Next-to-leading-order corrections to exclusive processes in k_T factorization

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We calculate next-to-leading-order corrections to exclusive processes in the k_T factorization theorem, taking $\pi\gamma^* \rightarrow \gamma$ as an example. Partons off shell by k_T^2 are considered in both the quark diagrams from full QCD and the effective diagrams for the pion wave function. The gauge dependences in the above two sets of diagrams cancel, when deriving the k_T-dependent hard kernel as their difference. The gauge invariance of the hard kernel is then proven to all orders by induction. The light-cone singularities in the k_T-dependent pion wave function are regularized by rotating the Wilson lines away from the light cone. This regularization introduces a factorization-scheme dependence into the hard kernel, which can be minimized in the standard way. Both the large double logarithms $\ln^2 k_T$ and $\ln^2 x$, x being a parton momentum fraction, arise from the loop correction to the virtual photon vertex, the former being absorbed into the pion wave function and organized by the k_T resummation and the latter absorbed into a jet function and organized by the threshold resummation. The next-to-leading-order corrections are found to be only a few percent for $\pi\gamma^* \rightarrow \gamma$, if setting the factorization scale to the momentum transfer from the virtual photon.

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I. INTRODUCTION

The k_T factorization theorem [1–6], as a fundamental tool of perturbative QCD (PQCD), has been widely applied to inclusive and exclusive processes. It has been pointed out that the k_T factorization theorem is appropriate for processes dominated by contributions from small parton momentum factions x [7]. Its application to exclusive B meson decays has led to the PQCD approach [8–12], which is free of the singularities from the end-point regions of x that usually appear in the collinear factorization theorem [13–18]. Several aspects of the k_T factorization theorem have been studied. For example, a naive definition of k_T-dependent hadron wave functions, in which the coordinate of a quark field is simply shifted by a transverse distance, contains light-cone divergences [19]. Modified definitions to remove these divergences have been proposed in Refs. [19–21]. The B meson wave function defined in the k_T factorization theorem is normalizable [20], while the B meson distribution amplitude in the collinear factorization theorem is not [22,23], when evolution effects are taken into account. The Sudakov resummation [4,8,24,25] of the large double logarithm $\ln^2 k_T$ is essential for improving perturbative expansion in the k_T factorization theorem [26,27].

The current application of the k_T factorization theorem to exclusive processes is made mainly at leading order (LO) in the strong coupling constant α_s [28]: The important logarithms in hadron wave functions have been organized to all orders, but hard kernels are still evaluated at tree level. To demonstrate that the k_T factorization theorem is a systematical tool, higher-order calculations of hard kernels are demanded. In this paper, we shall elucidate the framework for these calculations, deriving the next-to-leading-order (NLO) hard kernel for the scattering process $\pi\gamma^* \rightarrow \gamma$ as an example. The point is that partons in both the quark diagrams from full QCD and the effective diagrams for the pion wave function, carrying the momentum $k = (k^+, 0, \mathbf{k}_T)$, are off mass shell by k_T^2. The difference between the two sets of diagrams defines the hard kernel in the k_T factorization theorem, a procedure similar to the derivation of Wilson coefficients in an effective field theory. This is the way to obtain a k_T-dependent hard kernel without breaking gauge invariance, since the gauge dependences cancel between the above two sets of diagrams. A physical quantity is expressed as a convolution of a hard kernel with model wave functions, which are determined by methods beyond a perturbation theory, such as lattice QCD and QCD sum rules, or extracted from experimental data. A gauge-invariant hard kernel then leads to gauge-invariant predictions from the k_T factorization theorem.

We emphasize that the above prescription for computing a k_T-dependent gauge-invariant hard kernel has not yet been fully recognized. Several NLO calculations, which include the transverse momentum dependence via on shell partons carrying $k = (k^+, k^-, \mathbf{k}_T)$, $k^- = k_T^2/(2k^+)$, have been performed in the literature [21,29,30]. In these calculations, both quark diagrams and effective diagrams are gauge-invariant, and so are hard kernels. However, the considered parton momentum is not a configuration described by the nonlocal matrix elements associated with k_T-dependent hadron wave functions, because the minus

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component k^- should have been integrated out. Another subtlety is that the NLO hard kernel for the process $\pi\gamma^* \to \gamma$ obtained in the above formalism turns out to be k_T-independent [30]. The parton transverse degrees of freedom in the pion wave function are then integrated out, and the formalism reduces to the collinear factorization theorem. Moreover, we shall explain that the additional nonperturbative soft function introduced in Ref. [30] is not necessary for the k_T factorization theorem, since the infrared logarithms can be absorbed into the pion wave function completely.

As stated before, the light-cone singularities [19] in the naive definition for k_T-dependent hadron wave functions must be regularized. These singularities, not present in the quark diagrams, are not physical. If not regularized, higher-order hard kernels, computed as the difference of the quark diagrams and the effective diagrams, will be divergent. In this paper, we shall adopt the modified definition, in which the Wilson lines involved in the nonlocal matrix elements for hadron wave functions are rotated away from the light cone. After the subtraction of the singularities, a hard kernel depends on regularization schemes unavoidably, which can, nevertheless, be regarded as part of the factorization-scheme dependence. This dependence, usually minimized by adhering to a fixed prescription for deriving hard kernels, does not cause a problem. The removal of the light-cone singularities from wave functions and the gauge invariance of hard kernels are the two essential ingredients for making physical predictions from the k_T factorization theorem.

We shall demonstrate that the higher-order quark diagrams for $\pi\gamma^* \to \gamma$ generate two types of double logarithms $\ln^2(Q^2/k_T^2)$ and $\ln^2 x$, Q^2 being the large momentum transfer squared, from the loop correction to the virtual photon vertex. The former does not appear in the collinear factorization theorem, but the latter does [31,32]. It is found that the effective diagrams reproduce the same double logarithm $\ln^2(Q^2/k_T^2)$, which is then absorbed into the pion wave function and organized by k_T resummation [4,8,24,25]. The remaining double logarithm $\ln^2 x$ can be absorbed into the jet function and organized by threshold resummation [33]. Eventually, the hard kernel is free of any double logarithm, and its perturbative expansion is improved. It will be shown that the NLO corrections are only a few percent for the pion transition form factor involved in the scattering process $\pi\gamma^* \to \gamma$, if setting the factorization scale to the momentum transfer.

In Sec. II, we calculate the $O(\alpha_s)$ quark diagrams from full QCD, the $O(\alpha_s)$ effective diagrams for the pion wave function, and the $O(\alpha_s)$ jet function and then take their difference to obtain the $O(\alpha_s)$ hard kernel for $\pi\gamma^* \to \gamma$ in the k_T factorization theorem. The gauge invariance of the k_T-dependent hard kernel is proven to all orders in α_s by induction in Sec. III. Section IV is the conclusion.

II. $O(\alpha_s)$ k_T FACTORIZATION

In this section, we set up the framework for computing the hard kernel for the pion transition form factor in the k_T factorization theorem. The momentum P_1 of the pion and the momentum P_2 of the outgoing on shell photon are chosen as

$$P_1 = (P_1^+, 0, \mathbf{0}_T), \qquad P_2 = (0, P_2^-, \mathbf{0}_T). \tag{1}$$

The LO quark diagram, in which the antiquark $\bar{q}$ carries the on shell fractional momentum $k = (xP_1^+, 0, \mathbf{0}_T)$ and the internal quark carries $P_2 - k$, leads to the amplitude

$$G^{(0)}(x, Q^2) = \frac{\mathrm{tr}[\not\epsilon(\not P_2 - \not k)\gamma_\mu \not P_1 \gamma_5]}{(P_2 - k)^2} = -\frac{\mathrm{tr}[\not\epsilon \not P_2 \gamma_\mu \not P_1 \gamma_5]}{xQ^2}, \tag{2}$$

with the leading spin structure $\not P_1 \gamma_5$ of the pion and $Q^2 \equiv 2P_1 \cdot P_2$. We have suppressed other constant factors, such as the electric charge, the color number, and the pion decay constant, which are irrelevant in the following discussion.

The trivial factorization of Eq. (2) reads [7]

$$G^{(0)}(x, Q^2) = \int dx' d^2k'_T \Phi^{(0)}(x; x', k'_T) H^{(0)}(x', Q^2, k'_T),$$
$$\Phi^{(0)}(x; x', k'_T) = \delta(x - x')\delta(\mathbf{k}'_T),$$
$$H^{(0)}(x, Q^2, k_T) = -\frac{\mathrm{tr}[\not\epsilon \not P_2 \gamma_\mu \not P_1 \gamma^5]}{xQ^2 + k_T^2}. \tag{3}$$

Once we concentrate on the small x region, the treatment of the parton k_T differs from that in the collinear factorization theorem: k_T^2 in the denominator of Eq. (3) is not small compared to xQ^2, and the internal quark propagator should not be expanded into a power series in k_T^2 [26,34]. k_T in the numerator, being power suppressed by $1/Q$, is combined with three-parton meson wave functions to form a gauge-invariant set of higher-twist contributions as in the collinear factorization theorem. This special treatment of the parton k_T characterizes the distinction between the k_T and collinear factorizations [28]. Because of the zeroth-order wave function $\Phi^{(0)} \propto \delta(\mathbf{k}'_T)$, the LO hard kernel $H^{(0)}$ does not depend on the parton transverse momentum actually.

The $O(\alpha_s)$ quark diagrams corresponding to Eq. (2) from full QCD are displayed in Fig. 1, in which the upper line represents the q quark. The factorization of the collinear divergences from these radiative corrections is referred to [7]:

$$G^{(1)}(x, Q^2) = \int dx' d^2k'_T [\Phi^{(1)}(x; x', k'_T) H^{(0)}(x', Q^2, k'_T) + \Phi^{(0)}(x; x', k'_T) H^{(1)}(x', Q^2, k'_T)], \tag{4}$$

where the $O(\alpha_s)$ effective diagrams $\Phi^{(1)}$ are defined by the leading-twist quark-level wave function [7,35]

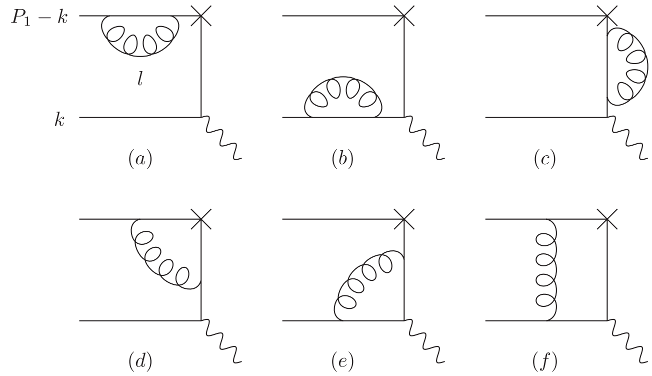

FIG. 1. $O(\alpha_s)$ quark diagrams for $\pi\gamma^* \to \gamma$ with $\times$ representing the virtual photon vertex.

$$\Phi(x; x', k'_T) = \int \frac{dy^-}{2\pi i} \frac{d^2 y_T}{(2\pi)^2} e^{-ix' P_1^+ y^- + i\mathbf{k}'_T \cdot \mathbf{y}_T} \langle 0 | \bar{q}(y) W_y(n)^\dagger \times I_{n;y,0} W_0(n) \not{n}_- \gamma_5 q(0) | q(P_1 - k) \bar{q}(k) \rangle, \quad (5)$$

with $y = (0, y^-, \mathbf{y}_T)$ being the coordinate of the antiquark field $\bar{q}$, $n_- = (0, 1, \mathbf{0}_T)$ a null vector along P_2, and $|q(P_1 - k)\bar{q}(k)\rangle$ the leading Fock state of the pion.

The factor $W_y(n)$, with $n^2 \neq 0$, denotes the Wilson line operator:

$$W_y(n) = P \exp\left[-ig \int_0^\infty d\lambda n \cdot A(y + \lambda n) \right]. \quad (6)$$

The two Wilson lines $W_y(n)$ and $W_0(n)$ are connected by a link $I_{n;y,0}$ at infinity in this case [7,36]. Equation (5) contains additional collinear divergences from the region with a loop momentum parallel to n_-, as the Wilson line direction approaches the light cone, i.e., as $n \to n_-$ [19]. It will be shown that n^2 serves as an infrared regulator for the light-cone singularities and that the wave function depends on the additional scale $\zeta^2 \equiv 4(n \cdot P_1)^2/|n^2|$, i.e., on the external kinematic variable. Besides, Φ also depends on the factorization scale μ_f, which is not shown explicitly. Note that Eq. (5) does not reduce to the distribution amplitude in the collinear factorization theorem directly, when integrated over k_T, but a convolution of a hard kernel with the distribution amplitude [37].

With one-gluon exchange, the outgoing partons from $\Phi^{(1)}$, i.e., the partons participating in the hard scattering, carry the transverse momenta, so that $H^{(0)}$ in Eq. (4) depends on k'_T nontrivially in the first-order factorization. Being convoluted with $\Phi^{(0)}$, the partons entering the NLO hard kernel $H^{(1)}$ are still on shell. To acquire the nontrivial k_T dependence, $H^{(1)}$ must be convoluted with the higher-order wave functions $\Phi^{(i)}$, $i \geq 1$: The gluon exchanges in $\Phi^{(i)}$ render the incoming partons of $H^{(1)}$, i.e., the incoming partons of the quark diagrams $G^{(1)}$ and the effective diagrams $\Phi^{(1)}$ off shell by k_T^2 [7]. We thus derive $H^{(1)}(x, Q^2, k_T)$ according to the formula

$$H^{(1)}(x, Q^2, k_T) = G^{(1)}(x, Q^2, k_T) - \int dx' d^2 k'_T \Phi^{(1)} \times (x, k_T; x', k'_T) H^{(0)}(x', Q^2, k'_T), \quad (7)$$

where $\Phi^{(1)}(x, k_T; x', k'_T)$ is defined by Eq. (5) but with the $\bar{q}$ quark momentum $k = (xP_1^+, 0, \mathbf{k}_T)$. As stated in the introduction, the gauge dependences of $G^{(1)}$ and $\Phi^{(1)}$ cancel in the above expression, such that $H^{(1)}(x, Q^2, k_T)$ turns out to be gauge-invariant.

A. Quark diagrams

The loop integrals associated with the $O(\alpha_s)$ quark diagrams in Figs. 1(a)–1(f), where the $\bar{q}$ quark carries the momentum $k = (xP_1^+, 0, \mathbf{k}_T)$ and the q quark carries $\bar{k} \equiv P_1 - k$, are written, in the Feynman gauge, as

$$G_a^{(1)}(x, Q^2, k_T) = \frac{-i}{2} g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \mathrm{tr}\left[\not{\epsilon} \frac{\not{P}_2 - \not{k}}{(P_2 - k)^2} \gamma_\mu \frac{\not{\bar{k}}}{\bar{k}^2} \gamma^\nu \frac{\not{\bar{k}} - \not{l}}{(\bar{k} - l)^2} \gamma_\nu \not{P}_1 \gamma_5 \right] \frac{1}{l^2}, \quad (8)$$

$$G_b^{(1)}(x, Q^2, k_T) = \frac{-i}{2} g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \mathrm{tr}\left[\gamma^\nu \frac{\not{k} - \not{l}}{(k - l)^2} \gamma_\nu \frac{\not{k}}{k^2} \not{\epsilon} \frac{\not{P}_2 - \not{k}}{(P_2 - k)^2} \gamma_\mu \not{P}_1 \gamma_5 \right] \frac{1}{l^2}, \quad (9)$$

$$G_c^{(1)}(x, Q^2, k_T) = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \mathrm{tr}\left[\not{\epsilon} \frac{\not{P}_2 - k}{(P_2 - k)^2} \gamma^\nu \frac{\not{P}_2 - \not{k} - \not{l}}{(P_2 - k - l)^2} \gamma_\nu \frac{\not{P}_2 - \not{k}}{(P_2 - k)^2} \gamma_\mu \not{P}_1 \gamma_5 \right] \frac{1}{l^2}, \quad (10)$$

$$G_d^{(1)}(x, Q^2, k_T) = -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \mathrm{tr}\left[\not{\epsilon} \frac{\not{P}_2 - \not{k}}{(P_2 - k)^2} \gamma^\nu \frac{\not{P}_2 - \not{k} + \not{l}}{(P_2 - k + l)^2} \gamma_\mu \frac{\not{\bar{k}} + \not{l}}{(\bar{k} + l)^2} \gamma_\nu \not{P}_1 \gamma_5 \right] \frac{1}{l^2}, \quad (11)$$

$$G_e^{(1)}(x, Q^2, k_T) = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \mathrm{tr}\left[\gamma_\nu \frac{\not{k} - \not{l}}{(k - l)^2} \not{\epsilon} \frac{\not{P}_2 - \not{k} + \not{l}}{(P_2 - k + l)^2} \gamma^\nu \frac{\not{P}_2 - \not{k}}{(P_2 - k)^2} \gamma_\mu \not{P}_1 \gamma_5 \right] \frac{1}{l^2}, \quad (12)$$

$$G_f^{(1)}(x, Q^2, k_T) = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \mathrm{tr}\left[\gamma^\nu \frac{\not k - \not l}{(k-l)^2} \not\epsilon \frac{\not P_2 - \not k + \not l}{(P_2 - k + l)^2} \gamma_\mu \frac{\bar{\not k} + \not l}{(\bar k + l)^2} \gamma_\nu \not P_1 \gamma_5\right] \frac{1}{l^2}. \quad (13)$$

The coefficients $1/2$ in Eqs. (8) and (9) arise from the definition of the self-energy corrections to external particles. C_F is a color factor, and μ is the renormalization scale.

We work in the dimensional reduction [38] to simplify the calculation and to avoid the ambiguity from handling γ_5 in arbitrary dimensions. The results for the self-energy corrections are

$$G_a^{(1)}(x, Q^2, k_T) = -\frac{\alpha_s}{8\pi} C_F\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2}{k_T^2 e^{\gamma_E}} + 2\right) \times H^{(0)}(x, Q^2, k_T), \quad (14)$$

$$G_b^{(1)}(x, Q^2, k_T) = -\frac{\alpha_s}{8\pi} C_F\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2}{k_T^2 e^{\gamma_E}} + 2\right) \times H^{(0)}(x, Q^2, k_T), \quad (15)$$

$$G_c^{(1)}(x, Q^2, k_T) = -\frac{\alpha_s}{4\pi} C_F\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2 e^{-\gamma_E}}{xQ^2 + k_T^2} + 2\right) \times H^{(0)}(x, Q^2, k_T), \quad (16)$$

where $1/\epsilon$ denotes the ultraviolet pole, and γ_E is the Euler constant. Since the external partons are off shell by k_T^2, the collinear divergences in Figs. 1(a) and 1(b) are represented by the infrared logarithms $\ln k_T^2$ in Eqs. (14) and (15), respectively. The internal quark in Fig. 1(c) is off shell by the invariant mass squared $xQ^2 + k_T^2$, which then replaces the argument k_T^2 in the infrared logarithm.

In the small x region, we drop terms suppressed by powers of x or k_T^2/Q^2. The loop correction to the virtual photon vertex gives

$$G_d^{(1)}(x, Q^2, k_T) = \frac{\alpha_s}{4\pi} C_F\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2}{k_T^2 e^{\gamma_E}} - 2\ln\frac{Q^2}{k_T^2} \times \ln\frac{Q^2}{xQ^2 + k_T^2} + 2\ln\frac{Q^2}{xQ^2 + k_T^2} + \ln\frac{Q^2}{k_T^2} - \frac{2\pi^2}{3} + \frac{3}{2}\right) H^{(0)}(x, Q^2, k_T). \quad (17)$$

At small x, the q quark in Fig. 1(d) is energetic, implying the existence of the collinear logarithmic enhancement $\ln(Q^2/k_T^2)$, and the internal quark is close to mass shell, implying the soft enhancement $\ln[Q^2/(xQ^2 + k_T^2)]$. Their overlap then leads to the double logarithm $\ln(Q^2/k_T^2) \times \ln[Q^2/(xQ^2 + k_T^2)]$ in Eq. (17). In the region with $x \sim O(1)$, the internal quark becomes off shell by $O(Q^2)$, the soft enhancement disappears as $\ln[Q^2/(xQ^2 + k_T^2)] \sim O(1)$, and the double logarithm reduces to a single logarithm. The result of $G_d^{(1)}$ clearly exhibits the transition of the double logarithm in the small x region to the single logarithm in the large x region.

The above double logarithm deserves more discussion, which can be reexpressed as

$$-2\ln\frac{Q^2}{k_T^2}\ln\frac{Q^2}{xQ^2 + k_T^2} = -\ln^2\frac{Q^2}{k_T^2} - \ln^2\frac{Q^2}{xQ^2 + k_T^2} + \ln^2\frac{xQ^2 + k_T^2}{k_T^2}. \quad (18)$$

The first term is known as the Sudakov logarithm [4,24], which will be absorbed into the pion wave function as stated before. The Sudakov effect from resumming this double logarithm suppresses the contribution from the small k_T region, i.e., the region with a large impact parameter [5]. The second term exists even in the collinear factorization theorem without taking into account k_T [31,39], $\ln[Q^2/(xQ^2 + k_T^2)] \sim \ln^2 x$, which cannot be factorized into the pion wave function. This threshold logarithm is important at small x, where the internal quark approaches mass shell. Hence, a jet function has been introduced to absorb $\ln^2 x$, and its resummation effect suppresses contributions from the small x region [33]. The third term, being of $O(1)$, does not require an all-order organization.

The loop correction to the outgoing on shell photon vertex is written as

$$G_e^{(1)}(x, Q^2, k_T) = \frac{\alpha_s}{4\pi} C_F\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2}{k_T^2 e^{\gamma_E}} + \ln\frac{xQ^2 + k_T^2}{k_T^2} + \frac{3}{2}\right) \times H^{(0)}(x, Q^2, k_T), \quad (19)$$

which does not contain a double logarithm for the following reason. In the large x region the internal quark is off shell by $O(Q^2)$, and the soft enhancement disappears. In the small x region, the $\bar q$ quark becomes soft, and the associated collinear enhancement is diminished by the limited phase space for the loop momentum. Therefore, there is a lack of overlap of the collinear and soft enhancements, and only the $O(1)$ single logarithm exists.

Finally, the evaluation of the box diagram in Fig. 1(f) is simple, giving a power-suppressed contribution at small x. In the region with $x \sim O(1)$, i.e., $k^+ \sim O(Q)$, the internal quark in Fig. 1(f) is off shell by $1/[P_2 \cdot (k-l)] \sim 1/Q^2$ for either a collinear loop momentum $l^+ \sim O(Q)$ or an ultraviolet loop momentum $l^\mu \sim O(Q)$, the same as $1/(P_2 \cdot k) \sim 1/Q^2$ in the LO amplitude. Namely, the radiative correction from the box diagram does not change the LO power-law behavior, and its contribution is finite. In the region with small $x \sim O(\Lambda)$, Λ being a hadronic scale, the LO amplitude scales like $1/(P_2 \cdot k) \sim 1/(Q\Lambda)$, while the internal quark in Fig. 1(f) remains off shell by $1/[P_2 \cdot$

$(k-l)] \sim 1/Q^2$ for either collinear or ultraviolet l. Thus, the contribution from the box diagram becomes power suppressed and negligible, and we have $G_f^{(1)}(x, Q^2, k_T) = 0$ at the leading power. The above observation is consistent with the corresponding NLO analysis in the collinear factorization theorem [31], which indicates the vanishing of the box-diagram contribution in the small x region explicitly.

The sum of the radiative corrections from the quark diagrams in Figs. 1(a)–1(f) gives

$$G^{(1)}(x, Q^2, k_T) = \sum_{i=a}^{f} G_i^{(1)}(x, Q^2, k_T) = -\frac{\alpha_s}{4\pi} C_F \left(2\ln\frac{Q^2}{k_T^2} \ln\frac{Q^2}{xQ^2 + k_T^2} - 3\ln\frac{Q^2}{k_T^2} + 1 + \frac{2\pi^2}{3} \right) H^{(0)}(x, Q^2, k_T). \quad (20)$$

It is observed that all of the ultraviolet poles cancel and the μ dependence disappears completely, a consequence of the conservation of the current that defines the pion transition form factor. It will be demonstrated in the next subsection that the effective diagrams for the pion wave function generate the same infrared logarithms $\ln k_T^2$.

B. Effective diagrams

We first explain the appearance of the nonphysical light-cone divergences in the naive definition for k_T-dependent hadron wave functions. To factor out the collinear gluons in Figs. 1(d) and 1(e), the following approximation for the product of the two internal quark propagators has been employed [35]:

$$\frac{2P_2^\nu}{(P_2 - k)^2 (P_2 - k + l)^2} \approx \frac{n_-^\nu}{n_- \cdot l} \left[-\frac{1}{xQ^2 + k_T^2} + \frac{1}{(x - l^+/P_1^+)Q^2 + |\mathbf{k}_T - \mathbf{l}_T|^2} \right], \quad (21)$$

where $2P_2^\nu$ comes from the contraction of $\not{P}_2$ and γ^ν in the numerators of Eqs. (11) and (12). The factor $n_-^\nu / n_- \cdot l$ is exactly the Feynman rule associated with the Wilson line along the light cone, which is necessary for the gauge invariance of the nonlocal matrix element in the pion wave function. The first (second) term in the above splitting corresponds to the case without (with) the loop momentum l flowing through the hard scattering. It is easy to see that the right-hand side of Eq. (21) is well-defined in the $n_- \cdot l = l^+ \to 0$ limit, if the transverse momenta k_T^2 and $|\mathbf{k}_T - \mathbf{l}_T|^2$ are dropped. That is, collinear factorization can be made gauge-invariant and free of the light-cone singularities. However, singularities from $l^+ \to 0$ are developed when the transverse momenta are included, implying that the factorization of collinear gluons should be performed more carefully in the k_T factorization theorem. This is the reason the naive definition is modified into Eq. (5) with the nonlightlike vector n, which makes finite $n \cdot l$ as $l^+ \to 0$.

The explicit expressions for the $O(\alpha_s)$ effective diagrams displayed in Figs. 2(a)–2(g) are written, following Eq. (5), as

$$\Phi_a^{(1)}(x, k_T; x', k_T') = -\frac{i}{8} g^2 C_F \mu_f^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \times \mathrm{tr}\left[\gamma_5 \not{n}_- \frac{\bar{\not{k}}}{\bar{k}^2} \gamma^\nu \frac{\bar{\not{k}} - \not{l}}{(\bar{k} - l)^2} \gamma_\nu \not{n}_+ \gamma_5 \right] \times \frac{1}{l^2} \delta(x - x') \delta(\mathbf{k}_T - \mathbf{k}_T'), \quad (22)$$

$$\Phi_b^{(1)}(x, k_T; x', k_T') = -\frac{i}{8} g^2 C_F \mu_f^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \times \mathrm{tr}\left[\gamma^\nu \frac{\not{k} - \not{l}}{(k - l)^2} \gamma_\nu \frac{\not{k}}{k^2} \gamma_5 \not{n}_- \not{n} \gamma_5 \right] \times \frac{1}{l^2} \delta(x - x') \delta(\mathbf{k}_T - \mathbf{k}_T'), \quad (23)$$

$$\Phi_c^{(1)}(x, k_T; x', k_T') = \frac{i}{4} g^2 C_F \mu_f^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \times \left[\gamma^\nu \frac{\not{k} - \not{l}}{(k - l)^2} \gamma_5 \not{n}_- \frac{\bar{\not{k}} + \not{l}}{(\bar{k} + l)^2} \gamma_\nu \not{n}_+ \gamma_5 \right] \times \frac{1}{l^2} \delta\left(x - x' - \frac{l^+}{P_1^+} \right) \delta(\mathbf{k}_T - \mathbf{k}_T' - \boldsymbol{l}_T), \quad (24)$$

$$\Phi_d^{(1)}(x, k_T; x', k_T') = -\frac{i}{4} g^2 C_F \mu_f^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \times \mathrm{tr}\left[\gamma_5 \not{n}_- \frac{\bar{\not{k}} + \not{l}}{(\bar{k} + l)^2} \gamma_\nu \not{n}_+ \gamma_5 \right] \times \frac{1}{l^2} \frac{n^\nu}{n \cdot l} \delta(x - x') \delta(\mathbf{k}_T - \mathbf{k}_T'), \quad (25)$$

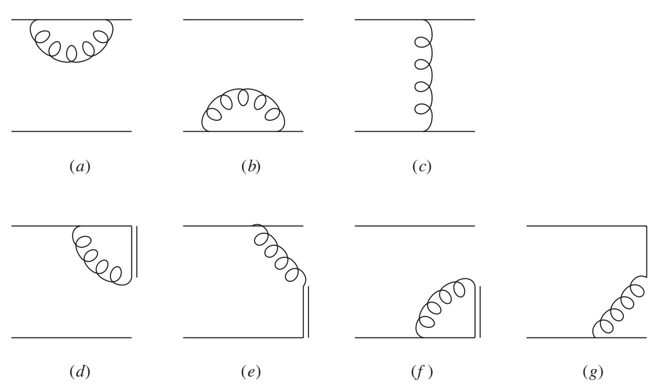

FIG. 2. $O(\alpha_s)$ effective diagrams for the pion wave function.

$$\Phi_e^{(1)}(x, k_T; x', k'_T) = \frac{i}{4} g^2 C_F \mu_f^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \times \mathrm{tr}\left[\gamma_5 \not{n}_- \frac{\bar{\not{k}} + \not{l}}{(\bar{k}+l)^2} \gamma_\nu \not{n}_+ \gamma_5\right] \frac{1}{l^2} \frac{n^\nu}{n \cdot l} \times \delta\left(x - x' - \frac{l^+}{P_1^+}\right) \delta(\mathbf{k}_T - \mathbf{k}'_T - \mathbf{l}_T), \tag{26}$$

$$\Phi_f^{(1)}(x, k_T; x', k'_T) = \frac{i}{4} g^2 C_F \mu_f^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \times \mathrm{tr}\left[\gamma_\nu \frac{\not{k} - \not{l}}{(k-l)^2} \gamma_5 \not{n}_- \not{n}_+ \gamma_5\right] \frac{1}{l^2} \frac{n^\nu}{n \cdot l} \times \delta(x - x') \delta(\mathbf{k}_T - \mathbf{k}'_T), \tag{27}$$

$$\Phi_g^{(1)}(x, k_T; x', k'_T) = -\frac{i}{4} g^2 C_F \mu_f^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \times \mathrm{tr}\left[\gamma_\nu \frac{\not{k} - \not{l}}{(k-l)^2} \gamma_5 \not{n}_- \not{n}_+ \gamma_5\right] \frac{1}{l^2} \frac{n^\nu}{n \cdot l} \times \delta\left(x - x' - \frac{l^+}{P_1^+}\right) \delta(\mathbf{k}_T - \mathbf{k}'_T - \mathbf{l}_T), \tag{28}$$

where $n_+ = (1, 0, \mathbf{0}_T)$ is a null vector along the pion momentum P_1, and the arguments μ_f and ζ^2 of $\Phi^{(1)}$ are not exhibited for brevity. Note that the zeroth-order wave function is given by $\Phi^{(0)} = \delta(x - x')\delta(\mathbf{k}_T - \mathbf{k}'_T)$ here.

We compute the convolution of $\Phi^{(1)}$ with the LO hard kernel $H^{(0)}$ in Eq. (3) over the integration variables x' and k'_T, denoted by $\otimes$ below:

$$\Phi_i^{(1)} \otimes H^{(0)} \equiv \int dx' d^2 k'_T \Phi_i^{(1)}(x, k_T; x', k'_T) H^{(0)}(x', Q^2, k'_T). \tag{29}$$

The self-energy corrections in Figs. 2(a) and 2(b) are similar to the quark diagrams in Figs. 1(a) and 1(b), respectively, and the results are

$$\Phi_a^{(1)} \otimes H^{(0)} = -\frac{\alpha_s}{8\pi} C_F \left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2}{k_T^2 e^{\gamma_E}} + 2\right) H^{(0)}(x, Q^2, k_T), \tag{30}$$

$$\Phi_b^{(1)} \otimes H^{(0)} = -\frac{\alpha_s}{8\pi} C_F \left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2}{k_T^2 e^{\gamma_E}} + 2\right) H^{(0)}(x, Q^2, k_T). \tag{31}$$

Similarly, the contribution from the box diagram in Fig. 2(c) is power suppressed in the small x region, and we have $\Phi_c^{(1)} \otimes H^{(0)} = 0$.

When evaluating Eqs. (25)–(28), the sign of the plus component n^+ of the vector n is arbitrary, which could be positive or negative (n^- has a positive sign, the same as that of P_2^-). Choosing $n^+ < 0$, i.e., $n^2 < 0$ as in Refs. [5,8,11,12], Fig. 2(d) leads, in the small x region, to

$$\Phi_d^{(1)} \otimes H^{(0)} = \frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2}{k_T^2 e^{\gamma_E}} - \ln^2\frac{\zeta^2}{k_T^2} + \ln\frac{\zeta^2}{k_T^2} + 2 - \frac{\pi^2}{3}\right) H^{(0)}(x, Q^2, k_T), \tag{32}$$

which reproduces the Sudakov logarithm $\ln^2(Q^2/k_T^2)$ from Fig. 1(d) in Eq. (18), noticing the scale $\zeta^2 = |n^-/n^+|Q^2$. The light-cone divergences are regularized in the price that the universality of a wave function is lost, for it depends on the external kinematic variable through ζ^2. This problem can be alleviated by extracting the evolution in ζ^2 from Eq. (5) [19], i.e., by resumming $\ln^2(\zeta^2/k_T^2)$ in Eq. (32) into the Sudakov factor [24,40]. The initial condition of the evolution is universal, like a distribution amplitude in the collinear factorization theorem. We stress that the Sudakov resummation, accurate up to fixed loops, does not remove the ζ^2 dependence of a wave function completely. That is, nonfactorizability may occur at a subleading level in the k_T factorization of the pion transition form factor.

The hard kernel associated with $\Phi_e^{(1)}$, i.e., the second term in Eq. (21), demands the physical range of l^+ to be $-\bar{k}^+ \leq l^+ \leq k^+$, which corresponds to the range of the parton momentum fraction $1 \geq x' \geq 0$. As when computing the convolution of $\Phi_e^{(1)}$ with $H^{(0)}$, this fact should be taken into account. Moreover, we assume $\zeta^2 \sim Q^2$ by choosing $|n^+| \sim n^-$ to avoid creating the additional large logarithm $\ln(\zeta^2/Q^2)$. The leading-power expression for Fig. 2(e) is then given, in the small x region, by

$$\Phi_e^{(1)} \otimes H^{(0)} = \frac{\alpha_s}{4\pi} C_F \ln^2\frac{\zeta^2(xQ^2 + k_T^2)}{Q^2 k_T^2} H^{(0)}(x, Q^2, k_T), \tag{33}$$

where terms vanishing with $k_T^2 \to 0$ have been dropped. It is found that Fig. 2(e) does not generate a large double logarithm with $\zeta^2 \sim Q^2$.

It is interesting to obtain the results corresponding to $n^+ > 0$ for Figs. 2(d) and 2(e). One simply analytically continues Eqs. (32) and (33) into the region with $n^2 > 0$ by means of the principle-value prescription:

$$P\left[\ln^2\frac{(n \cdot P_1)^2}{n^2}\right] = \ln^2\frac{(n \cdot P_1)^2}{|n^2|} - \pi^2, \qquad P\left[\ln\frac{(n \cdot P_1)^2}{n^2}\right] = \ln\frac{(n \cdot P_1)^2}{|n^2|}. \tag{34}$$

We then derive from Eq. (32)

$$\Phi_d^{(1)} \otimes H^{(0)} = \frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2}{k_T^2 e^{\gamma_E}} - \ln^2\frac{\zeta^2}{k_T^2} + \ln\frac{\zeta^2}{k_T^2} + 2 - \frac{4\pi^2}{3}\right) H^{(0)}(x, Q^2, k_T), \tag{35}$$

which can be confirmed by calculating the loop integral in Eq. (25) directly for $n^2 > 0$. It shows that the choices $n^2 > 0$ and $n^2 < 0$ lead to expressions different only by a constant term. Because the n-dependent double logarithms cancel in the summation

$$(\Phi_d^{(1)} + \Phi_e^{(1)}) \otimes H^{(0)} = \frac{\alpha_s}{4\pi} C_F \left[\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2}{k_T^2 e^{\gamma_E}} - 2\ln\frac{\zeta^2}{k_T^2} \times \ln\frac{Q^2}{xQ^2 + k_T^2} + \ln^2\frac{Q^2}{xQ^2 + k_T^2} + \ln\frac{\zeta^2}{k_T^2} + 2 - \frac{\pi^2}{3}\right] H^{(0)}(x, Q^2, k_T), \tag{36}$$

$(\Phi_d^{(1)} + \Phi_e^{(1)}) \otimes H^{(0)}$ does not depend on the sign of n^2 actually.

Applying the variable change $l \to -l$, and the transformations $n \to -n$ and $k \to \bar{k}$, Eq. (27) becomes identical to Eq. (25). Therefore, the result from Fig. 2(f) is the same as that of Fig. 2(d), but with the replacement of $\bar{k} \approx P_1$ by k, i.e., ζ by $x\zeta$. Keeping terms which do not vanish with $k_T^2 \to 0$, we have

$$\Phi_f^{(1)} \otimes H^{(0)} = \frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2}{k_T^2 e^{\gamma_E}} - \ln^2\frac{x^2\zeta^2}{k_T^2} + \ln\frac{x^2\zeta^2}{k_T^2} + 2 - \frac{\pi^2}{3}\right) H^{(0)}(x, Q^2, k_T), \tag{37}$$

where the double logarithm, being large in the region of $x \sim O(1)$, attenuates with the decrease of x. It should disappear, after combined with the contribution from Fig. 2(g), since such a double logarithm is absent in the corresponding quark diagram in Fig. 1(e) in any region of x. The same variable transformation relating $\Phi_f^{(1)}$ to $\Phi_d^{(1)}$ is not applicable to $\Phi_g^{(1)}$, for the latter involves the nontrivial convolution with $H^{(0)}$. Hence, $\Phi_g^{(1)} \otimes H^{(0)}$ is expected to have an expression different from $\Phi_e^{(1)} \otimes H^{(0)}$. Retaining terms which are finite as $k_T \to 0$, Fig. 2(g) leads, in the small x region with $xQ^2 \gg x^2\zeta^2$, to

$$\Phi_g^{(1)} \otimes H^{(0)} = \frac{\alpha_s}{4\pi} C_F \ln^2\frac{x^2\zeta^2}{k_T^2} H^{(0)}(x, Q^2, k_T). \tag{38}$$

The cancellation of the double logarithms in the summation of Eqs. (37) and (38) is obvious. For a similar reason, $(\Phi_f^{(1)} + \Phi_g^{(1)}) \otimes H^{(0)}$ is independent of the sign of n^2.

Summing all of the above $O(\alpha_s)$ quark-level wave functions, we derive

$$\Phi^{(1)} \otimes H^{(0)} = \sum_{i=a}^{g} \Phi_i^{(1)} \otimes H^{(0)} = \frac{\alpha_s}{4\pi} C_F \left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2}{k_T^2 e^{\gamma_E}} - \ln^2\frac{\zeta^2}{k_T^2} + \ln^2\frac{\zeta^2(xQ^2 + k_T^2)}{Q^2 k_T^2} + \ln\frac{\zeta^2}{k_T^2} + \ln\frac{x^2\zeta^2}{k_T^2} + 2 - \frac{2\pi^2}{3}\right) H^{(0)}(x, Q^2, k_T). \tag{39}$$

In contrast to Eq. (20), which is independent of the renormalization scale μ, the above expression depends on the factorization scale μ_f. The Sudakov resummation and the renormalization-group method can be applied to organize the logarithms $\ln^2(\zeta^2/k_T^2)$ and $\ln(\mu_f^2/k_T^2)$ to all orders, respectively [8].

C. $O(\alpha_s)$ hard kernel

We renormalize Eq. (39) in the modified minimal subtraction scheme and then take the difference of Eqs. (20) and (39) to obtain the $O(\alpha_s)$ hard kernel for the pion transition form factor. It is easy to find that the hard kernels $H_{a,b}^{(1)} \equiv G_{a,b}^{(1)} - \Phi_{a,b}^{(1)} \otimes H^{(0)}$, $H_c^{(1)} \equiv G_c^{(1)}$, $H_d^{(1)} \equiv G_d^{(1)} - (\Phi_d^{(1)} + \Phi_e^{(1)}) \otimes H^{(0)}$, $H_e^{(1)} \equiv G_e^{(1)} - (\Phi_f^{(1)} + \Phi_g^{(1)}) \otimes H^{(0)}$, and $H_f^{(1)} \equiv G_f^{(1)} - \Phi_c^{(1)} \otimes H^{(0)} = 0$ associated with Figs. 1(a)–1(f) are all free of the infrared logarithms $\ln k_T^2$ as claimed before. Compared to Ref. [30], we do not need the additional soft function S to achieve this cancellation. The difference is that the self-energy corrections to the Wilson lines have been included into the set of effective diagrams for the pion wave function in Ref. [30]. Hence, S must be introduced to remove these artificially included infrared divergences. We stress that the self-energy corrections to the Wilson lines do not exist, because such diagrams are not generated in the derivation of the factorization theorem using the diagrammatic approach [7]. This observation is consistent with the postulation that the gauge fields appearing in the Wilson lines in Eq. (6) are regarded as bare fields [19].

After subtracting the effective diagrams from the quark diagrams, the resultant hard kernel depends on the factorization scheme that defines the renormalization of Eq. (39). The quark diagrams do not have such a scheme dependence, as shown in Eq. (20). When making a physical prediction from the factorization theorem, one convolutes the hard kernel with a model for the pion wave function (not with the effective diagrams), so that the scheme dependence in the hard kernel remains. As stated in the introduction, the scheme dependence of physical predictions is usually minimized by adhering to a fixed prescription for deriving hard kernels, which will be elucidated below. The sum of the $O(\alpha_s)$ hard kernels is written as

$$H^{(1)}(x, Q^2, k_T) = \sum_{i=a}^{f} H_i^{(1)}(x, Q^2, k_T)$$
$$= \frac{\alpha_s}{4\pi} C_F\left(-\ln\frac{\mu_f^2}{xQ^2+k_T^2} + 2\ln\frac{\zeta^2}{Q^2} \times \ln\frac{Q^2}{xQ^2+k_T^2} - \ln^2\frac{Q^2}{xQ^2+k_T^2} + 2\ln\frac{Q^2}{x\zeta^2} + \ln\frac{Q^2}{xQ^2+k_T^2} - 3\right) \times H^{(0)}(x, Q^2, k_T). \quad (40)$$

The Sudakov logarithm $\ln^2(Q^2/k_T^2)$ in Eq. (18) for $G_d^{(1)}$ has been cancelled by that in Eq. (32) for $\Phi_d^{(1)} \otimes H^{(0)}$, but the threshold logarithm $\ln^2[Q^2/(xQ^2 + k_T^2)]$ remains in $H^{(1)}$. The large threshold logarithm can be absorbed into a jet function [33], so that the perturbative expansion of the hard kernel is further improved. At small x, a collinear enhancement arises from the region with the loop momentum parallel to the internal quark momentum $P_2 - k \approx P_2$. To factorize this collinear gluon into the jet function, we replace the q quark by the eikonal line in some direction u [40,41], as shown in Fig. 3(a). Similarly, we choose $u^2 \neq 0$ to avoid other infrared divergences, such as those from l parallel to P_1, which have been absorbed into the pion wave function. Including the self-energy correction to the internal quark [Fig. 3(b)], we arrive at the complete set of diagrams for the jet function at $O(\alpha_s)$.

Figure 3 has been evaluated in Ref. [33], focusing only on the double-logarithm piece $\ln^2 x$. Here we work out the single-logarithm and constant pieces, too. The explicit expression of the loop integral $J_a^{(1)}$ associated with Fig. 3(a) is referred to [33]. We obtain, for $u^2 < 0$,

$$J_a^{(1)} H^{(0)} = \frac{\alpha_s}{4\pi} C_F\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2 e^{-\gamma_E}}{xQ^2+k_T^2} - \ln^2\frac{\zeta_u^2}{xQ^2+k_T^2} + \ln\frac{\zeta_u^2}{xQ^2+k_T^2} + 2 - \frac{\pi^2}{3}\right) H^{(0)}(x, Q^2, k_T), \quad (41)$$

with the scale $\zeta_u^2 = 4(u \cdot P_2)^2/|u^2|$. Figure 3(b) gives a result identical to Eq. (16) for Fig. 1(c):

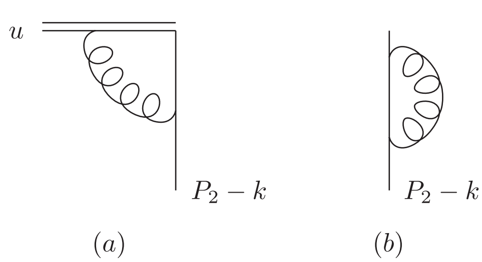

FIG. 3. $O(\alpha_s)$ diagrams for the jet function.

$$J_b^{(1)} H^{(0)} = -\frac{\alpha_s}{4\pi} C_F\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2 e^{-\gamma_E}}{xQ^2+k_T^2} + 2\right) H^{(0)}(x, Q^2, k_T). \quad (42)$$

Note that the sum $J^{(1)} = J_a^{(1)} + J_b^{(1)}$ is free of ultraviolet divergences and μ-independent. That is, the factorization of the jet function does not modify the renormalization-group behavior of the hard kernel. As expected, the jet function is characterized by the invariant mass of the internal quark.

Define $\zeta^2 = \nu Q^2$ and $\zeta_u^2 = \nu_u Q^2$, with ν and ν_u being constants of $O(1)$. The hard kernel, after subtracting the $O(\alpha_s)$ jet function, is given by

$$[H/J]^{(1)}(x, Q^2, k_T) \equiv H^{(1)}(x, Q^2, k_T) - J^{(1)}(x, Q^2, k_T) H^{(0)}(x, Q^2, k_T)$$
$$= -\frac{\alpha_s}{4\pi} C_F\left[\ln\frac{\mu_f^2}{xQ^2+k_T^2} - 2(\ln\nu + \ln\nu_u) \times \ln\frac{Q^2}{xQ^2+k_T^2} + 2\ln x 3 - \frac{\pi^2}{3} - \ln^2\nu_u + \ln\nu_u + 2\ln\nu\right] H^{(0)}(x, Q^2, k_T), \quad (43)$$

in which the double logarithms have been completely removed. Different values of ν and ν_u correspond to different factorization schemes. Adopting $\nu = 1$, i.e., $\zeta^2 = Q^2$ as in Ref. [5], and $\nu_u = 1$, Eq. (43) reduces to

$$[H/J]^{(1)}(x, Q^2, k_T) = -\frac{\alpha_s}{4\pi} C_F\left(\ln\frac{\mu_f^2}{xQ^2+k_T^2} + 2\ln x + 3 - \frac{\pi^2}{3}\right) H^{(0)}(x, Q^2, k_T). \quad (44)$$

Employing the factorization scale $\mu_f = Q$ and the asymptotic model of the pion wave function, the same as in the LO analysis in the k_T factorization theorem [42], the NLO corrections are found to be only 5%. That is, the NLO corrections are not expected to affect much the LO results for $\pi\gamma^* \to \gamma$. Our conclusion is drawn under the specific factorization scheme with $\nu = \nu_u = 1$. It requires an examination whether NLO corrections are also negligible under the same scheme in other exclusive processes containing pions, such as the pion form factor involved in $\pi\gamma^* \to \pi$.

III. GAUGE INVARIANCE

In this section, we prove the gauge invariance of the k_T-dependent hard kernel for the pion transition form factor by induction. We first show that the k_T factorization constructed in the Feynman gauge [7,35] holds in an arbitrary covariant gauge $\partial \cdot A = 0$ with the gauge parameter λ, in which the gluon propagator is given by

$(-i/l^2)N^{\mu\nu}(l)$ with the tensor

$$N^{\mu\nu}(l) = g^{\mu\nu} - (1-\lambda)\frac{l^\mu l^\nu}{l^2}. \quad (45)$$

It has been argued that the replacement

$$g^{\mu\nu} \to \frac{n_-^\mu l^\nu}{n_-\cdot l} = g^{\mu\alpha}\frac{n_{-\alpha} l^\nu}{n_-\cdot l} \quad (46)$$

for a collinear gluon propagator in the Feynman gauge extracts collinear divergences correctly [7,35]. In the arbitrary covariant gauge, we just need to modify the above replacement into

$$N^{\mu\nu}(l) \to \frac{n_-^\mu l^\nu}{n_-\cdot l} - (1-\lambda)\frac{l^\mu l^\nu}{l^2} = N^{\mu\alpha}(l)\frac{n_{-\alpha} l^\nu}{n_-\cdot l}, \quad (47)$$

and then the procedures for deriving the factorization theorem in Refs. [7,35] follow: The Ward identity is applied to all of the contractions of l^ν, leading to the factorization of the collinear gluon. The factor $n_{-\alpha}/n_-\cdot l$ explains how the Wilson lines are generated in factorizing hadron wave functions. For more details, refer to Refs. [7,35].

The k_T dependence in a hard kernel implies that the partons entering the quark diagrams and the effective diagrams for the pion wave function are off shell by k_T^2. The LO hard kernel $H^{(0)}(x, Q^2, k_T)$ in Eq. (3), which does not contain a gluon, is independent of the gauge parameter λ. Beyond LO, the gauge invariance of a hard kernel is a consequence of the gauge-dependence cancellation between the above two sets of diagrams. Assuming that the hard kernels defined by

$$H^{(j)}(x, Q^2, k_T) = G^{(j)}(x, Q^2, k_T) - \sum_{i=1}^{j}\int dx' d^2k'_T \times \Phi^{(i)}(x, k_T; x', k'_T)H^{(j-i)}(x', Q^2, k'_T) \quad (48)$$

are gauge-invariant for $j = 1, 2, \cdots N$, we shall prove the gauge invariance of the $O(\alpha_s^{N+1})$ hard kernel

$$H^{(N+1)}(x, Q^2, k_T) = G^{(N+1)}(x, Q^2, k_T) - \sum_{i=1}^{N+1}\int dx' d^2k'_T \times \Phi^{(i)}(x, k_T; x', k'_T)H^{(N+1-i)}(x', Q^2, k'_T), \quad (49)$$

using the method proposed in Ref. [43]. Note that the external quark spinors in the nonlocal matrix element in Eq. (5) have absorbed half of the self-energy corrections. Another half goes into the higher-order wave functions, giving the coefficients 1/2 in Eqs. (22) and (23). The same explanation applies to the appearance of 1/2 in Eqs. (8) and (9) for the $O(\alpha_s)$ quark diagrams. To discuss the gauge dependence, we consider the full self-energy corrections to the quark diagrams G and to the effective diagrams Φ.

Applying the differential operator $\lambda d/d\lambda$ to $H^{(N+1)}$, it acts only on the gluon propagators in $G^{(N+1)}$ and $\Phi^{(i)}$ on the right-hand side of Eq. (49), leading to

$$\lambda\frac{d}{d\lambda}N^{\mu\nu} = \lambda\frac{l^\mu l^\nu}{l^2} = v_\alpha(l^\mu N^{\alpha\nu} + N^{\mu\alpha}l^\nu), \quad (50)$$

with the special vertex $v_\alpha = l_\alpha/(2l^2)$. The derivatives $\lambda dH^{(N+1-i)}/d\lambda$ vanish due to the gauge-invariant assumption associated with Eq. (48). The loop momentum l^μ (l^ν) in Eq. (50) contracts with vertices in the diagrams of $G^{(N+1)}$ and $\Phi^{(i)}$, which are then replaced by the special vertex v_α. Summing all of the quark diagrams with various differentiated gluons and employing the Ward identity, only those in which the special vertex is located at the outer ends of the valence quark lines are left [35,43], as shown in Fig. 4(a). These diagrams come from the second terms in the following Ward identities associated with the quark and the antiquark, respectively:

$$\begin{aligned} &\frac{i(\bar{\not k}+\not l)}{(\bar k + l)^2}(-i\not l)\not P_1\gamma_5 = \not P_1\gamma_5 - \frac{\bar{\not k}+\not l}{(\bar k+l)^2}\bar{\not k}\not P_1\gamma_5, \\ &\not P_1\gamma_5(-i\not l)\frac{i(\not l-\not k)}{(l-k)^2} = \not P_1\gamma_5 + \not P_1\gamma_5\not k\frac{\not l-\not k}{(l-k)^2}, \end{aligned} \quad (51)$$

where $\not P_1\gamma_5$ is the leading spin structure appearing in the expressions for the quark diagrams. We have the similar Ward identities for the effective diagrams with $\not n_+\gamma_5$ being substituted for $\not P_1\gamma_5$. The first term is cancelled by one of the two terms from the contraction of l with the adjacent vertex. If all of the external quarks were on mass shell, i.e., $k_T = 0$, the second terms also vanish due to $\bar{\not k}\not P_1 = \not P_1\not k = 0$, implying $\lambda dG^{(N+1)}/d\lambda = 0$ and $\lambda d\Phi^{(i)}/d\lambda = 0$. That

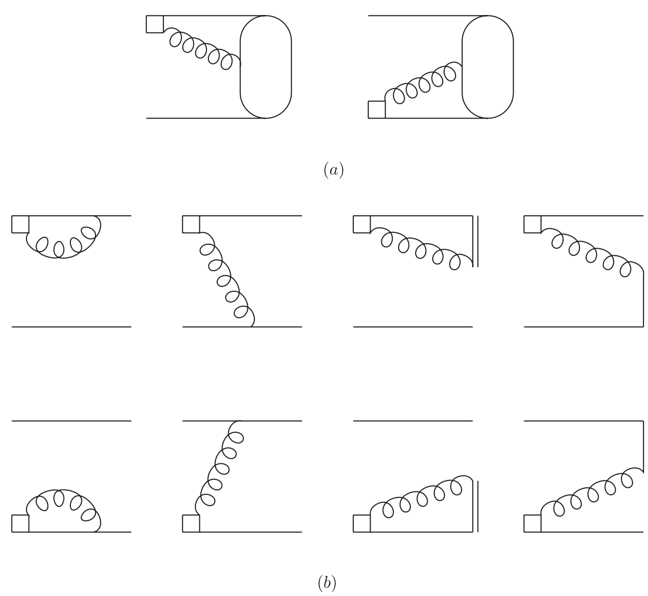

FIG. 4. (a) Diagrams for $\lambda dG^{(N+1)}/d\lambda$, where the bubbles represent $G^{(N)}$ and the squares contain the special vertex v_α. (b) Diagrams for $\lambda d\Phi^{(1)}/d\lambda$.

is, the quark diagrams from full QCD and the effective diagrams for the wave function with on shell partons are gauge-invariant.

For the differentiated quark diagrams $G^{(N+1)}$ in Fig. 4(a), the gluon emitting from the special vertex attaches all of the lines inside $G^{(N)}$. Adopting Eq. (47) and the procedures in Ref. [35], $\lambda dG^{(N+1)}/d\lambda$ is factorized into the convolution of $G^{(N)}$ with the differentiated $\Phi^{(1)}$ at leading power in $1/Q$:

$$\lambda \frac{d}{d\lambda} G^{(N+1)}(x, Q^2, k_T) = \int dx' d^2k'_T \lambda \frac{d}{d\lambda} \Phi^{(1)}(x, k_T; x', k'_T) \times G^{(N)}(x', Q^2, k'_T). \tag{52}$$

For illustration, we display the effective diagrams for $\lambda d\Phi^{(1)}/d\lambda$ in Fig. 4(b) explicitly. We repeat the above steps for the differentiated wave function $\Phi^{(i)}(x, k_T; x', k'_T)$ and obtain

$$\lambda \frac{d}{d\lambda} \Phi^{(i)}(x, k_T, x'', k''_T) = \int dx' d^2k'_T \lambda \frac{d}{d\lambda} \Phi^{(1)}(x, k_T; x', k'_T) \times \Phi^{(i-1)}(x', k'_T; x'', k''_T). \tag{53}$$

Combining Eqs. (52) and (53), the differentiation of the $O(\alpha_s^{N+1})$ hard kernel gives

$$\lambda \frac{d}{d\lambda} H^{(N+1)}(x, Q^2, k_T) = \int dx' d^2k'_T \lambda \frac{d}{d\lambda} \Phi^{(1)}(x, k_T; x', k'_T) \times \left[G^{(N)}(x', Q^2, k'_T) - \sum_{i=0}^{N} \int dx'' d^2k''_T \Phi^{(i)}(x', k'_T; x'', k''_T) \times H^{(N-i)}(x'', Q^2, k''_T) \right], \tag{54}$$

where the term in the square brackets diminishes because of Eq. (48) for $j = N$. We then prove the gauge invariance of the $O(\alpha_s^{N+1})$ hard kernel. The hard kernels and the resultant predictions from the k_T factorization theorem are thus gauge-invariant to all orders by induction.

IV. CONCLUSION

In this paper, we have elucidated the framework for the higher-order calculations in the k_T factorization theorem, which is appropriate for QCD processes dominated by contributions from small momentum fractions. The point is that partons in both the quark diagrams from full QCD and the effective diagrams for hadron wave functions are off mass shell by k_T^2. Their difference gives the gauge-invariant k_T-dependent hard kernels, since the gauge dependences cancel between the two sets of diagrams. The gauge invariance of the hard kernels for the scattering process $\pi\gamma^* \to \gamma$ in the k_T factorization theorem has been proven to all orders by induction. The proof can be easily generalized to other processes. We have explained that the light-cone divergences in a naive definition of k_T-dependent hadron wave functions are regularized by rotating the Wilson lines away from the light cone. This procedure introduces a regularization-scheme dependence, which, however, can be regarded as part of the factorization-scheme dependence and minimized by adhering to a fixed prescription for deriving hard kernels. The gauge invariance of a hard kernel and the removal of the light-cone singularities are the two essential ingredients for making physical predictions from the k_T factorization theorem.

We have calculated the NLO k_T-dependent hard kernel for $\pi\gamma^* \to \gamma$ in the region with a large momentum transfer Q^2 and a small momentum fraction x. We have demonstrated that the infrared logarithms $\ln k_T^2$, reflecting the collinear divergences, cancel between the quark diagrams and the effective diagrams exactly. Hence, there is no need to introduce the additional nonperturbative soft function in the k_T factorization theorem. The quark diagrams generate the double logarithms $\ln^2(Q^2/k_T^2)$ and $\ln^2 x$ from the loop correction to the virtual photon vertex. It has been shown that the former is absorbed into the pion wave function and the latter into the jet function, confirming the observations made in our previous works [8,33]. Note that the factorization of the jet function does not alter the renormalization-group behavior of the hard kernel. Eventually, the NLO corrections in $\pi\gamma^* \to \gamma$ amount only to 5% under a specific factorization scheme with the factorization scale set to the momentum transfer. NLO corrections under the same factorization scheme in other processes, such as the pion form factor and heavy-to-light transition form factors, will be examined elsewhere. Finally, we mention that the off-light-cone effects from $y_T \neq 0$ have been found to be sizable in some heavy-to-light correlators based on a QCD-sum-rule analysis recently [44].

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