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Diquark and light four-quark states

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Four-quark states with different internal clusters are discussed within the constituent quark model. It is pointed out that the diquark concept is not meaningful in the construction of a tetraquark interpolating current in the QCD sum-rule approach, and hence existing sum-rule studies of four-quark states are incomplete. An updated QCD sum-rule determination of the properties of diquark clusters is then used as input for the constituent quark model to obtain the masses of light 0^{++} tetraquark states (i.e. a bound state of two diquark clusters). The results support the identification of $\sigma(600)$, $f_0(980)$, and $a_0(980)$ as the 0^{++} light tetraquark states, and seem to be inconsistent with the tetraquark state interpretation of the new BES observations of the near-threshold $p\bar{p}$ enhancements, $X(1835)$ and $X(1812)$, with the possible exception that $X(1576)$ may be an exotic first orbital excitation of $f_0(980)$ or $a_0(980)$.

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I. INTRODUCTION

In the quark model, a meson consists of a quark and an antiquark, while a baryon consists of three quarks. Exotic hadrons are those beyond these naive $q\bar{q}$ mesons and qqq baryons. Among the exotic hadrons, four-quark state scenarios are of both theoretical and experimental interest. A four-quark state was predicted to exist early in a consistent description of the hadron scattering amplitudes [1], and their properties have been studied in many models [2–15].

Quarks/antiquarks in four-quark states may have complicated correlations and form different clusters. According to the spatial extension of the clusters, four-quark states may be composed of a diquark and an antidiquark [1,2,4], or be composed of two $q\bar{q}$ clusters (including the lightly bounded "molecular states") [5–8,11]. It could be expected that the properties of these two kinds of four-quark states are different because of their different internal structures though the dynamics among quarks in the four-quark state is still unclear.

So far, no four-quark state has been confirmed experimentally. However, there are some four-quark candidates in experiments, including $f_0(600)$ (or σ), $f_0(980)$, $a_0(980)$, and the unconfirmed $\kappa(800)$. They have a long history of being interpreted as four-quark states [2,16]. Recently, $X(3872)$ has been observed by Belle [17] in exclusive B decays and $Y(4260)$ has been observed by *BABAR* [18] in initial-state radiation events. These newly observed states have also been interpreted as four-quark states in some literature [11–13,19–22].

In particular, the BES collaboration has reported some new observations in the low energy region. Apart from the near-threshold $p\bar{p}$ enhancement [23], $X(1835)$ was observed by BES [24] in the decay

$$J/\Psi \to \gamma\pi^+\pi^-\eta',$$

with $M = 1833.7 \pm 6.1(\text{stat}) \pm 2.7(\text{syst})$ MeV, $\Gamma = 67.7 \pm 20.3 \pm 7.7$ MeV. $X(1812)$ was observed by BES [25] in the doubly Okubo-Zweig-Iizuka (OZI)-suppressed decay

$$J/\Psi \to \gamma\omega\phi$$

with $M = 1812^{+19}_{-26}(\text{stat}) \pm 18(\text{syst})$ MeV, $\Gamma = 105 \pm 20 \pm 28$ MeV. Most recently, $X(1576)$ was observed by BES at the K^+K^- invariant mass in the decay [26]

$$J/\Psi \to K^+K^-\pi^0$$

with pole position $1576^{+49}_{-55}(\text{stat})^{+98}_{-91}(\text{syst})$ MeV $- i(409^{+11}_{-12}(stat)^{+32}_{-67})$ MeV. The J^{PC} of this broad peak is 1^{--}. Whether these observations by BES will be confirmed or not by other experimental groups in the future, four-quark candidates interpretations [27–30] to them have appeared.

Diquarks are relevant to the understanding of four-quark states in constituent quark models. Diquarks were first mentioned by Gell-Mann [31] and have been applied successfully to many strong-interaction phenomena [32–34]. Interest in diquarks has been revived by a variety of new experimental observations [35–38]. As pointed out in Refs. [12,38,39], diquark clusters in hadrons are in fact a kind of strong correlation between pairs of quarks; this diquark correlation may be most important for the light multiquark states.

Motivated by these theoretical and experimental developments, we construct light four-quark states via diquark clusters. The diquarks are regarded as hadronic constituents, with their masses determined via QCD sum rules. As an approximation, the masses determined in this way may be regarded as the constituent masses of diquarks. Accordingly, masses of these light tetraquark states are then obtained within the constituent quark models. As outlined below, this approach obviates the difficulties in a sum-rule approach based purely on local interpolating currents for four-quark/tetraquark states. Based on these results, the tetraquark state possibility of the new observations by BES is analyzed. We emphasize that the existence of diquark clusters within exotic states is an open question. Our work explores the implications of diquarks for the mass spectrum of the tetraquark states.

II. FOUR-QUARK STATES AND DIQUARK CLUSTERS

As mentioned above, four-quark states have complex internal color, flavor, and spin structure. First, we consider the color, flavor, and spin correlation of the $(\bar{q}\,\bar{q})(qq)$ four-quark state (tetraquark) and the $(\bar{q}q)(\bar{q}q)$ four-quark state.

In the $(\bar{q}\,\bar{q})(qq)$ configuration, the quark q is in the color fundamental representation 3, while the antiquark $\bar{q}$ is in the color representation $\bar{3}$. Two quarks give $3 \otimes 3 = 6 \oplus \bar{3}$, and two antiquarks give $\bar{3} \otimes \bar{3} = \bar{6} \oplus 3$. Therefore, the final color singlet four-quark state may be produced from $\bar{3} \otimes 3$ or $\bar{6} \times 6$.

In the $(\bar{q}q)(\bar{q}q)$ configuration, a quark and an antiquark give the color representations $3 \otimes \bar{3} = 8 \oplus 1$, other quark antiquark pairs give the same result. The final color singlet is therefore produced from the $1 \otimes 1$ or 8×8. The former $1 \otimes 1$ four-quark state is usually called a "molecule."

It is difficult to distinguish between these internal color configurations because the final state is a color singlet. For the same reason, it is difficult to distinguish between four-quark and normal $q\bar{q}$ mesons. An understanding of the internal dynamics of the quarks in the hadrons would be necessary to distinguish between these various configurations.

In the flavor $SU(3)$ approximation, both the $(\bar{q}\,\bar{q})(qq)$ and $(\bar{q}q)(\bar{q}q)$ configurations give the same flavor multiplets: $(3 \otimes 3) \otimes (\bar{3} \otimes \bar{3}) = (3 \otimes \bar{3}) \otimes (3 \otimes \bar{3}) = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 8 \oplus 8 \oplus 1 \oplus 1$. Obviously, the flavor structure of $(qq)(\bar{q}\,\bar{q})$ and $(\bar{q}q)(\bar{q}q)$ four-quarks cannot be distinguished. Except for the flavor octet or singlet (crypto-exotic), other flavor structures in the four-quark state do not exist in a normal $q\bar{q}$ meson. The crypto-exotic four-quark states will mix with normal $q\bar{q}$ mesons, while the four-quark state may exhibit its exotic flavor explicitly in a different way from that in the normal $q\bar{q}$ meson. Accordingly, experiments could be designed to detect four-quark states due to their exotic flavor structures. The spin in the four-quark state will couple (correlate) and give complicated representations as those mentioned in Ref. [2].

In fact, the correlation of color, flavor, and spin in the hadron is interrelated since their total correlation has to obey some symmetry constraints. In the $(\bar{q}\,\bar{q})(qq)$ configuration, according to Refs. [35,38,39], the two quarks correlate antisymmetrically in color, flavor, and spin, separately, and thereby attract one another forming a "good" diquark cluster. In other words, the two quarks are most possible in the color, flavor, and spin representations $\bar{3}$, $\bar{3}$, and 0 and form the good diquark. This antisymmetric diquark cluster will be denoted as $[qq]$ in the following.

The composition of light tetraquark states with different isospin are $[\bar{q}\,\bar{q}][qq], [\bar{s}\,\bar{q}][qq], [\bar{s}\,\bar{q}][sq], \ldots$ as in Refs. [12,40] with $q = u, d$. The orbital excitation between the diquark and antidiquark may make different kinds of four-quark states.

III. DIQUARK FOUR-QUARK STATES AND NEW OBSERVATIONS BY BES

QCD sum rules [41] is believed to be a good tool to study hadron physics and nonperturbative QCD interactions. With this approach, four-quark states have been investigated. In the investigations, different interpolating currents (operators) have been used, e.g., $(\bar{q}q)(\bar{q}q)$ currents were used in Ref. [42], $(\bar{q}\,\bar{q})(qq)$ diquark antidiquark currents were employed in Refs. [43–45], and both currents were studied in Ref. [46]. These investigations are interesting and may give some hints to our understanding of four-quark states. However, some of the conclusions related to the diquark concept based on these studies are not definitive. The diquark may be the reality in constituent quark models, but a diquark interpretation is not meaningful in the framework of QCD sum rules.

As is well known, each hadron is in color singlet. Internal color configurations in $(\bar{q}q)(\bar{q}q)$ and $(\bar{q}\,\bar{q})(qq)$ could not be detected directly through those interpolating currents except through a special observable which may detect those different couplings of currents. Unfortunately, no such observable has yet been formulated. Moreover, these color configurations cannot be distinguished from the normal $\bar{q}q$ mesons either, which implies that the mixing between the four-quark state and normal mesons may be crucial.

From the analyses in the last section, internal constituent flavor configurations in $(\bar{q}q)(\bar{q}q)$ and $(\bar{q}\,\bar{q})(qq)$ give the same representations. Their internal flavor configurations therefore could not be detected directly through those interpolating currents either except for flavor exotic currents.

This point is much more easy to be realized in other ways. On one hand, the Fierz transformation will turn the $(\bar{q}q)(\bar{q}q)$ current into the $(\bar{q}\,\bar{q})(qq)$ current and vice versa. On the other hand, these two kinds of currents can mix with each other under renormalization. Therefore, it may not be meaningful to talk about diquarks in the framework of sum rules based on local interpolating currents. In order to construct precise four-quark state sum rules, both $(\bar{q}q) \times (\bar{q}q)$ and $(\bar{q}\,\bar{q})(qq)$ currents have to be used. Similar situations occurred in the study of baryons with sum rules [47,48]. We emphasize that the inability to distinguish between the local $(\bar{q}q)(\bar{q}q)$ interpolating currents $(\bar{q}\,\bar{q}) \times (qq)$ in the QCD sum-rule context does not occur, for example, in potential models [49].

In principle, there is no direct way to turn the operator picture into the constituent quark picture [50], and the internal constituent quark structures cannot be detected directly through the couplings of local interpolating currents to hadrons [51].

To make the sum-rule analyses reliable and predictable, it is better to use flavor exotic currents. For normal interpolating currents, the mixing has to be incorporated in the sum rule. For example, a renormalization invariant mixed current may be required, and the saturation of the spectral density with mixed hadrons should be taken into account. To avoid such complexities, we will not study the four-quark state in terms of four-quark currents in this article. Instead, we will determine the mass of the two-quark cluster via QCD sum rules, and subsequently construct the four-quark state in terms of these diquark constituents.

From a practical viewpoint, an analysis of tetraquark states that employs QCD sum rules for the diquark clusters to obtain inputs for the constituent quark model is likely the only feasible approach to the study of tetraquark states. Correlators of tetraquark/four-quark interpolating currents have only been calculated to leading order in α_s [42–46]. Obtaining the α_s corrections that would be needed to establish a reliable QCD sum-rule prediction would involve four-loop calculations and the renormalization of the dimension-six composite operators. Although the loop calculations are feasible in the chiral limit, the renormalization of these operators is unknown beyond a single quark flavor [52] and extensions to more complicated flavor structures would be exceptionally difficult, particularly with heavy flavors.

Lattice investigations pointed out that the force between colored clusters is universal [53,54]. Since the diquark and the antidiquark clusters in the $(\bar{q}\,\bar{q})(qq)$ tetraquark state have the same color representations as $\bar{q}$ and q in the normal meson $(3 \otimes \bar{3} \to 1)$, they are expected to have similar strong dynamics as the normal constituent quarks. It is thus reasonable to expect that these diquark and antidiquark clusters will form a tetraquark state just as the constituent quark and antiquark construct a normal meson. A four-quark state with constituent configuration $(\bar{q}q)(\bar{q}q)$ has no correspondence to existing hadrons, and will not be considered in our analysis.

Though the diquark is not an isolated cluster in the hadron, it may be approximately regarded a bound state composed of two quarks and may be used as a degree of freedom. Historically, the effective diquark masses and couplings were derived with the QCD sum-rules approach in the study of weak decays in Refs. [33,55].

Before proceeding, we give a simple description for the dynamics in the tetraquark state in our model. The diquark cluster is regarded as a constituent similar to a constituent quark. The strong dynamics between the diquark and the antidiquark in tetraquark states is supposed to be similar to that between the quark and the antiquark in mesons [53,54]. As an approximation, the masses of diquarks obtained with the sum-rule approach are regarded as the constituent masses in the constituent quark model. Therefore the spectrum of the four-quark states could be obtained as in Refs. [13,56].

Since the diquark and antidiquark are supposed to be in the 0^+ good configuration, the P-parity and the C-parity of the neutral tetraquark states are the same $(-1)^L$, where L is the orbital angular momentum between the diquark and the antidiquark. Accordingly, possible J^{PC} of these kinds of tetraquark states are [57] $0^{++}(L=0), 1^{--}(L=1), 2^{++}(L=2), \ldots$ Thus the mass of the $0^{++}(L=0)$ tetraquark is [12]

$$M_{4q} = m_d + m_{\bar{d}} - 3(\kappa_{qq})_{\bar{3}}, \tag{1}$$

where $m_d = m_{qq}$, $m_{\bar{d}} = m_{\bar{q}\,\bar{q}}$ are the constituent masses of the diquark and antidiquark, respectively. Similarly, the mass of the $1^{--}(L=1)$ and the $2^{++}(L=2)$ tetraquark is

$$M_{4q} = m_d + m_{\bar{d}} + B_{d\bar{d}}\frac{L(L+1)}{2}, \tag{2}$$

where $B_{d\bar{d}} = B'_q,\ B'_{1s},\ B'_{2s}$. $B'_q > B'_{1s} > B'_{2s}$ [13,56], and they denote the coefficients with zero, one, and two strange quarks, respectively. If $B_{d\bar{d}} \propto \alpha_s^2 M$, they are very sensitive to $\Lambda_{\rm QCD}$.

In Refs. [33,55], the flavor combinations $(qq)(q = u, d)$, (sq), and (ss) were used to construct different flavor contents of the diquark to simplify calculation. The flavor (sq) diquark current was taken to be [33,55]

$$j_i(x) = \epsilon_{ijk} s_j^T(x) C O q_k(x), \tag{3}$$

where i, j, k are color indices, C is the charge conjugation matrix, and $O = \gamma_5,\ 1,\ \gamma_\mu,\ \gamma_\mu\gamma_5$ are the Lorentz structures corresponding to quantum numbers $J^P = 0^+, 0^-, 1^+$, and 1^-, respectively.

After constructing a gauge invariant correlator, theoretical expressions of $\Pi(Q^2 = -q^2)$ for $(sq)_i$ were computed in Refs. [33,55]

$$\begin{aligned}\Pi(Q^2) = {}& \frac{3}{4\pi^2}\left(1 + 2\frac{m_s^2}{Q^2} + \frac{17}{6}\frac{\alpha_s}{\pi} - \frac{1}{2}\frac{\alpha_s}{\pi}\ln\frac{Q^2}{\mu^2}\right)Q^2\ln\frac{Q^2}{\mu^2} \\ & - (2m_s - m_q)\frac{\langle\bar{q}q\rangle}{Q^2} - (2m_q - m_s)\frac{\langle\bar{s}s\rangle}{Q^2} \\ & + \frac{1}{8}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle\frac{1}{Q^2} + 8\pi\kappa\alpha_s\frac{\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{Q^4} \\ & - \frac{16\pi}{27}\kappa\alpha_s\frac{\langle\bar{q}q\rangle^2 + \langle\bar{s}s\rangle^2}{Q^4},\end{aligned} \tag{4}$$

where $\langle\bar{q}q\rangle$, $\langle\bar{s}s\rangle$, $\langle\frac{\alpha_s}{\pi}G^2\rangle$ are two-quark and two-gluon condensates, and κ denotes the deviations from factorization approximation of four-quark condensates ($\kappa = 1$ for ideal factorization).

In this paper, we perform an updated sum-rule study of the $J^P = 0^+$ good diquark. Within the single pole and continuum approximation, the mass of the diquark is able to be obtained from $m_{0^+}^2 = -\frac{\partial}{\partial\tau}\ln R_k(\tau, s_0)$ with

$$R_k(\tau, s_0) = \frac{1}{\tau}\hat{L}\left[(-Q^2)^k\left\{\Pi(Q^2) - \sum_{k=0}^{n-1} a_k(-Q^2)^k\right\}\right] - \frac{1}{\pi}\int_{s_0}^{+\infty} s^k e^{-s\tau}\,\mathrm{Im}\Pi^{\{\mathrm{pert}\}}(s)ds = \frac{1}{\pi}\int_0^{s_0} s^k e^{-s\tau}\,\mathrm{Im}\Pi(s)ds. \tag{5}$$

$R_0(\tau, s_0)$ for $(sq)_i$ with renormalization group improvement follows from Refs. [33,55]

$$\begin{aligned} R_0 = {}& \left(\frac{\alpha_s(\mu^2)}{\alpha_s(1/\tau)}\right)^{-4/9}\frac{3}{4\pi^2}\frac{1}{\tau^2}\left\{\left(1+\frac{17}{6}\frac{\alpha_s(1/\tau)}{\pi}\right)\right. \\ & \times[1-(1+s_0\tau)e^{-s_0\tau}] \\ & \left.-(1-\gamma_E)\frac{\alpha_s(1/\tau)}{\pi}\Phi(s_0\tau) - 2m_s^2\tau(1-e^{-s_0\tau})\right\} \\ & -[(2m_s - m_q)\langle\bar{u}u\rangle + (2m_q - m_s)\langle\bar{s}s\rangle] \\ & +\frac{1}{8}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \left[8\pi\kappa\alpha_s\langle\bar{u}u\rangle\langle\bar{s}s\rangle\right. \\ & \left.-\frac{16\pi}{27}\kappa\alpha_s(\langle\bar{u}u\rangle^2 + \langle\bar{s}s\rangle^2)\right]\tau, \end{aligned} \tag{6}$$

respectively, where γ_E is the Euler constant, and $\Phi(x)$ was given in Ref. [55]. The result of the (qq) diquark could be obtained by the replacement of the s quark with the q quark in previous formulas.

To obtain an updated numerical result, the input parameters are chosen as those in Refs. [16,58,59]: $m_u = m_d = m_q = 5$ MeV, $m_s = 120$ MeV, $\langle\bar{q}q\rangle = -(260\text{ MeV})^3$, $\langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$, $\langle\frac{\alpha_s}{\pi}GG\rangle = 0.012\text{ GeV}^4$, $\kappa\alpha_s\langle\bar{q}q\rangle^2 = 5.8\times10^{-4}\text{ GeV}^6$, $\Lambda = 375$ MeV, $\alpha_s(\mu^2) = 4\pi/9\ln\frac{\mu^2}{\Lambda^2}$, and renormalization scale $\mu \sim 1$ GeV.

The sum rules for (qq) and (sq) diquarks are good. To obtain a reliable prediction, the continuum s_0 is chosen as low as possible, and to make the Borel window as wide as possible in the case of keeping R_k positive. The variation of $m_{0^+}^2$ for $(qq)_i$ and $(sq)_i$ with Borel variable τ are, respectively, shown in Figs. 1 and 2 for the resulting value $s_0 = 1.2\text{ GeV}^2$. The quantity $m_{0^+}^2$ increases slowly but monotonically with the increase of s_0; the s_0 dependence of $m_{0^+}^2$ for $(qq)_i$ and $(sq)_i$ is shown in Figs. 3 and 4. The most suitable m_{qq} and m_{sq} are, respectively, found to be around 400 MeV and 460 MeV. They are roughly comparable in scale to the constituent quark. The results obtained here are consistent with the fit of Maiani *et al.* [13], where the $m_{[ud]} = 395$ MeV and $m_{[sq]} = 590$ MeV.

Now that the properties of the diquark cluster are determined, it is possible to construct a four-quark provided that the quark dynamics in hadrons is known. There exist many works which address internal quark dynamics [2,3,7,60–66]. Unfortunately, there is no one approach directly from

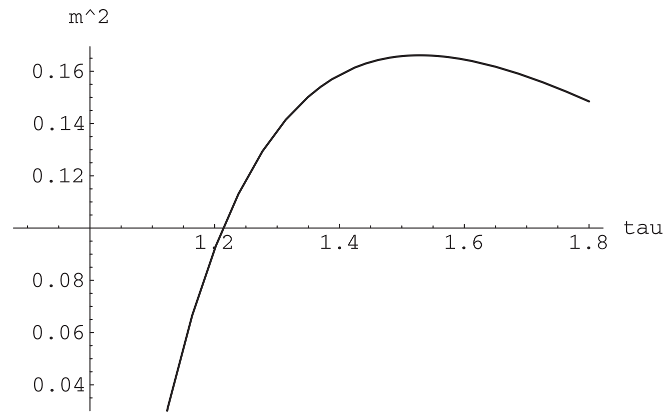

FIG. 1. τ dependence of $m_{0^+}^2$ for qq good diquark with $s_0 = 1.2\text{ GeV}^2$.

QCD—a very difficult task. In this paper, the prescription in Refs. [7,13,60] is used. Hadron masses were obtained from constituent quarks and their spin-dependent interactions

$$H = \sum_i m_i + \sum_{i<j} 2\kappa_{ij}(S_i \cdot S_j), \tag{7}$$

where the κ_{ij} are coefficients, and the sum runs over all the constituents. The resulting masses for four-quark states formed via diquark clusters are given in Eqs. (1) and (2).

In terms of the $(\kappa_{qq})_{\bar{3}} = 103$ MeV and $(\kappa_{sq})_{\bar{3}} = 64$ MeV [13], the masses of 0^{++} tetraquark states $[\bar{q}\,\bar{q}]\times[qq]$, $[\bar{q}\,\bar{q}][sq]$ ($[\bar{s}\,\bar{q}][qq]$) and $[\bar{s}\,\bar{q}][sq]$ are found via (1) to be ~490 MeV, ~610 MeV, and ~730 MeV, respectively.

As a more "accurate" prescription, the masses of tetraquark states could be obtained with constituent diquarks (with masses obtained by sum rules) in the Coulombic and linear confinement potentials, an approach which is beyond our scope of this article. Taking into account the decay features, in the approximation we used, it is reasonable to identify $f_0(600)$ (or σ), $f_0(980)$, $a_0(980)$, and the unconfirmed $\kappa(800)$ as the 0^{++} light tetraquark states. This

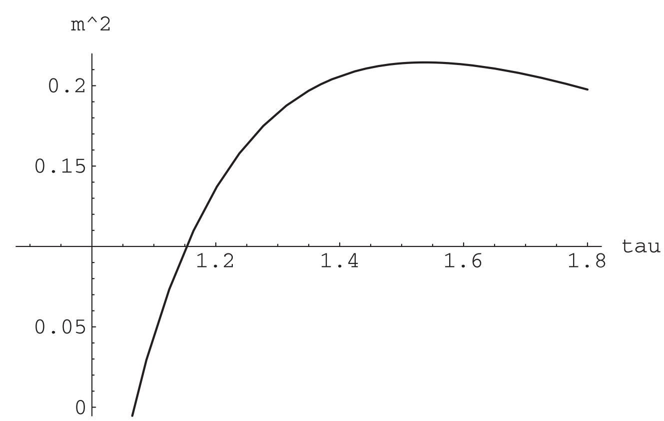

FIG. 2. τ dependence of $m_{0^+}^2$ for sq good diquark with $s_0 = 1.2\text{ GeV}^2$.

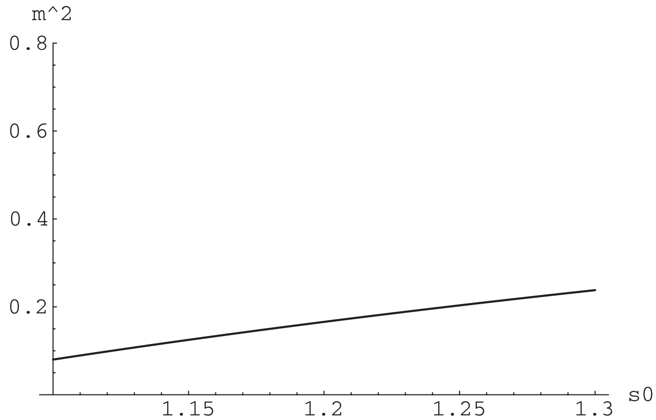

FIG. 3. s_0 dependence of $m^2_{0^+}$ for qq good diquark with $\tau = 1.5\ \mathrm{GeV}^{-2}$.

identification agrees with the conclusions of analyses based on chiral Lagrangians [14] and chiral perturbation theory [67].

Masses of the 1^{--} orbital excited $[\bar{q}\,\bar{q}][qq]$, $[\bar{s}\,\bar{q}][qq]$, and $[\bar{s}\,\bar{q}][sq]$ are, respectively, determined via (2):

$$\sim 490 + B'_q\ \mathrm{MeV}, \qquad \sim 610 + B'_{1s}\ \mathrm{MeV}, \qquad \sim 730 + B'_{2s}\ \mathrm{MeV}.$$

$B_{d\bar{d}}$ are very sensitive to Λ_{QCD}. It is not reliable to obtain the masses of orbital excited tetraquark states with them. Therefore theoretical estimates of the masses of these tetraquark states are not given here. However, if a tetraquark state is confirmed in the future, some constraints on these $B_{d\bar{d}}$ could then be derived.

Now we turn our attention to the tetraquark state possibility of the new observations by BES mentioned above. The $p\bar{p}$ threshold enhancement was also seen by Belle [68], while other observations have not been seen by any other experimental group. The P-parity of $p\bar{p}$ is likely to be $C = +1$, and its quantum number assignment is consistent with either $J^{PC} = 0^{-+}$ or 0^{++}. $X(1835)$ is found consistent with expectations for the state that produces the strong $p\bar{p}$ mass threshold enhancement. $X(1812)$ favors $J^P = 0^+$.

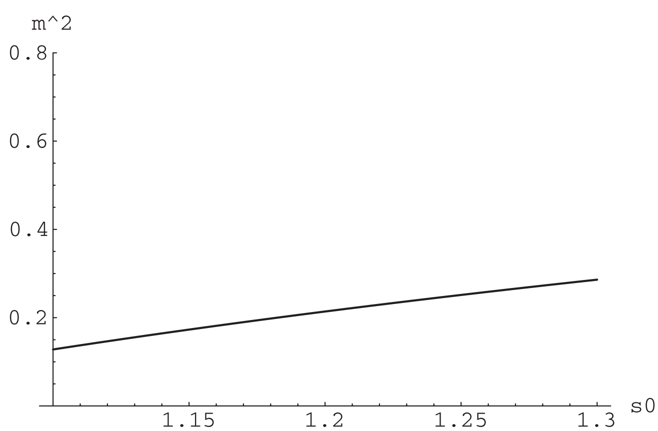

FIG. 4. s_0 dependence of $m^2_{0^+}$ for sq good diquark with $\tau = 1.5\ \mathrm{GeV}^{-2}$.

These three signals are very unlikely to be the tetraquark states. As analyzed above, if they have definite C parity, possible J^{PC} for tetraquark state interpretation is constrained. These 0^{++} or 0^{-+} observations lie above the predicted 0^{++} or 0^{-+} light tetraquark states (assuming that the mass difference between the 0^{++} and the 0^{-+} is not very large). A 0^{-+} tetraquark state requires the bad diquark.

It was argued that the light (orbital angular momentum between the diquark and the antidiquark $L = 0$) $q^2\bar{q}^2$ states may decay into meson-meson channels, while the heavier ones ($L \geq 1$) decay mainly into baryon-antibaryon channels [2], and may decay into another light tetraquark state ($L = 0$) [3]. $X(1576)$ has large decay width, and its isospin has not been determined. $X(1576)$ behaves unlike a normal meson and may be a 1^{--} tetraquark state [29,30,69]. If $X(1576)$ is a 1^{--} four-quark state, it is likely to be the $(\bar{s}\,\bar{q})(sq)$ orbital excited tetraquark state. It may be the first orbital excitation of $a_0(980)$ if its isospin $I = 1$, and it may be the first orbital excitation of $f_0(980)$ if its isospin $I = 0$.

If this suggestion is true, the $B'_{2s} \sim 586$ MeV. This B'_{2s} is large compared to those for normal mesons, which may imply an "exotic" orbital excitation. In the meantime, other 1^{--} orbital excited tetraquark states corresponding to $(\bar{q}\,\bar{q})(qq)$ (~ 1400 MeV) and $(\bar{s}\,\bar{q})(qq)$ (~ 1500 MeV) are expected. Both of them may have broad widths. In this case, their radiative decays into the scalars would be explicitly indications.

IV. CONCLUSIONS AND DISCUSSIONS

The intrinsic color, flavor, and spin configurations of two different kinds of spatially extended four-quark states are discussed in the constituent quark model. It is obvious that the $(\bar{q}\,\bar{q})(qq)$ and $(\bar{q}q)(\bar{q}q)$ states may mix with each other. It is hard to distinguish their internal color and flavor configurations unless the quark dynamics is well known or a special observable is established. The case for the crypto-exotic four-quark state is more complicated for the mixture with normal $q\bar{q}$ mesons.

QCD sum rules are a powerful technique for studying hadrons. To get a reliable prediction, suitable currents are required. However, there is no direct correspondence between the constituent quark picture and the operator (current) quark picture. The diquark concept is therefore not meaningful in the framework of a sum-rule approach.

The masses of good diquarks are obtained through an updated sum-rule analysis. As an approximation, the resulting masses may be identified as the masses of the corresponding constituent diquark. In terms of these constituent diquarks, the masses of light tetraquark states are estimated. Based upon these results, it is reasonable to identify $f_0(600)$ (or σ), $f_0(980)$, $a_0(980)$, and the unconfirmed $\kappa(800)$ as the 0^{++} light tetraquark states. However, further study of the internal quark dynamics is necessary to determine whether diquark clusters actually exist within exotic states.

The new observations by BES are qualitatively analyzed. The $p\bar{p}$ enhancements, $X(1835)$ and $X(1812)$ are unlikely to be tetraquark states. $X(1576)$ may be the 1^{--} tetraquark state (first orbital excitations of $a_0(980)$ or $f_0(980)$) with an exotic large orbital excitation. If this hypothesis is true, a whole family of 1^{--} tetraquark states corresponding to orbital excitations of $f_0(600)$ and $\kappa(800)$ may also exist, and it will be interesting to study their radiative decays into these scalars.

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