \tau') = \tfrac{1}{2}\langle a|\{R_2^f(\tau), R_2^f(\tau')\}|a\rangle, \tag{7}$$

$$\chi^A(\tau, \tau') = \tfrac{1}{2}\langle a|[R_2^f(\tau), R_2^f(\tau')]|a\rangle, \tag{8}$$

and those of the field are defined as

$$C^F(x(\tau), x(\tau')) = \tfrac{1}{2}\langle 0|\{\phi^f(x(\tau)), \phi^f(x(\tau'))\}|0\rangle, \tag{9}$$

$$\chi^F(x(\tau), x(\tau')) = \tfrac{1}{2}\langle 0|[\phi^f(x(\tau)), \phi^f(x(\tau'))]|0\rangle. \tag{10}$$

C^A is the symmetric correlation function of the atom in the state $|a\rangle$, and χ^A its linear susceptibility. C^F and χ^F are the Hadamard function and Pauli-Jordan or Schwinger function of the field, respectively. The explicit forms of the statistical functions of the atom are given by

$$C^A(\tau, \tau') = \frac{1}{2}\sum_b |\langle a|R_2^f(0)|b\rangle|^2 (e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')}), \tag{11}$$

$$\chi^A(\tau, \tau') = \frac{1}{2}\sum_b |\langle a|R_2^f(0)|b\rangle|^2 (e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')}), \tag{12}$$

where $\omega_{ab} = \omega_a - \omega_b$ and the sum runs over a complete set of atomic states.

III. SPONTANEOUS EXCITATION OF STATIC ATOMS OUTSIDE A BLACK HOLE

In the exterior region of the Schwarzschild black hole, a complete set of normalized basis functions for the massless scalar field that satisfy the Klein-Gordon equation is given by

$$\vec{u}_{\omega lm} = (4\pi\omega)^{-(1/2)} e^{-i\omega t} \vec{R}_l(\omega|r) Y_{lm}(\theta, \varphi), \tag{13}$$

$$\overleftarrow{u}_{\omega lm} = (4\pi\omega)^{-(1/2)} e^{-i\omega t} \overleftarrow{R}_l(\omega|r) Y_{lm}(\theta, \varphi), \tag{14}$$

where $Y_{lm}(\theta, \varphi)$ are the spherical harmonics and the radial functions have the following asymptotic forms [16]:

$$\vec{R}_l(\omega|r) \sim \begin{cases} r^{-1} e^{i\omega r_*} + \vec{A}_l(\omega) r^{-1} e^{-i\omega r_*} & r \to 2M, \\ B_l(\omega) r^{-1} e^{i\omega r_*} & r \to \infty, \end{cases} \tag{15}$$

$$\overleftarrow{R}_l(\omega|r) \sim \begin{cases} B_l(\omega) r^{-1} e^{-i\omega r_*} & r \to 2M, \\ r^{-1} e^{-i\omega r_*} + \overleftarrow{A}_l(\omega) r^{-1} e^{i\omega r_*} & r \to \infty, \end{cases} \tag{16}$$

with

$$r_* = r + 2M \ln\left(\frac{r}{2M} - 1\right) \tag{17}$$

being the Regge-Wheeler tortoise coordinate. The physical interpretation of these modes is that $\vec{u}$ represents modes emerging from the past horizon and $\overleftarrow{u}$ denotes those coming in from infinity. With the basics of the scalar field modes given above, we now apply the formalism outlined in the preceding section to examine the spontaneous excitation of the static atoms in three vacuum states of the quantum scalar fields, respectively.

A. Boulware vacuum

The Boulware vacuum is defined by requiring normal modes to be positive frequency with respect to the Killing vector $\partial/\partial t$. One can show that the Wightman function for massless scalar fields in this vacuum state is given by [17,18]

$$D_B^+(x, x') = \frac{1}{4\pi}\sum_{lm} |Y_{lm}(\theta, \varphi)|^2 \times \int_0^{+\infty} \frac{d\omega}{\omega} e^{-i\omega\Delta t} [|\vec{R}_l(\omega|r)|^2 + |\overleftarrow{R}_l(\omega|r)|^2], \tag{18}$$

and the corresponding Hadamard function and Pauli-Jordan or Schwinger function of the field are, respectively,

$$C^F(x(\tau), x(\tau')) = \frac{1}{8\pi}\sum_{lm} |Y_{lm}(\theta, \varphi)|^2 \times \int_0^{+\infty} \frac{d\omega}{\omega} (e^{(i\omega\Delta\tau)/(\sqrt{1-2M/r})} + e^{-[(i\omega\Delta\tau)/(\sqrt{1-2M/r})]}) \times [|\vec{R}_l(\omega|r)|^2 + |\overleftarrow{R}_l(\omega|r)|^2] \tag{19}$$

and

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{8\pi}\sum_{lm} |Y_{lm}(\theta, \varphi)|^2 \times \int_0^{+\infty} \frac{d\omega}{\omega} (e^{-[(i\omega\Delta\tau)/(\sqrt{1-2M/r})]} - e^{(i\omega\Delta\tau)/(\sqrt{1-2M/r})}) \times [|\vec{R}_l(\omega|r)|^2 + |\overleftarrow{R}_l(\omega|r)|^2], \tag{20}$$

where use has been made of

$$\Delta\tau = \Delta t\sqrt{1 - \frac{2M}{r}}. \tag{21}$$

Substituting the above results into Eqs. (5) and (6), extending the integration range for τ to infinity for sufficiently long times $\tau - \tau_0$, and performing the double integration, we obtain the contribution of the vacuum fluctuations to the rate of change of the mean atomic energy for an atom held static at a distance r from the black hole,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm vf} = -\frac{\mu^2}{4\pi}\left[\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 P(\omega_{ab}, r) - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 P(-\omega_{ab}, r)\right], \tag{22}$$

and that of radiation reaction,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm rr} = -\frac{\mu^2}{4\pi}\left[\sum_{\omega_a>\omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 P(\omega_{ab}, r) + \sum_{\omega_a<\omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 P(-\omega_{ab}, r)\right]. \tag{23}$$

Here we have defined

$$P(\omega_{ab}, r) = \vec{P}(\omega_{ab}, r) + \overleftarrow{P}(\omega_{ab}, r), \tag{24}$$

$$\vec{P}(\omega_{ab}, r) = \frac{\pi}{\omega_{ab}^2} \sum_{lm} |Y_{lm}(\theta, \phi)|^2 \left| \vec{R}_l\left(\omega_{ab} r \sqrt{1 - \frac{2M}{r}}\right)\right|^2 = \frac{1}{\omega_{ab}^2} \sum_{l=0}^{\infty} \frac{2l+1}{4} \left| \vec{R}_l\left(\omega_{ab} r \sqrt{1 - \frac{2M}{r}}\right)\right|^2, \tag{25}$$

and

$$\overleftarrow{P}(\omega_{ab}, r) = \frac{\pi}{\omega_{ab}^2} \sum_{lm} |Y_{lm}(\theta, \phi)|^2 \left| \overleftarrow{R}_l\left(\omega_{ab} r \sqrt{1 - \frac{2M}{r}}\right)\right|^2 = \frac{1}{\omega_{ab}^2} \sum_{l=0}^{\infty} \frac{2l+1}{4} \left| \overleftarrow{R}_l\left(\omega_{ab} r \sqrt{1 - \frac{2M}{r}}\right)\right|^2. \tag{26}$$

The following property of the spherical harmonics,

$$\sum_{m=-l}^{l} |Y_{lm}(\theta, \varphi)|^2 = \frac{2l+1}{4\pi}, \tag{27}$$

has been utilized in Eqs. (25) and (26). Adding up two contributions, we obtain the total rate of change of the mean atomic energy,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} = -\frac{\mu^2}{2\pi} \sum_{\omega_a>\omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 P(\omega_{ab}, r). \tag{28}$$

It follows that, for a static atom in the ground state ($\omega_a < \omega_b$), the contribution of the vacuum fluctuations and that of radiation exactly cancel, since each term in $\langle\frac{dH_A(\tau)}{d\tau}\rangle_{\rm vf}$ is canceled exactly by the corresponding term in $\langle\frac{dH_A(\tau)}{d\tau}\rangle_{\rm rr}$. Therefore, although both contributions to the rate of change of the mean atomic energy are modified by the presence of the factor $P(\omega_{ab}, r)$ as compared to the Minkowski vacuum case [7], the balance between them remains and the static ground-state atom in the Boulware vacuum is still stable. It should be pointed out, however, that the spontaneous emission rate of a static atom outside a Schwarzschild black hole in the Boulware vacuum is different from that of an inertial atom in the Minkowski vacuum in an unbounded flat space because of the presence of the factor $P(\omega_{ab}, r)$ in Eq. (28). In this sense, the Boulware vacuum is not equivalent to the usual Minkowski vacuum. However, a comparison of Eq. (28) with Eq. (23) in Ref. [8], which gives the rate of change of the mean atomic energy for an inertial atom in a flat space with a reflecting boundary, shows that the two rates are quite similar, and the appearance of $P(\omega_{ab}, r)$ in Eq. (28) can be understood as a result of backscattering of the vacuum field modes off the space-time curvature of the black hole in much the same way as the reflection of the field modes at the reflecting boundary in a flat space-time. In order to gain more understanding, let us now analyze the behavior of $P(\omega_{ab}, r)$ both in the asymptotic region and at the event horizon. Using the following asymptotic properties of the radial functions,

$$\sum_{l=0}^{\infty}(2l+1)|\vec{R}_l(\omega|r)|^2 \sim \begin{cases} \frac{4\omega^2}{1-\frac{2M}{r}} & r \to 2M, \\ \frac{1}{r^2}\sum_{l=0}^{\infty}(2l+1)|B_l(\omega)|^2 & r \to \infty, \end{cases} \tag{29}$$

$$\sum_{l=0}^{\infty}(2l+1)|\overleftarrow{R}_l(\omega|r)|^2 \sim \begin{cases} \frac{1}{4M^2}\sum_{l=0}^{\infty}(2l+1)|B_l(\omega)|^2 & r \to 2M, \\ 4\omega^2 & r \to \infty, \end{cases} \tag{30}$$

we obtain

$$\vec{P}(\omega_{ab}, r) \sim \begin{cases} 1 & r \to 2M, \\ \frac{1}{4r^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(\omega_{ab})|^2 & r \to \infty, \end{cases} \tag{31}$$

$$\overleftarrow{P}(\omega_{ab}, r) \sim \begin{cases} \frac{1}{16M^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(0)|^2 & r \to 2M, \\ 1 & r \to \infty, \end{cases} \tag{32}$$

and this leads to

$$P(\omega_{ab}, r) \sim \begin{cases} 1 + \frac{1}{16M^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(0)|^2 & r \to 2M, \\ 1 + \frac{1}{4r^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(\omega_{ab})|^2 & r \to \infty. \end{cases} \tag{33}$$

So, when $r \to \infty$, we have

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} \approx -\frac{\mu^2}{2\pi} \sum_{\omega_a>\omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \times \left[1 + \frac{1}{4r^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(\omega_{ab})|^2\right], \tag{34}$$

and when $r \to 2M$,

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm tot} \approx -\frac{\mu^2}{2\pi}\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2 \times\left[1+\frac{1}{16M^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(0)|^2\right]. \tag{35}$$

These asymptotic forms tell us that the rate of change of the mean atomic energy for a static atom outside a Schwarzschild black hole interacting with massless scalar fields in the Boulware vacuum gets enhanced as compared to the case of an inertial atom in the Minkowski vacuum in an unbounded flat space, and it reduces to the result of an inertial atom in the Minkowski vacuum at infinity and behaves normally at the event horizon. The normal behavior of the rate of change of the mean atomic energy near the horizon is in sharp contrast to the response rate of an Unruh detector [18].

B. Unruh vacuum

For the Unruh vacuum, the Wightman function for the massless scalar fields is given by [17,18]

$$D_U^+(x,x') = \frac{1}{4\pi}\sum_{lm}|Y_{lm}(\theta,\varphi)|^2\int_{-\infty}^{+\infty}\frac{d\omega}{\omega}\left[\frac{e^{-i\omega\Delta t}}{1-e^{-2\pi\omega/\kappa}}|\vec{R}_l(\omega|r)|^2+\theta(\omega)e^{-i\omega\Delta t}|\overleftarrow{R}_l(\omega|r)|^2\right], \tag{36}$$

where $\kappa = 1/4M$ is the surface gravity of the black hole. Then the statistical functions of the scalar field readily follow,

$$C^F(x(\tau),x(\tau')) = \frac{1}{8\pi}\sum_{lm}|Y_{lm}(\theta,\varphi)|^2\int_{-\infty}^{+\infty}\frac{d\omega}{\omega}(e^{(i\omega\Delta\tau)/(\sqrt{1-2M/r})}+e^{-[(i\omega\Delta\tau)/(\sqrt{1-2M/r})]})\left(\frac{|\vec{R}_l(\omega|r)|^2}{1-e^{-2\pi\omega/\kappa}}+\theta(\omega)|\overleftarrow{R}_l(\omega|r)|^2\right), \tag{37}$$

$$\chi^F(x(\tau),x(\tau')) = \frac{1}{8\pi}\sum_{lm}|Y_{lm}(\theta,\varphi)|^2\int_{-\infty}^{+\infty}\frac{d\omega}{\omega}(e^{-[(i\omega\Delta\tau)(\sqrt{1-2M/r})]}-e^{(i\omega\Delta\tau)/(\sqrt{1-2M/r})})\left[\frac{|\vec{R}_l(\omega|r)|^2}{1-e^{-2\pi\omega/\kappa}}+\theta(\omega)|\overleftarrow{P}_l(\omega|r)|^2\right]. \tag{38}$$

Similarly, we can compute the contributions of vacuum fluctuations and radiation reaction to the rate of change of the mean atomic energy to get

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm vf} = -\frac{\mu^2}{4\pi}\Bigg\{\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[\left(1+\frac{1}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right)\vec{P}(\omega_{ab},r)+\frac{\vec{P}(-\omega_{ab},r)}{e^{(2\pi\omega_{ab})/\kappa_r}-1}+\overleftarrow{P}(\omega_{ab},r)\right] - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[\left(1+\frac{1}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\right)\vec{P}(-\omega_{ab},r)+\frac{\vec{P}(\omega_{ab},r)}{e^{(2\pi|\omega_{ab}|)/\kappa_r-1}}+\overleftarrow{P}(-\omega_{ab},r)\right]\Bigg\} \tag{39}$$

and

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm rr} = -\frac{\mu^2}{4\pi}\Bigg\{\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[\left(1+\frac{1}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right)\vec{P}(\omega_{ab},r)-\frac{\vec{P}(-\omega_{ab},r)}{e^{(2\pi\omega_{ab})/\kappa_r}-1}+\overleftarrow{P}(\omega_{ab},r)\right] + \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[\left(1+\frac{1}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\right)\vec{P}(-\omega_{ab},r)-\frac{\vec{P}(\omega_{ab},r)}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}+\overleftarrow{P}(-\omega_{ab},r)\right]\Bigg\}, \tag{40}$$

where we have defined

$$\kappa_r = \frac{\kappa}{\sqrt{1-\frac{2M}{r}}}. \tag{41}$$

From the above results, one can see that both contributions are altered due to the appearance of thermal terms, as compared to the case of the Boulware vacuum. If we add up two contributions, we find the total rate

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm tot} = -\frac{\mu^2}{2\pi}\Bigg\{\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[\left(1+\frac{1}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right)\vec{P}(\omega_{ab})+\overleftarrow{P}(\omega_{ab})\right] - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\frac{P(\omega_{ab})}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\Bigg\}. \tag{42}$$

This reveals that the delicate balance no longer exists between the vacuum fluctuations and radiation reaction that ensures the stability of ground-state atoms held static at a radial distance r from the black hole in the Boulware vacuum. There is a positive contribution from the second term ($\omega_a < \omega_b$ term), and therefore transitions of ground-state atoms to excited states could spontaneously occur in the Unruh vacuum outside the black hole.

When the atom is held close to the event horizon, i.e., when $r \to 2M$, the total rate becomes

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} \approx -\frac{\mu^2}{2\pi}\left\{ \sum_{\omega_a>\omega_b} |\langle a|R_2^f(0)|b\rangle|^2 \omega_{ab}^2 \left[\left(1 + \frac{1}{16M^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(0)|^2\right) + \frac{1}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right] - \sum_{\omega_a<\omega_b} |\langle a|R_2^f(0)|b\rangle|^2 \omega_{ab}^2 \frac{1}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\right\}. \tag{43}$$

In comparison to Eq. (35), the corresponding result in the Boulware vacuum case, one sees the appearance of thermal terms which may be considered as resulting from the contribution of thermal radiation emanating from the black hole at a temperature

$$T = \frac{\kappa_r}{2\pi} = \frac{\kappa}{2\pi}\frac{1}{\sqrt{1-\frac{2M}{r}}} = (g_{00})^{-1/2}T_H, \tag{44}$$

where $T_H = \kappa/2\pi$ is the usual Hawking temperature of the black hole. Actually, this is the well-known Tolman relation [19] which gives the proper temperature as measured by a local observer. Notice that T, being always larger than the Hawking temperature, and reducing to it only at infinity, however diverges as the event horizon is approached. This can be understood as a result of the fact that the atom must be in acceleration relative to the local free-falling frame to maintain at a fixed distance from the black hole, and this acceleration, which blows up at the horizon, gives rise to additional thermal effect.

If the atom is far away from the black hole in the asymptotic region, that is, when $r \to \infty$, one then finds

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} \approx -\frac{\mu^2}{2\pi}\left\{ \sum_{\omega_a>\omega_b} |\langle a|R_2^f(0)|b\rangle|^2 \omega_{ab}^2 \left[1 + f(\omega_{ab}, r) + \frac{f(\omega_{ab}, r)}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right] - \sum_{\omega_a<\omega_b} |\langle a|R_2^f(0)|b\rangle|^2 \omega_{ab}^2 \frac{f(\omega_{ab}, r)}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\right\}, \tag{45}$$

where

$$f(\omega_{ab}, r) = \frac{1}{4r^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(\omega_{ab})|^2. \tag{46}$$

The appearance of $f(\omega_{ab}, r)$ in the thermal terms can now be envisaged as a result of backscattering of outgoing thermal flux from the event horizon off the space-time curvature. The backscattering results in the depletion of part of the outgoing flux. The influence of the thermal flux becomes weaker as the atom is placed farther away.

C. Hartle-Hawking vacuum

Let us now turn briefly to the case of the Hartle-Hawking vacuum. The Wightman function for the massless scalar fields now becomes [17,18]

$$D_H^+(x, x') = \frac{1}{4\pi}\sum_{lm}|Y_{lm}(\theta,\varphi)|^2 \int_{-\infty}^{+\infty}\frac{d\omega}{\omega}\left[\frac{e^{-i\omega\Delta t}}{1-e^{-2\pi\omega/\kappa}}|\vec{R}_l(\omega|r)|^2 + \frac{e^{-i\omega\Delta t}}{1-e^{-2\pi\omega/\kappa}}|\overleftarrow{R}_l(\omega|r)|^2\right], \tag{47}$$

which leads to the statistical functions of the scalar field in the Hartle-Hawking vacuum as follows:

$$C^F(x(\tau), x(\tau')) = \frac{1}{8\pi}\sum_{lm}|Y_{lm}(\theta,\varphi)|^2 \int_{-\infty}^{+\infty}\frac{d\omega}{\omega}\left(e^{(i\omega\Delta\tau)/(\sqrt{1-2M/r})} + e^{-[(i\omega\Delta\tau)/(\sqrt{1-2M/r})]}\right)\left(\frac{|\overleftarrow{R}_l(\omega|r)|^2}{1-e^{-2\pi\omega/\kappa}} + \frac{|\vec{R}_l(\omega|r)|^2}{e^{2\pi\omega/\kappa}-1}\right) \tag{48}$$

and

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{8\pi}\sum_{lm}|Y_{lm}(\theta, \varphi)|^2 \int_{-\infty}^{+\infty}\frac{d\omega}{\omega}(e^{(i\omega\Delta\tau)/(\sqrt{1-2M/r})} - e^{-[(i\omega\Delta\tau)/(\sqrt{1-2M/r})]})\left(\frac{|\vec{R}_l(\omega|r)|^2}{e^{2\pi\omega/\kappa}-1} - \frac{|\vec{R}_l(\omega|r)|^2}{1-e^{-2\pi\omega/\kappa}}\right). \tag{49}$$

By using the above results and Eqs. (5) and (6), the contribution of the vacuum fluctuations to the rate of change of the mean atomic energy can be found for an atom held static at a distance r from the black hole,

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm vf} = -\frac{\mu^2}{4\pi}\left\{\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[\frac{P(-\omega_{ab}, r)}{e^{(2\pi\omega_{ab})/\kappa_r}-1} + \left(1+\frac{1}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right)P(\omega_{ab}, r)\right]\right.$$
$$\left. - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[\frac{P(\omega_{ab}, r)}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1} + \left(1+\frac{1}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\right)P(-\omega_{ab}, r)\right]\right\}, \tag{50}$$

and that of radiation reaction,

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm rr} = -\frac{\mu^2}{4\pi}\left\{\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[-\frac{P(-\omega_{ab}, r)}{e^{(2\pi\omega_{ab})/\kappa_r}-1} + \left(1+\frac{1}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right)P(\omega_{ab}, r)\right]\right.$$
$$\left. - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left[\frac{P(\omega_{ab}, r)}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1} - \left(1+\frac{1}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\right)P(-\omega_{ab}, r)\right]\right\}. \tag{51}$$

Consequently, the total rate of change of the mean atomic energy follows,

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm tot} = -\frac{\mu^2}{2\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2 P(\omega_{ab}, r)\left(1+\frac{1}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right)\right.$$
$$\left. - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2 P(\omega_{ab}, r)\frac{1}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\right]. \tag{52}$$

Once again, with the existence of the $\omega_a < \omega_b$ term, for static atoms in the Hartle-Hawking vacuum, transitions from the ground state to the excited states can occur spontaneously in the exterior region of the black hole. In the spatial asymptotic region, the total rate can be written as

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm tot} \approx -\frac{\mu^2}{2\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{1}{4r^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(\omega_{ab})|^2\right)\left(1+\frac{1}{e^{(2\pi\omega_{ab})/\kappa}-1}\right)\right.$$
$$\left. - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{1}{4r^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(\omega_{ab})|^2\right)\frac{1}{e^{(2\pi|\omega_{ab}|)/\kappa}-1}\right]. \tag{53}$$

For an atom at spatial infinity ($r \rightarrow \infty$), $P(\omega_{ab}, r) \rightarrow 1$, and the temperature as perceived by the atom, T, approaches T_H; the total rate of change of the mean atomic energy becomes what one would get if the atom is immersed in a thermal bath at the temperature T_H. Therefore, a static atom in the spatial asymptotic region outside the black hole would spontaneously excite as if in a thermal bath of radiation at the Hawking temperature. This is consistent with our understanding gained from the calculations of expectation values of the energy-momentum tensor [18] that the Hartle-Hawking vacuum is not a state that is empty at infinity but corresponds instead to a thermal distribution of (Minkowski-type) quanta at the Hawking temperature, and therefore it describes a black hole in equilibrium with an infinite sea of blackbody radiation. On the other hand, when the atom is held near the event horizon, i.e., when $r \rightarrow 2M$, we have

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm tot} \approx -\frac{\mu^2}{2\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{1}{16M^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(0)|^2\right)\left(1+\frac{1}{e^{(2\pi\omega_{ab})/\kappa_r}-1}\right)\right.$$
$$\left. - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{1}{16M^2\omega_{ab}^2}\sum_{l=0}^{\infty}(2l+1)|B_l(0)|^2\right)\frac{1}{e^{(2\pi|\omega_{ab}|)/\kappa_r}-1}\right]. \tag{54}$$

Here one can see that, close to the horizon, in addition to the contribution that can be accounted for by the outgoing thermal radiation emanating from the horizon [refer to Eq. (43)], there is another contribution (the thermal term multiplied by the term containing B_l) that can be regarded as resulting from the incoming radiation from the sea of thermal radiation at infinity; this incoming radiation is, however, deflected by the space-time geometry. Notice that the difference between the rate of change of the mean atomic energy in the Unruh vacuum and that in the Hartle-Hawking vacuum is not a simple factor 2 as in the $1 + 1$ dimensional case [14]. The reason is that, in the four dimensional case, there are backscatterings by the space-time curvature so that the outgoing thermal radiation from the event horizon cannot travel through the space-time unaffected and nor can the incoming thermal radiation from infinity.

IV. SUMMARY

Using the DDC formalism, we have studied the spontaneous excitation of a two-level atom held static outside a Schwarzschild black hole and in interaction with a massless scalar field in the Boulware, Unruh, and Hartle-Hawking vacuums, respectively, and calculated the contributions of the vacuum fluctuations and radiation reaction to the rate of change of the mean atomic energy.

In the Boulware vacuum case, spontaneous excitation cannot occur so the ground-state atoms are stable. However, the spontaneous emission rate for excited atoms in the Boulware vacuum is not the same as that in the usual Minkowski vacuum, but is very similar to that in the vacuum in a flat space-time with a reflecting boundary. A noteworthy feature here is that the rate of change of the mean atomic energy is well behaved at the event horizon, in sharp contrast to the response rate of an Unruh detector [18].

Both for the Unruh vacuum and the Hartle-Hawking vacuum, our results show that an atom held static at a radial distance r from a Schwarzschild black hole would spontaneously excite. For the Unruh vacuum, it spontaneously excites as if there were an outgoing thermal flux of radiation (backscattered by the space-time geometry though) at a temperature characterized by the Tolman relation. For the Hartle-Hawking vacuum, the spontaneous excitation occurs as if the atom were in a thermal bath of radiation at a proper temperature which reduces to the Hawking temperature in the spatial asymptotic region, except for a frequency response distortion caused by the backscattering of the field modes off the space-time curvature.

ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China under Grant No. 10575035, the Program for New Century Excellent Talents in University (NCET, No. 04-0784), and the National Basic Research Program of China under Grant No. 2003CB71630.

[1] T. A. Welton, Phys. Rev. **74**, 1157 (1948).
[2] G. Compagno, R. Passante, and F. Persico, Phys. Lett. **98A**, 253 (1983).
[3] J. R. Ackerhalt, P. L. Knight, and J. H. Eberly, Phys. Rev. Lett. **30**, 456 (1973).
[4] P. W. Milonni, Phys. Scr. **T21**, 102 (1988); P. W. Milonni and W. A. Smith, Phys. Rev. A **11**, 814 (1975).
[5] J. Dalibard, J. Dupont-Roc, and C. Cohen-Tannoudji, J. Phys. (France) **43**, 1617 (1982).
[6] J. Dalibard, J. Dupont-Roc, and C. Cohen-Tannoudji, J. Phys. (France) **45**, 637 (1984).
[7] J. Audretsch and R. Müller, Phys. Rev. A **50**, 1755 (1994).
[8] H. Yu and S. Lu, Phys. Rev. D **72**, 064022 (2005); **73**, 109901 (2006).
[9] Z. Zhu, H. Yu, and S. Lu, Phys. Rev. D **73**, 107501 (2006).
[10] H. Yu and Z. Zhu, Phys. Rev. D **74**, 044032 (2006).
[11] Z. Zhu and H. Yu, Phys. Lett. B **645**, 459 (2007).
[12] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[13] J. Hartle and S. Hawking, Phys. Rev. D **13**, 2188 (1976).
[14] H. Yu and W. Zhou, Phys. Rev. D **76**, 027503 (2007).
[15] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[16] B. S. DeWitt, Phys. Rep. **19**, 295 (1975).
[17] S. M. Christensen and S. A. Fulling, Phys. Rev. D **15**, 2088 (1977).
[18] P. Candelas, Phys. Rev. D **21**, 2185 (1980).
[19] R. Tolman, Phys. Rev. **35**, 904 (1930); R. Tolman and P. Ehrenfest, Phys. Rev. **36**, 1791 (1930).