CP violation and arrows of time: Evolution of a neutral *K* or *B* meson from an incoherent to a coherent state

Ch. Berger

I. Physikalisches Institut der RWTH, Aachen, Germany

L. M. Sehgal[*]

Institut für Theoretische Physik (E) der RWTH, Aachen, Germany

We study the evolution of a neutral *K* meson prepared as an incoherent equal mixture of K^0 and $\bar{K}^0$. Denoting the density matrix by $\rho(t) = \frac{1}{2}N(t)[\mathbb{1} + \vec{\zeta}(t) \cdot \vec{\sigma}]$, the norm of the state $N(t)$ is found to decrease monotonically from one to zero, while the magnitude of the Stokes vector $|\vec{\zeta}(t)|$ increases monotonically from zero to one. This property qualifies these observables as arrows of time. Requiring monotonic behavior of $N(t)$ for arbitrary values of γ_L, γ_S, and Δm yields a bound on the *CP*-violating overlap $\delta = \langle K_L|K_S\rangle$, which is similar to, but weaker than, the known unitarity bound. A similar requirement on $|\vec{\zeta}(t)|$ yields a new bound, $\delta^2 < \frac{1}{2}(\frac{\Delta\gamma}{\Delta m})\sinh(\frac{3\pi}{4}\frac{\Delta\gamma}{\Delta m})$ which is particularly effective in limiting the *CP*-violating overlap in the B^0-$\bar{B}^0$ system. We obtain the Stokes parameter $\zeta_3(t)$ which shows how the average strangeness of the beam evolves from zero to δ. The evolution of the Stokes vector from $|\vec{\zeta}| = 0$ to $|\vec{\zeta}| = 1$ has a resemblance to an order parameter of a system undergoing spontaneous symmetry breaking.

　　　　PACS numbers: 11.10.St, 11.30.Er, 13.20.Eb, 13.20.He

I. INTRODUCTION

We examine in this paper the time evolution of a neutral *K* meson prepared as an equal incoherent mixture of K^0 and $\bar{K}^0$. Such a state is easily obtained in a reaction such as $e^+e^- \rightarrow \phi(1020) \rightarrow K^0\bar{K}^0$, when only one of the kaons in the final state is observed. [Our considerations apply equally to *B* mesons produced in $e^+e^- \rightarrow \Upsilon(4s) \rightarrow B^0\bar{B}^0$.] An incoherent beam of this type is characterized by a density matrix, which we write in the K^0-$\bar{K}^0$ basis as

$$\rho(t) = \tfrac{1}{2}N(t)[\mathbb{1} + \vec{\zeta}(t) \cdot \vec{\sigma}].$$

The evolution is described by a normalization function $N(t)$, which is the intensity of the beam at time *t*, and a Stokes vector $\vec{\zeta}$ which characterizes the polarization state of the system with respect to strangeness. The beam, which has $|\vec{\zeta}(0)| = 0$ at the time of production evolves ultimately into a pure state corresponding to the long-lived *K* meson K_L, with a Stokes vector of unit length: $|\vec{\zeta}(\infty)| = 1$. In this sense, the system can be regarded as possessing two dynamical functions: $N(t)$ which varies from one to zero, and $|\vec{\zeta}(t)|$ which goes from zero to one. This evolution touches on interesting issues such as the role of *CP* violation, and the extent to which the functions $N(t)$ and $|\vec{\zeta}(t)|$ define arrows of time. The requirement that these functions are monotonic yields constraints on the *CP*-violating parameter $\delta = \langle K_L|K_S\rangle$. The fact that the incoherent initial state is completely neutral with respect to strangeness and *CP* quantum numbers is of significance in this regard. In addition, the component $\zeta_3(t)$ of the Stokes vector de-

scribes the manner in which the strangeness of the state evolves from zero to final value $\delta = 3.27 \times 10^{-3}$ and serves as a model for flavor genesis induced by *CP* violation in a decaying system. Finally, the evolution of the system from an initial "amorphous" state with $\vec{\zeta}(0) = 0$ to a final "crystalline" state described by a three-dimensional Stokes vector $\vec{\zeta}(t)$, with unit length, is suggestive of a phase transition, with $\vec{\zeta}(t)$ playing the role of an order parameter of a system undergoing spontaneous symmetry breaking.

II. DENSITY MATRIX

An arbitrary state of the *K* meson can be described by a 2×2 density matrix which we write, in the K^0-$\bar{K}^0$ basis, as [1–3]

$$\rho(t) = \tfrac{1}{2}N(t)[\mathbb{1} + \vec{\zeta}(t) \cdot \vec{\sigma}]. \tag{1}$$

Here $N(t)$ is the intensity or norm of the state at time *t*, calculated from the trace of ρ,

$$N(t) = \mathrm{tr}\rho(t) \tag{2}$$

and $\vec{\zeta}(t)$ is the Stokes vector, whose components can be expressed as

$$\zeta_i(t) = \mathrm{tr}[\rho(t)\sigma_i]/\mathrm{tr}\rho(t). \tag{3}$$

An initial state which is a 1:1 incoherent mixture of K^0 and $\bar{K}^0$ has the density matrix

$$\rho(t) = \frac{1}{2}|K^0\rangle\langle K^0| + \frac{1}{2}|\bar{K}^0\rangle\langle \bar{K}^0| = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

which corresponds to an initial Stokes vector $\vec{\zeta}(0) = 0$. To determine the time evolution, we note that [4]

───────
[*]sehgal@physik.rwth-aachen.de

$$|K^0\rangle \underset{t}{\rightarrow} |\psi(t)\rangle = \frac{1}{2p}[|K_S\rangle e^{-\lambda_S t} + |K_L\rangle e^{-\lambda_L t}]$$

$$|\bar{K}^0\rangle \underset{t}{\rightarrow} |\bar{\psi}(t)\rangle = \frac{1}{2q}[|K_S\rangle e^{-\lambda_S t} - |K_L\rangle e^{-\lambda_L t}], \tag{5}$$

where we have introduced the eigenstates

$$|K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle \qquad |K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$$

$$(|p|^2 + |q|^2 = 1) \tag{6}$$

with eigenvalues

$$\lambda_{L,S} = \tfrac{1}{2}\gamma_{L,S} + im_{L,S}. \tag{7}$$

The overlap of the states $|K_L\rangle$ and $|K_S\rangle$ is given by the CP-violating parameter

$$\delta = \langle K_L|K_S\rangle = (|p|^2 - |q|^2)/(|p|^2 + |q|^2)$$

$$= 3.27 \times 10^{-3}. \tag{8}$$

The resulting density matrix at time t is

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & \rho_{12}(t) \\ \rho_{21}(t) & \rho_{22}(t) \end{pmatrix} \tag{9}$$

with

$$\rho_{11}(t) = \frac{1}{4(1-\delta)}[e^{-\gamma_S t} + e^{-\gamma_L t}$$
$$- 2\delta e^{-(1/2)(\gamma_S+\gamma_L)t}\cos\Delta mt]$$

$$\rho_{22}(t) = \frac{1}{4(1+\delta)}[e^{-\gamma_S t} + e^{-\gamma_L t}$$
$$+ 2\delta e^{-(1/2)(\gamma_S+\gamma_L)t}\cos\Delta mt] \tag{10}$$

$$\rho_{12}(t) = \frac{1}{8p^*q}[e^{-\gamma_S t} - e^{-\gamma_L t}$$
$$+ i2\delta e^{-(1/2)(\gamma_S+\gamma_L)t}\sin\Delta mt]$$

$$\rho_{21}(t) = \rho_{12}^*(t).$$

Using Eqs. (2) and (3), we derive

$$N(t) = \frac{1}{2(1-\delta^2)}[e^{-\gamma_S t} + e^{-\gamma_L t}$$
$$- 2\delta^2 e^{-(1/2)(\gamma_S+\gamma_L)t}\cos\Delta mt] \tag{11}$$

and

$$\zeta_1(t) = \frac{2[\mathrm{Re}(pq^*)(e^{-\gamma_S t} - e^{-\gamma_L t}) - \mathrm{Im}(pq^*)2\delta e^{-(1/2)(\gamma_S+\gamma_L)t} \cdot \sin\Delta mt]}{e^{-\gamma_S t} + e^{-\gamma_L t} - 2\delta^2 e^{-(1/2)(\gamma_S+\gamma_L)t}\cos\Delta mt}$$

$$\zeta_2(t) = \frac{-2[\mathrm{Im}(pq^*)(e^{-\gamma_S t} - e^{-\gamma_L t}) + \mathrm{Re}(pq^*)2\delta e^{-(1/2)(\gamma_S+\gamma_L)t} \cdot \sin\Delta mt]}{e^{-\gamma_S t} + e^{-\gamma_L t} - 2\delta^2 e^{-(1/2)(\gamma_S+\gamma_L)t}\cos\Delta mt} \tag{12}$$

$$\zeta_3(t) = \delta\frac{e^{-\gamma_S t} + e^{-\gamma_L t} - 2e^{-(1/2)(\gamma_S+\gamma_L)t}\cos\Delta mt}{e^{-\gamma_S t} + e^{-\gamma_L t} - 2\delta^2 e^{-(1/2)(\gamma_S+\gamma_L)t}\cos\Delta mt}.$$

Note that the components $\zeta_{1,2}(t)$ involve $\mathrm{Re}(pq^*)$ and $\mathrm{Im}(pq^*)$ where p and q are the coefficients in the definition of $K_{L,S}$ in Eq. (6). These are convention dependent, since the relative phase of p and q can be changed by a phase transformation $|K^0\rangle \rightarrow e^{i\alpha}|K^0\rangle$, $|\bar{K}^0\rangle \rightarrow e^{-i\alpha}|\bar{K}^0\rangle$. A quantity independent of phase convention is

$$\zeta_1^2 + \zeta_2^2 = (1-\delta^2)$$

$$\times\frac{(e^{-\gamma_S t} - e^{-\gamma_L t})^2 + 4\delta^2 e^{-(\gamma_S+\gamma_L)t}\sin^2\Delta mt}{[e^{-\gamma_S t} + e^{-\gamma_L t} - 2\delta^2 e^{-(1/2)(\gamma_S+\gamma_L)t}\cos\Delta mt]^2}. \tag{13}$$

Thus, the length of the Stokes vector is

$$|\vec{\zeta}(t)| = [\zeta_1^2(t) + \zeta_2^2(t) + \zeta_3^2(t)]^{1/2}$$

$$= \frac{1}{2N(t)(1-\delta^2)}[\delta^2\{e^{-\gamma_S t} + e^{-\gamma_L t} - 2e^{-(1/2)(\gamma_S+\gamma_L)t}$$

$$\times \cos\Delta mt\}^2 + (1-\delta^2)\{(e^{-\gamma_S t} - e^{-\gamma_L t})^2$$

$$+ 4\delta^2 e^{-(1/2)(\gamma_S+\gamma_L)t} \cdot \sin^2\Delta mt\}]^{1/2}$$

$$= \left[1 - \frac{1}{N(t)^2}e^{-(\gamma_S+\gamma_L)t}\right]^{1/2}. \tag{14}$$

This equation provides a simple relation between the magnitude of the Stokes vector $|\vec{\zeta}(t)|$ and the normalization function $N(t)$.

In the CP-invariant limit, $\delta \rightarrow 0$, the density matrix reduces to

$$\rho(t)\underset{\delta\rightarrow 0}{\longrightarrow}\frac{1}{4}\begin{bmatrix} e^{-\gamma_S t} + e^{-\gamma_L t} & e^{-\gamma_S t} - e^{-\gamma_L t} \\ e^{-\gamma_S t} - e^{-\gamma_L t} & e^{-\gamma_S t} + e^{-\gamma_L t} \end{bmatrix} \tag{15}$$

and the limiting form of $N(t)$ and $\vec{\zeta}(t)$ is

$$N(t) \rightarrow \tfrac{1}{2}[e^{-\gamma_S t} + e^{-\gamma_L t}]$$

$$\zeta_{12} \equiv [\zeta_1^2(t) + \zeta_2^2(t)]^{1/2}$$

$$\rightarrow (e^{-\gamma_L t} - e^{-\gamma_S t})/(e^{-\gamma_S t} + e^{-\gamma_L t})$$

$$\zeta_3(t) \rightarrow 0. \tag{16}$$

The behavior of $N(t)$ and dN/dt for the K^0-$\bar{K}^0$ system is shown in Fig. 1.

The behavior of the functions $\zeta_{12}^2(t)$, $\zeta_3^2(t)$, and $\zeta^2(t) = \zeta_{12}^2(t) + \zeta_3^2(t)$ and their time derivatives is shown in Fig. 2. The function ζ_3^2 is clearly nonmonotonic, and its derivative has a number of zeros (e.g. $t/\tau_S = 4.95, 11.6, 18.2, \ldots$). By comparison, the derivative of ζ_{12}^2 has a distant zero at
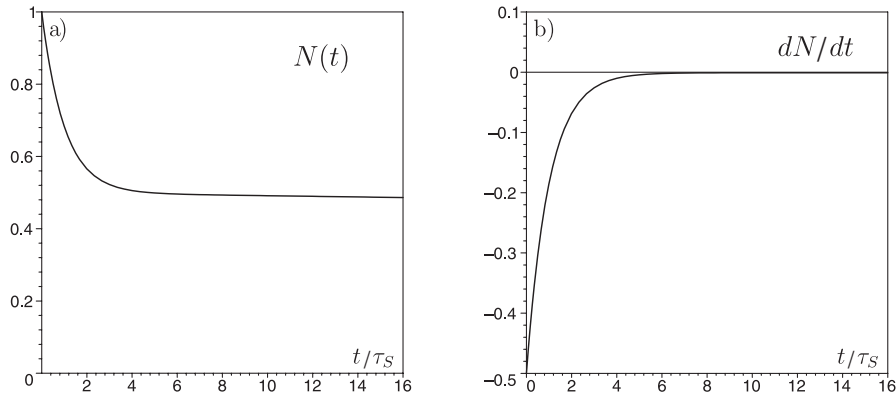
FIG. 1.   Normalization function $N(t)$ (a) and its time derivative (b) as function of time in units of τ_S for a state prepared as incoherent mixture of K^0 and $\bar{K}^0$.
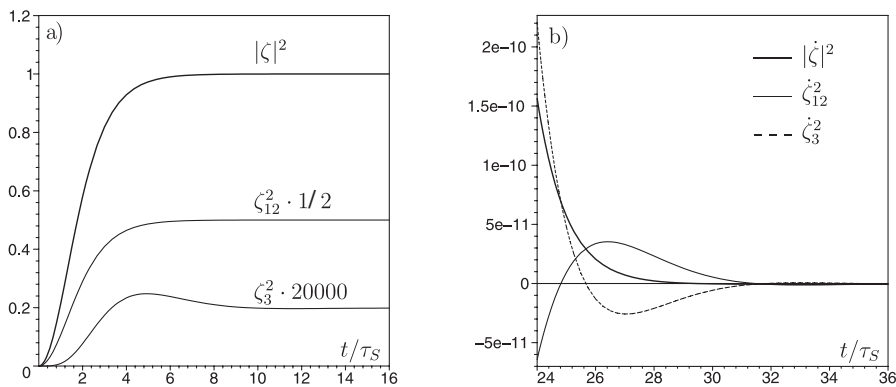

FIG. 2.   Behavior of $|\zeta|^2$, ζ_{12}^2, and ζ_3^2 (a) and their time derivatives (b) as a function of time. Note the different scale for ζ_{12}^2 and ζ_3^2 in (a). In (b) only the tail of the time dependence is shown.

$t/\tau_S = 25.7$. As seen in Fig. 2(b), these two nonmonotonic functions combine to produce a Stokes vector $\zeta^2(t)$ which is strictly monotonic, the asymptotic values being $\zeta_{12}^2 \to (1 - \delta^2)$, $\zeta_3^2 \to \delta^2$, $\zeta^2 \to 1$.

III. ARROWS OF TIME

A. The normalization arrow $N(t)$

The normalization of the kaon state is given in Eq. (11). As seen in Fig. 1, this function is indeed monotonic for the parameters of the K meson system. This monotonic (unidirectional) property implies that $N(t)$ behaves as an arrow of time. In the absence of *CP* violation ($\delta = 0$), the function $N(t)$ is simply the sum of two exponentials $(e^{-\gamma_S t} + e^{-\gamma_L t})/2$, and the monotonic decrease is ensured by the requirement γ_S, $\gamma_L > 0$ (positivity of the decay matrix). The third term in Eq. (11), appearing when $\delta \neq 0$, indicates a K_L-K_S interference effect. It implies that an incoherent K^0-$\bar{K}^0$ mixture does not evolve like an incoherent K_L-K_S mixture. Notice however, that the coefficient of the interference term is quadratic in δ, so that the function $N(t)$ is *CP*-even, remaining unchanged under $\delta \to -\delta$. Nevertheless the presence of the δ^2 term is decisive in

determining whether or not $N(t)$ is monotonic, and hence an arrow of time. If we require the function $N(t)$ to be monotonic ($dN/dt < 0$) then we have from (11) (see also [5]) that

$$\frac{dN}{dt} = \frac{-1}{2(1 - \delta^2)}\left[\gamma_S e^{-\gamma_S t} + \gamma_L e^{-\gamma_L t}\right.$$
$$- 2\delta^2 e^{-(1/2)(\gamma_S + \gamma_L)t}\left\{\frac{\gamma_S + \gamma_L}{2}\cos\Delta mt\right.$$
$$\left.\left. + \Delta m \sin\Delta mt\right\}\right] < 0, \tag{17}$$

from which it follows, as a sufficient condition, that

$$\delta^2 \leq \left(\frac{\gamma_S \gamma_L}{(\gamma_S + \gamma_L)^2/4 + \Delta m^2}\right)^{1/2} \quad \text{or}$$
$$\delta^2 \leq \left(\frac{r}{(1 + r)^2/4 + \mu^2}\right)^{1/2}, \tag{18}$$

where we have introduced the notation $r = \gamma_L/\gamma_S$, $\mu = \Delta m/\gamma_S$. This constraint is analogous to, but weaker than, the unitarity constraint derived in [6,7], which reads
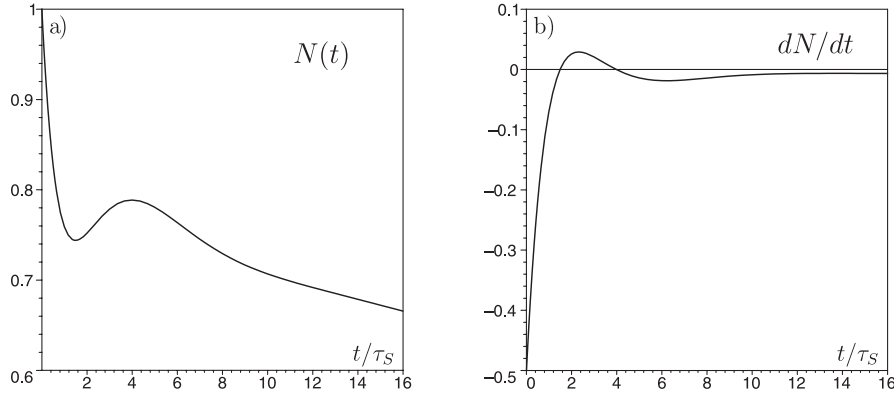
FIG. 3.   Behavior of $N(t)$ (a) and dN/dt (b) for values of parameters $\mu = \mu_K$, $r = 0.01$, $\delta = 0.6$ violating the monotonicity bound (18).

$$\delta^2_{\text{unit}} \leq \frac{r}{(1 + r)^2/4 + \mu^2}. \quad (19)$$

It is clear that the unitarity bound is interesting for a system like K^0-$\bar{K}^0$, where $r = \gamma_L/\gamma_S$ is small, but does not provide a useful constraint for B^0-$\bar{B}^0$, where r is close to 1.

To see what happens if the parameters δ, r, and μ are allowed to vary, we show in Fig. 3 the behavior of $N(t)$ and dN/dt for $\delta = 0.6$, $r = 0.01$, keeping μ at its standard K meson value, $\mu_K = 0.47$. The function $N(t)$ shows fluctuations, and the derivative dN/dt changes sign. Such a behavior results from the violation of the bound (18). The fluctuations in $N(t)$ may be regarded as fluctuations in the direction of the time arrow (we call this phenomenon "Zeitzitter"), and can occur when the CP-violating parameter δ^2 exceeds the limit (18). This is the manner in which CP violation impacts on the time arrow, even though the function $N(t)$ is CP-even.

From the point of view of an observer monitoring the intensity of the kaon beam (for example by measuring the rate of leptonic decays $\pi^{\mp}\ell^{\pm}\nu$), the fluctuation in N would appear as an inexplicable enhancement or suppression of the beam intensity in certain intervals of time. The effect can be regarded equivalently as a violation of unitarity or a flutter in the arrow of time.

B. The Stokes arrow

The magnitude of the Stokes vector $|\vec{\zeta}(t)|^2$, calculated in Eq. (14), is a measure of the coherence of the state, and is plotted in Fig. 2 for the physical K-meson parameters. One sees that the function $|\vec{\zeta}|^2$ evolves monotonically from 0 to 1, and its derivative remains positive at all times. Thus the Stokes parameter $|\vec{\zeta}(t)|$ qualifies as an arrow of time. To see how this arrow is affected if the parameters δ, r, and μ are allowed to vary, we look at the derivative of the function $\zeta(t)$. Writing

$$|\vec{\zeta}(t)| = \left[1 - \frac{e^{-(\gamma_S + \gamma_L)t}}{N(t)^2}\right]^{1/2}, \quad (20)$$

we find that the monotonicity condition $d|\vec{\zeta}(t)|/dt > 0$ is equivalent to the condition

$$\left(\frac{dN}{dt} + \frac{1}{2}(\gamma_S + \gamma_L)N\right) \leq 0, \quad (21)$$

which implies

$$(e^{\Delta\gamma t/2} - e^{-\Delta\gamma t/2}) + 4\delta^2\frac{\Delta m}{\Delta\gamma}\sin\Delta mt \geq 0, \quad (22)$$

where $\Delta\gamma = \gamma_S - \gamma_L$. From this we derive a new upper bound on δ^2:

$$\delta^2 < \frac{1}{2}\left(\frac{\Delta\gamma}{\Delta m}\right)\sinh\left(\frac{3\pi}{4}\frac{\Delta\gamma}{\Delta m}\right) \quad \text{or}$$
$$\delta^2 < \frac{1}{2}\left(\frac{1 - r}{\mu}\right)\sinh\left(\frac{3\pi}{4}\frac{(1 - r)}{\mu}\right). \quad (23)$$
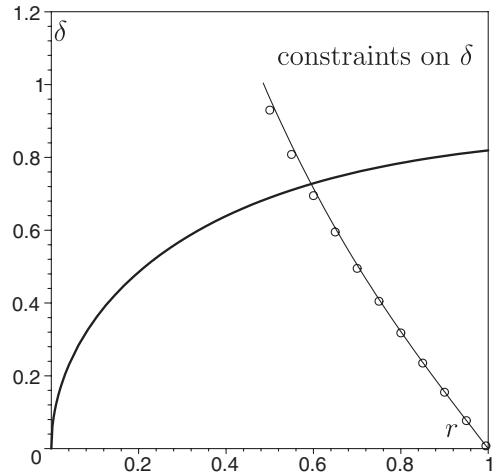

FIG. 4.   Constraints on δ in the δ-r plane resulting from unitarity and monotonicity of $|\zeta(t)|$ for a B^0-like system with $\mu = 0.7$. The thick line represents the unitarity bound (19) and the thin line our new bound evaluated from (23). The numerical evaluation of (22) (open circles) yields values very close to the approximation given in Eq. (23).
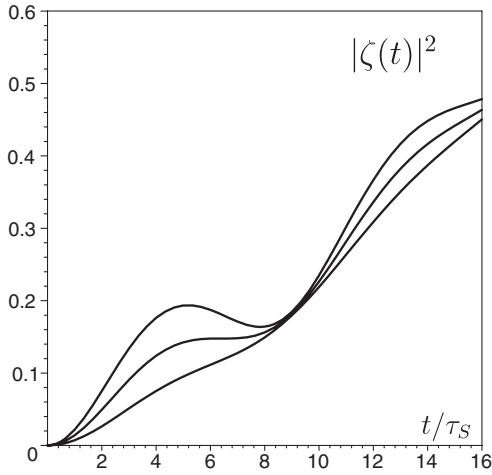
FIG. 5. Evolution of the Stokes vector $|\zeta(t)|^2$ for a B^0-like system with parameters $\mu_B = 0.7$, $r = 0.9$. The middle curve corresponds to $\delta_{\mathrm{crit}} = 0.156$ obtained from the bound (23). The nonmonotonic upper curve is obtained for $\delta = 0.2$ and the monotonic lower curve for $\delta = 0.1$.

This bound is obtained from the requirement that $|\vec{\zeta}(t)|$ be monotonic (an arrow of time) just as the bound in Eq. (18) was derived from the monotonicity of $N(t)$. The bound (23) is particularly effective in constraining the value of the overlap parameter in the B^0-$\bar{B}^0$ system, in which the decay widths of the two eigenstates are close together, $r \to 1$. In this respect, the bound in Eq. (23) is complementary to the unitarity bound in Eq. (19) which is effective when $r \to 0$. The contrast between the two bounds is highlighted in Fig. 4. Taking the parameters of the B^0-$\bar{B}^0$ system to be $r = 0.99$, $\mu = 0.7$, we obtain from (23)

$$\delta_B = \langle B^0_S | B^0_L \rangle \lesssim 0.0155. \tag{24}$$

We wish to stress that for a B^0-like system the bound in (23) is not just a sufficient condition for monotonic behavior of $|\zeta(t)|^2$, but almost a critical value separating the monotonic and nonmonotonic domains. As an illustration, we show in Fig. 5 the transition in the behavior of $|\zeta(t)|^2$ for a system with parameters $\mu = 0.7$, $r = 0.9$, as δ is

varied from a value 0.1, below the critical value of $\delta_{\mathrm{crit}} = 0.156$, to a value 0.2 above δ_{crit}.

The fluctuations in $|\zeta(t)|^2$, shown in Fig. 5, are the analog of the fluctuations in $N(t)$, shown in Fig. 3, which arise when the parameters of the system violate the bound in Eq. (18). Whereas the fluctuation in $N(t)$ would reveal itself as an inexplicable Zitter in the beam intensity, the fluctuation in $|\vec{\zeta}(t)|$ would show up as an unaccountable Zitter in the coherence of the beam. In both cases, the effect results from a breakdown in the monotonicity of a function, associated with a loss of directionality in an arrow of time.

IV. EVOLUTION OF STRANGENESS

The component $\zeta_3(t)$ of the Stokes vector has a special significance: it is the expectation value of σ_3, which can be identified with the strangeness operator with eigenvalues $+1$ for K^0 and -1 for $\bar{K}^0$. Thus, a measurement of $\zeta_3(t)$ is simply a measurement of the decay asymmetry into the channels $\pi^- \ell^+ \nu$ and $\pi^+ \ell^- \bar{\nu}$:

$$\zeta_3(t) = \frac{\Gamma(\pi^- \ell^+ \nu; t) - \Gamma(\pi^+ \ell^- \bar{\nu}; t)}{\Gamma(\pi^- \ell^+ \nu; t) + \Gamma(\pi^+ \ell^- \bar{\nu}; t)}. \tag{25}$$

Referring to Eq. (12), we observe that $\zeta_3(t)$ is a pure *CP*-violating observable, since it changes sign under $\delta \to -\delta$. [By contrast, the functions $N(t)$ and $|\vec{\zeta}(t)|$, are invariant under $\delta \to -\delta$.] Writing $\zeta_3(t)$ explicitly as

$$\zeta_3(t) = \delta \frac{e^{-\gamma_S t} + e^{-\gamma_L t} - 2e^{-(1/2)(\gamma_L + \gamma_S)t} \cos\Delta m t}{2(1 - \delta^2)N(t)}, \tag{26}$$

we note that it is a quotient of a function that contains an oscillating term and a monotonic function $N(t)$. The average strangeness $\zeta_3(t)$ is thus clearly not a monotonic function of time. This is visible in Fig. 6, where we also show the derivative $d\zeta_3(t)/dt$. We have here an explicit example of a *CP*-odd observable emerging from an initial state that has no preferred *CP* direction. Such observables are not monotonic, and cannot be associated with an arrow of time.
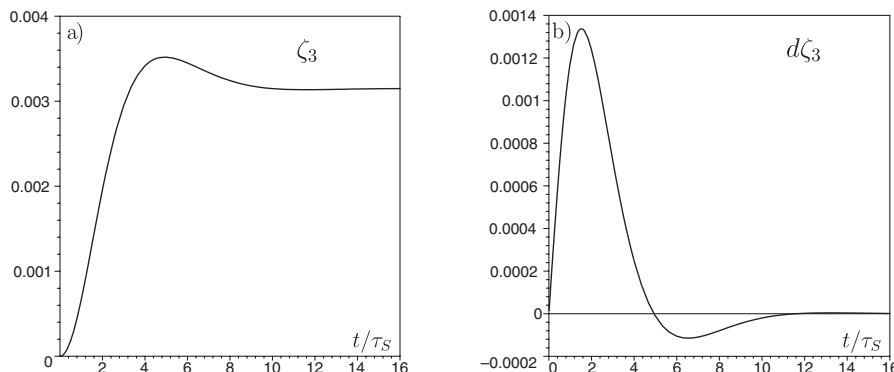

FIG. 6. Evolution of $\zeta_3(t)$ (a) and its derivative (b) in the K^0-$\bar{K}^0$ system.

V. SUMMARY

(1) We have shown that the evolution of an incoherent K^0-$\bar{K}^0$ mixture is characterized by two time-dependent functions, the norm $N(t)$ and the magnitude of the Stokes vector $|\vec{\zeta}(t)|$ both of which evolve monotonically and may therefore be associated with microscopic arrows of time. It should be stressed that we are discussing here *conditional* arrows of time, whose existence depends on the degree of *CP* violation, and not simply on the positivity of the decay widths $\gamma_{L,S}$.

(2) If the parameters γ_L, γ_S, Δm, and $\delta = \langle K_L | K_S \rangle$ are allowed to vary, the requirement of monotonic behavior of $N(t)$ leads to the bound in Eq. (18), which is similar to, but weaker than, the unitarity bound (19), derived in [6,7]. The requirement of monotonicity for $|\vec{\zeta}(t)|$ leads to a new bound on δ^2 given in Eq. (23), which is complementary to the unitarity bound (19), and far more restrictive for systems such as B^0-$\bar{B}^0$ with $r = \gamma_L / \gamma_S$ close to unity.

A violation of the bounds in Eq. (18) and (23) leads to fluctuations in $N(t)$ and $\zeta(t)$ associated with fluctuations in the arrow of time (Zeitzitter) and a violation of unitarity.

(3) It is worth noting that the product $N^2(1 - |\vec{\zeta}|^2)$ is equal to $e^{-(\gamma_L + \gamma_S)t}$ and therefore monotonic for all values of δ, r, and μ. This product is just 4 times the determinant of the density matrix $\rho(t)$.

(4) The time dependence of $\zeta_3(t)$ describes the evolution of strangeness in a beam that is initially an equal mixture of K^0 and $\bar{K}^0$. It is an example of flavor genesis induced by *CP* violation in a decaying system.

(5) The emergence of a nonzero three-dimensional Stokes vector $\vec{\zeta}(t)$ from a state that is initially amorphous ($\vec{\zeta}(0) = 0$) is suggestive of a phase transition. The evolution of the Stokes vector from zero to unit length is reminiscent of an order parameter for a system undergoing spontaneous symmetry breaking.

(6) All our considerations have been in the framework of ordinary quantum mechanics and *CPT* invariance. Discussions that involve violation of quantum mechanics and/or *CPT* symmetry may be found, for example, in [8]. An early discussion of the arrow of time in connection with K meson decays is given in [9]. Broader issues connected with the arrow of time are discussed, for instance, in [10]. Finally, experimental investigations of discrete symmetries in the decays of K mesons and B mesons produced in e^+e^- or $p\bar{p}$ collisions are described in [11].

ACKNOWLEDGMENTS

[1] U. Fano, Rev. Mod. Phys. **29**, 74 (1957).

[2] R. G. Sachs, *Physics of Time Reversal* (University of Chicago Press, Chicago, 1987).

[3] L. M. Sehgal, Tata Institute Report, TIFR-TH-70-35, 1970.

[4] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

[5] L. M. Sehgal, "Decays of Neutral K Mesons Produced in $p\bar{p}$ Annihilation. A Comment," Aachen Report, 1973 (unpublished).

[6] T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, B1490 (1965).

[7] J. S. Bell and J. Steinberger, in Proceedings of the Oxford International Conference on Elementary Particles, 1965, pp. 195–222.

[8] P. Huet and M. E. Peskin, Nucl. Phys. **B434**, 3 (1995); J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, Phys. Lett. B **293**, 142 (1992).

[9] A. Aharony, Ann. Phys. (N.Y.) **67**, 1 (1971); **68**, 163 (1971); A. Aharony and Y. Ne'eman, Int. J. Theor. Phys. **3**, 437 (1970).

[10] *Physical Origins of Time Asymmetry*, edited by Jose Angel Sanchez Asiain *et al.* (Cambridge University Press, Cambridge, England, 1996).

[11] F. Ambrosino *et al.* (KLOE Collaboration), Phys. Lett. B **642**, 315 (2006); B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **96**, 251802 (2006); E. Nakano *et al.* (Belle Collaboration), Phys. Rev. D **73**, 112002 (2006); M. Carrol *et al.* (CPLEAR Collaboration), Nucl. Phys. **A626**, 157c (1997); K. Kleinknecht, *Uncovering CP Violation* (Springer-Verlag, Berlin, 2003).