$|V_{ub}|$ from exclusive semileptonic $B \to \pi$ decays revisited

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We update the extraction of $|V_{ub}|$ from exclusive semileptonic $B \to \pi$ decays, combining experimental partial branching fraction information with theoretical form-factor calculations, using the recently revised HPQCD results for the form factors f_+ and f_0. We use Omnès representations to provide the required parametrizations of the form factors. The extracted value is $|V_{ub}| = (3.47 \pm 0.29 \pm 0.03) \times 10^{-3}$, in striking agreement with $|V_{ub}|$ extracted using all other inputs in CKM fits and showing some disagreement with $|V_{ub}|$ extracted from inclusive semileptonic $B \to \pi$ decays.

In this short paper we update our extraction of $|V_{ub}|$ from combined experimental and theoretical information on exclusive semileptonic $B \to \pi$ decays in light of the recently revised values for the form factors f_+ and f_0 from the lattice QCD calculation by the HPQCD Collaboration [1]. Our analysis procedure and inputs are fully described in [2,3]. We combine experimental partial branching fraction information with theoretical calculations of both form factors, using Omnès representations to provide parametrizations of the form factors. The Omnès representation for $f_+(q^2)$ takes into account the existence of the B^* pole as described in [3].

We have used experimental partial branching fraction data from the tagged analyses of CLEO [4], Belle [5] and *BABAR* [6], and from the untagged analysis of *BABAR* [7,8]. When computing partial branching fractions, we have used $\tau_{B^0} = 1/\Gamma_{\text{Tot}} = 1.527 \pm 0.008 \times 10^{-12}$ s [9] for the B^0 lifetime. For theoretical form-factor inputs we use the light cone sum rule (LCSR) result $f_+(0) = f_0(0) = 0.258 \pm 0.031$ [10] and lattice QCD results from FNAL-MILC [11–14] [using the three $f_+(q^2)$ values quoted in [15] and reading off three values for $f_0(q^2)$ at the same q^2 points from [12]]. The lattice QCD results from HPQCD have recently been revised [1], and we note the updated HPQCD form-factor values in Table I.[1]

Our fit uses four evenly spaced Omnès subtraction points for each form factor, covering the range $0 \le q^2 \le q_{\text{max}}^2 = (m_B - m_\pi)^2$, together with the value of $|V_{ub}|$. The best-fit parameters are

$$|V_{ub}| = (3.47 \pm 0.29) \times 10^{-3}, \qquad f_+(0) = f_0(0) = 0.245 \pm 0.023, \qquad f_+(q_{\text{max}}^2/3) = 0.475 \pm 0.046,$$

$$f_+(2q_{\text{max}}^2/3) = 1.07 \pm 0.08, \qquad f_+(q_{\text{max}}^2) = 7.73 \pm 1.29, \qquad f_0(q_{\text{max}}^2/3) = 0.338 \pm 0.089, \tag{1}$$

$$f_0(2q_{\text{max}}^2/3) = 0.520 \pm 0.041, \qquad f_0(q_{\text{max}}^2) = 1.06 \pm 0.26.$$

The fit has $\chi^2/\text{dof} = 0.74$ for 24 degrees of freedom, while the Gaussian correlation matrix of fitted parameters is

$$\begin{pmatrix}
1 & -0.39 & -0.92 & -0.83 & -0.56 & -0.02 & 0.00 & 0.00 \\
& 1 & 0.19 & 0.48 & -0.04 & 0.06 & 0.00 & -0.01 \\
& & 1 & 0.77 & 0.59 & 0.01 & 0.00 & 0.00 \\
& & & 1 & 0.38 & 0.03 & 0.00 & 0.00 \\
& & & & 1 & 0.00 & 0.00 & 0.00 \\
& & & & & 1 & 0.25 & 0.83 \\
& & & & & & 1 & 0.21 \\
& & & & & & & 1
\end{pmatrix}. \tag{2}$$

In Fig. 1 we show the fitted form factors, the differential decay rate calculated from our fit, and the quantities $\log[(m_{B^*}^2 - q^2)f_+(q^2)/m_{B^*}^2]$ and $P\phi f_+$ [17] where the details of the fit and inputs can better be seen. The dashed magenta curve in the $P\phi f_+$ plot is a cubic polynomial fit in z to the output from our analysis.[2] The sum of squares of the coefficients in this polynomial is $A = \sum_n a_n^2 = 0.012$ which is consistent with being of order $(\Lambda/m_b)^3$ [19], where Λ is a hadronic scale and m_b is the b-quark mass, and safely satisfies the dispersive constraint $A \le 1$ [15].

From our fit we calculate the total branching fraction

$$B(B^0 \to \pi^- l^+ \nu) = (1.37 \pm 0.08 \pm 0.01) \times 10^{-4} \tag{3}$$

where the first uncertainty is from our fit and the second is from the uncertainty in the experimental B^0 lifetime. We

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[1]The changes to the results for f_0 are relatively small so we do not expect large effects on analyses based on these values alone, for example, using f_0 input to extract phase-shift information for s-wave elastic $B\pi$ scattering [16].

[2]Expressions for P, ϕ, and z can be found in [15]. We set $t_0 = s_{\text{th}}(1 - \sqrt{1 - q_{\text{max}}^2/s_{\text{th}}})$, where $s_{\text{th}} = (m_B + m_\pi)^2$, which is the "preferred choice," labeled BGLa, in [18]. This choice for t_0 ensures that $|z| \le 0.3$ for $0 \le q^2 \le q_{\text{max}}^2$.

TABLE I. Revised HPQCD results for the form factors f_+ and f_0 [1]. The error shown is statistical only: the systematic error for each input form factor value is 10%.

q^2/GeV^2	$f_+(q^2)$	$f_0(q^2)$
17.34	1.101 ± 0.053	0.561 ± 0.026
18.39	1.273 ± 0.099	0.600 ± 0.021
19.45	1.458 ± 0.142	0.639 ± 0.023
20.51	1.627 ± 0.185	0.676 ± 0.041
21.56	1.816 ± 0.126	0.714 ± 0.056

evaluate

$$\frac{1}{m_B} \frac{f_+(q^2_{\mathrm{max}})}{f_0(q^2_{\mathrm{max}})}\Big|_{B\pi} = 1.4 \pm 0.4 \ \mathrm{GeV}^{-1} \qquad (4)$$

to be compared to the corresponding quantity in $D \to \pi$ exclusive semileptonic decays, $1.4 \pm 0.1 \ \mathrm{GeV}^{-1}$ extracted from the unquenched lattice QCD results in [20]. We also calculate the combination,

$$|V_{ub}|f^+(0) = (8.5 \pm 0.8) \times 10^{-4}. \qquad (5)$$

A model-independent extraction of this combination can be performed by applying soft collinear effective theory (SCET) to $B \to \pi\pi$ decays and deriving a factorization result [21]. Our result compares well with $|V_{ub}|f^+(0) = 7.6 \pm 1.9 \times 10^{-4}$ quoted in [22] using the SCET/factorization approach.

We have assumed that the lattice input form-factor data have independent statistical errors and fully correlated systematic errors (but no correlations linking f_+ and f_0). Since we do not know these correlations, we have also performed fits with no correlations in the lattice inputs and assuming correlated systematic errors linking f_+ and f_0. We find that the central fitted value for $|V_{ub}|$ shifts by less than 0.03×10^{-3}, which we will apply as a systematic error for our extracted value:

$$|V_{ub}| = (3.47 \pm 0.29 \pm 0.03) \times 10^{-3}. \qquad (6)$$

This value differs by more than 1 standard deviation from
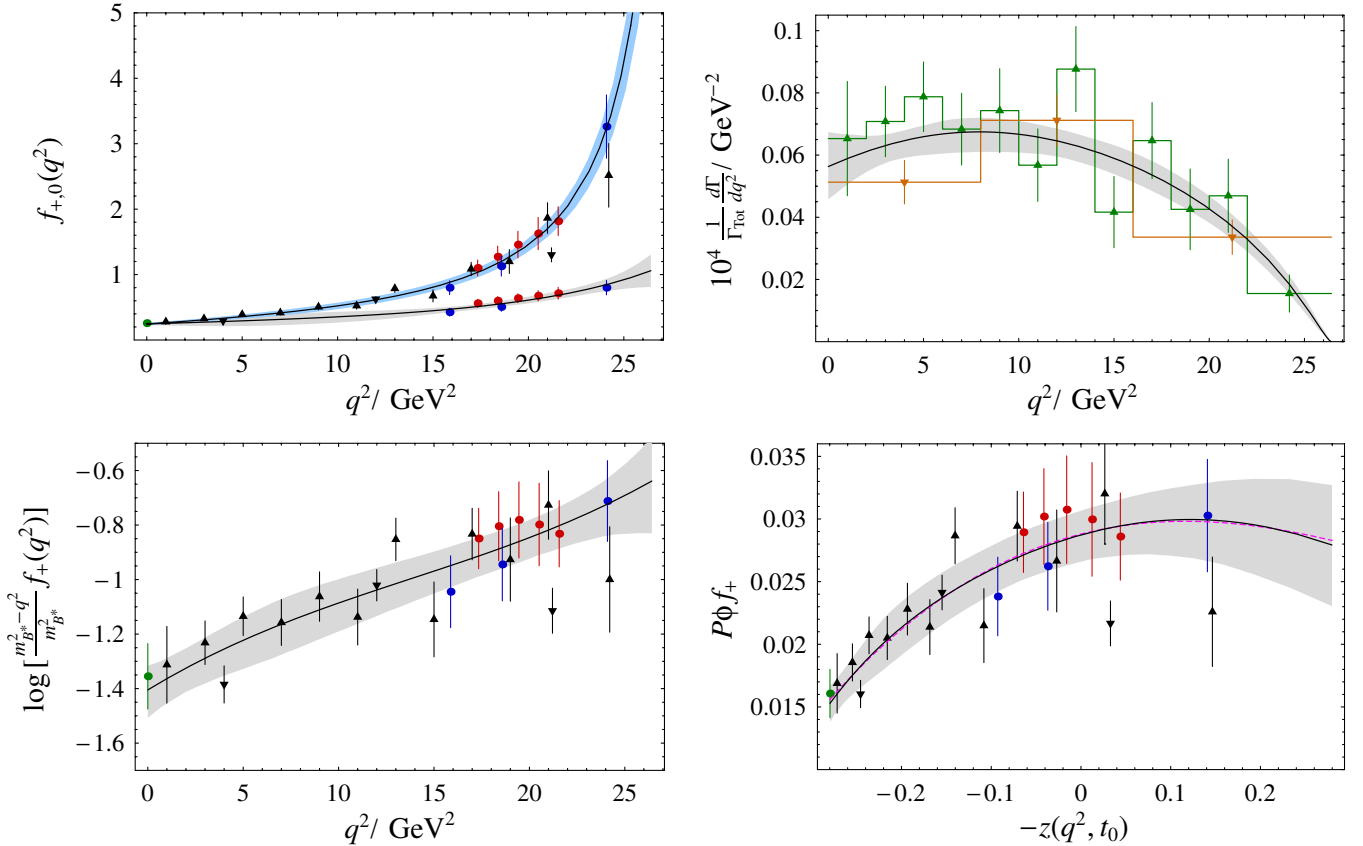

FIG. 1 (color online). Results obtained from the fit to experimental partial branching fraction data and theoretical form-factor calculations. The top left plot shows the two form factors with their error bands, the lattice and LCSR input points (dots: green LCSR, red HPQCD, blue FNAL-MILC), and "experimental" points (black triangles, upward pointing for tagged and downward pointing for untagged data) constructed by plotting at the center of each bin the constant form factor that would reproduce the partial branching fraction in that bin. The top right plot shows the differential decay rate together with the experimental inputs. The bottom plots provide more details of the inputs and fits by showing on the left $\log[(m^2_{B^*} - q^2)f_+(q^2)/m^2_{B^*}]$ as a function of q^2, and on the right $P\phi f_+$ as a function of $-z$. The dashed magenta curve in the bottom right plot is a cubic polynomial fit in z to the Omnès curve.

the $|V_{ub}|$ values extracted from inclusive semileptonic $B \rightarrow \pi$ decays and quoted in [9]. However, using the inclusive determinations with the highest efficiency and best theoretical control leads to $|V_{ub}| = (4.10 \pm 0.30_{exp} \pm 0.29_{th}) \times 10^{-3}$ [23] which is consistent with the value found here.

The result is in very good agreement with values for $|V_{ub}|$ coming from CKM fits using inputs apart from $|V_{ub}|$ itself. For example, the angles-only fit in [24] leads to $|V_{ub}| = (3.67 \pm 0.24) \times 10^{-3}$, while the UTfit Collaboration's result for $|V_{ub}|$ determined from all other inputs, including Winter 2007 updated information [25], is $|V_{ub}| = (3.44 \pm 0.16) \times 10^{-3}$.

The revised HPQCD results are in closer agreement with the FNAL-MILC results and lead to smaller $|V_{ub}|$. These groups use different methods for treating heavy quarks in their simulations, so the agreement is very encouraging. However, since they both use the same input gauge field ensembles, it remains very important that the outputs are confirmed by independent simulations. Both lattice QCD and light cone sum rules calculations of the $B \rightarrow \pi$ form factors, when combined with experimental partial branching fraction information, now agree on values of $|V_{ub}|$ around 3.5×10^{-3} or so [see Eq. (6) and also [15,18]], in striking agreement with the value obtained using all other inputs in global CKM fits. The hints of a disagreement with inclusive determinations of $|V_{ub}|$ are strengthened.

[1] E. Dalgic *et al.* (HPQCD Collaboration), Phys. Rev. D **73**, 074502 (2006); **75**, 119906(E) (2007).
[2] J. M. Flynn and J. Nieves, Phys. Rev. D **75**, 013008 (2007).
[3] J. M. Flynn and J. Nieves, Phys. Lett. B **649**, 269 (2007).
[4] S. B. Athar *et al.* (CLEO Collaboration), Phys. Rev. D **68**, 072003 (2003).
[5] T. Hokuue *et al.* (Belle Collaboration), Phys. Lett. B **648**, 139 (2007).
[6] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **97**, 211801 (2006).
[7] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **98**, 091801 (2007).
[8] See EPAPS Document No. E-PRVDAQ-76-R03715 for tables of systematic errors and error matrices. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[9] E. Barberio *et al.* (Heavy Flavor Averaging Group), arXiv:0704.3575.
[10] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005).
[11] M. Okamoto *et al.*, Nucl. Phys. B, Proc. Suppl. **140**, 461 (2005).
[12] M. Okamoto, Proc. Sci., LAT2005 (2006) 013.
[13] P. B. Mackenzie *et al.* (Fermilab Lattice, MILC, and HPQCD Collaboration), Proc. Sci., LAT2005 (2006) 207.
[14] R. S. Van de Water and P. Mackenzie, Proc. Sci., LAT2006 (2006) 097.
[15] M. C. Arnesen, B. Grinstein, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. Lett. **95**, 071802 (2005).
[16] J. M. Flynn and J. Nieves, Phys. Rev. D **75**, 074024 (2007).
[17] R. J. Hill, arXiv:hep-ph/0606023.
[18] P. Ball, Phys. Lett. B **644**, 38 (2007).
[19] T. Becher and R. J. Hill, Phys. Lett. B **633**, 61 (2006).
[20] C. Aubin *et al.* (Fermilab Lattice Collaboration), Phys. Rev. Lett. **94**, 011601 (2005).
[21] C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004).
[22] I. W. Stewart, in Proceedings of CKM2006: 4th International Workshop on the CKM Unitarity Triangle, Nagoya, Japan, 2006.
[23] M. Neubert, in Proceedings of FPCP 2007: 5th Conference on Flavor Physics and *CP* Violation, Bled, Slovenia, 2007.
[24] M. Bona *et al.* (UTfit Collaboration), J. High Energy Phys. 10 (2006) 081.
[25] UTfit Collaboration, http://utfit.roma1.infn.it/ckm-results/ckm-results.html. CKM fits including Winter 2007 updates.