Z boson propagator correction in technicolor theories with extended technicolor effects included

Masafumi Kurachi,[1] Robert Shrock,[1] and Koichi Yamawaki[2]

[1]*C.N. Yang Institute for Theoretical Physics State University of New York, Stony Brook, New York 11794, USA*
[2]*Department of Physics Nagoya University, Nagoya 464-8602, Japan*

We calculate the *Z* boson propagator correction, as described by the *S* parameter, in technicolor theories with extended technicolor interactions included. Our method is to solve the Bethe-Salpeter equation for the requisite current-current correlation functions. Our results suggest that the inclusion of extended technicolor interactions has a relatively small effect on *S*.

I. INTRODUCTION

The origin of electroweak symmetry breaking (EWSB) is an outstanding unsolved question in particle physics. An interesting possibility is that electroweak symmetry breaking is driven by an asymptotically free, vectorial non-Abelian gauge interaction, technicolor (TC) [1], with a coupling that becomes strong at the electroweak scale. The EWSB is produced by the formation of bilinear condensates of technifermions. To communicate this symmetry breaking to the standard-model fermions (which are technisinglet), one embeds technicolor in a larger, extended technicolor (ETC) theory [2]. In order to account for the generational structure of the standard-model (SM) fermion masses, the ETC gauge symmetry is envisioned to break sequentially in stages at the respective mass scales Λ_j, $j = 1, 2, 3$, corresponding to these generations, finally yielding the technicolor gauge symmetry as an exact, unbroken subgroup. Because these scales enter as inverse powers in the resultant expressions for quark and lepton masses, the highest ETC symmetry-breaking scale, Λ_1, corresponds to the first generation, and so forth for the others. The scales Λ_j and the corresponding masses of the ETC gauge bosons must be large in order to satisfy constraints from flavor-changing neutral current processes. In current, reasonably ultraviolet-complete, ETC models, these ETC breaking scales are $\Lambda_1 \sim 10^3$ TeV, $\Lambda_2 \sim$ 50–100 TeV, and $\Lambda_3 \sim$ few TeV [3–6]. Modern technicolor theories feature a large but slowly running ("walking") technicolor gauge coupling, g_{TC} [7–19]. While some early studies of walking assumed an ultraviolet (UV) fixed point in models with U(1) gauge symmetry, modern walking technicolor theories are based on the fact that walking can result naturally from the presence of an approximate infrared (IR) fixed point in the TC renormalization-group equations of the non-Abelian technicolor gauge theory. This occurs at a value $\alpha = \alpha_*$ [where $\alpha = g_{TC}^2/(4\pi)$] which is close to, but slightly larger than, the minimal value α_{cr} for which the technifermion condensates form. An important property of a technicolor theory with walking behavior is that the anomalous dimension of the bilinear technifermion operator is $\gamma_{\bar{\psi}\psi} \simeq 1$, so that the

momentum-dependent dynamical technifermion mass $\Sigma(p)$ falls off as p^{-1} rather than p^{-2} for an extended range of Euclidean momenta p. This produces the requisite strong enhancement of SM fermion masses, relative to the values that they would have in a quantum chromodynamics (QCD)-like (nonwalking) theory, which is needed in order to fit experiment. In the pure technicolor theory, the chiral symmetry breaking occurs if $\alpha C_2(R)$ exceeds a number of order unity, where $C_2(R)$ denotes the quadratic Casimir invariant for the technifermions, which transform according to the representation R of the TC gauge group.

Technicolor theories are severely constrained by the corrections that they induce in precision electroweak quantities, in particular, corrections to the *Z* and *W* boson propagators, conveniently represented by the S, $T = \Delta\rho/\alpha_{em}(m_Z)$, and U [20] or equivalent [21] parameters, where $\rho = m_W^2/(m_Z^2\cos^2\theta_W)$ and $\Delta\rho$ is the deviation of ρ from unity due to new physics beyond the standard model. Experimentally allowed regions in these parameters are given in Ref. [22]. Since the $SU(2)_L \times U(1)_Y$ gauge couplings are small at the TeV scale, the condensates of $T_3 = 1/2$ and $T_3 = -1/2$ technifermions can naturally be approximately equal, so that technicolor contributions to $\Delta\rho$ are small (e.g., [23]). Further contributions to $\Delta\rho$ may arise from ETC effects, as discussed below. From the point of view of low-energy effective field theory, since the technicolor sector arises as the low-energy residue of the ETC theory, a reasonable first approximation for calculating *S* is to consider the technicolor theory by itself, without any higher-dimension operators arising from ETC interactions. Calculations of this type have been performed in QCD-like and walking technicolor theories [20,24–34] (see also [35,36]). In particular, it has been found that technicolor contributions to *S* may be suppressed in a TC theory with walking behavior [25–35].

The question then arises as to what influence the exchange of strongly coupled massive ETC gauge bosons, and the resultant effective local four-fermion current-current interaction in the technicolor theory, have on the technicolor correction to the *Z* boson propagator, as described by the *S* parameter. Since the ETC gauge coupling is strong at the TeV mass scale, the exchange of the lightest

massive ETC vector bosons generates four-fermion operators that could, *a priori*, have a significant effect on the chiral symmetry breaking in the technicolor theory and on the approximate IR fixed point.

Accordingly, in this paper we study the effects of ETC-induced four-fermion operators on S. As a first step, we map out the chiral phase boundary via the solution of a Schwinger-Dyson equation for the technifermion propagator, including both massless technigluon exchange and the leading massive ETC gauge boson exchange. In the context of a walking technicolor model, we then solve a Bethe-Salpeter equation for the requisite derivative of the current-current correlation function that yields S [cf. Equation (4.7) below].

The effects of various types of four-fermion operators on dynamical chiral symmetry breaking have been studied in many contexts in the past. In a pioneering work, Nambu and Jona-Lasinio (NJL) showed that a four-fermion operator with a sufficiently strong coupling can induce dynamical chiral symmetry breaking [37]. An early approach to nonperturbative generation of fermion masses starting with a massless fermion in electrodynamics (QED) was Ref. [38]. Using large-N methods, Ref. [39] showed that a certain four-fermion operator in a $(1 + 1)$-dimensional model produced dynamical chiral symmetry breaking. In QCD with $N_f = 2$ massless quark flavors, the effective instanton-induced operator is a four-fermion operator [40], and this was shown to contribute importantly to the formation of a bilinear quark condensate and associated dynamical chiral symmetry breaking [41]. There have also been studies of the chiral phase transition in both Abelian and non-Abelian gauge theories with various types of four-fermion operators [42–58]. We note that the four-fermion operators that we consider are ETC-induced and differ from four-fermion operators directly involving top quark condensates, such as appear in topcolor models.

The paper is organized as follows. Section II is devoted to a review of some pertinent material on technicolor and extended technicolor theories. In Sec. III we discuss the calculation of the dynamical technifermion mass via the solution of the Schwinger-Dyson equation and the mapping of the chiral phase boundary. Section IV contains the equations expressing S in terms of current-current correlation functions. In Sec. V we discuss our solution of the Bethe-Salpeter equation and our results for S. Section VI contains our conclusions. In an Appendix we comment on the similarities and differences between the ETC-induced four-fermion interaction that we study and the four-fermion interaction used in the Nambu-Jona Lasino model and its gauged extensions.

II. SOME PROPERTIES OF TECHNICOLOR AND EXTENDED TECHNICOLOR THEORIES

In this section we discuss some properties of the technicolor and extended technicolor theories that will be used in our present study. We begin with the pure technicolor sector and then include ETC. The technicolor theory is a vectorial, asymptotically free gauge theory with a gauge group that we take to be $SU(N_{\text{TC}})$, with gauge coupling g_{TC}. We choose $N_{\text{TC}} = 2$, as in recent TC/ETC model building [3–6], for several reasons, including the fact that this choice (i) minimizes technicolor contributions to the S parameter as compared with larger values of N_{TC}, (ii) can naturally produce a walking technicolor theory in a one-family model, and (iii) makes possible a mechanism to explain light neutrino masses [4]. To discuss the features of this theory, we shall revert to general N_{TC} for some formulas. The theory contains N_f massless Dirac technifermions, and we assume that these transform according to the fundamental representation of $SU(N_{\text{TC}})$ [59]. The renormalization group (RG) equation for the running technicolor gauge coupling squared, $\alpha(\mu) \equiv \alpha_{\text{TC}}(\mu)$, is

$$\beta = \mu \frac{d\alpha(\mu)}{d\mu} = -\frac{\alpha(\mu)^2}{2\pi}\left(b_0 + \frac{b_1}{4\pi}\alpha(\mu) + O(\alpha(\mu)^2)\right),$$
(2.1)

where μ is the momentum scale, and b_0 and b_1 are known coefficients. The two terms listed are scheme independent. The next two higher-order terms have also been calculated but are scheme dependent; their inclusion does not significantly affect our results. Since the technicolor theory is asymptotically free, $b_0 > 0$. For sufficiently large N_f, $b_1 < 0$, so that the technicolor beta function has a second zero (approximate infrared fixed point of the renormalization group) at a certain α_*, given, to this order, by $\alpha_* = -4\pi b_0/b_1$. As the number of technifermions, N_f, increases, α_* decreases. In a walking technicolor theory, one arranges so that α_* is slightly greater than the critical value, α_{cr}, for the formation of the bilinear technifermion condensate. As N_f increases toward $N_{f,\text{cr}}$, α_* decreases toward α_{cr} [60]. In the one-gluon exchange approximation, the Schwinger-Dyson equation for the inverse propagator of a technifermion transforming according to the representation R of the technicolor gauge group yields a nonzero solution for the dynamically generated fermion mass (which is an order parameter for spontaneous chiral symmetry breaking) if $\alpha \gtrsim \alpha_{\text{cr}}$, where α_{cr} is given by

$$\frac{3\alpha_{\text{cr}}C_2(R)}{\pi} = 1.$$
(2.2)

For the case at hand, where the technifermion transforms according to the fundamental representation of $SU(N_{\text{TC}})$, this is $C_2(\text{fund}) \equiv C_{2F} = (N_{\text{TC}}^2 - 1)/(2N_{\text{TC}})$, so that, with $N_{\text{TC}} = 2$, $\alpha_{\text{cr}} \simeq 1.4$. To estimate $N_{f,\text{cr}}$, one solves the equation $\alpha_* = \alpha_{\text{cr}}$, yielding the result [14,19] $N_{f,\text{cr}} = 2N_{\text{TC}}(50N_{\text{TC}}^2 - 33)/[5(5N_{\text{TC}}^2 - 3)]$. For $N_{\text{TC}} = 2$ this gives $N_{f,\text{cr}} \simeq 7.9$. This estimate is clearly rough, in view of the strongly coupled nature of the physics. Moreover, the coupling α_* is only an approximate IR fixed point of

the renormalization group, since the technifermions gain dynamical masses Σ and are integrated out in the effective field theory for energies below Σ, where the technicolor beta function consequently has the form for a pure gauge theory. Effects of higher-order gluon exchanges have been studied in [12]. From earlier studies in QCD, it is known that instantons (which are not directly included in the above-mentioned Schwinger-Dyson equation) contribute to the formation of a bilinear fermion condensate in a vectorial gauge theory [41]. The effect of instantons on the (zero-temperature) chiral transition in a vectorial gauge theory as a function of N_f has been studied in Refs. [17,18]. In principle, lattice gauge simulations should provide a way to determine $N_{f,\text{cr}}$, but the groups that have studied this have not reached a consensus [61]. Thus, we shall use the value $N_{f,\text{cr}} \simeq 8$ for $N_{\text{TC}} = 2$ but note that there is some uncertainty in the determination of this number, as is expected in view of the strongly coupled nature of the physics.

We shall focus on technicolor models in which the technifermions comprise one family with respect to the standard-model gauge group, $G_{\text{SM}} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ (e.g., [43]). In such a one-family technicolor model, the technifermions transform as

$$Q_L: (N_{\text{TC}}, 3, 2)_{1/3,L} \qquad U_R: (N_{\text{TC}}, 3, 1)_{4/3,R}$$
$$D_R: (N_{\text{TC}}, 3, 1)_{-2/3,R} \qquad L_L: (N_{\text{TC}}, 1, 2)_{-1,L} \qquad (2.3)$$
$$N_R: (N_{\text{TC}}, 1, 1)_{0,R} \qquad E_R: (N_{\text{TC}}, 1, 1)_{-2,R},$$

where the numbers in the parentheses refer to the representations of $\text{SU}(N_{\text{TC}}) \times \text{SU}(3)_c \times \text{SU}(2)_L$ and the subscripts refer to weak hypercharge, Y. Hence, with $N_w = 2$, this type of model contains $N_f = N_w(N_c + 1) = 8$ technifermions. Given the above-mentioned fact that $N_{\text{TC}} = 2 \Rightarrow N_{f,\text{cr}} \simeq 8$, it follows that, to within the accuracy of the two-loop beta function analysis, this technicolor model can naturally exhibit walking behavior. Since there are

$$N_D = (N_c + 1) \qquad (2.4)$$

$\text{SU}(2)_L$ doublets for each technicolor index, the total number of electroweak doublets, $N_{D,\text{tot}}$, of technifermions is

$$N_{D,\text{tot}} = N_D \dim(R) = (N_c + 1)N_{\text{TC}} = 8, \qquad (2.5)$$

where R denotes the technifermion representation of $\text{SU}(N_{\text{TC}})$, with $R = $ fund here.

We next discuss the embedding of this technicolor theory in extended technicolor. Although much of our analysis of chiral symmetry breaking and the S parameter is rather general, it will be useful to have a specific class of ETC models in mind as a theoretical framework. A natural formulation of ETC gauges the fermion generational index and combines it with the technicolor gauge index, so that, for an $\text{SU}(N_{\text{ETC}})$ gauge group, one has

$$N_{\text{ETC}} = N_{\text{gen}} + N_{\text{TC}} = 3 + N_{\text{TC}}. \qquad (2.6)$$

With $N_{\text{TC}} = 2$, this then leads to SU(5) for the ETC gauge group. The SM fermions and corresponding technifermions transform according to the representations

$$Q_L: (5, 3, 2)_{1/3,L}, \qquad u_R: (5, 3, 1)_{4/3,R},$$
$$d_R: (5, 3, 1)_{-2/3,R} \qquad (2.7)$$

and

$$L_L: (5, 1, 2)_{-1,L}, \qquad e_R: (5, 1, 1)_{-2,R}, \qquad (2.8)$$

where the subscripts denote Y. For example, writing out the components of e_R, one has $e_R \equiv (e^1, e^2, e^3, e^4, e^5)_R \equiv (e, \mu, \tau, E^4, E^5)_R$, where the last two entries are the charged technileptons. There are also SM-singlet, ETC-nonsinglet fields in various representations of $\text{SU}(5)_{\text{ETC}}$ such that the overall ETC theory is a chiral gauge theory. The right-handed, SM-singlet, neutrino fields and corresponding technineutrinos arise as certain components of these SM-singlet, ETC representations, as discussed in Refs. [3–6]. In this type of ETC theory, because the SM fermions and the technifermions in each of the ETC multiplets (2.7) and (2.8) transform in the same way under the SM gauge group G_{SM}, it follows that the ETC gauge bosons are SM singlets, and $[G_{\text{SM}}, G_{\text{ETC}}] = 0$. It may be noted that one could also consider a technicolor theory with a single electroweak doublet of technifermions, so that $N_{D,\text{tot}} = N_{\text{TC}}$. Although this model, by itself, does not exhibit walking behavior, this can be achieved by adding the requisite number of SM-singlet technifermions, as, e.g., in [62]. To avoid electroweak gauge anomalies, the technifermion electroweak doublet must have weak hypercharge $Y = 0$, and consequently, the ETC gauge bosons that transform SM fermions to these technifermions carry hypercharge and charge (and, for some, also color), so that $[G_{\text{SM}}, G_{\text{ETC}}] \neq 0$. Consequently, an analysis of electroweak corrections in this type of theory is more complicated than the analysis in a model based on one-family technicolor, and we do not pursue this here.

The overall ETC theory is constructed to be asymptotically free, so that as the energy scale decreases from large values, the ETC gauge coupling grows, and produces various bilinear condensates. Since the ETC theory is chiral, these condensates generically break the ETC gauge symmetry, and this can be arranged to occur in stages. Explicit, reasonably ultraviolet-complete ETC models of this type were studied in Refs. [3–6]. The ETC gauge bosons may be denoted V_j^i, where $1 \leq i$, $j \leq 5$. It is convenient to use the notation V_{dj}, $j = 1, 2, 3$ for the ETC gauge bosons corresponding to the diagonal Cartan generators T_{dj}. For the sake of generality, we display the T_{dj} for arbitrary N_{TC}. With the canonical normalization $\text{Tr}(T_iT_j) = (1/2)\delta_{ij}$ and in our basis with ETC indices ordered as $i = 1, 2, 3$ for generations and $i = \tau = 4, 5, \ldots, N_{\text{ETC}}$ for technicolor, these Cartan generators are

$$T_{d1} = \nu_1 \, \text{diag}(-(N_{\text{TC}} + 2), 1, 1, \{1\}), \qquad (2.9)$$

$$T_{d2} = \nu_2 \, \text{diag}(0, -(N_{\text{TC}} + 1), 1, \{1\}), \qquad (2.10)$$

and

$$T_{d3} = \nu_3 \, \text{diag}(0, 0, -N_{\text{TC}}, \{1\}), \qquad (2.11)$$

where $\{1\}$ denotes a string of N_{TC} 1's,

$$\nu_1 = [2(N_{\text{TC}} + 2)(N_{\text{TC}} + 3)]^{-1/2}, \qquad (2.12)$$

$$\nu_2 = [2(N_{\text{TC}} + 1)(N_{\text{TC}} + 2)]^{-1/2}, \qquad (2.13)$$

and, most importantly for our purposes,

$$\nu_3 = [2N_{\text{TC}}(N_{\text{TC}} + 1)]^{-1/2}. \qquad (2.14)$$

In particular, $\nu_3 = 1/(2\sqrt{3})$ for $N_{\text{TC}} = 2$.

At the scale $\Lambda_1 \simeq 10^3$ TeV, the $SU(5)_{\text{ETC}}$ gauge symmetry is envisioned to break to $SU(4)_{\text{ETC}}$, with the nine ETC gauge bosons in the coset $SU(5)_{\text{ETC}}/SU(4)_{\text{ETC}}$, V_j^1, $V_1^j = (V_j^1)^\dagger$, $2 \leq j \leq 5$, and V_{d1}, picking up masses $M_1 \simeq \Lambda_1$. Similarly, at the scale $\Lambda_2 \simeq 50$–100 TeV, $SU(4)_{\text{ETC}}$ breaks to $SU(3)_{\text{ETC}}$, with the seven ETC gauge bosons in the coset $SU(4)_{\text{ETC}}/SU(3)_{\text{ETC}}$, V_j^2, $V_2^j = (V_j^2)^\dagger$, $3 \leq j \leq 5$, and V_{d2}, picking up masses $M_2 \simeq \Lambda_2$. Finally, at the scale $\Lambda_3 \simeq$ few TeV, $SU(3)_{\text{ETC}}$ breaks to the residual exact technicolor gauge group $SU(2)_{\text{TC}}$, and the five ETC gauge bosons in the coset $SU(3)_{\text{ETC}}/SU(2)_{\text{TC}}$, V_j^3, $V_3^j = (V_j^2)^\dagger$, $j = 4, 5$, and V_{d3} gain masses $M_3 \simeq \Lambda_3$. Henceforth, we shall denote technicolor indices as $\tau = 4, 5$ to distinguish them from generational indices $i = 1, 2, 3$. In principle, other strongly coupled gauge symmetries such as topcolor might also be present, but we will not consider these here.

The most important ETC contributions to the technicolor sector arise from the exchange of the lowest-lying massive ETC gauge bosons, namely, those with mass M_3. Exchanges of more massive ETC gauge bosons make contributions that are strongly suppressed by factors $M_3^2/M_j^2 \ll 1$, where $j = 1, 2$. In the technicolor theory, as a low-energy effective field theory, the exchange of massive ETC gauge bosons produce local four-fermion operators of the current-current form. There are two types of corrections to the propagator of a technifermion due to the emission and reabsorption of virtual ETC gauge bosons of mass M_3, namely, those involving (i) V_{d3} and (ii) V_3^τ. Thus a one-loop technifermion propagator correction involving the exchange (i) has the same technifermion on the internal fermion line, while that involving the exchange (ii) has the corresponding third-generation SM fermion on the internal line. In a viable ETC model, the exchange (ii) must make a smaller contribution to the dynamical mass Σ_F of a technifermion F than the exchange (i) because it violates custodial symmetry. This is a consequence of the fact that the emission and reabsorption of a virtual V_3^τ yields a one-loop diagram in which a U techniquark transforms to a

virtual t quark and back, while a D techniquark transforms to a b quark and back. This correction to the dynamical mass Σ of the techniquark therefore introduces a dependence on m_t for U and m_b for D, so Σ_U would, in general, differ significantly from Σ_D, violating custodial symmetry in the technicolor sector.

Such violations must be small. Global fits to data yield allowed regions in (S, T) depending on a reference value of the SM Higgs mass, $m_{H,\text{ref}}$. The comparison of these with a technicolor theory is complicated by the fact that technicolor has no fundamental Higgs field. Sometimes one formally uses $m_{H,\text{ref}} \sim 1$ TeV for a rough estimate, since the standard model with $m_H \sim 1$ TeV has strong longitudinal vector boson scattering, as does technicolor. However, this may involve some double counting when one also includes contributions to S from technifermions, whose interactions and bound states (e.g., technivector mesons) are responsible for the strong scattering in the $W_L^+ W_L^-$ and other longitudinal vector-vector channels in a technicolor framework. The current allowed region in (S, T) disfavors values of $S \gtrsim 0.1$ and $T \gtrsim 0.4$ [22].

Because violations of custodial symmetry in the technicolor sector must be small for the theory to be viable, we shall focus on the corrections of type (i). This constraint also implies that $|\Sigma_U - \Sigma_D|/(\Sigma_U + \Sigma_D) \ll 1$, so we shall drop the subscripts on Σ_U and Σ_D. Indeed, since all SM interactions are small at the scale of technicolor mass generation, it is expected that the dynamical masses for all of the technifermions, Σ_U, Σ_D, Σ_N, and Σ_E, are approximately equal, and we shall therefore simply denote them as Σ.

In the ETC framework that we use, the V_{d3} exchange yields an effective local operator in the technicolor theory of the form

$$\mathcal{L}_{V_{d3}} = -\frac{\nu_3^2 g_{\text{ETC}}^2}{M_3^2} \sum_\psi \left[\sum_\tau \bar{\psi}_\tau \gamma_\mu \psi^\tau \right] \left[\sum_{\tau'} \bar{\psi}_{\tau'} \gamma^\mu \psi^{\tau'} \right],$$

$$(2.15)$$

where $\sum_\psi$ is over the technifermions ψ in the theory, and $\sum_\tau$ is over the technicolor indices. As is evident from Eqs. (2.14) and (2.15), the dimensionless coefficient $\nu_3 g_{\text{ETC}}^2$ is not independent of the technicolor theory and its coupling, g_{TC}, since these arise from the sequential breaking of the ETC theory. However, from an abstract field-theoretic point of view, it is of interest to investigate the (zero-temperature) chiral transition of an asymptotically free vectorial gauge theory in the presence of a four-fermion operator with a coupling that can be varied independently of the gauge coupling. In this context, one could map out the chiral phase boundary as a function of these two, *a priori* independent, dimensionless couplings. If, indeed, one really posited a four-fermion operator as a fundamental interaction in the theory, it would change the renormalization-group behavior of the couplings. However, in the physical ETC context, in which this

four-fermion operator is the low-energy remnant of the ETC theory, its effects on the technicolor gauge coupling can be considered to have already been taken into account at the higher scale where the ETC gauge degrees of freedom are dynamical, since the technicolor sector emerges as the low-energy effective field theory from this larger ETC theory.

By formally varying M_3 from zero to its physical nonzero value, and noting that the sign of the resultant contribution to the fermion propagator does not change, one infers that the effect of the ETC interaction is to enhance the spontaneous chiral symmetry breaking. Indeed, the NJL model showed that a four-fermion operator can induce dynamical chiral symmetry breaking by itself if its coupling strength is sufficiently great [37]. Note that ETC-induced four-fermion operator (2.15) is a product of two vector currents and is different from the four-fermion operator that appears in the gauged NJL model, namely, a sum of the form $VV - AA$, where V and A denote vector and axial-vector currents.

III. SCHWINGER-DYSON EQUATION AND MAPPING OF PHASE BOUNDARY

The first step in our analysis is the calculation of the dynamical technifermion mass and the mapping of the associated chiral phase boundary in the theory. Some early studies using Schwinger-Dyson equations and related methods to study nonperturbative fermion mass generation in Abelian and non-Abelian gauge theories are Refs. [38,63]. Past work on the (zero-temperature) chiral transition for Abelian and non-Abelian gauge models in the presence of four-fermion operator of various types is contained in Refs. [45–51]. Several of these studies used models with a (nonasymptotically free) U(1) gauge symmetry. In these studies it was assumed that the theory had a UV fixed point at $\alpha_{\text{cr}}^{(0)}$. This contrasts with modern TC/ETC models, in which the TC sector arises as the low-energy limit of an ETC theory, and both are based on asymptotically free, non-Abelian gauge symmetries, with $\alpha_{\text{cr}}^{(0)}$ being an approximate IR fixed point of the TC theory.

Our method is to use the Schwinger-Dyson (SD) equation for the technifermion propagator to calculate the dynamically induced mass Σ, taking into account the exchange of both massless technigluons and the massive V_{d3} ETC vector bosons. The dynamical technifermion mass serves as an order parameter for the chiral transition. This mass may be defined in terms of the momentum-dependent fermion mass evaluated at an appropriate Euclidean reference momentum. One choice would be $\Sigma = \Sigma(p_E = 0)$, but we shall actually use the related choice

$$\Sigma = \Sigma(p_E = \Sigma). \tag{3.1}$$

We recall that the analysis of the Schwinger-Dyson equation, by itself, does not give direct information about

whether or not the theory has confinement. Indeed, the original NJL model provides an example of dynamical chiral symmetry breaking via a four-fermion operator (with sufficiently large coupling of the correct sign) without confinement. In the pure gauge theory, for $N_f < N_{f,\text{cr}}$, i.e., for $\alpha_* > \alpha_{\text{cr}}^{(0)}$, one can argue persuasively that (although the area law behavior of the Wilson loop ceases to hold because of string breaking) the theory exhibits confinement, by a formal analytic continuation in N_f from $N_f = 0$ [60]. This is all that we will need for our present purposes. In the pure gauge theory, the chirally symmetric phase is expected to be a non-Abelian deconfined Coulombic phase for sufficiently weak coupling. If appropriate conditions were satisfied, there might possibly also be an intermediate phase with confinement but no spontaneous chiral symmetry breaking; the presence or absence of this phase will not be important for our work here. Since for physical reasons, viz. the necessity of a nonvanishing technifermion condensate for electroweak symmetry breaking, we must be in the confined phase.

The full inverse technifermion propagator can be written as

$$S_F(p)^{-1} = A(p^2)\not{p} - B(p^2). \tag{3.2}$$

The resultant SD equation has the form

$$S_f(p)^{-1} - \not{p} = I_{\text{TC}} + I_{\text{ETC}}, \tag{3.3}$$

where I_{TC} and I_{ETC} are the contributions from technigluons and V_{d3} ETC vector bosons, respectively. A graphical representation of these contributions is shown in Fig. 1. The first graph on the right is the (massless) technigluon exchange diagram, and the second represents the contribution of the effective local four-fermion operator resulting from the exchange of the massive ETC gauge boson V_{d3}. The dark blobs on the technifermion line signify that we use the full technifermion propagator. The first term in Eq. (3.3) is given by

$$I_{\text{TC}} = -C_{2F} \int \frac{d^4 q}{i(2\pi)^4} g_{\text{TC}}^2(p, q) D_{\mu\nu}^{\text{TC}}(p - q) \gamma^\mu S_F(q) \gamma^\nu, \tag{3.4}$$

where the technigluon propagator is

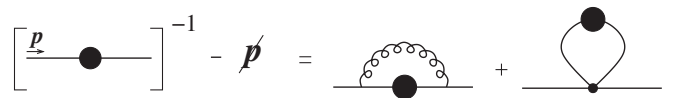$$D_{\mu\nu}^{\text{TC}}(k) = \frac{N_{\mu\nu}}{k^2} \tag{3.5}$$

FIG. 1. Pictorial representation of the Schwinger-Dyson equation for the technifermion. The first graph on the right represents technigluon exchange, and the second represents the contribution of the effective local four-fermion operator resulting from the exchange of the massive ETC gauge boson V_{d3}.

with numerator (depending on the gauge parameter ξ)

$$N_{\mu\nu} = -g_{\mu\nu} + \xi \frac{k_\mu k_\nu}{k^2}. \tag{3.6}$$

Our analysis is thus an improved ladder approximation, where the term ''improved'' refers to the fact that in Eq. (3.4) we take account of the momentum dependence of the running technicolor coupling, g_{TC}. We next explain several further approximations that will be made. In the pure technicolor theory, if N_f were greater than $N_{f,cr}$, i.e., if α_* were less than α_{cr}, so that this IR fixed point of the two-loop RG equation were exact, then one could make use of an exact solution to this equation in the entire energy region. With $b \equiv b_0/(2\pi)$, this solution is given by [64,65]

$$\alpha(\mu) = \alpha_*[W(e^{-1}(\mu/\Lambda)^{b\alpha_*}) + 1]^{-1}, \tag{3.7}$$

where $W(x) = F^{-1}(x)$, with $F(x) = xe^x$, is the Lambert W function, and Λ is a RG-invariant scale defined by [14]

$$\Lambda \equiv \mu \exp\left[-\frac{1}{b}\left\{\frac{1}{\alpha_*}\ln\left(\frac{\alpha_* - \alpha(\mu)}{\alpha(\mu)}\right) + \frac{1}{\alpha(\mu)}\right\}\right]. \tag{3.8}$$

However, since physically one must be in the phase where $\alpha > \alpha_{cr}$ so that the technifermion condensate forms and the electroweak symmetry is broken, the IR fixed point is only approximate rather than exact, as noted above. Hence, Eq. (3.7) is only applicable in an approximate manner to our case; for momenta much less than the dynamical fermion mass Σ, the fermions decouple, and in this very low-momentum region, with the fermions integrated out, the resultant α would increase above the value α_* at the approximate IR fixed point. But since $\Sigma \ll \Lambda$ in a walking theory, it follows that this lowest range of momenta makes a relatively small contribution to the integrals to be evaluated in our calculations. Hence, over most of the integration range for these integrals where the coupling α is large, it is approximately constant and equal to its fixed-point value, α_*. This means that one can use the approximation

$$\alpha(\mu) = \alpha_*\theta(\Lambda - \mu), \tag{3.9}$$

where θ is the step function. Moreover, we shall assume that

$$g_{TC}(p, q) = g_{TC}((p - q)^2) = g_{TC}(p_E^2 + q_E^2). \tag{3.10}$$

Since g_{TC} would naturally depend on the technigluon momentum squared, $(p - q)^2 = p^2 + q^2 - 2p \cdot q$, the functional form (3.10) amounts to dropping the scalar product term, $-2p \cdot q$. This is a particularly reasonable approximation in the case of a walking gauge theory because most of the contribution to the integral (3.4) comes from a region of Euclidean momenta where α is nearly constant. Hence, the shift upward or downward due to the $-2p \cdot q$ term in the argument of α has very little effect on the value of this coupling for the range of momenta that make the most important contribution to the integral. The

approximations (3.9) and (3.10) are the same as in our previous work [32,33,66,67].

Making a Euclidean rotation and performing the angular integration in I_{TC} then yields two equations, for $A(p_E^2)$ and $B(p_E^2)$. In Landau gauge, with gauge parameter $\xi = 1$, the solution to the equation for $A(p_E^2)$ is $A(p_E^2) = 1$, so that the dynamical mass of the technifermion, $\Sigma(p_E^2) = B(p_E^2)/A(p_E^2)$, takes the simple form $\Sigma(p_E^2) = B(p_E^2)$. This simplification motivates the use of Landau gauge, although physical results involving Σ are, of course, invariant under technicolor gauge transformations (e.g., [12,56]). For the integral I_{TC}, setting $x \equiv p_E^2$ and $y \equiv q_E^2$, we obtain

$$I_{TC} = \frac{3C_{2F}}{16\pi^2} \int_0^\infty y dy \frac{g_{TC}^2(x + y)\Sigma(y)}{\max(x, y)[y + \Sigma^2(y)]}. \tag{3.11}$$

Note that, although the upper limit on the integration is formally infinite, the integral is actually cut off at $y \simeq \Lambda^2$ because of Eq. (3.9). If the technigluon exchange were the only contribution in Eq. (3.3), then this equation would have a nonzero solution for $\Sigma(p_E^2)$ if $\alpha_{TC} > \alpha_{cr}^{(0)}$, where $\alpha_{cr}^{(0)}$ was given by Eq. (2.2). In our calculations, we consider the full nonlinear integral Eq. (3.11). However, for comparison with our numerical results, we recall in the pure technicolor theory (without the ETC-induced four-fermion interaction) if, as $\alpha_* \searrow \alpha_{cr}^{(0)}$, one neglects the momentum dependence of $\Sigma(y)$ in the denominator of Eq. (3.11), then the solution is [14–19]

$$\Sigma = \text{const}\Lambda \exp\left[-\pi\left(\frac{\alpha_*}{\alpha_{cr}} - 1\right)^{-1/2}\right]. \tag{3.12}$$

A numerical solution of the Schwinger-Dyson equation in a non-Abelian gauge theory found a rather similar result, $\Sigma \propto \Lambda \exp[-0.82\pi(\alpha_*/\alpha_{cr} - 1)^{-1/2}]$ [19]. A similar numerical solution of the Schwinger-Dyson equation for a range of values of α slightly larger than $\alpha_{cr}^{(0)}$ obtained results which were fit with the form (3.12) [66].

We next discuss the leading ETC contribution to Eq. (3.3), I_{ETC}. We again shall introduce some physically motivated approximations to simplify the calculation. In models with explicit specification of ETC dynamics (e.g., [3–6]), one finds that the walking regime typically extends from the TC scale to the lowest ETC symmetry-breaking scale, Λ_3. Hence, as far as the technicolor theory is concerned, the scale parameter Λ in Eq. (3.8) is of order Λ_3. At momentum scales $\mu \gtrsim \Lambda_3$, one is dealing with the full $SU(3)_{ETC}$ gauge interaction [and so forth on up to $SU(5)_{ETC}$ for $\mu \gtrsim \Lambda_1$]. Although the dynamical gauge degrees of freedom in the coset $SU(3)_{ETC}/SU(2)_{TC}$ are frozen out for momenta $\mu \lesssim \Lambda_3$, it will be convenient to use the Landau-gauge form of the ETC gauge boson propagator for our calculation, so that

$$I_{\text{ETC}} = -\nu_3^2 \int \frac{d^4q}{i(2\pi)^4} g_{\text{ETC}}^2(p,q) D_{\mu\nu}^{\text{ETC}}(p-q)\gamma^\mu S_F(q)\gamma^\nu,$$

(3.13)

where

$$D_{\mu\nu}^{\text{ETC}}(k) = \frac{N_{\mu\nu}}{k^2 - M_3^2}.$$

(3.14)

This choice maintains $A(p_E^2) = 1$. We can then combine the TC and ETC terms as $I = I_{\text{TC}} + I_{\text{ETC}}$, with

$$I = -\int \frac{d^4q}{i(2\pi)^4} \frac{\kappa(p,q)}{(p-q)^2} N_{\mu\nu}\gamma^\mu S_F(q)\gamma^\nu,$$

(3.15)

where

$$\kappa(p,q) = C_{2F} g_{\text{TC}}^2(p,q) + \nu_3^2 g_{\text{ETC}}^2(p,q)$$
$$\times \left[\frac{(p-q)^2}{(p-q)^2 - M_3^2}\right].$$

(3.16)

Since the dominant contribution to the momentum integration in I is from scales smaller than M_3, the momentum-dependent term in the denominator of the second term (3.8) is dropped relative to M_3^2. Since, as noted, the TC sector is the low-energy effective theory arising from ETC, the gauge couplings are closely related. As noted, in explicit ETC models, the walking regime for the TC theory typically extends up to the scale Λ_3 where the ETC symmetry breaks to the TC symmetry. Thus, the approximation (3.9) for $g_{\text{TC}}(\mu)$ means that the Euclidean momentum integration in the integrals is cut off at $\simeq \Lambda_3$. Below this scale, the gauge degrees of freedom in the coset $\text{SU}(N_{\text{TC}} + 1)/\text{SU}(N_{\text{TC}})$ are frozen out, and g_{ETC} does not run. Combining this with the fact that the TC gauge coupling inherits its magnitude from the ETC gauge coupling, we will approximate $g_{\text{ETC}}(\mu)$ by the same form as $g_{\text{TC}}(\mu)$, given in Eq. (3.9), namely $g_{\text{ETC}}(\mu) \simeq r g_{\text{TC}}(\mu)$, with the parameter $r \simeq 1$ introduced to account for a slight difference in magnitude between these couplings. We thus obtain (with $x = p_E^2$ and $y = q_E^2$)

$$\Sigma(x) = \frac{3}{16\pi^2} \int_0^\infty y dy \frac{\kappa(x+y)\Sigma(y)}{\max(x,y)[y + \Sigma(y)^2]}.$$

(3.17)

[Note again that the integral is cut off at $y \simeq \Lambda^2$ because of Eq. (3.9).] From the above, the explicit form for $\kappa(z)$ (where $z = x + y$) is

$$\kappa(z) = g_{\text{TC}}^2(z)\left[C_{2F} + r^2\nu_3^2 \frac{z}{M_3^2}\right].$$

(3.18)

Including the prefactor $3/(16\pi^2)$, we can write this as

$$\frac{3\kappa(z)}{16\pi^2} = \frac{\alpha_{\text{TC}}(z)}{4\alpha_{\text{cr}}^{(0)}} + \kappa_4(z)\frac{z}{M_3^2},$$

(3.19)

where the rescaled coefficient of the ETC-induced four-fermion coupling, $\kappa_4(z)$, is

$$\kappa_4(z) = \frac{3r^2\nu_3^2\alpha_{\text{TC}}(z)}{4\pi}.$$

(3.20)

With $N_{\text{TC}} = 2$ and hence $\nu_3^2 = 1/12$, if $\alpha_{\text{TC}} \simeq \alpha_{\text{cr}}^{(0)} = \pi/(3C_{2F})$, then, taking $r \simeq 1$, the value of the momentum-dependent κ_4 at the relevant scale of order a TeV, which we shall refer to simply as κ_4, is

$$\kappa_4 \simeq \frac{1}{48C_{2F}} \simeq 0.03.$$

(3.21)

Thus, the value of κ_4 in the type of ETC model considered here is rather small.

In addition to ETC-induced four-fermion interactions, certain four-fermion operators could be induced by the nonperturbative dynamics of the technicolor theory itself. In particular, as we discussed above, instanton effects produce effective local multifermion operators, and, upon contraction of technifermion fields, these yield a particular type of four-fermion operator, which has been shown to be important for chiral symmetry breaking in QCD [41]. These instanton-generated multifermion operators are soft, i.e., the mass that enters as an inverse square factor multiplying them in an effective Lagrangian is of order the QCD chiral symmetry-breaking scale. Indeed, NJL-type four-fermion interactions are commonly used in modern phenomenological models of chiral symmetry breaking in QCD at zero and finite-temperature [68]. Instanton effects on the chiral phase transition depending on N_f have been studied in Refs. [17,18]. In Refs. [57,58] it was suggested that a four-fermion interaction with a strength equivalent to our $\kappa_4 \simeq 1/4$ could be induced by the nonperturbative dynamics of walking technicolor itself. As with instanton-induced multifermion operators, the mass scale characterizing these four-fermion interactions is expected to be $\simeq \Sigma$. This is different from the type of ETC-induced four-fermion operator considered here, which is hard at the scale Σ (and become soft above Λ_3). Although there are no instantons in QED$_4$, the importance of four-fermion operators has also been discussed in connection with a possible UV fixed point in this theory in Ref. [42]. (This question was relevant to the interpretation of differing results from lattice gauge theory simulations concerning the continuum limit of 4D U(1) lattice gauge theory [69,70].) If the technicolor theory itself generates large four-fermion operators, then the properties of the theory, even in the absence of ETC effects, might correspond to an interval effectively equivalent to our $1/4 \lesssim \kappa_4 \leq 1$ in the phase diagram of Fig. 7 below. Apart from this, there is also interest, from a general field-theoretic point of view, in investigating the range of κ_4 values extending up to O(1), where the four-fermion coupling, by itself, would be sufficient to break the chiral symmetry. In accordance with our discussion above, we take M_3 to be equal to the scale Λ that effectively sets the upper limit of the walking regime for the technicolor theory.

One next discretizes the Schwinger-Dyson equation and solves it using iterative numerical methods, as described in Ref. [66] (which also contains references to the literature on Schwinger-Dyson and Bethe-Salpeter equations). In this analysis, one formally takes α and κ_4 to be independent, although, as discussed above, in the actual ETC context, they are related. In Fig. 2 we show the solution for the dynamical technifermion mass Σ (divided by Λ) as a function of $\alpha/\alpha_{\rm cr}^{(0)}$ for a range of values of κ_4. The effect of the local current-current interaction is clearly to enhance the spontaneous chiral symmetry breaking, so that the critical value of the technicolor coupling for the appearance of a nonzero Σ, which can be denoted $\alpha_{\rm cr}$, decreases monotonically as κ_4 increases from zero. Similarly, if one fixes the value of $\alpha/\alpha_{\rm cr}^{(0)}$ where $\Sigma(0)/\Lambda$ is nonzero, then the value of $\Sigma(0)/\Lambda$ increases monotonically as κ_4 increases. For $\kappa_4 = 0$, our results are, as expected, in agreement with the exponential vanishing in Eq. (3.12) as α decreases toward its critical value, $\alpha_{\rm cr}^{(0)}$. As κ_4 increases, this behavior changes. Although the detailed critical behavior as one approaches the chiral boundary is not the focus of our present work, we comment that if one were to make a fit of the form

$$\frac{\Sigma(0)}{\Lambda} \propto \left(\frac{\alpha}{\alpha_{\rm cr}} - 1\right)^{\beta_\Sigma} \quad \text{as} \quad \frac{\alpha}{\alpha_{\rm cr}} \rightarrow 1^+, \qquad (3.22)$$

where β_Σ is a critical exponent, analogous to the standard notation for the critical exponent for the order parameter in statistical mechanics, then the value of β_Σ decreases from its value of infinity for $\kappa_4 = 0$, corresponding to the essential zero in Eq. (3.12), through the value $\beta_\Sigma \simeq 1$ at $\kappa_4 = 0.5$, to the value $\beta_\Sigma \simeq 0.5$ at $\kappa_4 = 1$.

In Fig. 3 we display contour curves of equal $\Sigma(0)/\Lambda$ as a function of $\alpha/\alpha_{\rm cr}^{(0)}$ and κ_4. The chiral phase boundary is shown as the dashed curve. For $\kappa_4 < 0.25$, the critical point $\alpha_{\rm cr}$ for the chiral phase transition is essentially independent of κ_4, just as is the case in the gauged NJL model [45,46,52]. For larger values of κ_4, this critical point
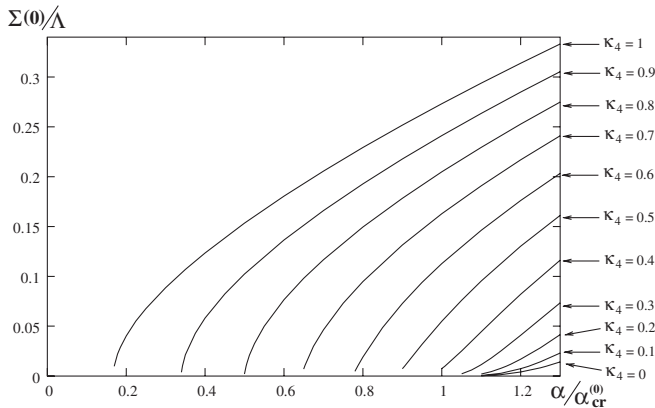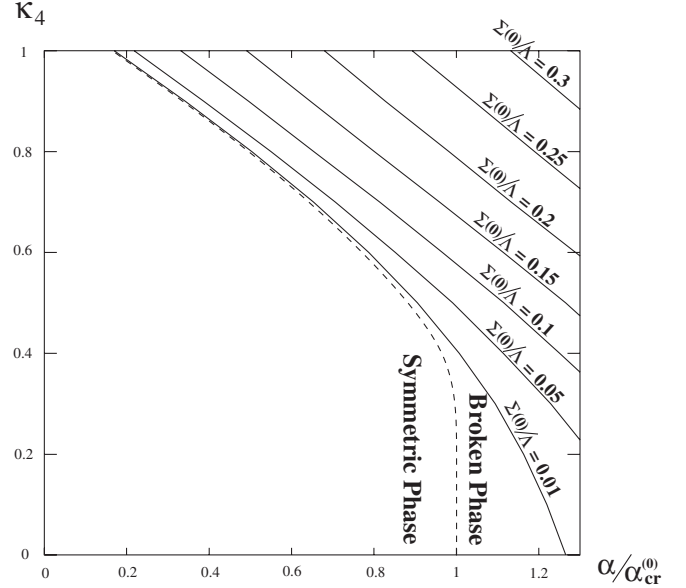

FIG. 3. Contours of constant $\Sigma(0)/\Lambda$ as a function of $\alpha/\alpha_{\rm cr}^{(0)}$ and κ_4. The dashed curve is the boundary between the phases with manifest and spontaneously broken chiral symmetry.

decreases, and, for $\kappa_4 \simeq 1.1$, this critical point occurs at $\alpha = 0$, i.e., for $\kappa_4 \gtrsim 1.1$, the local current-current interaction is sufficient, by itself, to produce a nonzero $\Sigma(0)$, without any technicolor gauge interaction. This is again qualitatively similar to the situation for the gauged NJL interaction.

IV. EXPRESSION FOR S IN TERMS OF CURRENT-CURRENT CORRELATION FUNCTIONS

In this section we review the formulas that we will use to calculate the S parameter in terms of (the derivative of) a certain combination of current-current correlation functions. As a measure of corrections to the Z propagator arising from heavy particles and new physics (NP) in theories beyond the standard model, S was originally defined as [20]

$$S = \frac{4s_W^2 c_W^2}{\alpha_{\rm em}(m_Z)} \frac{d\Pi_{ZZ}^{\rm (NP)}(q^2)}{dq^2}\bigg|_{q^2=0}, \qquad (4.1)$$

where $s_W^2 = 1 - c_W^2 = \sin^2\theta_W$, evaluated at m_Z. More recent analyses of precision electroweak data define S slightly differently, replacing the derivative at $q^2 = 0$ by a finite difference (in the $\overline{MS}$ scheme) [22]

$$S_{\rm PDG} = \frac{4s_W^2 c_W^2}{\alpha_{\rm em}(m_Z)} \left[\frac{\Pi_{ZZ}^{\rm (NP)}(m_Z^2) - \Pi_{ZZ}^{\rm (NP)}(0)}{m_Z^2}\right]. \qquad (4.2)$$

The difference between these definitions is small if the heavy fermion mass Σ satisfies $(2\Sigma/m_Z)^2 \gg 1$, as is the case in the technicolor models considered here. To make this quantitative, we recall that in the one-family techni-

FIG. 2. Plot of $\Sigma(0)/\Lambda$ as a function of $\alpha/\alpha_{\rm cr}^{(0)}$, for various values of κ_4.

color model, $m_W^2 = (g^2/4)f_{TC}^2(N_c + 1) = g^2 f_{TC}^2$, where g is the SU(2)$_L$ coupling and f_{TC} is the technicolor analogue of the pion decay constant in QCD. This yields $f_{TC} \simeq$ 125 GeV. We next use a rough scaling relation connecting the dynamical technifermion mass

$$\frac{\Sigma}{\Sigma_{QCD}} \simeq \frac{f_{TC}}{f_\pi}\left(\frac{N_c}{N_{TC}}\right)^{1/2}. \qquad (4.3)$$

With $f_\pi = 92.4$ MeV, $\Sigma_{QCD} \simeq m_N/N_c$, and $N_{TC} = 2$, one thus has $\Sigma \simeq 520$ GeV, so that $(2\Sigma/m_Z)^2 \simeq 1.3 \times 10^2$. For our purposes it will be convenient to use the original definition, Eq. (4.1).

Suppressing the SU(N_{TC}) gauge index, one can write the technifermions as a vector, $\psi = (\psi_i, \ldots, \psi_{N_f})$. One can then define vector and axial-vector currents as

$$V_\mu^a(x) = \bar{\psi}(x)T^a\gamma_\mu\psi(x) \qquad A_\mu^a(x) = \bar{\psi}(x)T^a\gamma_\mu\gamma_5\psi(x), \qquad (4.4)$$

where the $N_f \times N_f$ matrices T^a ($a = 1, \ldots, N_f^2 - 1$) are the generators of SU(N_f) with the standard normalization. In terms of these currents, the two-point current-current correlation functions Π_{VV} and Π_{AA} are defined via the equation

$$i \int d^4 x e^{iq\cdot x}\langle 0|T(J_\mu^a(x)J_\nu^b(0))|0\rangle$$
$$= \delta^{ab}\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_{JJ}(q^2), \qquad (4.5)$$

where $J_\mu^a(x) = V_\mu^a(x), A_\mu^a(x)$. Since SM gauge interactions are small at the technicolor scale of several hundred GeV, it follows that the contributions to S of each of the N_c techniquark electroweak doublets $\binom{U^a}{D^a}$, $a = 1, \ldots, N_c$ and from the technilepton electroweak doublet $\binom{N}{E}$ are essentially equal. It is therefore convenient to define a reduced quantity, $\hat{S}$, that represents the contribution to S from each such technifermion doublet, viz.,

$$\hat{S} = \frac{S}{N_D} \qquad (4.6)$$

where $N_D = N_c + 1 = 4$ for the one-family technicolor theory. Then, in terms of the current-current correlation functions defined above, S, as defined in Eq. (4.1), is given by

$$\hat{S} = 4\pi\frac{d}{dq^2}[\Pi_{VV}(q^2) - \Pi_{AA}(q^2)]|_{q^2=0}, \qquad (4.7)$$

The relation (4.7) is equivalent to the expression in terms of the integral over the vector and axial-vector spectral functions for the currents (4.4) [71],

$$\hat{S} = 4\pi\int_0^\infty \frac{ds}{s}[\rho_V(s) - \rho_A(s)], \qquad (4.8)$$

where

$$\rho_J(s) \equiv \frac{\text{Im}(\Pi_{JJ}(s))}{\pi s} \qquad (4.9)$$

for $J = V, A$.

V. CALCULATION OF S VIA BETHE-SALPETER EQUATION

The method that we use to calculate S is similar to our earlier work [32,33], except that now we use a scattering kernel for the Bethe-Salpeter equation that includes the exchange of not just the massless technigluons, but also the massive ETC vector boson V_{d3}. We define certain Bethe-Salpeter amplitudes $\chi_{\alpha\beta}^{(J)}(p; q, \epsilon)$ as

$$\delta_j^k(T^a)_f^{f'} \int \frac{d^4 p}{(2\pi)^4} e^{-ip\cdot r}\chi_{\alpha\beta}^{(J)}(p; q, \epsilon)$$
$$= \epsilon^\mu \int d^4 x e^{iq\cdot x}\langle 0|T(\psi_{\alpha f}^k(r/2)\bar{\psi}_{jf'\beta}(-r/2)J_\mu^a(x))|0\rangle, \qquad (5.1)$$

where $J = V$ or A, and (f, f'), (j, k), and (α, β) are, respectively, the flavor, gauge, and spinor indices. In Fig. 4 we show symbolically the terms contributing to the Bethe-Salpeter equation. Full propagators are used on the internal technifermion lines and the running gauge coupling is included.

Closing the fermion legs of the above three-point vertex function and taking the limit $r \rightarrow 0$, we can express the current-current correlation function in terms of these amplitudes as

$$\Pi_{JJ}(q^2) = \frac{1}{3}\left(\frac{N}{2}\right)\sum_\epsilon \int \frac{d^4 p}{i(2\pi)^4} \text{Tr}[(\epsilon \cdot G^{(J)})\chi^{(J)}(p; q, \epsilon)], \qquad (5.2)$$

where

$$G_\mu^{(V)} = \gamma_\mu, \qquad G_\mu^{(A)} = \gamma_\mu\gamma_5, \qquad (5.3)$$

and an average has been taken over the polarizations, so that $\Pi_{JJ}(q^2)$ does not depend on the polarization ϵ. We then calculate these amplitudes and evaluate the requisite combination to obtain $\hat{S}$. Since we include a running coupling in the calculation, we are again working in the improved ladder approximation.

In Fig. 5 we show our results for $\hat{S}$ as a function of $\alpha/\alpha_{cr}^{(0)}$ for various values of κ_4. As in the analysis of the Schwinger-Dyson equation, here again one formally varies α and κ_4 independently, but understands that in a given ETC theory, they are related, via Eqs. (2.14) and (3.20). For a given value of κ_4, the smallest value of $\alpha/\alpha_{cr}^{(0)}$ is close to $\alpha_{cr}/\alpha_{cr}^{(0)}$; i.e., the curve starts near to the chiral boundary. For fixed κ_4, $\hat{S}$ increases as a function of $\alpha/\alpha_{cr}^{(0)}$. This is the same trend that we found in Refs. [32,33] for the pure gauge theory without four-fermion interaction. For $\kappa_4 = 0$, the reason is clear; as $\alpha = \alpha_*$ increases, corresponding to a
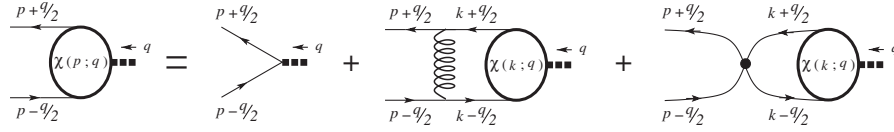
FIG. 4. Pictorial representation of the Bethe-Salpeter equation analyzed here. The first graph on the right is the bare current vertex, the second represents technigluon exchange, and the third represents the contribution of the effective local four-fermion operator resulting from the exchange of the massive ETC gauge boson V_{d3}. See text for further explanation.

decrease in N_f, one is moving away from the walking regime toward the more QCD-like regime, and the reduction in $\hat{S}$ associated with walking behavior is removed. This increase actually becomes more abrupt as one increases κ_4, which indicates that the departure from walking behavior is more abrupt as one moves away from the chiral phase boundary in the presence of a substantial four-fermion coupling. This is in agreement with what we found for the dynamical technifermion mass, namely, that as one moves into the chirally broken phase from the chiral phase boundary, the turn-on of Σ is more rapid (i.e., the critical exponent β_Σ is smaller) for larger values of κ_4, as the exponential suppression inherent in the form (3.12) is removed.

In Fig. 6 we show our results for $\hat{S}$ as a function of κ_4 for a range of values of $\alpha/\alpha_{\rm cr}^{(0)}$. In a walking technicolor theory a typical value of $\alpha/\alpha_{\rm cr}^{(0)}$ with $\alpha \simeq \alpha_*$ could be about 1.1. In both Fig. 5 and Fig. 6, the observed feature that increasing κ_4 at fixed α eventually increases $\hat{S}$ is understandable because the current-current interaction facilitates the formation of the technifermion condensate and associated appearance of the dynamical mass Σ. This is the same trend as the increase in $\hat{S}$ that results from increasing α_* (e.g., by decreasing the number of technifermions, N_f), moving one from the walking regime in the direction of more QCD-like behavior. From inspection of the curve for this value, it is evident that $\hat{S} \simeq 0.2$ for $0 \leq \kappa_4 \lesssim 0.25$,

with $\hat{S}$ increasing for larger values of κ_4. Since κ_4 is expected to be rather small [cf. Eq. (3.21)], we thus find that in the type of ETC model considered here, the inclusion of the ETC-induced four-fermion operator has little effect on $\hat{S}$ [72]. In Fig. 7 we plot curves of constant $\hat{S}$ as functions of κ_4 and $\alpha/\alpha_{\rm cr}^{(0)}$. The chiral boundary is again represented by the dashed curve.

It is also of interest to investigate how $\hat{S}$ behaves if one varies both $\alpha/\alpha_{\rm cr}^{(0)}$ and κ_4 (formally taken to be an independent couplings) in such a manner as to move along a contour of a fixed value of the ratio of physical scales $\Sigma(0)/\Lambda$. From Fig. 3, it follows that the condition of maintaining fixed $\Sigma(0)/\Lambda$ means that if one increases κ_4, then this should be compensated by a decrease in α. In models such as those of Refs. [5,6], where walking behavior extends up to the lowest ETC breaking scale, Λ_3, the ratio $\Sigma(0)/\Lambda$ is typically of order 0.1. The requirement to reduce S as much as possible provides motivation to consider a stronger degree of walking and hence a smaller value of $\Sigma(0)/\Lambda$. In Fig. 8 we present a plot of $\hat{S}$ for the case where one keeps this ratio fixed at the value $\Sigma(0)/\Lambda = 0.01$. From inspection of Fig. 3, one can see that the contour with $\Sigma(0)/\Lambda = 0.01$ intersects the vertical axis at about $\alpha/\alpha_{\rm cr}^{(0)} \simeq 1.27$ for $\kappa_4 = 0$, but moves very close to the chiral boundary as κ_4 increases past 0.4. This is in agreement with the fact that the turn-on of the chiral symmetry-breaking order parameter Σ is more abrupt as κ_4
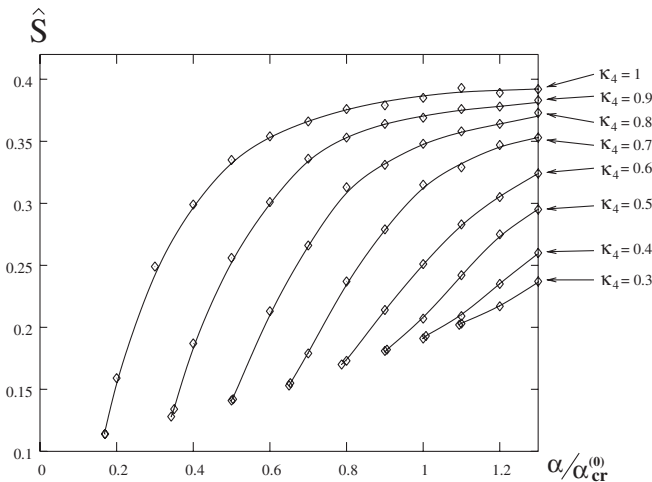

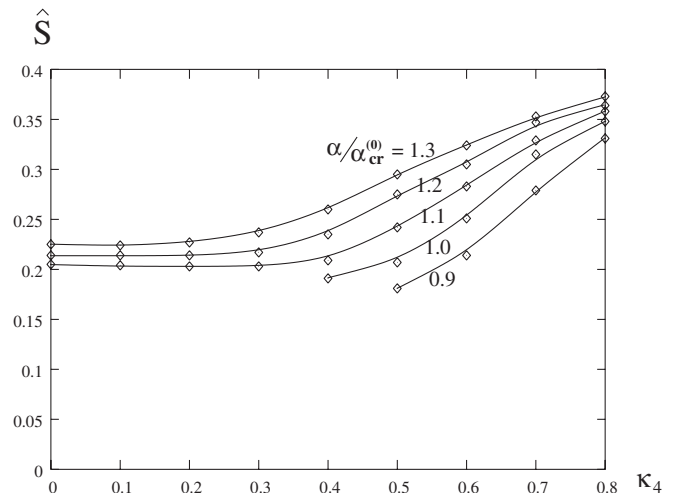FIG. 5. $\hat{S}$ as a function of $\alpha/\alpha_{\rm cr}^{(0)}$ for various values of κ_4.

FIG. 6. $\hat{S}$ as a function of κ_4 for various values of $\alpha/\alpha_{\rm cr}^{(0)}$.
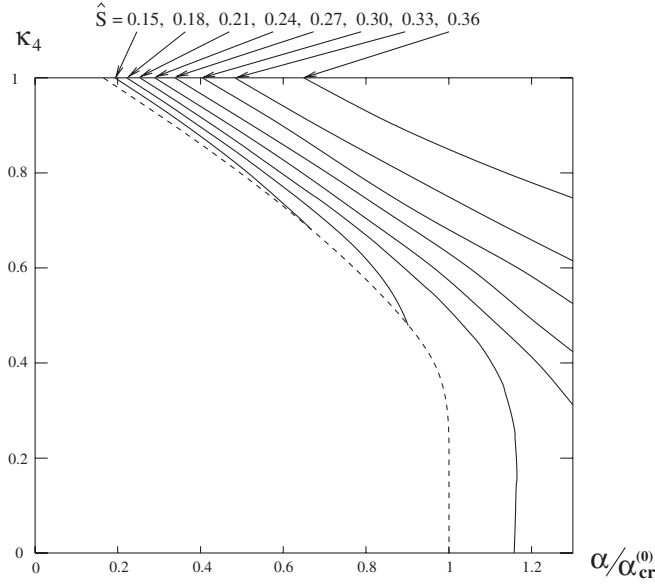
FIG. 7. Contours of constant $\hat{S}$, plotted as functions of κ_4 and $\alpha/\alpha_{\rm cr}^{(0)}$.

increases and the fact that $\hat{S}$ vanishes in the chirally symmetric phase. Our results in Fig. 8 show that $\hat{S}$ decreases as κ_4 increases, with $\alpha/\alpha_{\rm cr}^{(0)}$ reduced so as to keep $\Sigma(0)/\Lambda$ constant. A plausible interpretation of this is that, to remain on the contour of fixed $\Sigma(0)/\Lambda = 0.01$ as κ_4 increases, one is moving closer to the chiral boundary. It is also plausible that this decrease in $\hat{S}$ is associated with an increase in the anomalous dimension $\gamma_{\bar{\psi}\psi}$ for the technifermion bilinear $\bar{\psi}\psi$. We recall that $\gamma_{\bar{\psi}\psi} = 1$ in the walking limit of a technicolor gauge theory (and $\gamma_{\bar{\psi}\psi} \to 2$ for $\alpha = 0$, $\kappa_4 \to 1$). For the expected small value of κ_4, our Bethe-Salpeter
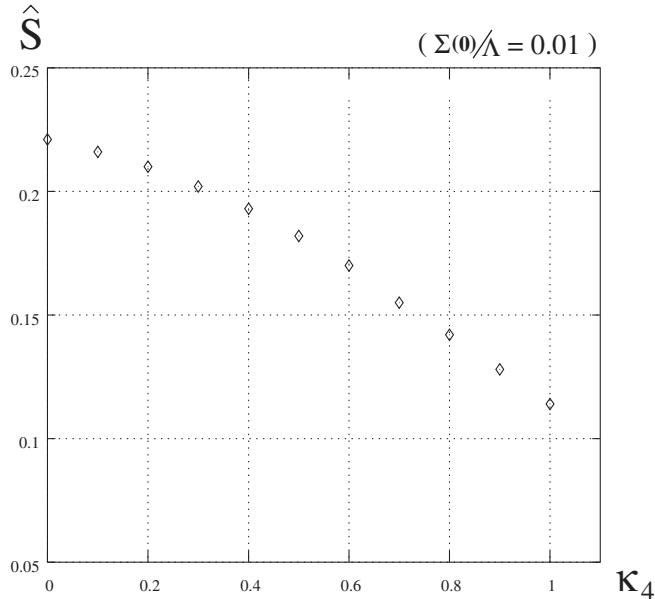

FIG. 8. $\hat{S}$ as a function κ_4 with α varying so as to maintain a fixed value of $\Sigma(0)/\Lambda = 0.01$.

calculation yields $\hat{S} \simeq 0.2$ on this contour, so that $S \simeq 0.8$ [using $N_D = 4$ as in Eq. (2.4)]. We recall again the possibility that the dynamics of the technicolor theory itself could produce large four-fermion operator effects which could reduce S; here we focus on the ETC-induced contributions. The above value of S is too large to agree with experimental limits on S, so to maintain the viability of this type of model, it is necessary to assume that there is a further reduction in S. We comment on this next.

Although one cannot use perturbation theory reliably to calculate S in a strongly coupled gauge theory, the perturbative formula is often employed for comparisons of different technicolor models. The one-loop perturbative calculation with degenerate fermions having effective masses satisfying $(2\Sigma/m_Z)^2 \gg 1$ yields the well-known result $S_{\rm pert} = N_{D,{\rm tot}}/(6\pi)$ where here $N_{D,{\rm tot}} = N_{\rm TC}N_D$, so that

$$\hat{S}_{\rm pert} = \frac{N_{\rm TC}}{6\pi}. \tag{5.4}$$

In QCD with just light quarks, and hence $N_D = N_f/2 = 1$ and hence $N_{D,{\rm tot,QCD}} = N_c N_D = 3$, this perturbative calculation would predict $S_{\rm QCD,pert} \simeq 1/(2\pi) \simeq 0.16$. To the extent that the experimental value of S in QCD is dominated by the contributions of light-quark hadrons in Eq. (4.8) [73], it follows that $N_D = 1$, so that $S \simeq \hat{S}$ for QCD. This experimental value of S in QCD is $S = 0.33 \pm 0.04$ [74], so that the perturbative estimate is about a factor of 2 smaller than the actual value. An approximate calculation of $\hat{S}$ was carried out using the ladder approximation to the Schwinger-Dyson and Bethe-Salpeter equations for QCD ($N = 3$) with $N_f = 2$ quarks of negligible mass [27]. Studies have also been done for the case where one neglects the strange quark mass m_s, i.e., $N = 3$, $N_f = 3$ [27,67]. Since for either of these values of N_f the two-loop beta function of the QCD theory does not exhibit an infrared fixed point, it was necessary in these calculations to cut off the growth of the strong coupling. For typical cutoffs, it was found that the calculations tended to yield too large a value of $\hat{S} = S$, namely $\hat{S} \simeq 0.45$–0.5 [27,67]. This suggests that this type of Bethe-Salpeter calculation may overestimate S. Our studies of $\hat{S}$ showed that in a walking theory, $\hat{S}$ is reduced, relative to its value in a QCD-like theory [32,33], in agreement with other studies of the effect of walking [25–30]. Combining this reduction with a plausible correction factor to compensate for the tendency of the Bethe-Salpeter calculation to overestimate $\hat{S}$ in QCD would yield a further reduction in $\hat{S}$. However, even with a reduction to $\hat{S} \simeq 0.1$, inserting the factor $N_D = 4$ yields $S \simeq 0.4$, which is sufficiently large to be of strong concern. In this context, it should be noted that the question of the value of $\hat{S}$ in walking technicolor has been investigated in recent analyses using holographic methods [35,36], and several authors have found evidence for a sizable reduction [35], although Ref. [36] did not. An

important task that merits further study is to relate these holographic methods to the sort of Schwinger-Dyson and Bethe-Salpeter methods used in previous works for pure walking gauge theories and here for a walking gauge theory with additional ETC-induced four-fermion interaction.

VI. SUMMARY

Technicolor and extended technicolor theories are very ambitious, since they aim to explain not only electroweak symmetry breaking and the associated masses for the W and Z bosons, but also the spectrum of quark, charged lepton, and neutrino masses. A successful model of this type would thus explain longstanding puzzles such as the value of the intergenerational lepton mass ratio m_e/m_μ and intragenerational mass ratios such as m_u/m_d and m_t/m_b. It is thus not surprising that no fully realistic ETC model has been constructed. However, since TC/ETC theories continue to provide an interesting theoretical framework complementary to the standard-model itself and to other approaches such as supersymmetry, it is worthwhile to investigate their properties further. Accordingly, in this paper, using approximate numerical solutions of the relevant Schwinger-Dyson and Bethe-Salpeter equations, we have calculated the correction to the Z boson propagator, as described by the S parameter, in a technicolor theory, taking account of both massless technigluon exchange and the dominant contribution from massive ETC gauge boson exchange. Our results suggest that, for the types of ETC models considered here, this additional contribution from massive ETC gauge boson exchange has a relatively small effect on S.

ACKNOWLEDGMENTS

APPENDIX

It is useful to compare and contrast the ETC-induced four-fermion operator whose effects we have analyzed here with the four-fermion operator studied in the work of Nambu and Jona-Lasinio [37] and to gauged NJL models. The NJL model for a single fermion ψ is described by the Lagrangian $\mathcal{L}_{\mathrm{NJL}} = \bar{\psi} i \not{\partial} \psi + \mathcal{L}_{\mathrm{int,NJL}}$, where the interaction term $\mathcal{L}_{\mathrm{int,NJL}}$ is given by

$$\mathcal{L}_{\mathrm{int,NJL}} = \frac{c}{2\Lambda^2}[[\bar{\psi}\psi]^2 - [\bar{\psi}\gamma_5\psi]^2] = \frac{2c}{\Lambda^2}[\bar{\psi}_L\psi_R][\bar{\psi}_R\psi_L],$$
(A1)

Here, Λ is a mass scale introduced to make the coupling c dimensionless. By a Fierz transformation, this is equivalent to

$$\begin{aligned}\mathcal{L}_{\mathrm{int,NJL}} &= -\frac{c}{4\Lambda^2}[[\bar{\psi}\gamma_\mu\psi][\bar{\psi}\gamma^\mu\psi] - [\bar{\psi}\gamma_\mu\gamma_5\psi] \\ &\quad \times [\bar{\psi}\gamma^\mu\gamma_5\psi]] \\ &= -\frac{c}{\Lambda^2}[\bar{\psi}_L\gamma_\mu\psi_L][\bar{\psi}_R\gamma^\mu\psi_R].\end{aligned}$$
(A2)

We define

$$\bar{c} \equiv \frac{c}{4\pi^2}.$$
(A3)

[This coupling $\bar{c}$ is identical to the coupling $G\Lambda^2/(4\pi^2)$ in Ref. [45] and to $\beta = G\Lambda^2/(2\pi^2)$ in Ref. [46].] This theory is invariant under a global chiral symmetry group $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$. An analysis of the Schwinger-Dyson equation for the fermion, with an ultraviolet cutoff of Λ imposed on the momentum integration, shows that for $\bar{c} > 1$, this equation has a nonzero solution for Σ, signifying dynamical chiral symmetry breaking. Associated with this is the formation of a bilinear fermion condensate $\langle\bar{\psi}\psi\rangle$, breaking $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ to the diagonal subgroup $\mathrm{U}(1)_V$. It is straightforward to generalize the model to an N-component fermion.

There has also been interest in considering a class of models in which ψ transforms as a nonsinglet under an Abelian $\mathrm{U}(1)$ or non-Abelian $\mathrm{SU}(N)$ gauge group, so that the kinetic term $\bar{\psi} i \not{\partial} \psi$ is replaced by $\bar{\psi} i \not{D} \psi$, where D_μ is the appropriate covariant derivative. In the Abelian case, the Fierz transformation relating the interaction in Eq. (A1) to that in Eq. (A2) applies in the same manner as for the original NJL model. In the non-Abelian case, the situation is more complicated. If the currents in the current-current product involve the full non-Abelian $\mathrm{SU}(N)$ gauge generators, then a Fierz transformation operates not just on the Dirac matrices but also on the matrices representing the gauge generators. As noted in the text, a number of studies of a gauged NJL model, particularly in the Abelian case, were performed and mapped out the chiral phase boundary for this type of model as a function of the gauge coupling g and the NJL coupling c (generically taken to be independent).

There are both similarities and differences between an ETC-based model of the type considered in our text and the non-Abelian gauged NJL model. First, the ETC interaction yields a product of two vectorial currents, denoted symbolically as VV, rather than the structure of the NJL interaction in Eq. (A2), which is $VV - AA$. Second, while the NJL model requires an ultraviolet cutoff, this is not necessary in the present case since we actually start with a reasonably ultraviolet-complete theory, from which the interaction (2.15) arises as part of the effective low-energy field theory. Third, while the general gauged NJL model treats c and the gauge coupling as independent, this is not the case in a TC/ETC theory, since the TC gauge group arises as a subgroup of the ETC gauge group and hence the

running TC gauge coupling at a given scale is determined by the ETC gauge coupling at the higher ETC scales where the ETC gauge degrees of freedom are still active.

In an approximation in which the Σ term in the denominator of the fermion propagator is neglected, thereby linearizing the Schwinger-Dyson equation, it was found that, as one increases the NJL coupling $\bar{c}$ from zero, the critical gauge coupling $\alpha_{\rm cr}$ remains unchanged for $0 \leq \bar{c} \leq 1/4$ and, for $\bar{c} > 1/4$, the chiral phase boundary can be described by the following functional relation for the coordinates of a point $(\alpha_{\rm cr}, \bar{c}_{\rm cr})$ on this boundary,

$$\bar{c}_{\rm cr} = \frac{1}{4}\left(1 + \sqrt{1 - \frac{\alpha_{\rm cr}}{\alpha_{\rm cr}^{(0)}}}\right)^2, \qquad {\rm (A4)}$$

or equivalently, $\alpha_{\rm cr}/\alpha_{\rm cr}^{(0)} = 4\sqrt{\bar{c}_{\rm cr}}(1 - \sqrt{\bar{c}_{\rm cr}})$. As one moves up along the phase boundary from the point $(\alpha, \bar{c}) = (\alpha_{\rm cr}^{(0)}, 0)$ to $(0, 1)$, the nature of the critical singularity also changes from the essential zero in Eq. (3.12) to the algebraic singularity (3.22) $\Sigma/\Lambda \sim (\bar{c} - 1)^{1/2}$ as $\bar{c} \to 1^+$.

In the gauged NJL model, the Schwinger-Dyson equation is, after Euclidean rotation and angular integration, in the notation of Sec. III,

$$\Sigma(x) = \frac{\alpha}{4\alpha_{\rm cr}^{(0)}} \int_0^{\Lambda^2} ydy \frac{\Sigma(y)}{\max(x, y)[y + \Sigma(y)^2]} + \frac{\bar{c}}{\Lambda^2}$$
$$\times \int_0^{\Lambda^2} ydy \frac{\Sigma(y)}{[y + \Sigma(y)^2]}. \qquad {\rm (A5)}$$

If one were formally to replace the factor $\kappa_4(x, y) \times (x + y)/\max(x, y)$ in our Eqs. (3.17), (3.18), and (3.19) by $\bar{c}$, then the Schwinger-Dyson equation for the ETC model analyzed in the present paper would be transformed into a structure equivalent to that studied in Refs. [45,46,52]. Since the factor $(x + y)/\max(x, y)$ takes the value unity for $y \to 0$ and $y \to \infty$ and takes the maximal value 2 (at $x = y$), one could anticipate that the solutions for the Schwinger-Dyson equation in the present paper should be formally similar to the results obtained in Refs. [45,46,52] for the same input values of α and similar values of κ_4 and $\bar{c}$.

[1] S. Weinberg, Phys. Rev. D **19**, 1277 (1979); L. Susskind, *ibid.* **20**, 2619 (1979); see also S. Weinberg, Phys. Rev. D **13**, 974 (1976).

[2] S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. B **90**, 125 (1980).

[3] T. Appelquist and J. Terning, Phys. Rev. D **50**, 2116 (1994).

[4] T. Appelquist and R. Shrock, Phys. Lett. B **548**, 204 (2002); Phys. Rev. Lett. **90**, 201801 (2003).

[5] T. Appelquist, M. Piai, and R. Shrock, Phys. Rev. D **69**, 015002 (2004); Phys. Lett. B **593**, 175 (2004); **595**, 442 (2004).

[6] T. Appelquist, N. Christensen, M. Piai, and R. Shrock, Phys. Rev. D **70**, 093010 (2004).

[7] B. Holdom, Phys. Lett. B **150**, 301 (1985).

[8] K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986).

[9] T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).

[10] T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **35**, 774 (1987); **36**, 568 (1987).

[11] M. Bando, T. Morozumi, H. So, and K. Yamawaki, Phys. Rev. Lett. **59**, 389 (1987).

[12] T. Appelquist, K. Lane, and U. Mahanta, Phys. Rev. Lett. **61**, 1553 (1988).

[13] A. Cohen and H. Georgi, Nucl. Phys. **B314**, 7 (1989).

[14] T. Appelquist, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. Lett. **77**, 1214 (1996).

[15] V. Miransky and K. Yamawaki, Phys. Rev. D **55**, 5051 (1997); **56**, 3768(E) (1997).

[16] R. S. Chivukula, Phys. Rev. D **55**, 5238 (1997).

[17] T. Appelquist and S. Selipsky, Phys. Lett. B **400**, 364 (1997).

[18] M. Velkovsky and E. Shuryak, Phys. Lett. B **437**, 398 (1998).

[19] T. Appelquist, A. Ratnaweera, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. D **58**, 105017 (1998).

[20] M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992).

[21] G. Altarelli and R. Barbieri, Phys. Lett. B **253**, 161 (1991); G. Altarelli, R. Barbieri, and F. Caravaglios, Int. J. Mod. Phys. A **13**, 1031 (1998).

[22] See http://pdg.lbl.gov and http://lepewwg.web.cern.ch.

[23] P. Sikivie, L. Susskind, M. Voloshin, and V. Zakharov, Nucl. Phys. **B173**, 189 (1980).

[24] B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990); M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991); H. Georgi, Nucl. Phys. **B363**, 301 (1991); R. Johnson, B.-L. Young, and D. McKay, Phys. Rev. D **43**, R17 (1991); R. Cahn and M. Suzuki, Phys. Rev. D **44**, 3641 (1991).

[25] T. Appelquist and G. Triantaphyllou, Phys. Lett. B **278**, 345 (1992).

[26] R. Sundrum and S. Hsu, Nucl. Phys. **B391**, 127 (1993).

[27] M. Harada and Y. Yoshida, Phys. Rev. D **50**, 6902 (1994).

[28] T. Appelquist, N. Evans, and S. Selipsky, Phys. Lett. B **374**, 145 (1996).

[29] T. Appelquist and F. Sannino, Phys. Rev. D **59**, 067702 (1999).

[30] S. Ignjatovic, L. C. R. Wijewardhana, and T. Takeuchi, Phys. Rev. D **61**, 056006 (2000).

[31] N. Christensen and R. Shrock, Phys. Rev. Lett. **94**, 241801 (2005).

[32] M. Harada, M. Kurachi, and K. Yamawaki, Prog. Theor.

Phys. **115**, 765 (2005); see also M. Kurachi, in *Proceedings of the 2004 Workshop on Dynamical Symmetry Breaking, 2004*, edited by M. Harada and K. Yamawaki (Physics Department, Nagoya University, Nagoya, 2004), p. 125.

[33] M. Kurachi and R. Shrock, Phys. Rev. D **74**, 056003 (2006).

[34] M. Kurachi and R. Shrock, J. High Energy Phys. 12 (2006) 034.

[35] D. K. Hong and H.-U. Yee, Phys. Rev. D **74**, 015011 (2006); J. Hirn and V. Sanz, Phys. Rev. Lett. **97**, 121803 (2006); J. High Energy Phys. 03 (2007) 100; M. Piai, arXiv:hep-ph/0608241.

[36] K. Agashe, C. Csáki, C. Grojean, and M. Reece, arXiv:0704.1821.

[37] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[38] K. Johnson, M. Baker, and R. Willey, Phys. Rev. **136**, B1111 (1964); M. Baker and K. Johnson, Phys. Rev. D **3**, 2516 (1971).

[39] D. Gross and A. Neveu, Phys. Rev. D **10**, 3235 (1974).

[40] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976); Phys. Rev. D **14**, 3432 (1976).

[41] D. Caldi, Phys. Rev. Lett. **39**, 121 (1977); C. Callan, R. Dashen, and D. Gross, Phys. Rev. D **17**, 2717 (1978); R. Carlitz, Phys. Rev. D **17**, 3225 (1978); R. Carlitz and Creamer, Ann. Phys. (N.Y.) **118**, 429 (1979).

[42] W. Bardeen, C. Leung, and S. Love, Phys. Rev. Lett. **56**, 1230 (1986); C. Leung, S. Love, and W. Bardeen, Nucl. Phys. **B273**, 649 (1986); **B323**, 493 (1989).

[43] E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981).

[44] Y. Nambu, in *Proceedings of the XI Warsaw Symposium on Elementary Particle Physics*, edited by Z. Ajduk *et al.* (World Scientific, Singapore, 1989), p. 1; , in *1988 International Workshop on Strongly Coupled Gauge Theories*, edited by M. Bando *et al.* (World Scientific, Singapore, 1989), p. 3.

[45] K.-I. Kondo, H. Mino, and K. Yamawaki, Phys. Rev. D **39**, 2430 (1989); K. Yamawaki, in Proceedings of the 12th Johns Hopkins Workshop on Current Problems in Particle Theory, 1988.

[46] T. Appelquist, M. Soldate, T. Takeuchi, and L. C. R. Wijewardhana, in Proceedings of the 12th Johns Hopkins Workshop on Current Problems in Particle Theory, 1988.

[47] V. A. Miransky and K. Yamawaki, Mod. Phys. Lett. A **4**, 129 (1989).

[48] V. A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. B **221**, 177 (1989).

[49] T. Appelquist, M. Einhorn, T. Takeuchi, and L. C. R. Wijewardhana, Phys. Lett. B **220**, 223 (1989).

[50] V. A. Miransky, T. Nonoyama, and K. Yamawaki, Mod. Phys. Lett. A **4**, 1409 (1989).

[51] B. Holdom, Phys. Lett. B **226**, 137 (1989).

[52] T. Takeuchi, Phys. Rev. D **40**, 2697 (1989).

[53] T. Appelquist, T. Takeuchi, M. Einhorn, and L. C. R. Wijewardhana, Phys. Lett. B **232**, 211 (1989).

[54] W. Bardeen, C. Hill, and M. Lindner, Phys. Rev. D **41**, 1647 (1990).

[55] T. Appelquist and O. Shapira, Phys. Lett. B **249**, 83 (1990).

[56] T. Appelquist, U. Mahanta, D. Nash, and L. C. R. Wijewardhana, Phys. Rev. D **43**, R646 (1991).

[57] K. I. Kondo, S. Shuto, and K. Yamawaki, Mod. Phys. Lett. A **6**, 3385 (1991).

[58] K. I. Aoki, K. Morikawa, J. I. Sumi, H. Terao, and M. Tomoyose, Prog. Theor. Phys. **102**, 1151 (1999).

[59] Models with technifermions transforming according to higher-dimensional representations of the technicolor gauge group have been considered, e.g., in [62,75,76]. The simplest case, in which the technifermions are in the fundamental representation, will suffice for our present study.

[60] Here and below, when we mention nonintegral values of N_f, it is implicitly understood that physical values of N_f are, of course, non-negative integers, and the nonintegral values are defined via an analytic continuation away from these physical values.

[61] Y. Iwasaki *et al.*, Phys. Rev. Lett. **69**, 21 (1992); Phys. Rev. D **69**, 014507 (2004); P. Damgaard, U. Heller, A. Krasnitz, and P. Olesen, Phys. Lett. B **400**, 169 (1997); R. Mawhinney, Nucl. Phys. B, Proc. Suppl. **83**, 57 (2000).

[62] N. Christensen and R. Shrock, Phys. Lett. B **632**, 92 (2006).

[63] J. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974); K. Lane, Phys. Rev. D **10**, 2605 (1974); T. Maskawa and H. Nakajima, Prog. Theor. Phys. **52**, 1326 (1974); **54**, 860 (1975); R. Fukuda and T. Kugo, Nucl. Phys. **B117**, 250 (1974).

[64] E. Gardi and M. Karliner, Nucl. Phys. **B529**, 383 (1998); E. Gardi, G. Grunberg, and M. Karliner, J. High Energy Phys. 07 (1998) 007.

[65] R. Corless, G. Gonnet, D. Hare, D. Jeffrey, and D. Knuth, Adv. Comput. Math. **5**, 329 (1996).

[66] M. Harada, M. Kurachi, and K. Yamawaki, Phys. Rev. D **68**, 076001 (2003).

[67] M. Harada, M. Kurachi, and K. Yamawaki, Phys. Rev. D **70**, 033009 (2004).

[68] Some reviews include U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991); S. Klevansky, Rev. Mod. Phys. **64**, 649 (1992); V. A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories* (World Scientific, Singapore, 1993).

[69] J. Kogut, E. Dagotto, and A. Kocic, Phys. Rev. Lett. **61**, 2416 (1988); **62**, 1001 (1989); Phys. Rev. D **43**, R1763 (1991); S. Hands, J. Kogut, and E. Dagotto, Nucl. Phys. **B333**, 551 (1990).

[70] M. Göckeler, R. Horsley, E. Laermann, P. Rakow, G. Schierholz, R. Sommer, and U. Wiese, Nucl. Phys. **B334**, 527 (1990); M. Göckeler, R. Horsley, E. Laermann, U. Wiese, P. Rakow, G. Schierholz, and R. Sommer, Phys. Lett. B **251**, 567 (1990); Nucl. Phys. **B371**, 713 (1992).

[71] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. **19**, 859 (1967).

[72] We note again the possibility that the dynamics of the technicolor theory itself could produce large four-fermion operator effects, which would affect S. Here we focus on the ETC-induced contributions.

[73] This is consistent with the fact that the contributions of the light-quark vector and axial-vector mesons ρ and a_1 largely saturate the spectral function integral for S,

Eq. (4.8), in QCD.

[74] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985); **B250**, 517 (1985); M. Harada and K. Yamawaki, Phys. Rep. **381**, 1 (2003).

[75] E. Eichten and K. Lane, Phys. Lett. B **222**, 274 (1988).

[76] D. Hong, S. Hsu, and F. Sannino, Phys. Lett. B **597**, 89 (2004); F. Sannino and K. Tuominen, Phys. Rev. D **71**, 051901 (2005).