Dynamics of "logamediate" inflation

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A computation of the inflationary observables n_s and r is made for logamediate inflation where the cosmological scale factor expands as $a = \exp(A(\ln t)^\lambda)$, and is compared to their predicted values in the intermediate inflationary theory, where $a = \exp(Bt^f)$. Both versions prove to be consistent with observational measurements of the cosmic background radiation. It is shown that the dynamics of a single inflaton field can be mimicked by a system of several fields in an analogous manner to that created by the joint evolution of the fields in assisted power-law inflation.

I. INTRODUCTION

In the light of the latest observations of the cosmic microwave background (CMB) radiation [1], a scenario of inflationary expansion in the early Universe stands out as a strong candidate to solve the horizon and flatness problems of the standard model of cosmology, as well as providing the seeds for the formation of large-scale structure with a spectrum of adiabatic and nearly scale-invariant Gaussian density perturbations. For a review of models of inflation see, e.g., Ref. [2].

In the particular scenario of "intermediate" inflation [3–9], the expansion scale factor of the Friedmann universe evolves as $a = \exp(At^f)$, where $A > 0$ and $1 > f > 0$ are constants; the expansion of the Universe is slower for standard de Sitter inflation, which arises when $f = 1$, but faster than in power-law inflation, $a = t^p$, with $p > 1$ constant. It has been shown that intermediate inflation satisfies the bounds on the spectral index n_s and ratio of tensor-to-scalar perturbations, r, as measured by the latest observations of the CMB [1]. To first order, an exact Harrison-Zeldovich spectrum [10] of fluctuations arises when $f = 2/3$ as well as when $f = 1$. For the construction by series of a potential which produces this spectrum to all orders, see Ref. [11].

In this work, we analyze another generalized version of inflation, which we call "logamediate inflation," with a scale factor of the form $a = \exp(A(\ln t)^\lambda)$, with $A > 0$, $\lambda > 1$ constants. When $\lambda = 1$, this model reduces to power-law inflation with $a = t^p$, where $p = A$, here. The logamediate inflationary form is motivated by considering a class of possible cosmological solutions with indefinite expansion which result from imposing weak general conditions on the cosmological model. In Ref. [12] it was shown that an application of the considerations of Hardy and Fowler to ordinary differential equations of the form $\ddot{a} = P(a, t)/Q(a, t)$, as $t \to \infty$ with polynomials P and Q, leads to eight possible asymptotic solutions of the cosmological

dynamics. Three of these give noninflationary expansions for $a(t)$ and three of the others give power-law, de Sitter, and intermediate inflationary expansions. The remaining two cases give an asymptotic expansion of the form $a = \exp(A(\ln t)^\lambda)$, and this is the only one of the allowed possibilities that, to date, has not been studied in detail with respect to the observational data. We note also that this form of inflation arises naturally in a number of scalar-tensor theories [13]. Here, we show that, for observationally viable models of logamediate inflation, the ratio of tensor-to-scalar perturbations, r, must be small and that the power spectrum can be either red or blue tilted, depending on the specific parameters of the model.

We will also study the dynamics when an ensemble of fields is present. We look for situations in which the dynamics allows the ratios of the individual kinetic energies of the fields to approach constant values. This situation has been studied in the literature in the context of power-law inflation and it was dubbed assisted inflation [14–21]. It has the interesting property that the cooperative evolution of all the fields can lead to inflation even if the individual logarithmic slopes of the fields are too steep to provide inflation when the fields are rolling in isolation. In other words, the effective p can become larger than unity if additional fields are included in the dynamics, even if the individual p_i are smaller than unity if the fields were rolling alone. In the case of logamediate inflation, as we shall see, we do not encounter this property as the condition for inflation is set by the value of λ alone; nonetheless, the fractional Hubble constant, A, becomes the effective quantity that is changed by the number of contributing fields.

II. LOGAMEDIATE INFLATION: $a = \exp(A(\ln t)^\lambda)$

We start by considering the evolution of the scale factor of a flat Friedmann universe to be

$$a(t) = \exp[A(\ln t)^\lambda], \tag{1}$$

and $t > 1$. The Hubble rate is

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$$H \equiv \frac{\dot{a}}{a} = A\lambda(\ln t)^{\lambda-1}\frac{1}{t}, \qquad (2)$$

hence, for an expanding universe we require $A\lambda > 0$. On the other hand,

$$\frac{\ddot{a}}{a} = \frac{A\lambda}{t^2}(\ln t)^{\lambda-1}[(\lambda-1)(\ln t)^{-1} - 1 + A\lambda(\ln t)^{\lambda-1}]. \quad (3)$$

Therefore, for an inflationary universe with $\ddot{a}/a > 0$, it is necessary that $\lambda > 1$, or if $\lambda = 1$, that $A > 1$.

A. Single-field inflation

Now assume the material content of the universe is a singe scalar field ϕ with potential $V(\phi)$. Since $\dot{H} = -\dot{\phi}^2/2$, at late times we have

$$\dot{\phi} = \sqrt{2}(A\lambda)^{1/2}(\ln t)^{(\lambda-1)/2}\frac{1}{t}, \qquad (4)$$

and the evolution of the field satisfies

$$\phi = \phi_0 + 2\frac{(A\lambda)^{1/2}}{\lambda+1}(\ln t)^{(\lambda+1)/2}, \qquad (5)$$

where ϕ_0 is constant. The scalar potential results from $V = 3H^2 + \dot{H}$. Substituting for H and $\dot{H}$ gives

$$V(\phi) = 3(A\lambda)^2(\ln t)^{2(\lambda-1)}\frac{1}{t^2} + A\lambda(\lambda-1)(\ln t)^{\lambda-2}\frac{1}{t^2}$$

$$- A\lambda(\ln t)^{\lambda-1}\frac{1}{t}. \qquad (6)$$

At late times, only the first term survives, hence, using solution (4) (setting $\phi_0 = 0$, without loss of generality), the potential can be written as

$$V(\phi) = V_0\phi^\alpha \exp(-\beta\phi^\gamma), \qquad (7)$$

where

$$V_0 = 3(A\lambda)^2 B^{2(\lambda-1)}, \qquad (8)$$

and

$$B = \left(\frac{\lambda+1}{2\sqrt{2}(A\lambda)^{1/2}}\right)^{2/(\lambda+1)}, \qquad (9)$$

and $\alpha = 4(\lambda-1)/(\lambda+1)$, $\beta = 2B$, and $\gamma = 2/(\lambda+1)$. This class of scalar potentials were studied in Refs. [22,23]. Note that we would have obtained the same form of the scalar potential had we assumed slow-roll inflation, $3H\dot{\phi} \approx -dV/d\phi$. Indeed, as the field rolls down the potential towards larger values, the slow-roll approximation becomes increasingly more accurate, hence the two different approaches give the same result.

In the Hamilton-Jacobi formalism we write the slow-roll parameters as

$$\epsilon = 2\left(\frac{H'}{H}\right)^2, \qquad \eta = 2\frac{H''}{H},$$

where the prime represents differentiation with respect to the scalar field ϕ. For our scalar potential these become
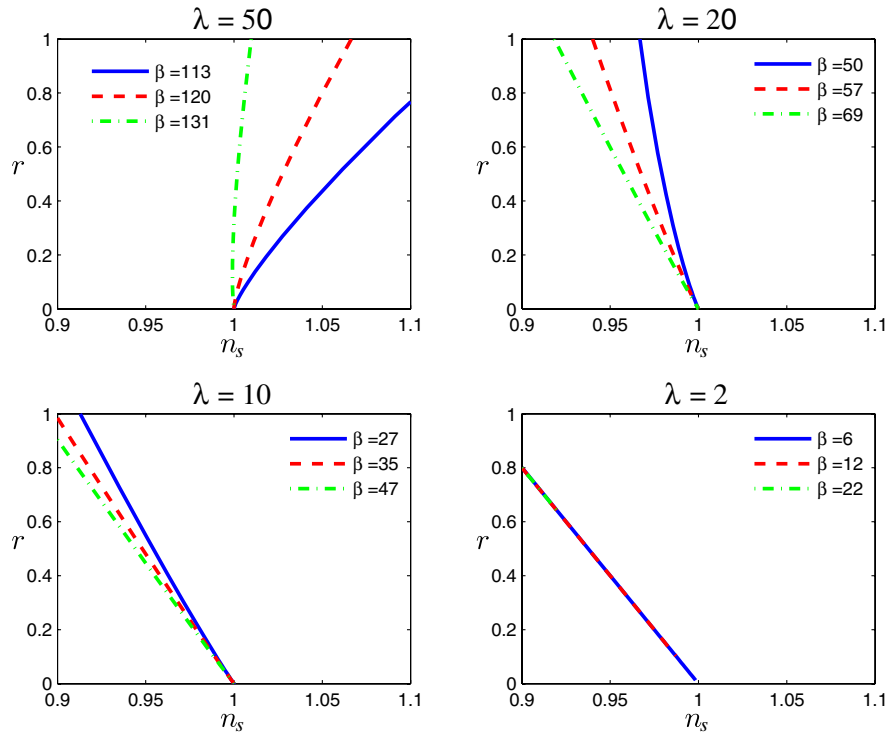

FIG. 1 (color online). Trajectories for different combinations of the parameters (λ, β) in the n_s-r plane. For these values of β, the parameter A ranges from $A = 1.5 \times 10^{-92}$ for $(\lambda, \beta) = (50, 131)$ up to $A = 2.1 \times 10^{-2}$ for $(\lambda, \beta) = (2, 6)$.

$$\epsilon = \frac{1}{2\phi^2}(\alpha - \beta\gamma\phi^\gamma)^2, \qquad (10)$$

$$\eta = -\frac{1}{\phi^2}\left[\alpha + \beta\gamma(\gamma - 1)\phi^\gamma - \frac{1}{2}(\alpha - \beta\gamma\phi^\gamma)^2\right]. \qquad (11)$$

The slow-roll parameter ϵ diverges when the field approaches zero, has a minimum at the maximum of the potential, peaks at some value ϕ_ϵ, and finally asymptotes to zero for large values of the field. We will focus on those cases where the peak occurs for $\epsilon > 1$, so that we can identify the moment when inflation begins with $\phi_1 \equiv \phi(\epsilon = 1)$. We are, therefore, limiting our analysis to the region of parameter space defined by

$$\beta > 2\left(\frac{1}{32}(\lambda + 1)^{(\lambda+3)}\right)^{1/(\lambda+1)}. \qquad (12)$$

The number of e-folds between two values of the field, ϕ_1 (defined to be the beginning of inflation) and ϕ_2 (when a given mode exits the horizon), is given by

$$N = -\int_{\phi_1}^{\phi_2} d\phi \frac{\phi}{\alpha - \beta\gamma\phi^\gamma}. \qquad (13)$$

The spectral index, n_s, and the ratio of tensor-to-scalar perturbations, r, can be expressed in terms of the slow-roll parameters as

$$n_s = 1 - 4\epsilon + 2\eta, \qquad (14)$$

$$r = 16\epsilon. \qquad (15)$$

For a given set of parameters, β and λ, and consequently ϕ_1, we have fixed a value ϕ_2, and then calculated the corresponding quantities N, ϵ, η, and $\xi^2 = \epsilon\eta - (2\epsilon)^{1/2}\eta'$ numerically. In Fig. 1 we show the trajectories in the n_s–r plane. Curiously, a scale-invariant spectrum with large r can be obtained for $(\lambda, \beta) = (50, 131)$. The second-order expression for the spectral index in terms of the slow-roll parameters is given by

$$n_s = 1 - 4\epsilon + 2\eta - [8(1 + C)\epsilon^2 - (6 + 10C)\epsilon\eta + 2C\xi^2], \qquad (16)$$

where $C = -0.73$. Comparing Figs. 1 and 2, we can conclude that the second-order correction to the spectral index is negligible.

In Fig. 3, we show the dependence of the spectral index on the number of e-folds of inflation, for the same range of values of the parameters (λ, β) of Fig. 1. We can observe that there is a range of values of n_s and r that is compatible with the WMAP3 analysis. For small numbers of e-folds of inflation, compatibility is assured for large values of the parameter λ.

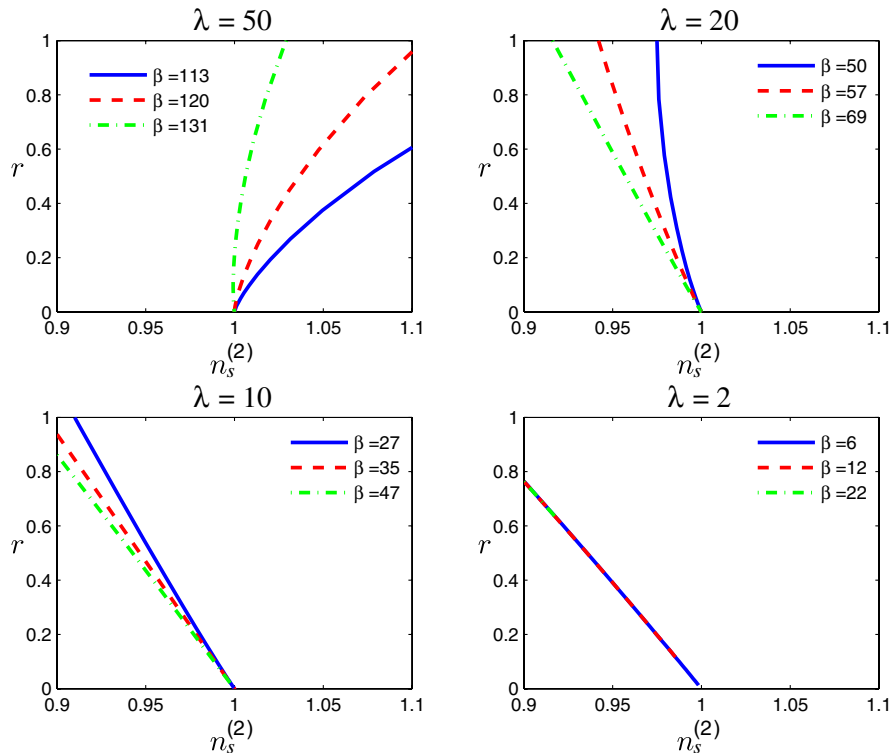The running of the spectral index, to lowest order in slow roll, is given by

FIG. 2 (color online).   Trajectories for different combinations of the parameters (λ, β) in the $n_s^{(2)}$–r plane, where $n_s^{(2)}$ is the second-order expansion of the spectral index in slow-roll parameters.
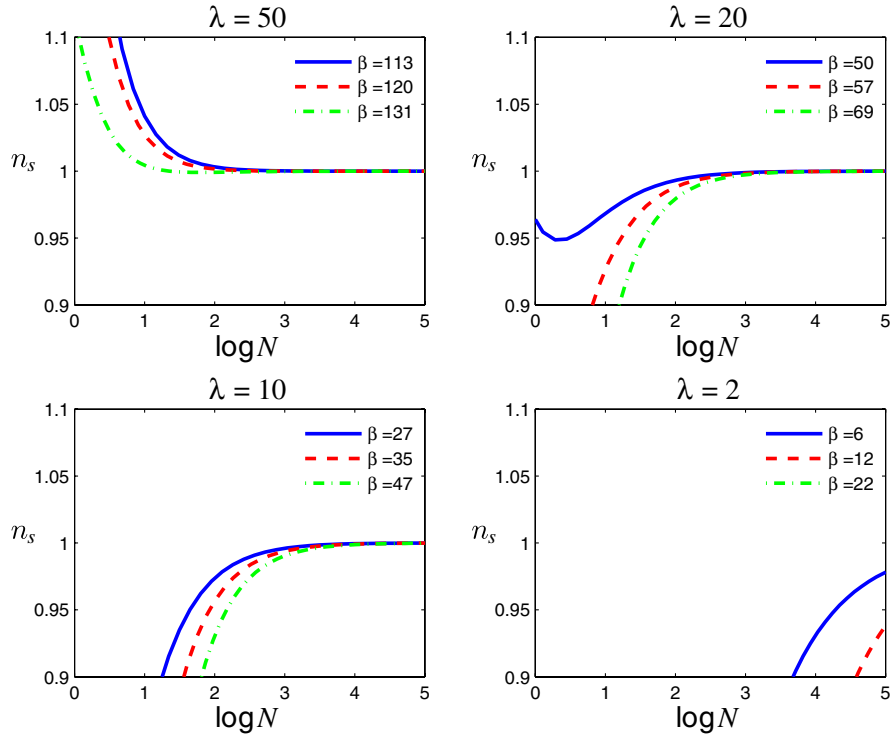
FIG. 3 (color online). Dependence of the spectral index on the number of e-folds of inflation for different combinations of the parameters (λ, β).

$$\frac{dn_s}{d\ln k} = -8\epsilon^2 + 10\epsilon\eta - 2\xi^2. \tag{17}$$

From Fig. 4, we observe that for certain combinations of the parameters (λ, β), the running of the spectral index can be negative, which is favored by WMAP3.

B. Multifield inflation

In the original model of assisted power-law inflation, with combinations of pure exponential potentials, the dynamics can be interpreted by performing a rotation in the field space decomposing the fields into a weighted mean field that sets the direction of motion, and a set of fields orthogonal to it. The potential for the orthogonal directions has a minimum, therefore assuring the stability of the scaling solution obtained in these models. We expect the same to be true in the case of logamediate inflation, although a scaling solution is not attained (i.e., the ratio between the potential and kinetic energies is *not* a constant throughout the evolution) because it is reasonable to assume that the ratios of the kinetic energies of the fields can reach a constant value as these potentials encounter a valley along each defined direction. Moreover, by increasing the number of fields we are increasing the Hubble damping in the equations of motion for these fields, hence preventing them from running away. Consequently, slow-roll evolution is reinforced. We will see now that the combined evolution of an ensemble of fields can be mim-

icked by one field, as in the previous section, where the parameter A depends on the steepness of the potential in all the original field-space directions.

We will now generalize the previous logamediate inflationary solution for the scalar potential to include an arbitrary number of scalar fields. To this end, and motivated by the single-field case, we will assume that at late times we can express the cosmic time as a weighted sum of m_q different scalar fields, ϕ_i, in the form

$$\ln t = \sum_i^{m_q} \alpha_{qi} \phi_i^{2/(\lambda+1)}, \tag{18}$$

so that when the potential is written in terms of time, t, it becomes

$$V = \sum_q k_q (\ln t)^{2(\lambda-1)} \frac{1}{t^2}, \tag{19}$$

where the k_q are constants that depend on the parameters α_{qi} and will be determined in what follows. Hence, we will assume that the general form of the scalar potential that has this behavior at late times can be written in terms of the fields as

$$V = \sum_q^n v_q \left[\sum_i^{m_q} \alpha_{qi} \phi_i^{2/(\lambda+1)} \right]^{2(\lambda-1)}$$

$$\times \exp\left[-2\sum_i^{m_q} \alpha_{qi} \phi_i^{2/(\lambda+1)} \right], \tag{20}$$
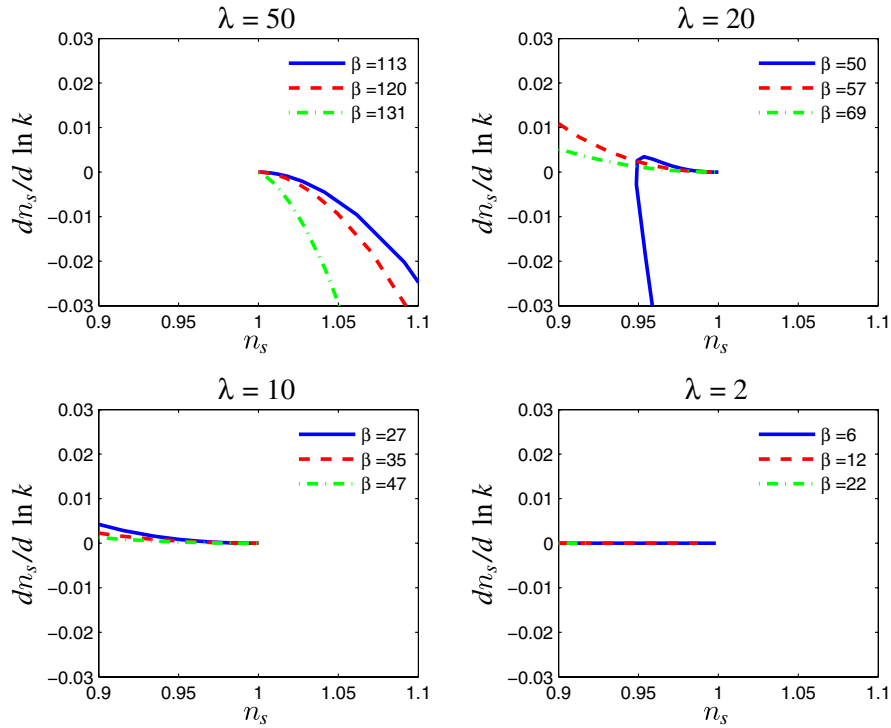
where the v_q are arbitrary constants.

FIG. 4 (color online).   Trajectories in the n_s–$(dn_s/d\ln k)$ plane, for different combinations of the parameters (λ, β).

We are mainly interested in those situations where all fields play an important role in the evolution. More specifically, we will focus on models for which the ratio of the kinetic energies of the fields reaches a constant value at late times. Since we must have

$$2\dot{H} = -\sum_i^m \dot{\phi}_i^2 = 2A\lambda(\lambda - 1)(\ln t)^{\lambda-2}t^{-2}, \quad (21)$$

where m is the total number of fields, we search for solutions where

$$\dot{\phi}_i = c_i(\ln t)^{(\lambda-1)/2}\frac{1}{t}, \quad (22)$$

such that

$$\sum_i^m c_i^2 = 2A\lambda. \quad (23)$$

The late-time solutions for the ϕ_i are obtained by integrating Eq. (22), so

$$\phi_i = \frac{2c_i}{\lambda + 1}(\ln t)^{(\lambda+1)/2}. \quad (24)$$

We can derive an additional condition for the c_i coefficients by combining Eqs. (18) and (24):

$$\sum_i^{m_q} \alpha_{qi}c_i^{2/(\lambda+1)} = \left(\frac{\lambda + 1}{2}\right)^{2/(\lambda+1)}. \quad (25)$$

We then go on to compute the various c_i and k_q in terms of the parameters α_{qi} in the potential. Substituting (24) in the

equations of motion for each of the fields, which satisfy

$$\ddot{\phi}_i + 3H\dot{\phi}_i + \frac{\partial V}{\partial \phi_i} = 0, \quad (26)$$

we obtain the set of relations

$$3A\lambda^2 c_i = 2\left(\frac{2}{\lambda + 1}\right)^{2/(\lambda+1)}\sum_q^n k_q\alpha_{qi}c_i^{-[(\lambda-1)/(\lambda+1)]}. \quad (27)$$

Multiplying by c_i, and using constraints (23) and (25) we obtain

$$\sum_q^n k_q = 3A^2\lambda^2,$$

as expected, by comparing Eqs. (6) and (20).

Equation (27) can also be rewritten as

$$c_i^{2/(\lambda+1)} = \left[\frac{2}{3A\lambda}\left(\frac{2}{\lambda + 1}\right)^{2/(\lambda+1)}\right]^{1/\lambda}\left(\sum_q^n k_q\alpha_{qi}\right)^{1/\lambda}. \quad (28)$$

Multiplying by α_{ri}, summing over all fields, and using relation (25), we obtain a set of constraints that must be satisfied by the various scales of the potential k_q:

$$\sum_i^m\left(\sum_q^n k_q\alpha_{qi}\right)^{1/\lambda}\alpha_{ri} = \left(\frac{\lambda + 1}{2}\right)^{2/\lambda}\left(\frac{3A\lambda}{2}\right)^{1/\lambda}. \quad (29)$$

In general, it is a difficult task to compute these quantities, but in the case where the fields only appear in one of the terms in the potential, for a given ϕ_i, all the α_{qi} vanish

except one. In this case, Eqs. (29) can be simplified to give

$$k_q = \left(\frac{\lambda + 1}{2}\right)^2 \left(\frac{3A\lambda}{2}\right)\left(\sum_i^{m_q} \alpha_{qi}^{(\lambda+1)/\lambda}\right)^{-\lambda}. \tag{30}$$

We have already seen that $\sum_q^n k_q = 3A^2\lambda^2$; hence, we can write the parameter A in terms of the coefficients α_{qi} in the potential (20):

$$A = \frac{1}{2\lambda}\left(\frac{\lambda + 1}{2}\right)^2 \sum_q^n \left(\sum_i^{m_q} \alpha_{qi}^{(\lambda+1)/\lambda}\right)^{-\lambda}. \tag{31}$$

From this expression we can see that the value of A increases if we increase the number of terms in the potential and decreases if we increase the number of scalar fields in each of the terms. This behavior is very similar to that encountered in assisted power-law inflation driven by a combination of pure exponential potentials [17].

From Eq. (28), we are now ready to compute the c_i:

$$c_i^{2\lambda/(\lambda+1)} = \left(\frac{\lambda + 1}{2}\right)^{2\lambda/(\lambda+1)} \alpha_{qi}\left(\sum_j^{m_q} \alpha_{qj}^{(\lambda+1)/\lambda}\right)^{-\lambda}. \tag{32}$$

In Fig. 5, we compare the effective value of A,

$$A_{\text{eff}} = \left(\frac{-H^2}{\dot{H}}\right)^\lambda \frac{1}{\lambda^\lambda N^{\lambda-1}}, \tag{33}$$

determined from the numerical integration of the equations of motion, with the value expected at late times using Eq. (31).
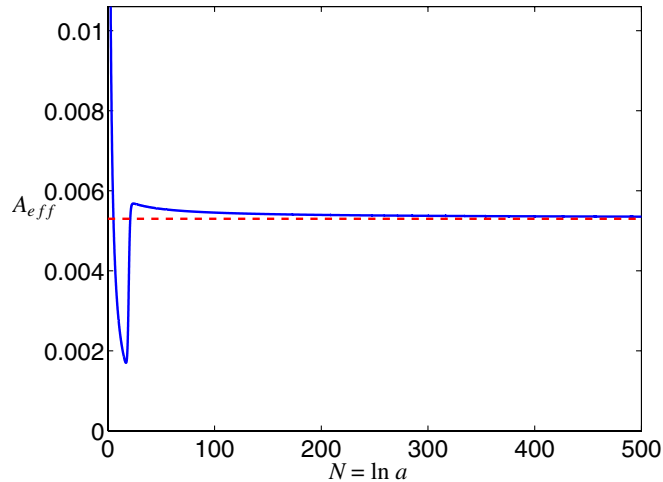

FIG. 5 (color online). Evolution of A_{eff} (solid line) compared with the late-time value expected from Eq. (31) (dashed line). The actual potential used is $V = \sum_q v_q (\sum_i \alpha_{qi}\phi_i^{2/(\lambda+1)})^{2(\lambda-1)} \times \exp[-2\sum_i \alpha_{qi}\phi_i^{2/(\lambda+1)}]$, with $\alpha_{11} = 3.6$, $\alpha_{12} = 3.9$, $\alpha_{23} = 3.3$, $\alpha_{24} = 4.2$, $\lambda = 2$, and $v_1 = v_2 = 1$.

III. INTERMEDIATE INFLATION: $a = \exp(At^f)$

Intermediate inflation is defined by a scale-factor evolution of the form $a = \exp(t^f)$, for which we have $H = Aft^{f-1}$. Hence, for an expanding universe, $Af > 0$. From the time derivative of the Hubble rate $\dot{H} = Af(f-1)t^{f-2}$, which must be negative, we conclude in addition that $f < 1$. Consequently, we impose $A > 0$ and $0 < f < 1$.

A. Single-field inflation

For a single scalar field, $\dot{H} = -\dot{\phi}^2/2$ and we have

$$\dot{\phi} = (2Af(1-f))^{1/2}t^{f/2-1}, \tag{34}$$

which gives the evolution of the field as

$$\phi = \left(8A\frac{1-f}{f}\right)^{1/2}t^{f/2} \tag{35}$$

and results in a time dependence of the potential given by

$$V = 3(Aft^{f-1})^2 - Af(1-f)t^{f-2}. \tag{36}$$

Substituting time for the value of ϕ, we obtain at late times that

$$V(\phi) = 48\frac{A^2(2A\beta)^{\beta/2}}{(\beta+4)^2}\phi^{-\beta}, \tag{37}$$

where $\beta = 4(1-f)/f$.

The relation between the spectral index and the ratio of scalar-to-tensor perturbations takes the form

$$n_s = 1 - \frac{\beta - 2}{8\beta}r, \tag{38}$$

and it was found that this model is in agreement with the latest WMAP data [9].

We will now determine the effective value of A when several fields are evolving.

B. Multifield inflation

As before, we can admit that the cosmic time can be written in terms of a combination of the fields such that

$$t = \sum_i^{m_q} \alpha_{qi}\phi_i^{2/f}, \tag{39}$$

and the potential can be written as

$$V = \sum_q^n v_q \left(\sum_i^{m_q} \alpha_{qi}\phi_i^{2/f}\right)^{2(f-1)}. \tag{40}$$

We must note, however, that v_q is dependent of α_{qi} and not free as in the previous model. Indeed, the potential can be rewritten in terms of the parameters $b_{qi} = v_q^{1/(2f-2)}\alpha_{qi}$ in the form

$$V = \sum_q^n \left(\sum_i^{m_q} b_{qi} \phi_i^{2/f} \right)^{2(f-1)}. \tag{41}$$

Scalar potentials of this form, however, have a ridge in field space rather than a valley as in the previous example; consequently, any solution for which the ratio of the kinetic energies of the fields is a constant is unstable. For this reason we will focus on the simplified class of potentials given by

$$V = \sum_i^m \alpha_i \phi_i^{4(f-1)/f}. \tag{42}$$

Admitting that the system attains a regime where all the fields are important in the evolution of the universe, we write

$$\phi_i = \frac{2}{f} c_i t^{f/2}, \tag{43}$$

and by requiring

$$2\dot{H} = -\sum_i^m \dot{\phi}_i^2 = 2Af(1-f)t^{f-2}, \tag{44}$$

we have the condition

$$\sum_i^m c_i^2 = 2Af(1-f). \tag{45}$$

Substituting Eq. (43) in the equations of motion results in the following set of relations:

$$3Afc_i = 4\frac{1-f}{f}\left(\frac{2}{f}\right)^{3-4/f} \alpha_i c_i^{3-4/f}. \tag{46}$$

Multiplying by c_i and using condition (45) we obtain

$$3A^2 f^2 = \left(\frac{2}{f}\right)^{4-4/f} \sum_i^m \alpha_i c_i^{4(f-1)/f}. \tag{47}$$

Using Eq. (46) to write c_i in terms of α_i and substituting into Eq. (47), we have that

$$A = \frac{1}{3^{f/2}f}\left(\frac{f^2}{8(1-f)}\right)^{1-f}\left(\sum_i^m \alpha_i^{f/(2-f)}\right)^{(2-f)/2}. \tag{48}$$

We see that also in intermediate inflation, the effective value of A increases by increasing the number of fields. Upon substitution back into Eq. (46), the coefficients c_i are given by

$$c_i^{2-4/f} = \frac{3^{(2-f)/2}}{f\alpha_i}\left(\frac{2}{f}\right)^{f-4+4/f}\left(4\frac{f-1}{f}\right)^{f-2}. \tag{49}$$

For a specific model, we can now compare the late-time numerical evolution of the effective A_{eff},

$$A_{\text{eff}} = \left(\frac{-\dot{H}}{H}\right)^f \frac{N}{(1-f)^f}, \tag{50}$$

against the expected value given by Eq. (48). An exercise of this kind would lead to an equivalent evolution to the one found in Fig. 5.

IV. CONCLUSIONS

We have analyzed a new inflationary scenario named "logamediate" inflation and revisited the scenario of intermediate inflation. We have demonstrated that both lead to phenomenologically viable models of inflation as there are wide regions of parameter space compatible with the latest CMB observations. Then, in each of these two scenarios, we generalized the solutions obtained for the scalar potentials to allow an ensemble of scalar fields to participate in the dynamics. As in the original model of assisted inflation for pure exponential potentials, we found that also in intermediate and logamediate inflation the fields behave as a group and the late-time dynamics is dictated by the steepness of the potential in all field-space directions. However, unlike the original assisted inflation case, the cooperative behavior does not determine if inflation is more or less probable as we increase the number of fields. The reason for this difference is that the parameters that affect the condition for acceleration of the universe are $\lambda > 1$ and $f < 1$, which are required to be fixed to a certain value for all fields. The only effective parameter that depends on the parameters of the potential is A, whose value, however, does not influence the criteria for accelerated expansion to occur.

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