

**Hard thermal effective action in QCD through the thermal operator**Ashok Das<sup>1,2</sup> and J. Frenkel<sup>3</sup><sup>1</sup>*Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627-0171, USA*<sup>2</sup>*Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta 700064, India*<sup>3</sup>*Instituto de Física, Universidade de São Paulo, São Paulo, SP 05315-970, Brazil*

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We derive in a simple way the well-known hard thermal effective action for QCD through the application of the thermal operator to the zero temperature retarded Green's functions. This derivation also clarifies the origin of important properties of the hard thermal effective action, such as the manifest Lorentz and gauge invariance of its integrand, by relating them directly to the properties of the corresponding zero temperature effective action in the hard regime.

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**I. INTRODUCTION**

The high temperature limit of thermal QCD is of much interest because of its relevance in the study of the physical properties of the quark-gluon plasma [1,2]. Several important features in this regime are embodied in the leading hard thermal effective action, whose overall coefficient is proportional to  $T^2$ , where  $T$  denotes the equilibrium temperature. Such leading contributions to the effective action arise at one loop from diagrams where the internal momentum is hard (being of the order  $T$ ) and is much larger than any external momenta. The hard thermal effective action in QCD, which enjoys various symmetry properties (at the integrand level) such as manifest Lorentz and gauge invariance [3,4], has also been studied from various points of view [5–10].

The purpose of this note is to present a simple derivation of this action which may explain the origin of these interesting symmetry properties. The derivation is based on an interesting relation between Feynman graphs at finite temperature and the corresponding zero temperature graphs [11,12], which holds both in the imaginary time as well as in the real time formalisms [1,2,13]. This relation, known as the thermal operator representation, arises as a consequence of the fact that the thermal propagator for a bosonic field can be related to the zero temperature one through a simple thermal operator which carries the entire temperature dependence and has the explicit form

$$O^{(T)}(E) = 1 + n_B(E)(1 - S(E)). \quad (1)$$

Here  $E = \sqrt{k^2 + m^2}$ ,  $S(E)$  is a reflection operator that takes  $E \rightarrow -E$ , and  $n_B(E)$  represents the bosonic distribution function. (For a fermionic field a similar relation holds with  $n_B \rightarrow -n_F$  [12].) This relation between the finite temperature Feynman graphs and the zero temperature ones is computationally quite useful and allows us to study directly many questions of interest such as the cutting rules at finite temperature [14]. Furthermore, the relation between the retarded thermal Green's functions and the forward scattering amplitudes for on shell thermal particles

[15] has been clarified through the application of the thermal operator representation to the corresponding zero temperature forward scattering amplitudes [16]. In this paper, we further demonstrate the simplicity and the utility of the thermal operator representation by deriving the hard thermal effective action for QCD from the zero temperature retarded amplitudes in the hard regime. This derivation directly associates the origin of the interesting properties of the hard thermal effective action, such as Lorentz and gauge invariance of the integrand, to those of the zero temperature effective action.

The paper is organized as follows. In Sec. II, we show that in the hard region at zero temperature, the forward scattering amplitudes are Lorentz and gauge covariant and obey simple Ward identities. These features, together with the fact that the leading one loop contributions are quadratic in the hard internal momentum, are sufficient to determine uniquely all the hard  $n$ -point gluon amplitudes in terms of the hard gluon self-energy. An explicit example of how this works for the 3-point gluon amplitude is discussed in more detail in the appendix. In Sec. III, we are thus able to write down a generating functional (effective action) for all the hard  $n$ -point gluon amplitudes at zero temperature. Through the application of the thermal operator, it is then immediate to arrive at the well-known form of the hard thermal effective action. In this way, the symmetry properties of the action, such as Lorentz and gauge invariance, can be directly understood in terms of the properties of the zero temperature retarded amplitudes. We conclude with a brief summary in Sec. IV.

**II. HARD FORWARD SCATTERING AMPLITUDES AT ZERO TEMPERATURE**

The forward scattering amplitude associated with the retarded gluon self-energy at one loop,  $\Pi_{\mu\nu}^{ab}(p)$ , can be described as in Fig. 1, where “R” denotes a retarded propagator while the cut line with momentum  $k$  represents an on shell particle scattering in the forward direction [16]. In the hard region where the internal momentum  $k \gg p$ , with  $k^2 = 0$ , we can expand the denominator (of the re-

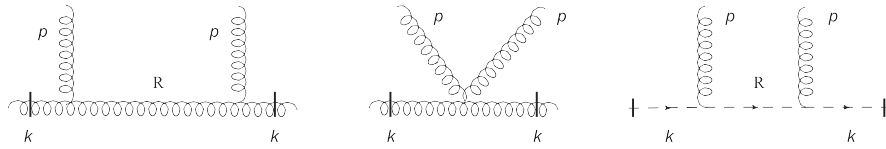


FIG. 1. Forward scattering graphs for the retarded gluon self-energy at one loop. Wavy and dashed lines denote, respectively, gluon and ghost particles. Color and Lorentz indices are suppressed for simplicity.

tarded propagator where  $k_0$  is to be understood as  $k_0 + i\epsilon$  in the graph as

$$\frac{1}{(k+p)^2} = \frac{1}{2k \cdot p} - \frac{p^2}{(2k \cdot p)^2} + \dots \quad (2)$$

The important thing to note here, for later use, is that this expansion does not involve any factor of  $p^2$  in the denominator.

Similarly, expanding the numerator of the graph in powers of the external momentum, we find that the leading hard contribution to the gluon self-energy,  $\Pi_{\mu\nu}^{ab}(p) = \delta^{ab}\Pi_{\mu\nu}(p)$ , in the  $SU(N)$  Yang-Mills theory can be written as [16]

$$\Pi_{\mu\nu}(p) = 2g^2N \int \frac{d^4k}{(2\pi)^3} \delta_+(k^2) \times \left( \eta_{\mu\nu} - \frac{k_\mu p_\nu + k_\nu p_\mu}{k \cdot p} + p^2 \frac{k_\mu k_\nu}{(k \cdot p)^2} \right), \quad (3)$$

where we have defined

$$\delta_+(k^2) = \theta(k_0)\delta(k^2). \quad (4)$$

There are several things to note from the structure of the gluon self-energy in (3). The quantity in the parenthesis is manifestly Lorentz covariant being of zero degree in the

internal momentum. Consequently, the integrand is quadratic in  $k$  for large values of the internal momentum and, as we will show shortly, this leads to the leading contribution of order  $T^2$  at high temperature. To apply the thermal operator, we need to carry out the integration over  $k_0$  [12] in which case (3) takes the form

$$\Pi_{\mu\nu}(p) = 2g^2N \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \times \left( \eta_{\mu\nu} - \frac{\hat{k}_\mu p_\nu + \hat{k}_\nu p_\mu}{\hat{k} \cdot p} + p^2 \frac{\hat{k}_\mu \hat{k}_\nu}{(\hat{k} \cdot p)^2} \right), \quad (5)$$

where  $E_k = |\vec{k}|$  and we have defined  $\hat{k}_\mu = (1, -\vec{k})$ . [We use a metric with signature  $(+, -, -, -)$ .] It is worth remarking here that the self-energy in (5) is gauge independent and satisfies the Ward identity

$$p^\mu \Pi_{\mu\nu}(p) = 0. \quad (6)$$

Next, let us consider the retarded  $n$ -point gluon forward scattering amplitude at one loop. As discussed in [16], this is described by graphs with a single on shell cut propagator, all other propagators in a graph being retarded/advanced. Thus, one can write such an amplitude in the form [compare with (3)]

$$\Gamma_{\mu_1 \mu_2 \dots \mu_n}^{a_1 a_2 \dots a_n}(p_1, p_2, \dots, p_n) = \int \frac{d^4k}{(2\pi)^3} \delta_+(k^2) \gamma_{\mu_1 \mu_2 \dots \mu_n}^{a_1 a_2 \dots a_n}(k, p_1, \dots, p_n), \quad (7)$$

where  $\gamma$  represents the forward scattering amplitude for an on shell particle of momentum  $k$ . For large values of  $k$ , we can again expand the denominators in  $\gamma$  [see (2)] in powers of  $p^2/(2k \cdot p)$ , where  $p$  denotes a linear combination of the external momenta. Furthermore, we can also expand the numerator in powers of  $p_{i,\mu}/|k|$ . The first term in the expansion comes from the leading term in each of the retarded propagators of the form  $1/(k \cdot p)$  and a numerator independent of  $p_i$ . The contributions to  $\gamma$  from these superleading terms, which are of degree one in  $k$ , however vanish by symmetry under  $k \rightarrow -k$ . The next term in the expansion which does provide the leading quadratic contribution to the  $n$ -point gluon amplitude comes from terms in  $\gamma$  which are functions of degree zero in  $k$  [15].

In the hard regime, to leading order, these amplitudes obey simple linear Ward identities. This follows as a consequence of the fact that diagrams with external (open) ghost lines, which appear in the Becchi-Rouet-Stora-

Tyutin identities, have one less power of  $k$  in the numerator compared with the corresponding diagrams involving only external gluon lines. As a result, the first term in the expansion of such a graphs is also naively quadratic in  $k$  much like the leading contribution for the gluon amplitude. However, in the case of graphs with external ghost lines, these leading contributions cancel by an eikonal identity when all graphs are added. Therefore, the contribution of the amplitude becomes subleading compared to the gluon amplitude. In the appendix, we illustrate how such a cancellation takes place for the one loop ghost-ghost-gluon (3-point) amplitude. As a consequence, the leading terms in the  $n$ -point gluon amplitude obey simple Ward identities. For example, to leading order, the 3-point gluon amplitude satisfies the relation

$$p_3^\lambda \Gamma_{\mu\nu\lambda}^{abc}(p_1, p_2, p_3) = igf^{abc}[\Pi_{\mu\nu}(p_1) - \Pi_{\mu\nu}(p_2)], \quad (8)$$

showing that it is related to the hard self-energy. Similarly, the Ward identities relate the 4-point gluon amplitude to the 3-point amplitude as

$$p_4^\rho \Gamma_{\mu\nu\lambda\rho}^{abcd}(p_1, p_2, p_3, p_4) = ig[f^{cde}\Gamma_{\mu\nu\lambda}^{abe}(p_1, p_2, p_3 + p_4) + f^{bde}\Gamma_{\mu\nu\lambda}^{cae}(p_1, p_2 + p_4, p_3) + f^{ade}\Gamma_{\mu\nu\lambda}^{bce}(p_1 + p_4, p_2, p_3)]. \quad (9)$$

The above properties are sufficient to determine uniquely to leading order all the  $n$ -point gluon amplitudes in terms of the hard gluon self-energy at zero temperature. This is discussed in detail in the appendix for the 3-point gluon amplitude, but similar arguments hold for all  $n$ -point gluon amplitudes. [We would like to emphasize here that, in general, the solution to the Ward identities such as (8) is not unique since the transverse part of the amplitude is not fully determined by the identity. However, in the hard region, the fact that the leading term has an integrand of zero degree in  $k$  and that the denominators have the form  $k \cdot p$  as in (2) is sufficient to determine all the amplitudes uniquely.]

### III. THE HARD EFFECTIVE ACTION AND ITS PROPERTIES

The simple Ward identities satisfied by the hard  $n$ -point gluon amplitudes can be written in a compact form as

$$D_\mu^{ab} \frac{\delta\Gamma[A]}{\delta A_\mu^b(x)} = (\delta^{ab}\partial_\mu - gf^{abc}A_\mu^c) \frac{\delta\Gamma[A]}{\delta A_\mu^b(x)} = 0, \quad (10)$$

where  $\Gamma[A]$  denotes the generating functional (effective action) and  $D_\mu^{ab}$  is the covariant derivative. Relation (10) implies that the effective action,  $\Gamma[A]$ , is invariant under an infinitesimal non-Abelian gauge transformation with parameter  $\omega^a(x)$ , namely, under

$$A_\mu^a \rightarrow A_\mu^{a(\omega)} = A_\mu^a + D_\mu^{ab}\omega^b, \quad \Gamma[A] \rightarrow \Gamma[A^{(\omega)}], \quad (11)$$

the infinitesimal change in the effective action is given by

$$\left. \frac{\delta\Gamma[A^{(\omega)}]}{\delta\omega^a} \right|_{\omega=0} = \frac{\delta A_\mu^{b(\omega)}}{\delta\omega^a} \frac{\delta\Gamma[A]}{\delta A_\mu^b} = -D_\mu^{ab} \frac{\delta\Gamma[A]}{\delta A_\mu^b} = 0, \quad (12)$$

where we have used a compact notation suppressing all the intermediate integrations.

Using (7) and performing the  $k_0$  integration, we can write the effective action to leading order in the form

$$\Gamma[A] = \int d^4x \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \gamma(A, k). \quad (13)$$

From our earlier discussion, we note that  $\gamma$  has the following characteristics:

- (1) It is gauge invariant.
- (2) It is a Lorentz invariant function of zero degree in  $k_\mu$

with  $k_0 = |\vec{k}|$ .

- (3) The integrand involves denominators with products of factors of the form  $k \cdot p$  where  $p$  is some linear combination of external momenta.

As we have discussed earlier, in the hard region, all the  $n$ -point gluon amplitudes are determined uniquely from the hard gluon self-energy. Therefore, any effective action satisfying the properties listed above and yielding the correct one loop hard gluon self-energy would correspond to the unique hard effective action. Comparing with the hard gluon self-energy in (5), it follows that the hard effective action at zero temperature can be written in the form

$$\Gamma[A] = \frac{g^2 N}{(2\pi)^3} \int d^4x \int \frac{d^3k}{2E_k} \mathcal{A}_\mu^a(x, \hat{k}) \mathcal{A}^{\mu a}(x, \hat{k}), \quad (14)$$

where the gauge covariant potential, introduced in [10], has the form

$$\mathcal{A}_\mu^a(x, \hat{k}) = \left( \frac{1}{\hat{k} \cdot D} \hat{k}^\nu F_{\nu\mu} \right)^a = A_\mu^a - \left( \frac{1}{\hat{k} \cdot D} \partial_\mu \hat{k} \cdot A \right)^a, \quad (15)$$

and  $F_{\mu\nu}^a$  denotes the non-Abelian field strength tensor

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c. \quad (16)$$

The gauge covariant non-Abelian potential,  $\mathcal{A}_\mu^a(x, \hat{k})$ , is in general a nonlocal function, where the nonlocality arises from retarded line integrals along the direction of  $\hat{k}$ .

Once the hard effective action (14) is determined at zero temperature, the hard thermal effective action can be determined by applying the thermal operator in the following manner. It is worth recalling [12,16] that the thermal operator acts on functions of energy in the integrand before the spatial momentum integrations are carried out. Furthermore, in spite of the fact that the perturbative expansion of the covariant potential in (15) involves retarded propagators, the thermal operator leaves such propagators invariant [16] so that the covariant gauge potential is unaffected by the application of the thermal operator. As a consequence, the only term in the integrand on which the thermal operator (1) acts nontrivially is the energy denominator (coming from the on shell cut propagator)

$$O^{(T)}(E_k) \frac{1}{2E_k} = \frac{1}{2E_k} (1 + 2n_B(E_k)). \quad (17)$$

Thus, the application of the thermal operator immediately leads to the temperature dependent hard thermal effective action as

$$\Gamma^{(\beta)}[A] = \frac{g^2 N}{(2\pi)^3} \times \int d^4x \int \frac{d^3k}{E_k} \frac{n_B(E_k)}{E_k} \mathcal{A}_\mu^a(x, \hat{k}) \mathcal{A}^{\mu a}(x, \hat{k}). \quad (18)$$

Since  $E_k = |\vec{k}|$ , the radial momentum integration can be carried out using the standard integral (with the Boltzmann constant set to unity)

$$\int_0^\infty dk k n_B(E_k) = \int_0^\infty dk \frac{k}{e^{k/T} - 1} = \frac{\pi^2 T^2}{6}, \quad (19)$$

which yields the hard thermal effective action

$$\Gamma^{(\beta)}[A] = \frac{g^2 T^2 N}{12} \int d^4x \int \frac{d\Omega}{4\pi} \mathcal{A}_\mu^a(x, \hat{k}) \mathcal{A}^{\mu a}(x, \hat{k}). \quad (20)$$

Here  $d\Omega$  represents the angular integration over the unit vector  $\hat{k}$ . Finally, integrating this expression by parts, we can recast it in the well-known form of the hard thermal effective action [3,4]

$$\Gamma^{(\beta)}[A] = \frac{m_{\text{gl}}^2}{2} \int d^4x \int \frac{d\Omega}{4\pi} F^{\mu\nu a} \left( \frac{\hat{k}_\nu \hat{k}^\lambda}{(\hat{k} \cdot D)^2} \right)^{ab} F_{\lambda\mu}^b, \quad (21)$$

where we have identified  $m_{\text{gl}}^2 = g^2 T^2 N/6$  as the square of the thermal gluon mass.

It is now straightforward to extend the above result for the pure Yang-Mills theory to QCD with quarks in the fundamental representation. To this end, we recall that the thermal operator representation works also for theories involving fermions [12]. In this case, as we have pointed out earlier, the thermal operator, relating the propagator at zero temperature to that at finite temperature, has a form similar to the one given in (1), with  $n_B(E) \rightarrow -n_F(E)$ , where  $n_F$  denotes the fermionic distribution function. Because of the linearity of the Ward identities in the hard regime, the contribution of the hard quark loops to an amplitude, which is additive, independently satisfies simple Ward identities like those given in Eqs. (8) and (9) [4]. As shown in the appendix, these identities together with the fact that the leading order terms in the integrand are Lorentz covariant functions of degree zero in the internal loop momentum are sufficient to determine uniquely all the higher  $n$ -point gluon amplitudes in the hard regime in terms of the corresponding gluon self-energy. Given the above properties of hard amplitudes as well as the transversality condition in (6), it follows that the integrand for the self-energy with a hard quark loop at zero temperature necessarily has a structure similar to the one given in (3), up to an overall multiplicative factor which can be calculated easily. Consequently, adding this contribution to that of the pure Yang-Mills theory in (3), the hard thermal effective action for QCD, which can be obtained by applying the thermal operator, has a similar form to the one given in (21) with the square of the thermal gluon mass given by

$$m_{\text{QCD}}^2 = \frac{g^2 T^2}{6} \left( N + \frac{1}{2} N_f \right), \quad (22)$$

where  $N_f$  denotes the number of quark flavors.

## IV. CONCLUSION

In this work, we have used the forward scattering description for the retarded amplitudes in QCD to construct the hard effective action at one loop at zero temperature. By applying the thermal operator to the zero temperature amplitudes, we have derived in a simple way the hard thermal effective action for QCD. This approach emphasizes that various relevant features of this action, such as Lorentz invariance of the integrand in (21) as well as its gauge invariance, simply arise because the leading hard zero temperature amplitudes (and the effective action) precisely have such properties. (The angular integration breaks Lorentz invariance at finite temperature, as expected, since the rest frame of the heat bath defines a preferred reference frame. An alternative form with a Lorentz noninvariant integrand has been given in [5].) The above properties of the hard thermal effective action turn out to be very convenient in the study of the energy-momentum tensor for the quark-gluon plasma, as well as in the analysis of the high temperature behavior of gauge field theories in a curved space-time [17]. Furthermore, the effective action (21) is also useful in implementing the resummation procedure [3] which is necessary for a consistent perturbative expansion in hard thermal QCD.

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## APPENDIX: HARD AMPLITUDES FROM WARD IDENTITIES

For definiteness, we discuss here in more detail some of the properties of the hard 3-point amplitudes at one loop. First, let us look at the ghost-ghost-gluon vertex,  $V_\mu^{abc}(p_1, p_2, p_3)$ , at one loop and show how the leading terms cancel in this case. Let us examine the forward scattering graphs associated with such a vertex correction at one loop shown in Fig. 2, where  $p_1 + p_2 + p_3 = 0$ .

In the hard regime (apart from an overall color factor  $f^{abc}$ ) these graphs have a common numerator  $(p_2 \cdot k)k_\mu$ . The presence of quadratic terms in  $k$  in the numerator means that the leading terms, as in (2), of the two retarded propagators (in the graph with  $k^2 = 0$ ) are sufficient to give us the leading terms of degree zero in  $k$ . However, when the three graphs are summed, this contribution cancels as a consequence of the eikonal identity

$$\frac{1}{(k \cdot p_1)(k \cdot p_2)} + \frac{1}{(k \cdot p_2)(k \cdot p_3)} + \frac{1}{(k \cdot p_3)(k \cdot p_1)} = 0. \quad (A1)$$

Therefore, the ghost-ghost-gluon amplitude has a subleading contribution compared with the 3-point gluon ampli-

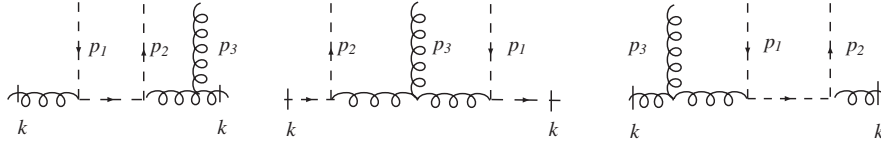


FIG. 2. Examples of forward scattering graphs associated with the retarded ghost-ghost-gluon vertex at one loop. Color and Lorentz indices are suppressed for simplicity.

tude, which as a result, satisfies the simpler Ward identity (8).

Let us next show that the Ward identity (8), together with the fact that to leading order the integrand of the 3-point gluon amplitude,  $\gamma_{\mu\nu\lambda}^{abc}(k, p_1, p_2, p_3)$ , is a function of zero degree in  $k$ , is sufficient to determine the hard 3-point gluon amplitude in terms of the hard gluon self-energy in (3). We note that if we factor out an overall color factor of  $f^{abc}$ , the Lorentz structure of the leading terms in  $\gamma_{\mu\nu\lambda}^{abc}$  (which are of degree zero in  $k$  and have the dimension of an inverse mass) can be parameterized in general as

$$Bk_{\mu}k_{\nu}k_{\lambda} + \sum_i (C_{1i}p_{i\mu}k_{\nu}k_{\lambda} + C_{2i}k_{\mu}p_{i\nu}k_{\lambda} + C_{3i}k_{\mu}k_{\nu}p_{i\lambda}) + (E_1k_{\mu}\eta_{\nu\lambda} + E_2k_{\nu}\eta_{\mu\lambda} + E_3k_{\lambda}\eta_{\mu\nu}), \quad (\text{A2})$$

where the coefficient functions  $B, C_{1i}, C_{2i}, C_{3i}, E_1, E_2, E_3$  are Lorentz invariant functions of  $k$  and  $p_i$ . The important point to note from (A2) is that it is at most linear in  $p_i$  as well as in the metric tensor. [It is worth remarking here that while a Lorentz structure such as  $D_{ij}p_{i\mu}p_{j\nu}k_{\lambda}$  is allowed in principle, for such a term to be of degree zero in  $k$  and have the inverse dimension of mass, the coefficient  $D_{ij}$  must contain a denominator of the form  $p^2$  where  $p$  denotes a linear combination of the external momenta. However, as we have pointed out earlier, such terms do not arise in the expansion (2) of the retarded propagator. This is why the numerator can at most be linear in the external momenta.]

Given the general Lorentz structure (A2), let us next consider the Ward identity (8) by contracting the amplitude with  $p_3^{\lambda}$ , which leads in the integrand to the Lorentz structure

$$\left( Bk \cdot p_3 + \sum_i C_{3i}p_i \cdot p_3 \right) k_{\mu}k_{\nu} + \sum_i (C_{1i}k \cdot p_3 + E_2\delta_{3i}) p_{i\mu}k_{\nu} + \sum_i (C_{2i}k \cdot p_3 + E_1\delta_{3i}) p_{i\nu}k_{\mu} + E_3k \cdot p_3\eta_{\mu\nu}. \quad (\text{A3})$$

We can now compare this structure to the ones on the right-hand side of the Ward identity in (8) coming from the structure of the self-energy in (3). Comparing the coefficients of the metric tensor on both sides, the last term in (A3), namely,  $E_3$ , is uniquely determined. Likewise, the Ward identity (8) where we contract with  $p_1^{\mu}$  or  $p_2^{\nu}$  determines respectively  $E_1$  and  $E_2$  uniquely. Similarly, matching the second and the third structures in (A3) with the corresponding structures coming from the self-energy (along with the other two identities) uniquely determines  $\sum_i C_{1i}p_{i\mu}$ ,  $\sum_i C_{2i}p_{i\nu}$ ,  $\sum_i C_{3i}p_{i\lambda}$ . Finally, comparing the  $k_{\mu}k_{\nu}$  terms on both sides of the Ward identity (8) then uniquely determines  $B$ . In this way, the leading 3-point gluon amplitude is uniquely determined in terms of the hard gluon self-energy. One can follow this argument to show that the leading higher point gluon amplitudes are recursively determined uniquely by the hard gluon self-energy. This fact allows us to determine the effective action for gluon amplitudes in the hard region in a unique form.

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