Towards closing the window on strongly interacting dark matter: Far-reaching constraints from Earth's heat flow

Gregory D. Mack,[1,2,*] John F. Beacom,[1,2,3,†] and Gianfranco Bertone[4,‡]

[1]*Department of Physics, Ohio State University, Columbus, Ohio 43210, USA*
[2]*Center for Cosmology and Astro-Particle Physics, Ohio State University, Columbus, Ohio 43210, USA*
[3]*Department of Astronomy, Ohio State University, Columbus, Ohio 43210, USA*
[4]*Institut d'Astrophysique de Paris, UMR 7095-CNRS, Université Pierre et Marie Curie, 98bis boulevard Arago, 75014 Paris, France*

(Received 11 June 2007; published 28 August 2007)

We point out a new and largely model-independent constraint on the dark matter scattering cross section with nucleons, which applies when this quantity is larger than for typical weakly interacting dark matter candidates. When the dark matter capture rate in Earth is efficient, the rate of energy deposition by dark matter self-annihilation products would grossly exceed the measured heat flow of Earth. This improves the spin-independent cross section constraints by many orders of magnitude and closes the window between astrophysical constraints (at very large cross sections) and underground detector constraints (at small cross sections). In the applicable mass range, from $\sim$1 to $\sim$$10^{10}$ GeV, the scattering cross section of dark matter with nucleons is then bounded from *above* by the latter constraints and hence must be truly weak, as usually assumed.

DOI: 10.1103/PhysRevD.76.043523 PACS numbers: 95.35.+d, 91.35.Dc, 95.30.Cq

I. INTRODUCTION

There is a large body of evidence for the existence of dark matter, but its basic properties—especially its mass and scattering cross section with nucleons—remain unknown. Assuming dark matter is a thermal relic of the early Universe, weakly interacting massive particles are prime candidates, suggested by constraints on the dark matter mass and self-annihilation cross section from the present average mass density [1]. However, as this remains unproven, it is important to systematically test the properties of dark matter particles using only late-Universe constraints. In 1990, Starkman, Gould, Esmailzadeh, and Dimopoulos [2] examined the possibility of strongly interacting dark matter, noting that it indeed had not been ruled out. Many authors since have explored further constraints and candidates. In this literature, "strongly interacting" denotes cross sections significantly larger than those of the weak interactions; it does not necessarily mean via the usual strong interactions between hadrons. We generally consider the constraints in the plane of dark matter mass m_χ and spin-independent scattering cross section with nucleons $\sigma_{\chi N}$.

Figure 1 summarizes the astrophysical, high-altitude balloon/satellite/rocket detector, and underground detector constraints in the $\sigma_{\chi N}$-m_χ plane. Astrophysical limits such as the stability of the Milky Way disk constrain very large cross sections [2,3]. Accompanying and comparable limits include those from cosmic rays and the cosmic microwave background [4,5]. Small cross sections are probed by cryogenic dark matter search (CDMS) and other underground detectors [6–9]. A dark matter (DM) particle can be directly detected if $\sigma_{\chi N}$ is strong enough to cause a

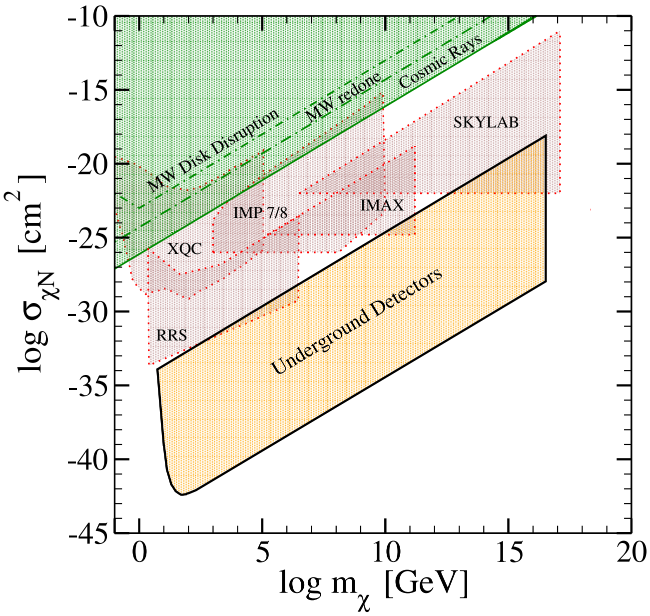

FIG. 1 (color online). Excluded regions in the $\sigma_{\chi N}$-m_χ plane, not yet including the results of this paper. From top to bottom, these come from astrophysical constraints (dark-shaded area) [2–5], reanalyses of high-altitude detectors (medium-shaded area) [2,10–12], and underground direct dark matter detectors (light-shaded area) [6–9]. The dark matter number density scales as $1/m_\chi$, and the scattering rates as $\sigma_{\chi N}/m_\chi$; for a fixed scattering rate, the required cross section then scales as m_χ. We will develop a constraint from Earth heating by dark matter annihilation to more definitively exclude the window between the astrophysical and underground constraints.

*gdmack@mps.ohio-state.edu
†beacom@mps.ohio-state.edu
‡bertone@iap.fr

 © 2007 The American Physical Society

nuclear recoil in the detector but only if it is weak enough to allow the DM to pass through Earth to the detector.

In between the astrophysical and underground limits is the window in which $\sigma_{\chi N}$ can be relatively large [2]. High-altitude detectors in and above the atmosphere have been used to exclude moderate-to-strong values of the cross section in this region [2,10–12]. However, there are still large gaps not excluded. There also is some doubt associated with these exclusions, as some of the experiments were not specifically designed to look for DM, nor were they always analyzed for this purpose by people associated with the projects. In fact, the exclusion from the x-ray quantum calorimetry experiment (XQC) was recently reanalyzed [12], and it changed substantially from earlier estimates [11]. If this intermediate region can be closed, then underground detectors would set the upper limit on $\sigma_{\chi N}$. That would mean that these detectors are generally looking in the right cross section range and that DM-nucleon scattering interactions are indeed totally irrelevant in astrophysics.

We investigate cross sections between the astrophysical and underground limits and show that $\sigma_{\chi N}$ is large enough for Earth to efficiently capture DM. Incoming DM will scatter off nucleons, lose energy, and become gravitationally captured once below Earth's escape velocity (Sec. IV). If this capture is maximally efficient, the rate is $2 \times 10^{25}\ (\mathrm{GeV}/m_\chi)\ \mathrm{s}^{-1}$. The gravitationally captured DM will drift to the bottom of the potential well, Earth's core. Self-annihilation results if the DM is its own antiparticle, and we assume standard model final state particles so that these products will deposit nearly all of their energy in the core.

Inside a region in the $\sigma_{\chi N}$-m_χ plane that will be defined, too much heat would be produced relative to the actual measured value of Earth's heat flow. The maximal heating rate obtained via macroscopic considerations is $\simeq 3330$ terawatts (TW) and follows the maximal capture rate, assuming that Earth is opaque with a geometric cross section. Note that the flux of DM scales as $1/m_\chi$, while the heat energy from annihilation scales as m_χ, *yielding a heat flow that is independent of DM mass*. The efficient capture we consider leads to a very similar heating rate, though it is based on a realistic calculation of microscopic DM-nucleon scattering, as discussed below. DM interactions with Earth have been previously studied in great detail, e.g., Refs. [2,13–21], but those investigations generally considered only weak cross sections for which capture is inefficient.

In our analysis, the $\sigma_{\chi N}$ exclusion region arises from the captured DM's self-annihilation energy exceeding Earth's internal heat flow. This region is limited below by the efficient capture of DM (Sec. IV) and above by being weak enough to allow sufficient time for the DM to drift to the core (Sec. V). These two limits define the region in which DM heating occurs. Why is it important? Earth's received solar energy is large, about 170 000 TW [22], but it is all reflected or reradiated. The internal heat flow is much less, about 44 TW (Sec. III) [23]. Inside this bounded region for $\sigma_{\chi N}$, DM heating would exceed the measured rate by about 2 orders of magnitude and therefore is not allowed. We will show that this appears to close the window noted above in Fig. 1, up to about $m_\chi \simeq 10^{10}$ GeV. In order to be certain of this, however, we call for new analyses of the aforementioned constraints, especially the exact region excluded by CDMS and other underground detectors. Our emphasis is not on further debate of the details of specific open gaps but rather on providing a new and independent constraint. In Table I, we summarize the heat values relevant to this paper. While the origin of Earth's heat flow is not completely understood, we emphasize that we are not trying to account for any portion of it with heating from DM.

There has been some previous work on the heating of planets by DM annihilation [13,24–30]. These papers have focused mostly on the Jovian planets, for which the internal heat flow values are deduced from their infrared radiation [31]. In some cases [13,24–27], DM annihilation was invoked to explain the anomalously large heat flow values of Jupiter and Saturn, while in other cases [28–30], the low heat flow value of Uranus was used to constrain DM annihilation. An additional reason for the focus on these large planets is that they will be able to stop DM particles of a smaller cross section than Earth can (Ref. [27] considered Earth but invoked an extreme DM clumping factor to overcome the weakly interacting cross section). However, as we argue below in Sec. VI, the more relevant criterion is how significant of an excess heat flow could be produced by DM annihilation, and this is much more favorable for Earth. (If this criterion is met, then the ranges of excluded cross sections will simply shift for different planets.) Furthermore, the detailed knowledge of Earth's properties gives much more robust results. In this paper, we are presenting the first detailed and systematic study of the broad exclusion region in the $\sigma_{\chi N}$-m_χ plane that is based on not overheating Earth.

Our constraints depend on DM being its own antiparticle, so that annihilation may occur (or, if it is not, that the DM-antiDM asymmetry not be too large). This is a mild

TABLE I. Relevant heat flow values. The top entries are measured, while the lower entries are the calculated potential effects of dark matter.

Heat source	Heating rate
Solar (received and returned)	170 000 TW
Internal (measured)	44.2 ± 1 TW
DM annihilation (opaque Earth)	3330 TW
DM annihilation (our assumptions)	3260 TW
DM kinetic heating	$\sim 3000 \times 10^{-6}$ TW

and common assumption. The heating due to kinetic energy transfer is negligible. Since the DM velocity is $\simeq 10^{-3}c$, kinetic heating is $\sim 10^{-6}$ that from annihilation and would provide no constraint (Sec. IV). The model-independent nature of our annihilation constraints arises from the nearly complete insensitivity to which standard model particles are produced from DM annihilation and at what energies. All final states except neutrinos will deposit all of their energy in Earth's core. (Above about 100 TeV, neutrinos will, too.) Since the possible heating rate ($>$ 3000 TW) is so large compared to the measured rate ($\sim$ 40 TW), in effect we require only that not more than $\sim$99% of the energy goes into low-energy neutrinos, which is an extremely modest assumption.

Some of the annihilation products will likely be neutrinos, and these may initiate signals in neutrino detectors, e.g., as upward-going muons [2,32–39]. While the derived cross section limits can be constraining, they strongly depend on the branching ratio to neutrinos and the neutrino energies. Comprehensive constraints based on neutrino fluxes for the full range of DM masses appear to be unavailable; most papers have concentrated on the 1–1000 GeV range, and a few have considered masses above 10^8 GeV. We note that the constraints for DM masses above about 10^{10} GeV may require annihilation cross sections above the unitarity bound, as discussed below. As this paper is meant to be a model-independent, direct approach to DM properties based on the DM density alone, we do not include these neutrino constraints.

We review the current DM constraints in Sec. II, review Earth's heat flow in Sec. III, calculate the DM capture, annihilation, and heating rates in Secs. IV and V, and close with discussions and conclusions in Sec. VI.

II. REVIEW OF PRIOR CONSTRAINTS

Figure 1 shows the current constraints in the $\sigma_{\chi N}$-m_χ plane. As we will show, the derived exclusion region found by the requirement of not overheating Earth using DM annihilation lies in the uncertain intermediate area between the astrophysical and underground constraints.

A. Indirect astrophysical constraints

If $\sigma_{\chi N}$ were too large, DM particles in a galactic halo would scatter too frequently with the baryonic disk of a spiral galaxy and would significantly disrupt it. Using the integrity of the Milky Way disk, Starkman *et al.* [2] restrict the cross section to $\sigma_{\chi N} < 5 \times 10^{-24}(m_\chi/\text{GeV})\ \text{cm}^2$. A more detailed study by Natarajan *et al.* [3] requires $\sigma_{\chi N} < 5 \times 10^{-25}(m_\chi/\text{GeV})\ \text{cm}^2$. Both of these limits consider DM scattering only with hydrogen. As shown below in Eq. (12), the spin-independent DM-nucleon cross section scales as A^4 for large m_χ, and though the number density of helium ($A = 4$) is about 10 times less than that of hydrogen ($A = 1$), taking it into account could improve these constraints by $\simeq 256/10 \simeq 25$. Chivukula *et al.* [40] showed that charged dark matter could be limited through its ionizing effects on interstellar clouds; this technique could be adapted for strongly interacting dark matter.

Strong scattering of DM and baryons would also affect the cosmic microwave background radiation. Adding stronger DM-baryon interactions increases the viscosity of the baryon-photon fluid [4]. A strong coupling of baryons and DM would generate denser clumps of gravitationally interacting matter, and the photons would not be able to push them as far apart. The peaks in the cosmic microwave background power spectrum would be damped, with the exception of the first one. The resulting constraint is $\sigma_{\chi N} < 3 \times 10^{-24}(m_\chi/\text{GeV})\ \text{cm}^2$ [4] and is not shown in Fig. 1. These results do take helium into account but do so only using A^2 instead of A^4. This possible change, along with the much more precise cosmic microwave background radiation data available currently, calls for a detailed reanalysis of this limit, which should strengthen it.

Cosmic ray protons interact inelastically with interstellar protons, breaking the protons and creating neutral pions that decay to high-energy gamma rays. A similar situation could occur with a cosmic ray beam on DM targets instead [5]. The fundamental interaction is between the quarks in the nucleon and the DM; it is very unlikely that all quarks will be struck equally, and the subsequent destruction of the nucleon creates pions. If the DM-nucleon cross section were high enough, the resulting gamma rays would be readily detectable. From this, Cyburt *et al.* [5] place an upper limit of $\sigma_{\chi N} < 7.6 \times 10^{-27}(m_\chi/\text{GeV})\ \text{cm}^2$. Improvements could probably be made easily with a more realistic treatment of the gamma-ray data.

B. Direct detection constraints

Underground detector experiments have played a large role in limiting DM that can elastically scatter nuclei, giving the nuclei small but measurable kinetic energies. Because of the cosmic ray background, this type of detector is located underground. The usual weakly interacting DM candidates easily pass through the atmosphere and Earth en route to the detector. However, for large $\sigma_{\chi N}$ the DM would lose energy through scattering before reaching the detector, decreasing detection rates.

Albuquerque and Baudis [7] have explored constraints at relatively large cross sections and large masses using results from CDMS and EDELWEISS. In Fig. 1, we present a crude estimate of the current underground detector exclusion region. The top line is defined by the ability of a DM particle to make it through the atmosphere [41] and Earth to the detector without losing too much energy [7]. The lower left corner and nearby points are taken from the official CDMS papers [6] with the aid of their website [42]. The right edge is taken from the dark matter scintillator detector (DAMA) [9]. As the mass of the DM increases, the number density (and hence the flux through

Earth) decreases. At the largest m_χ values, the scattering rate within a finite time vanishes. Finally, we have extrapolated each of these constraints to meet each other, connecting them consistently. We call for a complete and official analysis of the exact region that CDMS and other direct detectors exclude. Our focus is on the cross sections in between the underground detectors and astrophysical limits.

To investigate cross sections in this middle range, direct detectors must be situated above Earth's atmosphere, in high-altitude balloons, rockets, or satellites. Several such detectors have been analyzed for this purpose, though they were not all originally intended to study DM. Since these large $\sigma_{\chi N}$ limits have in some cases been calculated by people not connected with the original experiments, some caution is required. Nevertheless, in Fig. 1 we show the claimed exclusion regions, following Starkman *et al.* [2] and Rich, Rocchia, and Spiro [10], along with Wandelt *et al.* [11] and Erickcek *et al.* [12] (including the primary references [43–46]). We are primarily in accordance with Erickcek *et al.* These regions span masses of almost 0.1–10^{16} GeV and cross sections between roughly 10^{-33} and 10^{-11} cm^2. These include the Pioneer 11 spacecraft and Skylab, the interplanetary monitoring platform (IMP) 7/8 cosmic ray silicon detector satellite, XQC, and the balloon-borne isotope matter antimatter experiment (IMAX). These regions are likely ruled out, but not in absolute certainty, and there are gaps between them. The Pioneer 11 region is completely covered by the IMP 7/8 and XQC regions and is therefore not shown in Fig. 1. The region labeled RRS is Rich, Rocchia, and Spiro's analysis of a silicon semiconductor detector near the top of the atmosphere, truncated according to Starkman *et al.* and adjusted with the appropriate A scaling as in Eq. (12).

III. EARTH'S HEAT FLOW

Heat from the Sun warms Earth, but it is not retained. If all of the incident sunlight were absorbed by Earth, the heating rate would be about 170 000 TW [22]. Some of it is reflected by the atmosphere, clouds, and surface, and the rest is absorbed at depths very close to the surface and then reradiated [31]. Earth's blackbody temperature would be about 280 K, and it is observed to be between 250 and 300 K, supporting the idea of Earth-Sun heat equilibrium. Internal heating therefore has minimal effects on the overall heat of Earth [31].

Our focus is on this *internal* heat flow of Earth, as measured underground. Geologists have extensively studied Earth's internal heat for decades [47]. To make a measurement, a borehole is drilled kilometers deep into the ground. The temperature gradient in that borehole is recorded, and that quantity multiplied by the thermal conductivity of the relevant material yields a heat flux [47,48].

The deepest borehole is about 12 kilometers, which is still rather close to Earth's surface. Typical temperature gradients are between 10 and 50 K/km, but these cannot hold for lower depths. If they did, all rock in the deeper parts of Earth would be molten, in contradiction to seismic measurements, which show that shear waves can propagate through the mantle [48]. Current estimates place temperature gradients deep inside Earth between 0.6 and 0.8 K/km [48].

More than 20 000 borehole measurements have been made over Earth's surface. Averaging over the continents and oceans, there is a heat flux of 0.087 ± 0.002 W/m^2 [23,47]. Integrating this flux over the surface of Earth gives a heat flow of 44.2 ± 1 TW [23,47]. Again, the heat flux is directly measured underground, all over Earth, and is independent of the solar flux, Earth's atmosphere, and anything else above Earth's surface. Obviously, the possibility to make direct heat flow measurements under the surface is unique to Earth.

While the heat flow value is known well, the origin of the heat is not and in fact is undergoing much theoretical debate [49,50]. Some specific contributors are known, however. The decay of radioactive elements produces a significant amount; uranium and thorium decay in the crust generates about 40% of the total [23]. Potassium adds to this, though there is much less of it in the crust. However, there is potentially a large amount in the mantle and perhaps even the outer core [23]. The Kamioka liquid scintillator antineutrino detector (KamLAND) has a hint of detected neutrinos coming from uranium and thorium decays [51], and it (along with other detectors) could potentially help to make the heat contribution from them more accurate [52–61]. Larger concentrations of uranium and thorium are excluded by KamLAND, and theoretical predictions from the bulk silicate Earth model are consistent with the 40% value [55–57]. The remaining heat is due to processes in the core and perhaps even the mantle, although specific knowledge of Earth's interior is limited [62].

The residual heat flow, which we assume to be 20 TW [49,50], we use as the target limit for the heat flow from DM annihilation. Models give values of the core's heat output between 2.3 and 21 TW, supporting the conservative choice of 20 TW [62]. Annihilation scenarios creating heat flows greater than 20 TW are therefore excluded. In fact, if heating by DM annihilation is important at all, we show that it typically would exceed this value by more than 2 orders of magnitude. It is important to note that we are not trying to solve geological heat problems with DM, and in fact our analysis implies it is very unlikely that DM is contributing to Earth's internal heat flow, which is interesting in itself.

IV. DARK MATTER CAPTURE RATE OF EARTH

The DM mass density $\rho_\chi = n_\chi m_\chi$ in the neighborhood of the solar system is about 0.3 GeV/cm^3 [1]. Neither the mass nor the number density are separately known. The

DM is believed to follow a nonrelativistic Maxwell-Boltzmann velocity distribution with an average speed of about 270 km/s. If a DM particle scatters a sufficient number of times while passing through Earth, its speed will fall below the surface escape speed, 11.2 km/s. Having therefore been gravitationally captured, it will then orbit the center of Earth, losing energy with each subsequent scattering until it settles into a thermal distribution in equilibrium with the nuclei in the core. For the usual weak cross sections, Earth is effectively transparent, and scattering and capture are very inefficient. In contrast, we will consider only large cross sections for which capture is almost fully efficient. Note that, for our purposes, the scattering history is irrelevant as long as capture occurs; in particular, the depth in the atmosphere or Earth of the first scattering has no bearing on the results. The energies of the individual struck nuclei are also irrelevant, unlike in direct detection experiments. We just require that the DM is captured and ultimately annihilated.

A. Maximum capture rate

We begin by considering the maximum possible capture rate of DM in Earth, which corresponds to Earth being totally opaque. Although our final calculations will involve the microscopic scattering cross section of DM on nuclei, this initial example deals with just the macroscopic geometric cross section of Earth. The flux per solid angle of DM near Earth is $n_\chi v_\chi/4\pi$, where n_χ is the DM number density, and v_χ is the average DM velocity. Since Earth is taken to be opaque, the solid angle acceptance at each point on the surface is 2π sr. Thus the flux at Earth's surface is $n_\chi v_\chi/2$. The capture rate is then found by multiplying by Earth's geometric cross section: $\sigma_\oplus = 4\pi R_\oplus^2 \simeq 5.1 \times 10^{18}$ cm^2. Since n_χ is not known, this is $(\rho_\chi/m_\chi)\sigma_\oplus v_\chi$. For $v_\chi = 270$ km/s, this maximal capture rate is

$$\Gamma_C^{\max} = 2 \times 10^{25} \left(\frac{\text{GeV}}{m_\chi}\right) \text{s}^{-1}. \quad (1)$$

We will show that our results depend only logarithmically on the DM velocity and hence are insensitive to the details of the velocity distribution.

This maximal capture rate estimate is too simplistic, as it assumes that merely coming into contact with Earth, interacting with any thickness, will result in DM capture. Instead, we define opaqueness to be limited to path lengths greater than $0.2R_\oplus$, a value that incorporates the largest 90% of path lengths through Earth. This reduces the capture rate but only by about 2%. We therefore adopt $0.2R_\oplus$ as our minimum thickness to determine efficient scattering. This length, translated into a chord going through the spherical Earth, defines the new effective area for Earth. The midpoint of the chord lies at a distance of $0.99R_\oplus$ from Earth's center. Thus, practically speaking, nearly all DM passing through Earth will encounter sufficient material. The above requirements exclude glancing trajectories from consideration, for which there would be some probability of reflection from the atmosphere [28,29]; note also that the exclusion region in Sec. IV would be unaffected by taking this into account, since the DM heating of Earth would still be excessive.

The type of nucleus with which DM scatters depends on its initial trajectory through Earth. For a minimum path length of $0.2R_\oplus$, this trajectory runs through the crust, where the density is 3.6 g/cm^3 [63], and the most abundant element is oxygen [16]. Choosing this path length and density are conservative steps. Any larger path length would result in more efficient capture, and a higher density and heavier composition (corresponding to a larger chord and therefore a different target nucleus, such as iron, which is the most abundant element in the core) would as well. A more complex crust or mantle composition, such as 30% oxygen, 15% silicon, 14% magnesium, and smaller contributions from other elements [16], would stop DM $\sim$2 times more effectively.

B. Dark matter scattering on nuclei

When a DM particle (at $v_\chi \simeq 10^{-3}c$) elastically scatters with a nucleus/nucleon (at rest) in Earth, it decreases in energy and velocity. After one scattering with a nucleus of mass m_A, DM with mass m_χ and initial velocity v_i will have a new velocity of

$$\frac{v_f}{v_i} = \sqrt{1 - 2\frac{m_A m_\chi}{(m_\chi + m_A)^2}(1 - \cos\theta_{cm})}, \quad (2)$$

$$\xrightarrow[m_\chi \gg m_A]{} \sqrt{1 - 2\frac{m_A}{m_\chi}(1 - \cos\theta_{cm})}. \quad (3)$$

All quantities are in the lab frame, except the recoil angle θ_{cm}, which is most usefully defined in the center of mass frame (see Landau and Lifschitz [64]). Here and below, we give the large m_χ limit for demonstration purposes, but we use the full forms of the equations for our results. After scattering, the DM has a new kinetic energy,

$$KE_f^\chi = \frac{1}{2} m_\chi v_i^2 \left(1 - 2\frac{m_A m_\chi}{(m_\chi + m_A)^2}(1 - \cos\theta_{cm})\right) \quad (4)$$

$$\xrightarrow[m_\chi \gg m_A]{} KE_i^\chi \left(1 - 2\frac{m_A}{m_\chi}(1 - \cos\theta_{cm})\right). \quad (5)$$

The nucleus then obtains a kinetic energy of

$$\begin{aligned} KE^A &= KE_i^\chi - KE_f^\chi \\ &= \frac{1}{2} m_\chi v_i^2 \left(1 - 1 + 2\frac{m_A m_\chi}{(m_\chi + m_A)^2}(1 - \cos\theta_{cm})\right) \end{aligned} \quad (6)$$

$$\underset{m_\chi \gg m_A}{\longrightarrow} KE_i^\chi 2 \frac{m_A}{m_\chi}(1 - \cos\theta_{cm}) \quad (7)$$

$$= m_A v_i^2 (1 - \cos\theta_{cm}). \quad (8)$$

From the kinetic energy, the momentum transfer in the large m_χ limit is

$$KE = \frac{|\vec{q}|^2}{2m_A} = m_A v_i^2 (1 - \cos\theta_{cm}), \quad (9)$$

$$|\vec{q}|^2 = 2(m_A v_i)^2 (1 - \cos\theta_{cm}). \quad (10)$$

In order to maintain consistency with others, we work with n and σ in *nucleon* units even though the target we choose (oxygen) is a nucleus. This means that n_A (where A represents the mass number of the target) is

$$n_A = \frac{n}{A} = \frac{\rho}{m_N A}. \quad (11)$$

In turn, the cross section for spin-independent s-wave elastic scattering is represented as

$$\sigma_{\chi A} = A^2 \left(\frac{\mu(A)}{\mu(N)}\right)^2 \sigma_{\chi N} \underset{m_\chi \gg m_A}{\longrightarrow} A^4 \sigma_{\chi N}. \quad (12)$$

Here A is the mass number of the target nucleus, which equals m_A/m_N, and $\mu(A \text{ or } N)$ is the reduced mass of the DM particle and the target.

The A^2 factor arises because, at these low momentum transfers, the nucleus is not resolved, and the DM is assumed to couple coherently to the net "charge"—the number of nucleons. (If this coherence is somehow lost, a factor A would still remain for incoherent scattering.) The momentum transfer $q = \sqrt{2m_A KE^A} \simeq m_A v_i$ corresponds to a length scale of $\simeq 10$ fm for oxygen, much larger than the nucleus. We find that the corresponding nuclear form factor when the DM mass is comparable to the target mass is $\simeq 0.99$. The square of the reduced mass ratio arises from the Born approximation for scattering, which is based on the two-particle Schrödinger equation cast as a single particle with relative coordinates and reduced mass [65]. The spin-dependent scattering cross section does not have the A^2 factor in Eq. (12) [2]. Our constraints could be scaled to represent this case by also taking into account the relative abundance of target nuclei with nonzero spin in Earth, which is of order 1%.

Note that if $m_\chi = m_A$, and $\theta_{cm} = \pi$, the DM can transfer all of its momentum to the struck nucleus, losing all of its energy in a single scattering through this scattering resonance [16]. Taking this into account would make our constraints stronger over a small range of masses, but we neglect it. The nuclear recoil energy from this resonance is then $\frac{1}{2} m_\chi v_i^2$. Since v_i is on average 270 km/s, this means that the maximum energy transferred from a collision is $\sim 10^{-6}$ that of the annihilation energy $m_\chi c^2$.

C. Dark matter capture efficiency

From the full or approximate form of Eq. (4), we see that the DM kinetic energy is decreased by a multiplicative factor that is linear in $\cos\theta_{cm}$. If, in each independent scattering, we average over $\cos\theta_{cm}$, the average factor by which the kinetic energy is reduced in one or many scatterings will simply be that obtained by setting $\cos\theta_{cm} = 0$ throughout. (For s-wave scattering, the $\cos\theta_{cm}$ distribution is uniform.)

We will define efficient capture so that the heating is maximized. To be gravitationally trapped, a DM particle must be below the escape speed of Earth ($v_{\rm esc} = 11.2$ km/s) or, equivalently, its kinetic energy must be less than $\frac{1}{2} m_\chi v_{\rm esc}^2$. After one scattering event, the DM kinetic energy is reduced:

$$KE_f^\chi = KE_i^\chi f(m_\chi). \quad (13)$$

In successive collisions, this is compounded until

$$\tfrac{1}{2} m_\chi v_{\rm esc}^2 = \tfrac{1}{2} m_\chi v_i^2 [f(m_\chi)]^{N_{\rm scat}}. \quad (14)$$

Note that, for collinear scatterings, the velocity loss in Eq. (2) is also speed loss, leading to the same definition of $N_{\rm scat}$.

Therefore, on average, the number of scatterings required to gravitationally capture the DM is

$$N_{\rm scat} = \frac{-2 \ln(v_i/v_{\rm esc})}{\ln[1 - 2\frac{m_A m_\chi}{(m_\chi + m_A)^2}]} \quad (15)$$

$$\underset{m_\chi \gg m_A}{\longrightarrow} \frac{m_\chi}{m_A} \ln(v_i/v_{\rm esc}), \quad (16)$$

where we have set $\cos\theta_{cm} = 0$, since this corresponds to the average fractional change in the kinetic energy. Again, for simplicity the same element is taken to be the target each time. Note that, since the initial DM velocity is inside the logarithm, $N_{\rm scat}$ is insensitive to even large changes in the assumed initial velocity.

The number of scatterings for a given mass is large. A DM particle that has the same mass as the target nucleus will scatter about 10 times before it is captured. Note that the required $N_{\rm scat}$ scales as m_χ in the large mass limit, becoming very large: For m_χ above 16 TeV (10^3 times the target mass), $N_{\rm scat}$ is already larger than 3000. The actual energy losses in individual collisions are irrelevant for our analysis, as we require only that the DM is captured after many collisions. For large values of $N_{\rm scat}$, all scattering histories will be well-characterized by the average case.

So far, these equations have just been kinematics; the required $N_{\rm scat}$ for stopping has not yet been made specific to Earth. It becomes Earth-specific by relating $N_{\rm scat}$ to the path length in Earth L and the mean free path λ:

$$N_{\rm scat} = \frac{L}{\lambda} = L n_A \sigma_{\chi A}. \quad (17)$$

The column density of Earth then defines the required cross section to generate $N_{\rm scat}$ scatterings. The shortest path the particle could travel in is a straight line, so we use that as the minimum. Any other path would be longer and hence more effective at capture. This therefore defines the most conservative limit on $\sigma_{\chi A}$. Since we have fixed $\cos\theta_{cm}$ to be 0 on average, in fact the path will not be completely straight. However, the lab frame scattering angles are small.

For elastic collisions between two particles, the range of scattering angles in the lab frame depends on the two masses m_1 and m_2. There is a maximum scattering angle when one mass is initially at rest in the lab frame (in this case, m_2) [64]. If $m_1 < m_2$, there is no restriction on the scattering angle, which is defined in relation to m_1's initial direction (m_2 is at rest). However, if $m_1 > m_2$, then

$$\sin\theta_{\rm lab}^{\rm max} = m_2/m_1. \tag{18}$$

Our main focus is $m_1 = m_\chi > m_2 = m_A$. For m_χ somewhat greater than m_A, note that the DM scattering angle in the lab frame is always very forward.

Combining Eqs. (12), (15), and (17), the minimum required cross section to capture a DM particle is

$$\begin{aligned}\sigma_{\chi N}^{\rm min} &= \frac{m_N^2}{m_A(\frac{\mu(A)}{\mu(N)})^2\rho L} N_{\rm scat}(m_\chi)\\ &= \frac{-2\ln(v_i/v_{\rm esc})}{\ln[1-2\frac{m_A m_\chi}{(m_\chi+m_A)^2}]}\frac{m_N^2}{(m_A(\frac{\mu(A)}{\mu(N)})^2\rho L)}\\ &\xrightarrow[m_\chi\gg m_A]{} m_\chi\left(\frac{m_N}{m_A}\right)^4\frac{1}{\rho L}\ln(v_i/v_{\rm esc}).\end{aligned} \tag{19}$$

Again, we choose a path length of $0.2R_\oplus$, to select about 90% of the path lengths in Earth. Taking this length as a chord through Earth, the location corresponds to the crust, with an average density of 3.6 g/cm^3, where the most common element is oxygen. We also choose an incoming DM velocity of 500 km/s, which effectively selects the entire thermal distribution. A slower DM particle is more easily captured. These parameters give a required cross section of

$$\sigma_{\chi N}^{\rm min} = \frac{-1.8\times10^{-33}\ {\rm cm}^2(\frac{\mu(1)}{\mu(16)})^2}{\ln[1-2\frac{16\ {\rm GeV}m_\chi}{(m_\chi+16\ {\rm GeV})^2}]} \tag{20}$$

$$\xrightarrow[m_\chi\gg m_A]{} 2.2\times10^{-37}\ {\rm cm}^2\left(\frac{m_\chi}{\rm GeV}\right). \tag{21}$$

Note that we use the unapproximated version [Eq. (20)] for our figure and give the large m_χ limit in the equations for demonstrative purposes. When m_χ is comparable to m_A, $\sigma_{\chi N}^{\rm min}$ is different from the approximated, large m_χ case in an important way.

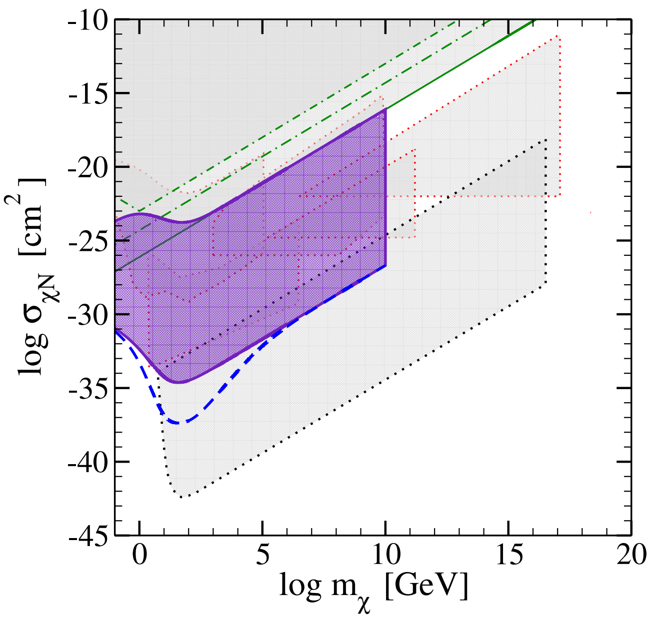

FIG. 2 (color online). Inside the heavily shaded region, dark matter annihilation would overheat Earth. Below the top edge of this region, dark matter can drift to Earth's core in a satisfactory time. Above the bottom edge, the capture rate in Earth is nearly fully efficient, leading to a heating rate of 3260 TW (above the dashed line, capture is only efficient enough to lead to a heating rate of $\gtrsim 20$ TW). The mass ranges are described in the text, and the light-shaded regions are as in Fig. 1.

The resulting curve for $\sigma_{\chi N}^{\rm min}$ is shown in Fig. 2, as the lower boundary of the heavily shaded exclusion region. The straight section of this constraint is easily seen from Eq. (21), as the required cross section for our efficient capture scenario scales as m_χ, due to the large number of collisions required for stopping, as in Eq. (16). At lower masses, the curved portion has its minimum at the mass of the target. DM masses close to that of the target can be captured with smaller cross sections because a greater kinetic energy transfer can occur for each collision. At very low masses, much less than the mass of the target, the DM mass dependence in the logarithm is approximated differently. In this limit, $\sigma_{\chi N}$ is $\simeq 10^{-32}$ (GeV$/m_\chi$) cm^2. As the DM mass decreases, it becomes increasingly more difficult for the DM to lose energy when it strikes a nucleus. As noted above, the cross section constraints in the spin-dependent case could be developed and would shift the results up by 3 or 4 orders of magnitude. All of the other limits that depend on this A^2 coherence factor would also shift accordingly.

The lower edge of the exclusion region is generally rather sharp, because of these parameters. For example, consider the case of large m_χ, where $N_{\rm scat}$ is also large. If the corresponding cross section is decreased by a factor δ, so is the number of scatterings, and by Eq. (14), the compounded fractional kinetic energy loss would only be the $1/\delta$ root of that required for capture. For small cross sections, as usually considered, the capture efficiency is very low. To efficiently produce heat, the minimum cross section must result in $\sim$90% DM capture. We stress again that we are not concerned with *where* the DM is captured in Earth, so long as it is. The probability for capture can, however, be decreased using Poisson statistics (shown in Fig. 2 as the dashed line with the accentuated dip at low masses) to yield just 20 TW of heat flow. This extension and the upper edge of the exclusion region are described below.

V. DARK MATTER ANNIHILATION AND HEATING RATES IN EARTH

A. Maximal annihilation and heating rates

Once it is gravitationally captured, DM will continue to scatter with nuclei in Earth, losing energy until drifting to the core. Once there, because of the large cross section, the DM will thermalize with the nuclei in the core. The number of DM particles $\mathcal{N}$ is governed by the relation between the capture (Γ_C) and annihilation ($\Gamma_{\mathcal{A}}$) rates [66]:

$$\Gamma_{\mathcal{A}} = \tfrac{1}{2}\mathcal{A}\mathcal{N}^2 = \tfrac{1}{2}\Gamma_C \tanh^2(t\sqrt{\Gamma_C \mathcal{A}}). \quad (22)$$

We neglect the possibility of evaporation [15] for the moment, which will affect our results for low m_χ and $\sigma_{\chi N}$, as we will explain further below. The variable t is the age of the system. $\mathcal{A}$ is related to the DM self-annihilation cross section $\sigma_{\chi\chi}$ by

$$\mathcal{A} = \frac{\langle \sigma_{\chi\chi} v\rangle}{V_{\rm eff}}, \quad (23)$$

where $V_{\rm eff}$ is the effective volume of the system [66]. For the relevant cross sections considered, equilibrium between capture and annihilation is generally reached (see below), so the annihilation rate is

$$\Gamma_{\mathcal{A}} = \tfrac{1}{2}\Gamma_C. \quad (24)$$

The effective volume is determined by the method of Griest and Seckel [66], which is essentially the volume of the DM distribution in the core. The number density of DM is assumed to be an exponentially decaying function $\exp(-m_\chi \phi/kT)$, like the Boltzmann distribution of molecules in the atmosphere. The temperature of the DM in thermal equilibrium is T. The variable ϕ is the gravitational potential, integrated out to a radius r, written as

$$\phi(r) = \int_0^r \frac{GM(\tilde{r})}{\tilde{r}^2} d\tilde{r}; \quad (25)$$

$$M(\tilde{r}) = 4\pi \int_0^{\tilde{r}} r'^2 \rho(r') dr'. \quad (26)$$

The resulting effective volume using the radius of Earth's outer core, approximately $0.4 R_\oplus$, a temperature of 5000 K, and a density of 9 g/cm^3 [63], is

$$V_{\rm eff} = 4\pi \int_0^{R_{\rm core}} r^2 e^{-(m_\chi \phi)/(kT)} dr \quad (27)$$

$$= 1.2 \times 10^{25}\ {\rm cm}^3 \left(\frac{100\ {\rm GeV}}{m_\chi}\right)^{3/2} \times \int_0^{0.5\sqrt{m_\chi/{\rm GeV}}} u^2 e^{-u^2} du \quad (28)$$

$$\xrightarrow[m_\chi \gg m_A]{} 5.3 \times 10^{24}\ {\rm cm}^3 \left(\frac{100\ {\rm GeV}}{m_\chi}\right)^{3/2}.$$

For increasing m_χ, the integral (without the prefactor) in Eq. (28) quickly reaches an asymptotic value of about 0.44.

At very large masses, the effective volume for annihilation becomes very small. For instance, at $m_\chi \gtrsim 10^{10}$ GeV, the radius of the effective volume is $\lesssim 0.1$ km. With such a large rate of energy injected in such a small volume, the core temperature would be increased, requiring a more careful treatment. However, in Sec. V B, we state how a limit on the annihilation cross section from the unitarity condition [67–69] makes us truncate our bound at $m_\chi = 10^{10}$ GeV, as reflected in Fig. 2. Therefore, this small effective volume is not a large concern for our exclusion region.

We assume that the DM annihilates into primarily standard model particles, which will deposit nearly all of their energy into Earth's core (with small corrections due to particle rest masses and the escape of low-energy neutrinos). When all of the DM captured is efficiently annihilated, as specified, the heating rate of Earth is in equilibrium with the capture rate:

$$\Gamma_{\rm heat} = \Gamma_C \times m_\chi = n_\chi \sigma_{\rm eff} v_\chi m_\chi$$
$$= \frac{\rho_\chi}{m_\chi} 2\pi (0.99 R_\oplus)^2 (270\ {\rm km/s}) m_\chi$$
$$= 3260\ {\rm TW}. \quad (29)$$

This heat flow is independent of DM mass, since the flux (and capture rate, when capture is efficient) scales as $1/m_\chi$, while each DM particle gives up m_χ in heat when it annihilates. The value is much larger than the measured rate of 44 TW we discussed in Sec. III.

B. Equilibrium requirements

Does the time scale of Earth allow for equilibrium between capture and annihilation for our scenario? In order for Eq. (22) to be in the equilibrium limit, $\tanh^2(t\sqrt{\Gamma_C \mathcal{A}})$ must be of order unity. This is true if $t\sqrt{\Gamma_C \mathcal{A}}$ has a value of a few or greater. Since Earth is about 4.5 Gyr old, we conservatively require that the time taken to reach equilibrium should be less than about 1 Gyr. From this, a realistic annihilation cross section is found. The condition

$$\tanh^2(t\sqrt{\Gamma_C \mathcal{A}}) = 1, \qquad t\sqrt{\Gamma_C \mathcal{A}} \simeq \text{few}, \tag{30}$$

allows the relation

$$\frac{\langle \sigma_{\chi\chi} v \rangle}{V_{\text{eff}}} = \mathcal{A} \gtrsim \frac{(\text{few})^2}{\Gamma_C t^2}, \tag{31}$$

$$\langle \sigma_{\chi\chi} v \rangle \gtrsim 10 \frac{V_{\text{eff}}}{\Gamma_C t^2}. \tag{32}$$

For an efficient capture rate [Eq. (1)], the time of 1 Gyr, and the limit of large m_χ, this requires an annihilation cross section for equilibrium of

$$\langle \sigma_{\chi\chi} v \rangle \gtrsim 10^{-30} \left(\frac{\text{GeV}}{m_\chi}\right)^{1/2} \text{cm}^3/\text{s}. \tag{33}$$

Since this required lower bound is much smaller than that of typical weakly interacting DM particles that are thermal relics ($\langle \sigma_{\chi\chi} v \rangle \simeq 10^{-26}$ cm^3/s [1]), it should be easily met. One expects large scattering cross sections to be accompanied by large annihilation cross sections, so that even the possibility of p-wave-only suppression of the annihilation rate should not be a problem.

For very large masses, the required annihilation cross section, while small, approaches a quantum mechanical limit. For example, for $m_\chi \gtrsim 10^{10}$ GeV the s-wave cross section exceeds the unitarity bound [67–69]. We note that this may also affect constraints on supermassive DM based on neutrinos from annihilation [32–34]. To be conservative, we therefore do not extend our constraints beyond this point, though they may still be valid.

The time scale also has to be long enough for DM to drift down to the core. If $\sigma_{\chi N}$ is too large, the DM will experience too many scatterings, will not settle into the core, and thus may not annihilate efficiently. Following Starkman *et al.* [2], we define the upper edge of our exclusion region to require a drift time of $\lesssim 1$ Gyr. This places a restriction of

$$\begin{aligned} \sigma_{\chi N} &\lesssim 7.7 \times 10^{-20}\ \text{cm}^2 \frac{(m_\chi/\text{GeV})}{A^2(\mu(A)/\mu(N))^2} \\ &\lesssim 7.7 \times 10^{-20}\ \text{cm}^2 \frac{m_\chi/\text{GeV}}{A^4(\frac{m_\chi+m_N}{m_\chi+m_A})^2}, \\ &\xrightarrow[m_\chi \gg m_A]{} 2.5 \times 10^{-23}\ \text{cm}^2 \left(\frac{m_\chi}{\text{GeV}}\right) \end{aligned} \tag{34}$$

for a target of iron. However, a more detailed calculation might relax this requirement. For example, Starkman *et al.* [2] show that, for large values of the capture cross section and certain other conditions, annihilation may be efficient enough to occur in a shell, before the DM reaches the core. This would generally still be subject to our constraint on heat from DM annihilation, and hence our exclusion region might extend to larger cross sections than shown. The two features of this drift line at low mass occur around the mass of the target and the mass of a nucleon, due to the various dominances of the mass-dependent term in the denominator. The details of the shape of this drift line at low masses are irrelevant, because the astrophysical constraints already exclude the corresponding regions.

Aside from drifting to the core, the question of whether heavy DM can actually get to Earth has been asked [70,71]. The low-velocity tail of the high-mass DM thermal distribution in the solar system may be driven into the Sun by gravitational capture processes [70,71], especially because this DM's velocity is on the order of the orbital speed of Earth in the solar system, which is about 30 km/s. However, this would affect only a tiny fraction of the full thermal distribution that we require to be efficiently captured.

C. Annihilation and heating efficiencies

We are not picking a specific model for the annihilation products, aside from considering only standard model particles, which will deposit their energy in Earth, with the exception of neutrinos. Our constraint thus has a very broad applicability. As noted, the calculated heat flow if DM annihilation is important is 3260 TW, which is very large compared to our adopted limit on an unconventional source of 20 TW (or even the whole measured rate of 44 TW). Typically then, either DM annihilation heating is overwhelming or it is negligible, inside or outside of the excluded region, respectively. As shown in Sec. IV, the kinetic energy transferred from DM scattering on nuclei is about 6 orders of magnitude less than the energy from DM annihilation. This contribution to Earth's heat is too low to be relevant for global considerations. However, it would be interesting to consider the more localized effect of the kinetic energy deposition in the atmosphere for very large cross sections.

There are circumstances in which the heating from DM annihilation can take a more intermediate value, including down to the chosen 20 TW number. As explained above, typically the number of scatterings required to gravitationally capture the DM is very large. Therefore, a small decrease in $\sigma_{\chi N}$ and the proportionate change in the expected number of scatterings mean that the compounded kinetic energy loss is nearly always insufficient. However, at low m_χ, the number is small enough that upward fluctuations relative to the expected number can lead to capture. If N_{scat} collisions typically lead to efficient capture for a cross section $\sigma_{\chi N}^{\text{min}}$, as defined above, a new and smaller N may be defined by the condition that the Poisson probability $\text{Prob}(N \geq N_{\text{scat}}) = 20/3260\,\text{TW} = 1/163$. With this N, and its proportionately smaller $\sigma_{\chi N}$, upward Poisson fluctuations in the number of scatterings lead to efficient capture for a fraction $1/163$ of the incoming flux. Note that this small capture fraction is not just the low-velocity tail of the DM thermal distribution, since we have defined these conditions for the highest incoming velocities $v_i = 500$ km/s.

The resulting constraint on $\sigma_{\chi N}$ is shown by the dashed line that dips below the main excluded region in Fig. 2. The enhanced valley around 16 GeV again arises from the ease of capture when the DM mass is near the target mass. Note that for each mass the required $\langle\sigma_{\chi\chi}v\rangle$ is increased by the same factor that decreased the original required $\sigma_{\chi N}^{\min}$. Since most of this exclusion region is already covered by underground detectors, its details may not be so important.

For low DM masses, evaporative losses of DM from the core due to upscattering by energetic iron nuclei may be relevant [15]. Simple kinematic estimates show that DM masses below $\simeq 5$ GeV might be affected. However, this is only potentially important if $\sigma_{\chi N}$ is small enough that scattering is very rare—since otherwise any upscattered DM will immediately downscatter. From the considerations above about Poisson fluctuations in the number of scatterings, we expect that this should be relevant only between the dark-shaded region and the dashed line.

VI. DISCUSSION AND CONCLUSIONS

A. Principal results

As summarized in Fig. 1, while very large DM-nucleon scattering cross sections are excluded by astrophysical considerations, and small cross sections are excluded by underground direct DM detection experiments, there is a substantial window in between that has proven very difficult to test, despite much effort [2,10–13,15–18,20,26,28,32,35–37,43–46,72,73]. High-altitude experiments have excluded only parts of this window. In this window, DM will be efficiently captured by Earth. We point out that the subsequent self-annihilation of DM in Earth's core would lead to an enormous heating rate of 3260 TW, compared to the geologically measured value of 44 TW.

We show that the conditions for efficient capture, annihilation, and heating are all quite generally met, leading to an exclusion of $\sigma_{\chi N}$ over about 10 orders of magnitude, which closes the window on strongly interacting DM between the astrophysical and direct detection constraints. These new constraints apply over a very large mass range, as shown in Fig. 2. We have been quite conservative, and so very likely an even larger region is excluded. These results establish that DM interactions with nucleons are bounded from above by the underground experiments and therefore that these interactions must be truly weak, as commonly assumed. This means that direct detection experiments are looking in the correct $\sigma_{\chi N}$ range when sited underground and motivates further theoretical study of weakly interacting DM [74–77]. Furthermore, it means that DM-nucleon scattering cannot have any measurable effects in astrophysics and cosmology, and this has many implications for models with strongly or moderately interacting DM [29,72,73,78] and other astrophysical constraints on the DM-nucleon interaction cross section [79,80]. This exclusion region also completely covers the cross section range in which strongly interacting dark matter might bind to nuclei [81].

To evade our constraints, extreme assumptions would be required: that DM is not its own antiparticle, or that there is a large ($\gtrsim 163$) particle-antiparticle asymmetry from DM, or that DM self-annihilation proceeds only to purely sterile non-standard model particles, at the level of $\gtrsim 100{:}1$. (Although in Ref. [69] such a large branching ratio to neutrinos was considered, it was emphasized that this was used only to set the most conservative bound on the DM annihilation cross section and not to be representative of a realistic model.) While our constraints are based on Earth's measured heat flow, it is important to emphasize that DM capture and annihilation generally cannot contribute measurably without being overwhelming and hence are excluded. Thus, in the ongoing debate over the unknown sources of Earth's heat flow, it seems that DM can play no role. The most important next step in refining our understanding of the known generators of the measured 44 TW will come from isolating the contribution from uranium and thorium decays by measuring the corresponding neutrino fluxes [51–61].

TABLE II. Comparison of potential dark matter constraints using various planets [31]. A greater difference between dark matter and internal heating rates gives greater certainty. The minimum cross section probed scales roughly with the surface gravity. Earth is the best for setting reliable and strong constraints.

Planet	DM max. heating (TW)	Internal heat (TW)	Surface gravity (units of ⊕)
Earth	3.3×10^3	0.044×10^3	1.00
Jupiter	420×10^3	400×10^3	2.74
Saturn	290×10^3	200×10^3	1.17
Uranus	53×10^3	$\leq 10^3$	0.94
Neptune	50×10^3	3×10^3	1.15

B. Comparison to other planets

Starkman *et al.* [2] calculated the efficient Earth capture line for DM but had only model-dependent results. We have now considered the consequences of annihilation in Earth and have shown that it gives a model-independent constraint. Other planets have been discussed, such as Jupiter and Uranus [13,24–30], but Earth is the best laboratory. It is the best understood planet, with internal heat flow data measured directly underground, from many locations, and Earth's composition and density profile are well known [23,47–50,62,63]. Importantly, the relative excess heat due to DM annihilation would be much greater for Earth than for the Jovian planets.

What about other planets? The maximal heating rate due to DM scales with surface area and can be compared with the internal heating rates estimated from infrared data [31]. If a constraint can be set, the minimum cross section $\sigma_{\chi N}^{\min}$ that can be probed scales with the planet's column density as $(nL)^{-1}$ [see Eq. (17)] up to nontrivial corrections for composition [see Eq. (12)]. Note that the column density nL is proportional to the surface gravity $\sim GM/R^2$, which varies little between the planets, as noted in Table II [31]. Because of its known (and heavy) composition and well-measured (and low) internal heat, the strongest and most reliable constraints will be obtained by considering Earth. As an interesting aside, it may then be unlikely that heating by DM could play a significant role in explaining the apparent overheating of some extrasolar planets ("hot Jupiters") [82–86].

C. Future directions

While it appears that the window on strongly interacting DM is now closed over a huge mass range, more detailed analyses are needed in order to be absolutely certain. Our calculations are conservative, and the true excluded region is likely to be larger. It would be especially valuable to have new analyses of the astrophysical limits and the underground detector constraints. This would give greater certainty that no sliver of the window is still open. For DM masses in the range 1–10^{10} GeV, the upper limits on the DM scattering cross section with nucleons from CDMS and other underground experiments have been shown to be true upper limits. Thus the DM does indeed appear to be very weakly interacting, and it will be challenging to detect it.

ACKNOWLEDGMENTS

We are grateful for discussions and advice from Nicole Bell, Eric Braaten, Matt Kistler, Jordi Miralda-Escudé, Jerry Newsom, Wendy Panero, Gary Steigman, Louie Strigari, Ralph von Frese, Terry Walker, and especially Laura Baudis, Andy Gould, Stefano Profumo, and Hasan Yüksel. G. D. M. was supported by the Department of Energy Grant No. DE-FG02-91ER40690, J. F. B. by the National Science Foundation CAREER Grant No. PHY-0547102, and G. B. during part of this project by the Helmholtz Association of National Research Centres under Project No. VH-NG-006.

[1] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996); L. Bergstrom, Rep. Prog. Phys. **63**, 793 (2000); G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005).

[2] G.D. Starkman, A. Gould, R. Esmailzadeh, and S. Dimopoulos, Phys. Rev. D **41**, 3594 (1990).

[3] P. Natarajan, A. Loeb, J. P. Kneib, and I. Smail, Astrophys. J. **580**, L17 (2002).

[4] X. L. Chen, S. Hannestad, and R. J. Scherrer, Phys. Rev. D **65**, 123515 (2002).

[5] R. H. Cyburt, B. D. Fields, V. Pavlidou, and B. D. Wandelt, Phys. Rev. D **65**, 123503 (2002).

[6] D. S. Akerib *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **559**, 390 (2006); D. S. Akerib *et al.* (CDMS Collaboration), Phys. Rev. Lett. **96**, 011302 (2006); Phys. Rev. D **73**, 011102 (2006).

[7] I. F. M. Albuquerque and L. Baudis, Phys. Rev. Lett. **90**, 221301 (2003); **91**, 229903(E) (2003).

[8] V. Sanglard *et al.* (EDELWEISS Collaboration), Phys. Rev. D **71**, 122002 (2005).

[9] R. Bernabei *et al.*, Phys. Rev. Lett. **83**, 4918 (1999).

[10] J. Rich, R. Rocchia, and M. Spiro, Phys. Lett. B **194**, 173 (1987).

[11] B. D. Wandelt, R. Dave, G. R. Farrar, P. C. McGuire, D. N. Spergel, and P. J. Steinhardt, in *Sources and Detection of Dark Matter and Dark Energy in the Universe*, edited by D. B. Cline (Springer-Verlag, New York, 2001).

[12] A. L. Erickcek, P. J. Steinhardt, D. McCammon, and P. C. McGuire, arXiv:0704.0794.

[13] L. M. Krauss, M. Srednicki, and F. Wilczek, Phys. Rev. D **33**, 2079 (1986).

[14] T. K. Gaisser, G. Steigman, and S. Tilav, Phys. Rev. D **34**, 2206 (1986).

[15] A. Gould, Astrophys. J. **321**, 560 (1987).

[16] A. Gould, Astrophys. J. **321**, 571 (1987).

[17] A. Gould, Astrophys. J. **328**, 919 (1988).

[18] A. Gould, J. A. Frieman, and K. Freese, Phys. Rev. D **39**, 1029 (1989).

[19] A. Gould, Astrophys. J. **368**, 610 (1991).

[20] A. Gould, Astrophys. J. **387**, 21 (1992).

[21] A. Gould and S. M. K. Alam, Astrophys. J. **549**, 72 (2001).

[22] http://marine.rutgers.edu/mrs/education/class/yuri/erb.html#solar.

[23] G. R. Beardsmore and J. P. Cull, *Crustal Heat Flow: A Guide to Measurement and Modelling* (Cambridge University Press, New York, 2001).

[24] M. Fukugita, P. Hut, and D. N. Spergel, Report No. IASSNS-AST-88-26, 1988 (unpublished).
[25] S. Dimopoulos, D. Eichler, R. Esmailzadeh, and G. D. Starkman, Phys. Rev. D **41**, 2388 (1990).
[26] M. Kawasaki, H. Murayama, and T. Yanagida, Prog. Theor. Phys. **87**, 685 (1992).
[27] S. Abbas and A. Abbas, Astropart. Phys. **8**, 317 (1998).
[28] S. Mitra, Phys. Rev. D **70**, 103517 (2004).
[29] G. Zaharijas and G. R. Farrar, Phys. Rev. D **72**, 083502 (2005).
[30] G. R. Farrar and G. Zaharijas, Phys. Rev. Lett. **96**, 041302 (2006).
[31] M. Zeilik and S. Gregory, *Introductory Astronomy and Astrophysics* (Saunder College, New York, 1998), 4th ed.
[32] I. F. M. Albuquerque, L. Hui, and E. W. Kolb, Phys. Rev. D **64**, 083504 (2001).
[33] P. Crotty, Phys. Rev. D **66**, 063504 (2002).
[34] I. F. M. Albuquerque, J. Lamoureux, and G. F. Smoot, Phys. Rev. D **66**, 125006 (2002).
[35] S. Desai *et al.* (Super-Kamiokande Collaboration), Phys. Rev. D **70**, 083523 (2004); **70**, 109901 (2004).
[36] A. Bottino, N. Fornengo, G. Mignola, and L. Moscoso, Astropart. Phys. **3**, 65 (1995).
[37] A. Achterberg *et al.* (AMANDA Collaboration), Astropart. Phys. **26**, 129 (2006).
[38] M. Cirelli, N. Fornengo, T. Montaruli, I. Sokalski, A. Strumia, and F. Vissani, Nucl. Phys. **B727**, 99 (2005).
[39] L. Bergstrom, J. Edsjo, and M. Kamionkowski, Astropart. Phys. **7**, 147 (1997).
[40] R. S. Chivukula, A. G. Cohen, S. Dimopoulos, and T. P. Walker, Phys. Rev. Lett. **65**, 957 (1990).
[41] H. Nelson, in Proceedings of the Lecture at SLAC Summer Institute, 2003, http://www.slac.stanford.edu/econf/C0307282/lec_notes/nelson.html#lec_1
[42] R. Gaitskell and V. Mandic, "Direct Detection of WIMP Dark Matter, Sensitivity Plots ... An Interactive Plotter for Experimental and Theoretical Data," http://dmtools.brown.edu.
[43] P. C. McGuire, Ph.D. thesis, University of Arizona, 1994, http://epsci.wustl.edu/~mcguire/cvw_pub.html.
[44] E. K. Shirk and P. B. Price, Astrophys. J. **220**, 719 (1978).
[45] D. P. Snowden-Ifft, S. W. Barwick, and P. B. Price, Astrophys. J. **364**, L25 (1990).
[46] J. D. Anderson, E. L. Lau, T. P. Krishner, D. A. Dicus, D. C. Rosenbaum, and V. L. Teplitz, Astrophys. J. **448**, 885 (1995).
[47] H. Pollack, S. Hurter, and J. Johnson, Rev. Geophys. **31**, 267 (1993).
[48] S. Karato, *The Dynamic Structure of the Deep Earth* (Princeton University Press, Princeton, NJ, 2003).
[49] R. von Frese (private communication).
[50] W. Panero (private communication).
[51] T. Araki *et al.*, Nature (London) **436**, 499 (2005).
[52] L. M. Krauss, S. L. Glashow, and D. N. Schramm, Nature (London) **310**, 191 (1984).
[53] C. G. Rothschild, M. C. Chen, and F. P. Calaprice, Geophys. Res. Lett. **25**, 1083 (1998).
[54] R. S. Raghavan, S. Schonert, S. Enomoto, J. Shirai, F. Suekane, and A. Suzuki, Phys. Rev. Lett. **80**, 635 (1998).
[55] G. Fiorentini, M. Lissia, F. Mantovani, and B. Ricci, Phys. Lett. B **629**, 77 (2005).
[56] G. Fiorentini, M. Lissia, F. Mantovani, and R. Vannucci, Phys. Rev. D **72**, 033017 (2005).
[57] B. D. Fields and K. A. Hochmuth, Earth Moon Planets **99**, 155 (2006).
[58] G. L. Fogli, E. Lisi, A. Palazzo, and A. M. Rotunno, Earth Moon Planets **99**, 111 (2006).
[59] S. T. Dye *et al.*, arXiv:hep-ex/0609041.
[60] L. Miramonti (BOREXINO Collaboration), arXiv:hep-ex/0609011.
[61] K. A. Hochmuth *et al.*, Astropart. Phys. **27**, 21 (2007).
[62] P. Olson, in *Earth's Core and Lower Mantle*, edited by C. Jones *et al.* (Taylor & Francis, New York, 2003).
[63] E. Lisi and D. Montanino, Phys. Rev. D **56**, 1792 (1997).
[64] L. Landau and E. Lifshitz, *Mechanics* (Pergamon, New York, 1960).
[65] H. Bethe and P. Morrison, *Elementary Nuclear Theory* (Wiley, New York, 1956).
[66] K. Griest and D. Seckel, Nucl. Phys. **B283**, 681 (1987); **B296**, 1034(E) (1988).
[67] K. Griest and M. Kamionkowski, Phys. Rev. Lett. **64**, 615 (1990).
[68] L. Hui, Phys. Rev. Lett. **86**, 3467 (2001).
[69] J. F. Beacom, N. F. Bell, and G. D. Mack, arXiv:astro-ph/0608090.
[70] A. Gould and S. M. Khairul Alam, Astrophys. J. **549**, 72 (2001).
[71] J. Lundberg and J. Edsjo, Phys. Rev. D **69**, 123505 (2004).
[72] K. Enqvist, A. Jokinen, T. Multamaki, and I. Vilja, Phys. Lett. B **526**, 9 (2002).
[73] T. Banks, J. D. Mason, and D. O'Neil, Phys. Rev. D **72**, 043530 (2005).
[74] J. D. Vergados, arXiv:hep-ph/0601064.
[75] L. Anchordoqui and F. Halzen, Ann. Phys. (N.Y.) **321**, 2660 (2006).
[76] D. Hooper and A. M. Taylor, J. Cosmol. Astropart. Phys. 03 (2007) 017.
[77] H. Baer, A. Mustafayev, E. K. Park, and S. Profumo, J. High Energy Phys. 07 (2005) 046.
[78] M. Y. Khlopov, Pis'ma Zh. Eksp. Teor. Fiz. **83**, 3 (2006) [JETP Lett. **83**, 1 (2006)]; arXiv:astro-ph/0607048.
[79] S. Furlanetto and A. Loeb, Astrophys. J. **565**, 854 (2002).
[80] L. Chuzhoy and A. Nusser, Astrophys. J. **645**, 950 (2006).
[81] R. N. Mohapatra and V. L. Teplitz, Phys. Rev. Lett. **81**, 3079 (1998); D. Javorsek, D. Elmore, E. Fischbach, D. Granger, T. Miller, D. Oliver, and V. Teplitz, Phys. Rev. D **65**, 072003 (2002); D. Javorsek, E. Fischbach, and D. Elmore, Phys. Rev. D **67**, 034015 (2003); V. Teplitz, D. Javorsek, E. Fischbach, and R. Mohapatra, Nucl. Phys. B, Proc. Suppl. **124**, 205 (2003).
[82] B. S. Gaudi, S. Seager, and G. Mallen-Ornelas, Astrophys. J. **623**, 472 (2005).
[83] G. A. Bakos *et al.*, Astrophys. J. **656**, 552 (2007).
[84] A. Burrows, I. Hubeny, J. Budaj, and W. B. Hubbard, Astrophys. J. **661**, 502 (2007).
[85] D. C. Fabrycky, E. T. Johnson, and J. Goodman, arXiv:astro-ph/0703418.
[86] J. Southworth, P. J. Wheatley, and G. Sams, arXiv:0704.1570.