Erratum: Minkowski space structure of the Higgs potential in the two-Higgs-doublet model [Phys. Rev. D 75, 035001 (2007)]

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The proof of proposition 10 in [1] is erroneous (I am thankful to C. Nishi who discovered the flaw). Nevertheless, the proposition still holds. Here is the corrected proof.

Proposition 10.—Tensor $\Lambda^{\mu\nu}$ is positive definite on the future light cone if and only if the following three conditions hold:

(1) $\Lambda^{\mu\nu}$ is diagonalizable by an $SO(1,3)$ transformation,

(2) the timelike eigenvalue Λ_0 is positive,

(3) all spacelike eigenvalues Λ_i are smaller than Λ_0.

Proof.—Obviously, if $\Lambda^{\mu\nu}$ satisfies conditions (1)–(3), then the positive definiteness follows immediately. So, one needs to prove that conditions (1)–(3) do follow from the positive definiteness.

Despite $\Lambda_{\mu\nu}$ being real and symmetric, its eigenvalues can be complex because of the non-Euclidean metric. The first step is to prove that the positive definiteness in the future light cone LC^+ implies that all the eigenvalues of $\Lambda_{\mu\nu}$ are real.

Indeed, suppose there is a pair of complex eigenvalues, λ and λ^*, with respective complex eigenvectors p^μ and q^μ:

$$\Lambda^\mu{}_\nu p^\nu = \lambda p^\mu, \qquad \Lambda^\mu{}_\nu q^\nu = \lambda^* q^\mu.$$

One can show that there can be only one pair of complex eigenvalues; thus, λ is nondegenerate. Since Λ^μ_ν is real, $q^\mu \propto p^{\mu *}$ (and can be taken equal to $p^{\mu *}$). These eigenvectors are orthogonal, $p^\mu q_\mu = 0$ (it follows from the standard argument due to $\lambda \neq \lambda^*$), and can be normalized so that $p^\mu p_\mu = q^\mu q_\mu = 1$.

Consider now a real vector r^μ,

$$r^\mu = c p^\mu + c^* p^{*\mu}.$$

Suppose that $r^\mu r_\mu = c^2 + c^{*2} = 2|c|^2\cos(2\phi_c) > 0$, so that either r^μ or $-r^\mu$ lies inside the forward light cone. Then, the corresponding quadratic form is

$$\Lambda_{\mu\nu} r^\mu r^\nu = \lambda c^2 + \lambda^* c^{*2} \equiv 2|\lambda||c|^2\cos(2\phi_c + \phi_\lambda).$$

Because of the phase shift $\phi_\lambda \neq 0$, one can always find ϕ_c such that $\cos(2\phi_c) > 0$ but $\cos(2\phi_c + \phi_\lambda) < 0$, i.e. one can always find an $r^\mu \in LC^+$ such that $\Lambda_{\mu\nu} r^\mu r^\nu < 0$, which contradicts the assumption.

Since all the eigenvalues of $\Lambda_{\mu\nu}$ are real, the eigenvectors can also be chosen all real and orthogonal. One can show that they can be normalized so that one of the eigenvectors has positive norm, $p_0^\mu p_{0\mu} = 1$, while the other three have negative norms $p_i^\mu p_{i\mu} = -1$ for each $i = 1, 2, 3$. Thus, the transformation matrix T that diagonalizes $\Lambda_{\mu\nu}$ is real, and after diagonalization $\Lambda_{\mu\nu}$ takes the form $\mathrm{diag}(\Lambda_0, -\Lambda_1, -\Lambda_2, -\Lambda_3)$. Note that transformation T also conserves norm, $r^\mu r_\mu =$ const. It means that T can be realized as a transformation from the proper Lorentz group.

Now, the requirement that $\Lambda^{\mu\nu}$ is positive definite in LC^+ means

$$\Lambda_0 - \rho(\Lambda_1 \sin\theta\cos\phi + \Lambda_2 \sin\theta\sin\phi + \Lambda_3\cos\theta) > 0$$

for all $0 < \rho < 1$, $0 \leq \theta \leq \pi$, and all ϕ. This holds when Λ_0 is positive and larger than all Λ_i.

[1] I. P. Ivanov, Phys. Rev. D **75**, 035001 (2007).

