Unparticle phase effects

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Unparticles proposed by Georgi carry CP conserving phases in their propagators. We demonstrate that these peculiar phases have an important impact on CP violation. Without including the strong QCD phases, we study the unparticle phase effects on the direct CP asymmetries in the exclusive decays of $\bar{B}_d \to \pi^+ \pi^-$ and $B \to \pi K$, in which the flavor changing neutral currents are forbidden at tree level but induced by one-loop diagrams. Interesting and consistent results comparing to the data are obtained. In addition, we find that unparticles will significantly enhance the differential branching ratio of $b \to s\ell^+\ell^-$ at the small invariant mass of $\ell^+\ell^-$. The forward-backward asymmetries for $b \to s\ell^+\ell^-$ due to unparticles are also explored.

I. INTRODUCTION

In Refs. [1,2] Georgi has suggested that a scale invariant sector with a nontrivial IR fixed point decoupled at a large scale is associated with unparticles, which could couple to the standard model (SM) particles at the TeV scale. Consequently, the unparticle physics phenomenology has been extensively explored in Refs. [1–21]. Moreover, in Ref. [2] Georgi has pointed out that the unparticle propagators in the timelike region are associated with some peculiar CP conserving phases depending on the nonintegral number of the scale dimension $d_\mathcal{U}$. He has shown that these phases can induce some unusual CP conserving interference effects between the timelike unparticle exchange amplitudes and the SM amplitudes in $e^+ e^- \to \mu^+ \mu^-$. The effect of the virtual unparticle propagation has also been noticed in Ref. [3].

Recently, in Ref. [5] we have demonstrated that the peculiar CP conserving phases in the unparticle propagators can also play very important roles in CP violation. We have explicitly examined the phase effects on the direct CP asymmetries (CPAs) in $B_d \to \pi^- \pi^+$ and $B_d \to \ell^- \ell^+$ decays based on operators with fermion flavor changing neutral currents (FCNCs) at tree level. We have found that the direct CPAs in both decays could be large. In this paper, we will examine the unparticle phase effects on CP violation with only flavor conserving operators at tree level. Specifically, we only consider those effects in which FCNCs are forbidden at tree level, like the SM, but they can be generated by one-loop diagrams including the penguin unparticle ones.

It is well known that in a decay process the direct CPA ($\mathcal{A}_{CP}$) depends on two types of phases, called weak (δ) and strong (ϕ) phases, which are CP violating and conserving, respectively. In particular, one has that

$$\mathcal{A}_{CP} \propto \sin\delta \sin\phi. \tag{1}$$

Clearly, to have a sizable value of $\mathcal{A}_{CP}$, both phases have to be nonzero and large. In the SM, the weak CP violating phase is the unique phase in the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [22], which has been fixed by experiments [23,24]. The CP conserving strong phase is process dependent, which is normally hard to be determined due to hadronic uncertainties. Since the unparticle phases appear in the propagators and conserve CP, it is interesting to speculate that these phases could act as the strong phases in some physical processes [5]. To explore this possibility, we will concentrate on B decays, as there are many experimental CP violating phenomena [24] from the current B factories as well as future super-B facilities. In particular, we will investigate direct CPAs in the decays of $B \to \pi\pi$ and $B \to \pi K$. It is clear that our study can be extended to other processes such as K and D decays.

The paper is organized as followed. In Sec. II, we present FCNCs which are induced by the unparticle penguin diagrams. We show the unparticle effects on charmless nonleptonic and semileptonic B decays in Sec. III. We give our numerical analysis in Sec. IV and conclude our results in Sec. V.

II. FLAVOR CHANGING NEUTRAL CURRENTS INDUCED BY UNPARTICLE PENGUIN DIAGRAMS

To study the low energy effects of unparticle physics, for simplicity, we assume that unparticles only couple to the flavor conserving fermion currents, described by [1,2]

$$\frac{1}{\Lambda_\mathcal{U}^{d_\mathcal{U}-1}} \bar{f}\gamma_\mu (C_L^f P_L + C_R^f P_R) f O_\mathcal{U}^\mu \tag{2}$$

where $P_{L(R)} = (1 \mp \gamma_5)/2$ and $O_\mathcal{U}$ is the vector unparticle operator. Clearly, at tree level, the fermion flavor is conserved. Similar to the SM, FCNCs such as $f \to f'\mathcal{U}$ can be
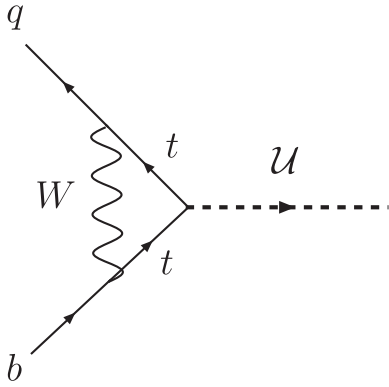
FIG. 1. Feynman diagram for $b \to q\,\mathcal{U}$.

induced by the charged weak currents at one-loop level. Because of the CKM mixing matrix element $V_{tb} \approx 1$ and the heavy top-quark enhancement, we will concentrate on B decays to illustrate some important physics phenomena involving unparticles. Our discussions can be straightforwardly generalized to K and D decays. The leading effective interaction for the flavor changing transition of $b \to q$ is induced by the unparticle penguin diagram shown in Fig. 1, which leads to

$$\mathcal{L}_{\mathcal{U}} = \frac{g^2}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} V_{tb} V_{tq}^* C_L^{qb} \bar{q} \gamma_\mu P_L b O_{\mathcal{U}}^\mu, \qquad (3)$$

where

$$C_L^{qb} = \frac{1}{(4\pi)^2} I(x_t),$$
$$I(x_t) = \frac{x_t(2C_R^t + C_L^t x_t)}{2(1 - x_t)^2}(-1 + x_t - \ln x_t) \qquad (4)$$

with $x_t = m_t^2/m_W^2$. Here, we have adopted the Feynman-'t Hooft gauge, and the contributions from the charged Goldstone boson have been included. For simplicity, in the following analysis, we will set $C_R^t \approx C_L^t$.

To obtain the unparticle-mediated effects, we need to know the unparticle propagator, which is given by [1,2]

$$\int d^4x e^{ip\cdot x}\langle 0|T(O_{\mathcal{U}}^\mu(x)O_{\mathcal{U}}^\nu(0))|0\rangle = i\Delta_{\mathcal{U}}(p^2)e^{-i\phi_{\mathcal{U}}}, \quad (5)$$

where

$$\Delta_{\mathcal{U}}(p^2) = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \frac{-g^{\mu\nu} + p^\mu p^\nu/p^2}{(p^2 + i\epsilon)^{2-d_{\mathcal{U}}}}, \qquad (6)$$
$$\phi_{\mathcal{U}} = (d_{\mathcal{U}} - 2)\pi,$$

with

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}. \qquad (7)$$

Note that in Eq. (5) the phase factor arises from $(-1)^{d_{\mathcal{U}}-2} = e^{-i\pi(d_{\mathcal{U}}-2)}$ and the vector operator is assumed to satisfy the transverse condition $\partial_\mu O_{\mathcal{U}}^\mu = 0$. In terms of

the effective interaction in Eq. (3) and the unparticle propagator in Eq. (5), the effective Hamiltonian for $b \to q f \bar{f}$ can be written as

$$\mathcal{H}_{\mathcal{U}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \tilde{\Delta}_{\mathcal{U}}(p^2)e^{-i\phi_{\mathcal{U}}} \bar{q}\gamma_\mu P_L b \bar{f}$$
$$\times (C_L^f P_L + C_R^f P_R)f, \qquad (8)$$

where $G_F = 8\sqrt{2}g^2/m_W^2$ is the Fermi constant and

$$\tilde{\Delta}_{\mathcal{U}}(p^2) = 8C_L^{qb} \frac{A_{d_{\mathcal{U}}}}{2\sin d_{\mathcal{U}}\pi} \frac{m_W^2}{p^2}\left(\frac{p^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}. \qquad (9)$$

We note that f can be neutrinos or charged leptons or quarks. We remark that, by replacing b by s in the effective Hamiltonian of Eq. (8), we may study $s \to q f \bar{f}$ decays.

III. CHARMLESS NONLEPTONIC AND SEMILEPTONIC B DECAYS

For the nonleptonic decays of $b \to qq''\bar{q}''$, we start with the explicit expression of the effective Hamiltonian in the SM [25],

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q'=u,c} \xi_{q'}^q \left[C_1(\mu)O_1^{(q)}(\mu) + C_2(\mu)O_2^{(q)}(\mu) \right.$$
$$\left. + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right], \qquad (10)$$

where $\xi_{q'}^q = V_{q'b}V_{q'q}^*$ denotes the product of the CKM matrix elements and the operators $O_1 - O_{10}$ are defined by

$$O_1^{(q)} = (\bar{q}_\alpha q'_\beta)_{V-A}(\bar{q}'_\beta b_\alpha)_{V-A},$$
$$O_2^{(q)} = (\bar{q}_\alpha q'_\alpha)_{V-A}(\bar{q}'_\beta b_\beta)_{V-A},$$
$$O_3 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q''}(\bar{q}''_\beta q''_\beta)_{V-A},$$
$$O_4 = (\bar{q}_\alpha b_\beta)_{V-A}\sum_{q''}(\bar{q}''_\beta q''_\alpha)_{V-A},$$
$$O_5 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q''}(\bar{q}''_\beta q''_\beta)_{V+A},$$
$$O_6 = (\bar{q}_\alpha b_\beta)_{V-A}\sum_{q''}(\bar{q}''_\beta q''_\alpha)_{V+A}, \qquad (11)$$
$$O_7 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q''}e_{q''}(\bar{q}''_\beta q''_\beta)_{V+A},$$
$$O_8 = \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q''}e_{q''}(\bar{q}''_\beta q''_\alpha)_{V+A},$$
$$O_9 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q''}e_{q''}(\bar{q}''_\beta q''_\beta)_{V-A},$$
$$O_{10} = \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q''}e_{q''}(\bar{q}''_\beta q''_\alpha)_{V-A},$$

with α and β being the color indices, $C_1 - C_{10}$ the Wilson coefficients (WCs), $e_{q''}$ the electric charge of q'', and

$(\bar{q}''q)_{V\pm A} = \bar{q}''\gamma^{\mu}(1 \pm \gamma_5)q''$. In Eq. (10), O_1–O_2 are from the tree level of weak interactions, O_3–O_6 are the so-called gluon penguin operators, and O_7–O_{10} are the electroweak penguin operators. Using the unitarity condition, the CKM matrix elements for the penguin operators O_3–O_{10} can also be related by

$$\xi_u^q + \xi_c^q = -\xi_t^q. \tag{12}$$

Comparing to Eq. (8), we clearly see that the structures of four-Fermi interactions with unparticle contributions are the same as those of O_3 and O_5. Consequently, we can easily get the unparticle contributions by replacing C_3 and C_5 in the SM with

$$C_3^{q\,\mathcal{U}}(p^2) = C_3 + \tfrac{1}{4}\tilde{\Delta}_{\mathcal{U}}(p^2)C_L^q e^{-i\phi_{\mathcal{U}}},$$
$$C_5^{q\,\mathcal{U}}(p^2) = C_5 + \tfrac{1}{4}\tilde{\Delta}_{\mathcal{U}}(p^2)C_R^q e^{-i\phi_{\mathcal{U}}}. \tag{13}$$

Accordingly, the associated effective WCs could be classified and reexpressed to be more useful forms by

$$a_1 = C_2 + \frac{C_1}{N_c},$$

$$a_2 = C_1 + \frac{C_2}{N_c},$$

$$a_3^{q\,\mathcal{U}} = C_3^{q\,\mathcal{U}} + \frac{C_4}{N_c} + \frac{3}{2}e_q\left(C_9 + \frac{C_{10}}{N_c}\right),$$

$$a_4^{q\,\mathcal{U}} = C_4 + \frac{C_3^{q\,\mathcal{U}}}{N_c} + \frac{3}{2}e_q\left(C_{10} + \frac{C_9}{N_c}\right), \tag{14}$$

$$a_5^{q\,\mathcal{U}} = C_5^{q\,\mathcal{U}} + \frac{C_6}{N_c} + \frac{3}{2}e_q\left(C_7 + \frac{C_8}{N_c}\right),$$

$$a_6^{q\,\mathcal{U}} = C_6 + \frac{C_5^{q\,\mathcal{U}}}{N_c} + \frac{3}{2}e_q\left(C_8 + \frac{C_7}{N_c}\right),$$

where $N_c = 3$ is the number of color.

A. $B \to \pi\pi$ decays

In this section, we are going to study the decays of $B \to \pi\pi$ dictated by $b \to dq\bar{q}$. Using the effective operators displayed in Eqs. (11) and (14), it is easy to see that the decays are tree dominated. However, it is clear that the unparticle effects on $B^- \to \pi^- \pi^0$ could be neglected if $C_{L(R)}^d \approx C_{L(R)}^u$, as the effects are always related to $-(a_3^{u\,\mathcal{U}} - a_3^{d\,\mathcal{U}}) + (a_5^{u\,\mathcal{U}} - a_5^{d\,\mathcal{U}})$, where the minus sign inside brackets is from the pion flavor wave function, $|\pi^0\rangle = (\bar{u}u - \bar{d}d)/\sqrt{2}$, and the other one outside the brackets is due to the pseudoscalar decay constant, defined by $\langle P(p)|\bar{f}'\gamma^{\mu}(1 \pm \gamma_5)f|0\rangle = \pm if_P p^{\mu}$. On the other hand, as no significant CPA for $B^- \to \pi^- \pi^0$ is found based on the current experimental world average, it should be a good scenario to take $C_{L(R)}^d \approx C_{L(R)}^u$. Unfortunately, since the unparticle contributions arise at one-loop level, we do not expect that we can solve the problem of the large branching ratio (BR) on $B_d \to \pi^0\pi^0$, in which the tree contribution

plays a dominant role. Hence, we will concentrate on the CPA of $B_d \to \pi^+\pi^-$.

It is known that the penguin effects on $B_d \to \pi^-\pi^+$ are significant even though the decay is tree dominated. In order to generate strong phases for the CPA, in the SM the annihilation topology from $O_6 \propto (V - A) \otimes (V + A)$ plays a very important role. With the Fierz transformation, since the corresponding QCD effects involve the timelike form factor denoted by $\langle\pi\pi|\bar{q}'(1 + \gamma_5)q|0\rangle$, the theoretical calculations are very uncertain. For instance, with the QCD factorization [26], to cure divergences one needs to introduce free parameters to parametrize the corresponding form factors. With the perturbative QCD approach [27], although singularities could be removed by transverse degrees of freedom, the dominant dynamical scale is close to the nonperturbative scale which is around 1 GeV [28]. Using the soft-collinear effective theory, it is found that, at the lowest order in α_s, the annihilation contributions are real [29]. In addition, these timelike form factors are all power suppressed in m_P/m_B, with m_P being the mass of the light pseudoscalar [30]. Hence, it still needs to make lots of efforts to fix the strong phases induced by the QCD effects.

As stated before, the CP-conserved phase in the unparticle propagator could provide a kind of strong phase needed for the CPA [5]. It has been realized that the phase could contribute to the CPA of $B_d \to \pi^-\pi^+$ with tree-allowed FCNCs [5]. In this study, we take the fermion flavor conservation at tree level like the SM. To examine the influence of the unparticle phase alone, we will neglect the uncertain strong phases induced by QCD interactions. By following the effective Hamiltonian in Eq. (10), we present the decay topologies in Fig. 2 where (a) [(b)] denotes the tree (loop) effects. Since $B_d \to \pi^-\pi^+$ is a color-allowed process, the factorization assumption is good enough to estimate the transition matrix elements. Consequently, the decay amplitude for $\bar{B}_d \to \pi^+\pi^-$ is given by

$$\mathcal{M}_{\pi^+\pi^-}^{\mathcal{U}} = \frac{G_F}{\sqrt{2}}f_\pi m_B^2 F_0^{B\pi}(m_\pi^2)[-V_{tb}V_{td}^*(a_4^{u\,\mathcal{U}} + 2r_\pi a_6^{u\,\mathcal{U}}) + V_{ub}V_{ud}^* a_1], \tag{15}$$

where the form factor $F_0^{B\pi}$ is defined by

$$\langle\pi(p)|\bar{b}\gamma_\mu u|\bar{B}(p_B)\rangle = \left[(p_B + p)_\mu - \frac{m_B^2}{q^2}q_\mu\right]F_1^{B\pi}(q^2)$$
$$+ \frac{m_B^2}{q^2}q_\mu F_0^{B\pi}(q^2) \tag{16}$$

with $q = p_B - p$ and $r_\pi = m_\pi^0/m_B$ associated with $\langle\pi|\bar{d}\gamma_5 u|0\rangle = if_\pi m_\pi^0$. For the light meson production in B decays, we take $F^{B\pi}(m_\pi^2) \approx F_0^{B\pi}(0)$. Hereafter, we will use $F_0^{B\pi}$ instead of $F_0^{B\pi}(0)$. As a result, the BR and CPA could be obtained by
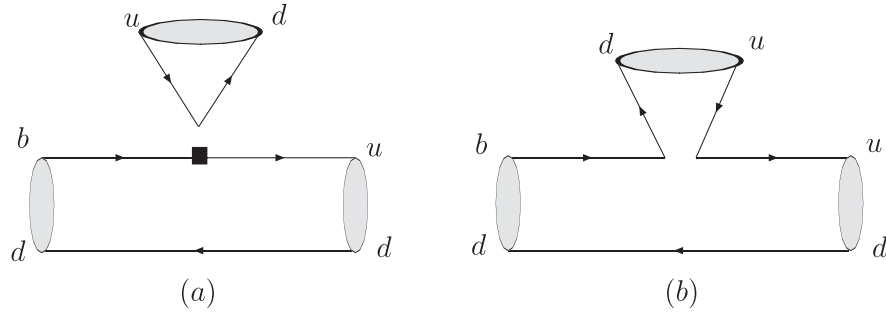
FIG. 2. Flavor diagrams from (a) tree and (b) penguin diagrams for $B_d \rightarrow \pi^- \pi^+$ decay, where the square symbol denotes the weak vertex.

$$\mathcal{B}(\bar{B}_d \rightarrow \pi^+ \pi^-) = \frac{\tau_{B_d}}{16\pi m_B} |\mathcal{M}^{\mathcal{U}}_{\pi^+ \pi^-}|^2,$$

$$\mathcal{A}_{CP} = \frac{\bar{\mathcal{B}}(\bar{B}_d \rightarrow \pi^+ \pi^-) - \mathcal{B}(B_d \rightarrow \pi^- \pi^+)}{\bar{\mathcal{B}}(\bar{B}_d \rightarrow \pi^+ \pi^-) + \mathcal{B}(B_d \rightarrow \pi^- \pi^+)},$$

(17)

where τ_{B_d} is the lifetime of B_d and the pion mass has been neglected. To be more clear, to see the relationship of the CPA with the unparticle phase $\phi_{\mathcal{U}}$, we rewrite the CPA for $\bar{B}_d \rightarrow \pi^+ \pi^-$ as

$$\mathcal{A}_{CP} = \frac{-2\chi^{\mathcal{U}}_{\pi\pi} \sin\alpha \sin\phi_{\mathcal{U}}}{|1 + \chi^{SM}_{\pi\pi} e^{i\alpha}|^2 + |\chi^{\mathcal{U}}_{\pi\pi}|^2 - 2\chi^{\mathcal{U}}_{\pi\pi} \cos\alpha \cos\phi_{\mathcal{U}}}$$

(18)

where $\alpha \equiv \beta + \gamma$ and

$$\chi^{\mathcal{U}}_{\pi\pi} = \frac{\tilde{\Delta}_{\mathcal{U}}(p^2)}{4N_c a_1} \frac{|\xi^d_t|}{|\xi^d_u|}(C^u_L + 2r_\pi C^u_R),$$

$$\chi^{SM}_{\pi\pi} = \frac{1}{a_1} \frac{|\xi^d_t|}{|\xi^d_u|}(a^u_4 + 2r_\pi a^u_6).$$

(19)

Note that $a^q_{4(6)}$ can be derived from Eq. (14) by setting $C^q_{L(R)} = 0$. Obviously, the CPA in $\bar{B}_d \rightarrow \pi^+ \pi^-$ depends on not only the weak phase α but also the CP-conserved unparticle phase $\phi_{\mathcal{U}}$.

B. $B \rightarrow \pi K$ decays

It is known that the decays of $b \rightarrow sq\bar{q}$ are penguin-dominant processes, as the tree contributions are suppressed by the CKM matrix elements of $V_{ub}V^*_{us}$. Since the unparticle effects are also induced from penguin loops, one expects that they should be significant. In $B \rightarrow \pi K$ decays, there are four specific decay modes. Since the BRs for $B \rightarrow \pi K$ and the CPA of $B_d \rightarrow \pi^- K^+$ are observed well in experiments, we have to discuss all modes in detail. We begin with our analysis on the decay of $B^- \rightarrow \pi^- \bar{K}^0$. According to the flavor diagram in Fig. 3(a), the decay corresponds to $q = d$. Hence, taking the same conditions as $B_d \rightarrow \pi^- \pi^+$, the decay amplitude for $B^- \rightarrow \pi^- \bar{K}^0$ can be expressed by

$$\mathcal{M}^{\mathcal{U}}_{\pi^- \bar{K}^0} = -\frac{G_F}{\sqrt{2}} V_{tb}V^*_{ts} f_K m^2_B F^{B\pi}_0 (a^{d\mathcal{U}}_4 + 2r_K a^{d\mathcal{U}}_6),$$

(20)

where f_K is the kaon decay constant and $r_K = m^0_K/m_B$, with m^0_K defined as m^0_π. Similar to $B^- \rightarrow \pi^- \bar{K}^0$, we can easily find the decay amplitude of $\bar{B}_d \rightarrow \pi^+ K^-$ by using $q = u$ instead of $q = d$, given by

$$\mathcal{M}^{\mathcal{U}}_{\pi^+ K^-} = \frac{G_F}{\sqrt{2}} f_K m^2_B F^{B\pi}_0 [-V_{tb}V^*_{ts}(a^{u\mathcal{U}}_4 + 2r_K a^{u\mathcal{U}}_6) + V_{ub}V^*_{us} a_1],$$

(21)

where we have included the tree contributions illustrated in Fig. 3(c).

Next, we analyze the decay of $B_d \rightarrow \pi^0 K^0$. Besides the flavor diagrams appearing in the decay of $B^- \rightarrow \pi^- \bar{K}^0$, there are new diagrams shown in Figs. 3(b) and 3(d). In the SM, these contributions correspond to the electroweak penguin and color-suppressed effects, respectively. Taking $q = u$ and d in Figs. 3(a) and 3(b), respectively, the decay amplitude for $\bar{B}_d \rightarrow \pi^0 \bar{K}^0$ is given by

$$\sqrt{2}\mathcal{M}^{\mathcal{U}}_{\pi^0 \bar{K}^0} = \frac{G_F}{\sqrt{2}} m^2_B \{V_{tb}V^*_{ts}[f_K F^{B\pi}_0 (a^{d\mathcal{U}}_4 + 2r_K a^{d\mathcal{U}}_6) - f_\pi \zeta F^{BK}_0] + V_{ub}V^*_{us} f_\pi a_2 F^{BK}_0\},$$

(22)

where $\zeta = a^{u\mathcal{U}}_3 - a^{d\mathcal{U}}_3 + a^{d\mathcal{U}}_5 - a^{u\mathcal{U}}_5$. We note that the new term $f_\pi \zeta F^{BK}_0$, corresponding to the contribution in Fig. 3(b), is opposite in sign to other terms. The reason comes from the flavor wave function of π^0 being $(\bar{u}u - \bar{d}d)/\sqrt{2}$. Note that Fig. 3(b) picks both components while Fig. 3(a) only takes the $\bar{d}d$ component. Since the tree contributions are color suppressed, the corresponding WC is a_2.

After introducing the decay amplitudes for $B^- \rightarrow \pi^- \bar{K}^0$, $\bar{B}_d \rightarrow \pi^+ K^-$, and $\bar{B}_d \rightarrow \pi^0 \bar{K}^0$, the amplitude of $B^- \rightarrow \pi^0 K^-$ could be immediately obtained as
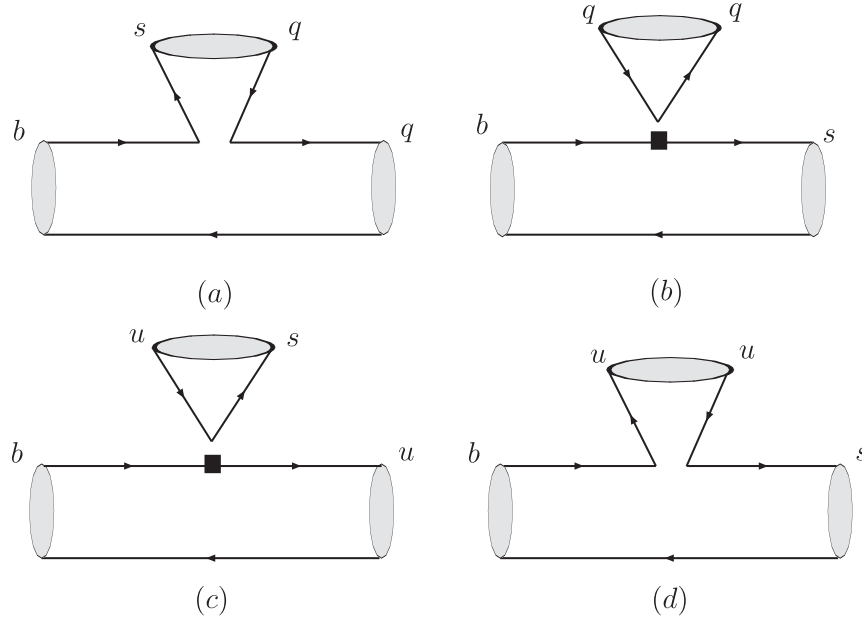
FIG. 3. Flavor diagrams for $B \to \pi K$ decays dictated by (a) [(b)] penguin and (c) [(d)] tree diagrams.

$$\sqrt{2}\mathcal{M}^{\mathcal{U}}_{\pi^0 K^-} = \frac{G_F}{\sqrt{2}} m_B^2 [V_{tb} V_{ts}^* (-f_K F_0^{B\pi}(a_4^u \mathcal{U} + 2 r_K a_6^u \mathcal{U})$$
$$- f_\pi \zeta F_0^{BK})$$
$$+ V_{ub} V_{us}^* (f_K a_1 F_0^{B\pi} + f_\pi a_2 F_0^{BK})]. \qquad (23)$$

Clearly, the amplitudes for the first three decay modes all appear in the decay of $B^+ \to \pi^0 K^+$. That is, once the first three decays are determined, the decay of $B^+ \to \pi^0 K^+$ is also fixed. We point out that, although Fig. 3(b) contributes to the modes of $\pi^0 \bar{K}^0$ and $\pi^0 K^+$, similar to the case in $B \to \pi\pi$, the unparticle effects in this topology will vanish if we take $C^d_{L(R)} \approx C^u_{L(R)}$. In our study, we also neglect their contributions. In summary, the BRs and CPAs for the $B \to \pi K$ decays can be found by the definitions in Eq. (17).

C. Inclusive semileptonic decays of $b \to q\bar{\ell}\ell$

If unparticles couple to leptons, we can apply the induced interactions for $b \to q\mathcal{U}$ to study the semileptonic decays of $b \to q\bar{\ell}\ell$. The corresponding Feynman diagram is presented in Fig. 4. It is easy to see that, due to the CKM suppression, the semileptonic decays with $b \to d$ are much less than those of $b \to s$. Hence, in the following discussions, we will concentrate on $b \to s\ell^+\ell^-$. Nevertheless, all discussions and formulas could be applied to $b \to d\ell^+\ell^-$ as well. It is also worth mentioning that, because the CKM matrix element V_{td} carries a CP violating phase, the system of $b \to d\ell^+\ell^-$ could be even more interesting on CP violation in the framework of unparticle physics.

Before including new physics interactions into $b \to s\ell^+\ell^-$, we write the effective Hamiltonian for the SM as

$$\mathcal{H}_{\text{eff}} = \frac{G_F \alpha_{\text{em}} \lambda_t}{\sqrt{2}\pi} [H_{1\mu} L^\mu + H_{2\mu} L^{5\mu}], \qquad (24)$$

with

$$H_{1\mu} = C_9^{\text{eff}}(\mu) \bar{s} \gamma_\mu P_L b - \frac{2 m_b}{q^2} C_7(\mu) \bar{s} i \sigma_{\mu\nu} q^\nu P_R b,$$
$$H_{2\mu} = C_{10} \bar{s} \gamma_\mu P_L b,$$
$$L^\mu = \bar{\ell} \gamma^\mu \ell, \qquad (25)$$
$$L^{5\mu} = \bar{\ell} \gamma^\mu \gamma_5 \ell,$$
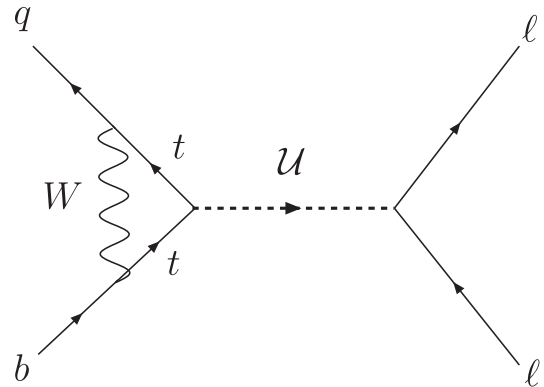

FIG. 4. $b \to q\ell^+\ell^-$ decays induced by an unparticle penguin diagram.

where α_{em} is the fine constant, $\lambda_t = V_{tb}V_{ts}^*$, C_9^{eff} and $C_{7,10}$ are the WCs with their explicit expressions given in Ref. [25] for the SM, m_b is the current b-quark mass, and q^2 is the invariant mass of the $\ell^+\ell^-$ pair. Although long-distance effects of $c\bar{c}$ bound states could contribute to C_9^{eff},

$$C_9^{eff}(\mu) = C_9(\mu) + (3C_1(\mu) + C_2(\mu))h(x, s),$$

$$h(z, s) = -\frac{8}{9}\ln\frac{m_b}{\mu} - \frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2 + x)|1 - x|^{1/2}\begin{cases}\ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi, & \text{for } x \equiv 4z^2/s < 1, \\ 2\arctan\frac{1}{\sqrt{x-1}}, & \text{for } x \equiv 4z^2/s > 1,\end{cases} \tag{26}$$

where $h(z, s)$ describes the one-loop matrix elements of operators $O_1^c = \bar{s}_\alpha\gamma^\mu P_L b_\beta \bar{c}_\beta\gamma_\mu P_L c_\alpha$ and $O_2^c = \bar{s}\gamma^\mu P_L b\bar{c}\gamma_\mu P_L c$ [25], with $z = m_c/m_b$ and $s = q^2/m_b^2$. Comparing to Eq. (8), we find that the operator structures of the unparticle contributions are the same as those of the SM. The unparticle effects with the SM contributions can be derived by using $C_9^{\mathcal{U}}$ and $C_{10}^{\mathcal{U}}$, defined by

$$C_9^{\mathcal{U}}(q^2) = C_9^{eff} + \frac{\pi}{\alpha_{em}}\frac{C_R^\ell + C_L^\ell}{2}\tilde{\Delta}_{\mathcal{U}}(q^2)e^{-i\phi_{\mathcal{U}}},$$

$$C_{10}^{\mathcal{U}}(q^2) = C_{10} + \frac{\pi}{\alpha_{em}}\frac{C_R^\ell - C_L^\ell}{2}\tilde{\Delta}_{\mathcal{U}}(q^2)e^{-i\phi_{\mathcal{U}}}, \tag{27}$$

instead of C_9^{eff} and C_{10}, respectively.

With Eq. (24) and the three-body phase space, the inclusive differential decay rate for $b \to s\ell^+\ell^-$ ($\ell = e, \mu$) can be expressed by

$$\frac{d\Gamma}{ds} = \frac{G_F^2 m_b^5 \alpha_{em}^2}{768\pi^5}|V_{ts}V_{tb}^*|^2(1 - s)^2 R(s),$$

$$R(s) = (|C_9^{\mathcal{U}}(s)|^2 + |C_{10}^{\mathcal{U}}(s)|^2)(1 + 2s) \tag{28}$$

$$+ 12\,\text{Re}(C_7^* C_9^{\mathcal{U}}(s)) + 4\left(1 + \frac{2}{s}\right)|C_7|^2,$$

where the lepton mass has been neglected. Besides the BRs, it has been known that the forward-backward asymmetry (FBA), defined by [31]

$$\frac{dA_{FB}}{ds} = \frac{\int_{-1}^1 d\cos\theta d\Gamma/dsd\cos\theta\,\text{sgn}(\cos\theta)}{\int_{-1}^1 d\cos\theta d\Gamma/dsd\cos\theta}, \tag{29}$$

could be a good candidate to probe new physics. By including the unparticle contributions, from Eqs. (28) and (29) we get

$$\frac{dA_{FB}}{ds} = -3\frac{s}{R(s)}\text{Re}\left[\left(C_9^{\mathcal{U}}(s) + \frac{2}{s}C_7\right)C_{10}^{\mathcal{U}*}(s)\right]. \tag{30}$$

We note that, as a whole, the decay of $b \to s\ell^+\ell^-$ is not sensitive to the CP-conserved phases carried by the unparticle propagators. However, it is important to point out that at the low q^2 regions the unparticle physics has large effects on these physical quantities, as the unparticle propagator is proportional to $(q^2)^{d_\mathcal{U}-2}$. In these region,

to study the behavior of unparticle physics in the semi-leptonic decays, for simplicity they are not included in the present study. On the other hand, the bound states could be excluded experimentally by cutting the phase space at the resonant regions. Explicitly, one has that [25]

we expect that both BR and FBA in $b \to s\ell^+\ell^-$ have significant deviations from the SM predictions.

IV. NUMERICAL ANALYSIS

To estimate the numerical values, we take the common parameters to be $G_F = 1.166 \times 10^{-5}$ GeV^{-2}, $f_\pi = 0.13$ GeV, $f_K = 0.16$ GeV, $m_K^0 = 2.4$ GeV, $m_\pi^0 = 1.7$ GeV, $V_{td} = 8.46 \times 10^{-3}e^{-i\beta}$ with $\beta = 25°$, $V_{ub} = 3.6 \times 10^{-3}e^{-i\gamma}$ with $\gamma = 72°$, and $\alpha_{em} = 1/129$ [24]. For the nonleptonic B decays, since we concentrate on the CPAs, the CP-averaged BRs are regarded as inputs and their world averages are adopted as [32]

$$\mathcal{B}(\bar{B}_d \to \pi^+\pi^-) = (5.16 \pm 0.22) \times 10^{-6},$$

$$\mathcal{B}(B^- \to \pi^-\bar{K}^0) = (23.1 \pm 1.0) \times 10^{-6},$$

$$\mathcal{B}(\bar{B}_d \to \pi^+ K^-) = (19.4 \pm 0.6) \times 10^{-6}, \tag{31}$$

$$\mathcal{B}(\bar{B}_d \to \pi^0\bar{K}^0) = (10.0 \pm 0.6) \times 10^{-6},$$

$$\mathcal{B}(B^- \to \pi^0 K^-) = (12.8 \pm 0.6) \times 10^{-6}.$$

For the semileptonic $b \to s\ell^+\ell^-$ decays, we use the world average as [24]

$$\mathcal{B}(b \to s\ell^+\ell^-) = (4.5 \pm 1.0) \times 10^{-6}. \tag{32}$$

We first calculate the unparticle contributions to the BR and CPA of $\bar{B}_d \to \pi^+\pi^-$. As mentioned before, to satisfy the indication of the experimental data on $B^- \to \pi^0\pi^-$, we will require that $C_{L(R)}^u \approx C_{L(R)}^d = C_{L(R)}^q$. Besides the scale dimension $d_\mathcal{U}$ and the unparticle scale $\Lambda_\mathcal{U}$ fixed to be 1 TeV, there are three extra unknown parameters from unparticle physics, i.e. C_L^t, C_L^q, and C_R^q. Since C_L^t is always associated with $C_{L(R)}^q$, in our numerical calculations, we will use the combined parameters of $\lambda_L^q = C_L^t C_L^q$ and $\lambda_R^q = C_L^t C_R^q$. In addition, we will set the available ranges for the variables to be $|\lambda_{L(R)}^q| < 0.5$. In terms of Eqs. (15) and (17), the numerical values of the BR and CPA for $\bar{B}_d \to \pi^-\pi^+$ versus the scale dimension $d_\mathcal{U}$ are presented in Fig. 5. Because the BR is input, the calculated values are all within 1σ errors. Recently, *BABAR* [33] and Belle [34] have reported the CPA measurements of
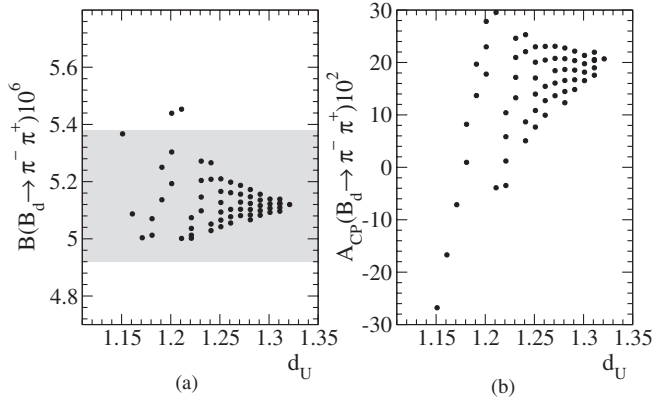
FIG. 5. (a) BR (in units of 10^{-6}) and (b) CPA (%) for $B_d \to \pi^- \pi^+$ versus the scale dimension $d_\mathcal{U}$ with $\Lambda_\mathcal{U} = 1$ TeV and $\lambda_{L(R)}^q < 0.5$, where the band in (a) denotes the world average with 1σ errors.

$$\mathcal{A}_{CP}(\bar{B}_d \to \pi^+ \pi^-) = (0.21 \pm 0.09 \pm 0.02) \quad (BABAR),$$
$$\mathcal{A}_{CP}(\bar{B}_d \to \pi^+ \pi^-) = (0.55 \pm 0.08 \pm 0.05) \quad (Belle),$$
$$(33)$$

with the average value being

$$\mathcal{A}_{CP}(\bar{B}_d \to \pi^+ \pi^-) = 0.38 \pm 0.18. \quad (34)$$

According to our results in Fig. 5(b), we see that, without any QCD phases, the unparticle-mediated FCNC with the peculiar *CP*-conserved phase induced by the penguin diagram could make the CPA of $\bar{B}_d \to \pi^+ \pi^-$ as large as 20%. Clearly, with more and more data accumulated at the *B*

factories, it is worth exploring whether the unparticle phase is the dominant source for dictating the CPA of $\bar{B} \to \pi^+ \pi^-$.

With the same set of parameters in $B \to \pi\pi$, we now study the decays of $B \to \pi K$. According to the formulas of the decay amplitudes introduced in Eqs. (20)–(23), the values of BRs through unparticle-mediated diagrams are presented in Fig. 6, where the band in each figure denotes the world average with 1σ errors. It is interesting to see that unparticle physics could make the BRs of $B \to \pi K$ consistent with data within 1σ world averages. We note that the data of $\mathcal{B}(\bar{B}_d \to \pi^+ \pi^-)$ have also been included to constrain the various unknown parameters.

Although the consistent results in the BRs have been impressive enough, to emphasize the importance of the magic phase in unparticle physics, one should pay attention to the CPA. According to our previous analysis in Eqs. (20) and (22), since the tree contributions are negligible (small) for $B^- \to \pi^- \bar{K}$ ($\bar{B}_d \to \pi^0 \bar{K}^0$), one can easily understand that the corresponding CPA should also be negligible (small). Nevertheless, we have to remark that the conclusions are correct only for the cases without including final state interactions (FSIs). Note that we have to exclude the discussions on FSIs since we can only control the short-distance effects. Here, we adopt that the assumption of color transparency dominates the processes in *B* decays [35]. We conclude that the interesting *CP* violating effects in $B \to \pi K$ decays are the CPAs for $\bar{B}_d \to \pi^+ K^-$ and $B^- \to \pi^0 K^-$.

From Eqs. (21) and (23), one finds that the penguin contributions are the same in both decays; the only difference is that there is an extra color-suppressed contribution
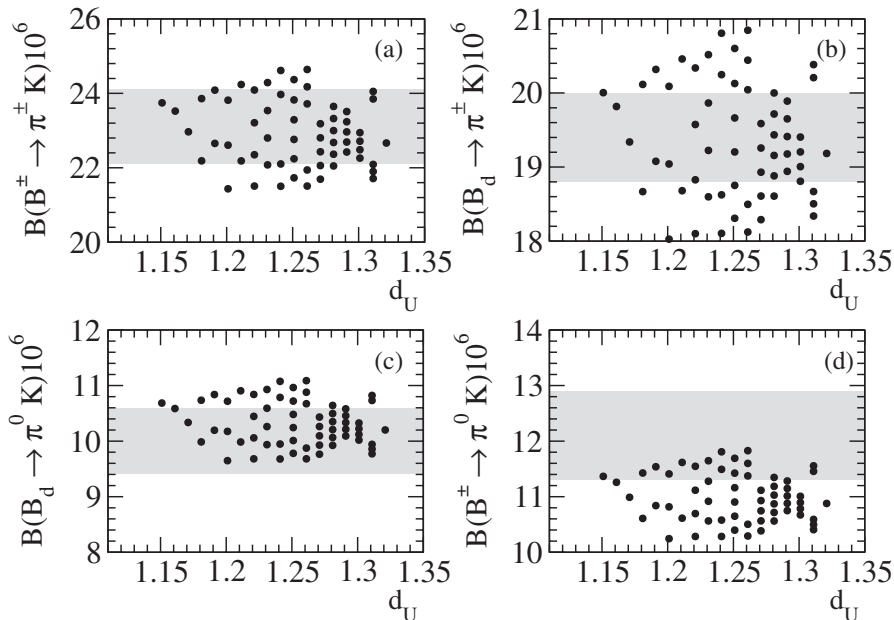

FIG. 6. BRs (in units of 10^{-6}) versus the scale dimension $d_\mathcal{U}$ for (a) $B^- \to \pi^- \bar{K}^0$, (b) $\bar{B}_d \to \pi^0 \bar{K}^0$, (c) $\bar{B}_d \to \pi^0 \bar{K}^0$, and (d) $B^- \to \pi^0 K^-$, where the band in each figure stands for the world average with 1σ errors.

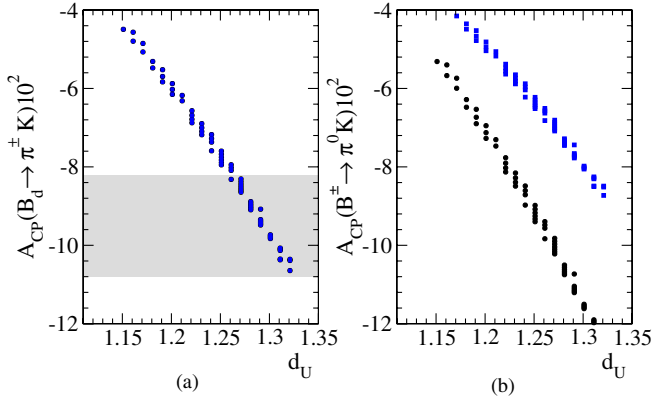FIG. 7 (color online). CPAs (%) for (a) $B_d \to \pi^\mp K^\pm$ and (b) $B^\mp \to \pi^0 K^\mp$ versus the scale dimension $d_\mathcal{U}$, where the band in (a) denotes the world average with 1σ errors and the circles (squares) in each figure represent $a_2 = 0.14\,(-0.14)$.

in $B^- \to \pi^0 K^-$. If the color-suppressed a_2 term in Eq. (23) is dropped, one expects that both modes should have the same CPAs. Therefore, the sign of a_2 will affect the CPA of $B^- \to \pi^0 K^-$. Using the parameters fitted by the BRs of $B \to \pi K$, we present the unparticle contributions versus the scale dimension $d_\mathcal{U}$ in Fig. 7, where the circles (squares) in each figure denote $a_2 = 0.14\,(-0.14)$. We see that the sign of a_2 has a significant influence on $\mathcal{A}_{CP}(B^- \to \pi^0 K^-)$. According to the current world average, given by [32]

$$\begin{aligned} \mathcal{A}_{CP}(\bar{B}_d \to \pi^+ K^-) &= -0.095 \pm 0.013, \\ \mathcal{A}_{CP}(B^- \to \pi^0 K^-) &= 0.047 \pm 0.026, \end{aligned} \tag{35}$$

it seems that somewhat different physics exists between the two modes. Plausibly, a_2 plays an important role in the CPA for $B^- \to \pi^0 K^-$. Since our focus is on the CPA in unparticle physics, further discussions on a_2 are given elsewhere. For detailed analysis, we refer the reader to Ref. [27]. From our results, we see that, with the unparticle phase, $\mathcal{A}_{CP}(B^- \to \pi^+ K^-)$ could be consistent with the current data. As the results in Eq. (35) are not conclusive yet, more precise data are needed to tell whether there is a deviation between $\bar{B}_d \to \pi^- K^+$ and $B^- \to \pi^0 K^-$ in the CPAs.

Finally, we study the unparticle effects on inclusive semileptonic decays of $b \to s\ell^+\ell^-$ with $\ell = e, \mu$. From Eq. (27), one finds that C_L^t is always associated with $(C_R^\ell + C_R^\ell)/2$ and $(C_R^\ell - C_L^\ell)/2$. To simplify our numerical analysis, we set $C_R^\ell = C_L^\ell$ or $C_R^\ell = -C_L^\ell$. We will redefine our parameters to be $C_L^t C_L^\ell = C_L^t C_R^\ell = \lambda_V^\ell$ and $C_L^t C_L^\ell = -C_L^t C_R^\ell = \lambda_A^\ell$ and discuss the constraints on $\lambda_{V(A)}^\ell$. From Eq. (27), we know that the one-loop matrix elements from operators O_1^c and O_2^c will generate a CP-conserved QCD phase, which, in principle, could interfere with the unparticle phase. However, the interference effect between the CP-conserved phases of the SM and unparticles is small

since the one-loop generated contributions are much smaller than $C_9 \sim -C_{10} \sim 4$.

Although $b \to s\ell^+\ell^-$ cannot be the candidate to probe the unique unparticle phase, we can utilize the decays to give strong constraints on the unparticle couplings to leptons, i.e. $\lambda_{V(A)}^\ell$. In terms of Eq. (28) and the values for the common parameters, the unparticle contributions to the BRs of $b \to s\ell^+\ell^-$ are presented in Fig. 8, where (a) [(b)] denotes the contributions of $\lambda_{V[A]}^\ell$, the horizontal thin lines are the SM contributions, and the thick solid, dashed, and dash-dotted lines correspond to $\lambda_{V[A]}^\ell = 0.005$, 0.01, and 0.05, respectively. The bands in the diagrams are the world average with 1σ errors. From the figure, we see clearly that, with a specific value for $\lambda_{V(A)}^\ell$, the BR of $b \to s\ell^+\ell^-$ is very sensitive to the scale dimension $d_\mathcal{U}$. To understand the sensitivity, we need to examine the behavior of the unparticle propagator and the unparticle couplings to fermions. With Eqs. (2) and (5), we know that the q^2 dependence in the BR will behave like

$$\left[\frac{1}{q^2} \left(\frac{q^2}{\Lambda_\mathcal{U}^2} \right)^{d_\mathcal{U}-1} \right]^2. \tag{36}$$

It is clear that, for the three-body $b \to s\ell^+\ell^-$ decays, the enhancement of unparticle effects is at the small invariant mass of $\ell^+\ell^-$. To be more clear, we display the differential BRs for $b \to s\ell^+\ell^-$ as functions of $s = q^2/m_b^2$ in Figs. 9(a) and 9(b), where the solid, dashed, and dash-dotted lines stand for $d_\mathcal{U}[\lambda_{V(A)}^\ell] = 1.1\,[0.005]$, 1.2 [0.01], and 1.4 [0.05], respectively. The large deviation at the small s region could confirm our argument.

For the FBA in $b \to s\ell^+\ell^-$, from Eq. (30), the numerical values of the unparticle contributions as functions of the invariant mass s are shown in Figs. 9(c) and 9(d).
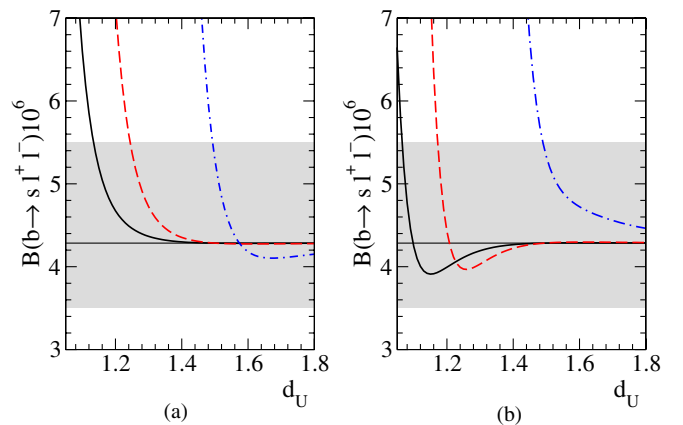

FIG. 8 (color online). BR (in units of 10^{-6}) for $b \to s\ell^+\ell^-$ with (a) $C_L^t C_R^\ell = C_L^t C_L^\ell = \lambda_V^\ell$ and (b) $C_L^t C_R^\ell = -C_L^t C_L^\ell = \lambda_A^\ell$ versus the scale dimension $d_\mathcal{U}$, where the thick solid, dashed, and dash-dotted lines correspond to $\lambda_{V(A)}^\ell = 0.005$, 0.01, and 0.05, respectively, while the horizontal lines are the SM contributions and the bands denote the world average with 1σ errors.
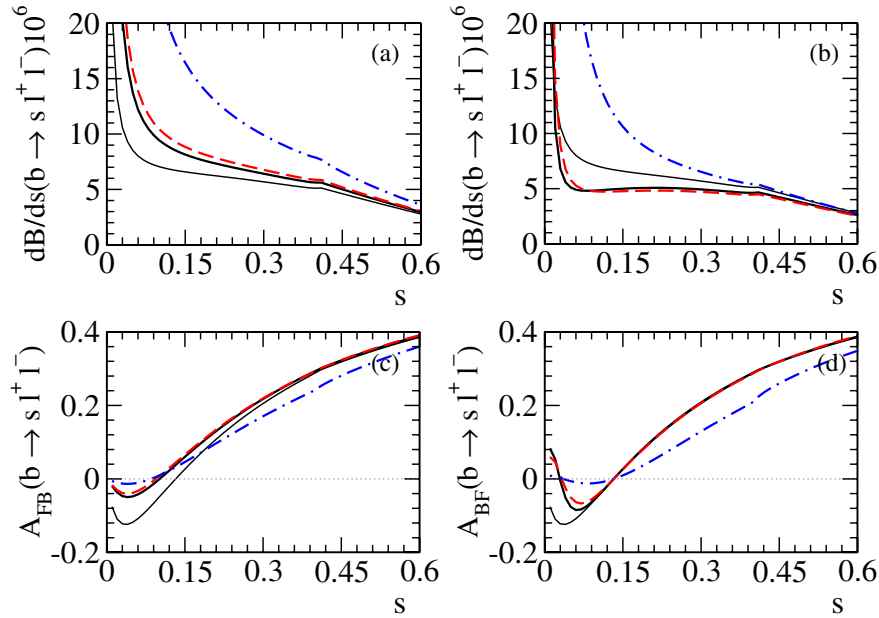
FIG. 9 (color online). (a) [(b)] Differential BR (in units of 10^{-6}) and (c) [(d)] FBA for $b \to s\ell^+\ell^-$ as functions of $s = q^2/m_b^2$, where the thick solid, dashed, and dash-dotted lines correspond to $d_{\mathcal{U}}(\lambda_{V[A]}^{\ell}) = 1.1(0.005), 1.2(0.01)$, and $1.4(0.05)$, respectively, while the thin lines are the SM contributions.

Clearly, the FBA at the small s region is also sensitive to the unparticle physics. In addition, one observes that the nonvanished λ_V^{ℓ} associated with $C_9^{\mathcal{U}}$ could shift the zero point of the FBA to be lower. However, the nonvanished λ_A^{ℓ} associated with $C_{10}^{\mathcal{U}}$ cannot change the zero-crossing point. The reason could be understood from Eq. (30), where the zero point can only happen at $C_9^{\mathcal{U}} + 2C_7/s = 0$.

V. CONCLUSIONS

We have studied the implications of the *CP* conserving phases in the unparticle propagators. We have demonstrated that these peculiar phases have an important impact on *CP* violation since they could act as the strong phases needed to induce the direct *CP* asymmetry. Without including the QCD phases, we have examined the unparticle phase effects on the direct *CP* asymmetries in the exclusive $\bar{B}_d \to \pi^+\pi^-$ and $B \to \pi K$ decays, in which FCNCs are forbidden at tree level but induced by one-loop unparticle penguin diagrams. We have obtained interesting and consistent results comparing to the experimental data. Moreover, we have found that the unparticle effects will significantly enhance the differential branching ratio of $b \to s\ell^+\ell^-$ at the small invariant mass of $\ell^+\ell^-$ so that the couplings of unparticles to leptons suffer strong constraints. The forward-backward asymmetries for the decays of $b \to s\ell^+\ell^-$ due to the unparticle effects have also been investigated.

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