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Observation of incipient black holes and the information loss problem

Tanmay Vachaspati and Dejan Stojkovic
CERCA, Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106-7079, USA

Lawrence M. Krauss
CERCA, Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106-7079 (permanent address) and Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee, USA
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We study the formation of black holes by spherical domain wall collapse as seen by an asymptotic observer, using the functional Schrödinger formalism. To explore what signals such observers will see, we study radiation of a scalar quantum field in the collapsing domain wall background. The total energy flux radiated diverges when backreaction of the radiation on the collapsing wall is ignored, and the domain wall is seen by the asymptotic observer to evaporate by nonthermal ''pre-Hawking radiation'' during the collapse process. Evaporation by pre-Hawking radiation implies that an asymptotic observer can never lose objects down a black hole. Together with the nonthermal nature of the radiation, this may resolve the black hole information loss problem.

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I. INTRODUCTION

Black holes embody the long-standing theoretical challenge of combining general relativity and quantum mechanics, with various proposals being advocated over the years to resolve paradoxes associated with black hole formation, evaporation, and information loss. Resolution of these issues has become even more timely with the possible formation and evaporation of black holes in particle accelerators in the framework of higher dimensional models that have recently garnered much attention. The process of black hole formation is generally discussed from the viewpoint of an infalling observer. However, in all physical settings it is the viewpoint of the asymptotic observer that is relevant. More concretely, if a black hole is formed in the Large Hadron Collider, it has to be observed by physicists sitting on the CERN campus. The physicists have clocks in their offices and they watch the process of formation and evaporation in this coordinate frame. They must address questions such as: At what time did a black hole form? Is any information lost into the black hole? How long did it take for the black hole to evaporate? What is the spectrum of the decay products?

The process of gravitational collapse has been studied extensively over the last few decades, from many different viewpoints, including $1 + 1$ dimensional models and modifications of general relativity (e.g. see [1]). Unlike a large subset of this work, our analysis is in $3 + 1$ dimensions and within conventional general relativity. We model the general problem by choosing to study a collapsing spherical shell of matter, more specifically a vacuum domain wall. The physical setup of the problem and the functional Schrödinger formalism are described in Sec. II.

A crucial aspect of our analysis is that we address the question of black hole formation and evaporation *as seen by an asymptotic observer*. Initially, when the domain wall is large, the space-time is described by the Schwarzschild metric, just as for a static star. From here on, the wall and the metric are evolved forward in time, always using the Schwarzschild time coordinate. We emphasize that all our discussion, unless explicitly stated, refers to the Schwarzschild time t and this defines the time slicing of the space-time. As is well known, the Schwarzschild coordinate system breaks down at a black hole horizon, and there is danger that our analysis will also break down at some point during the gravitational collapse. However, we do not encounter any such difficulties, suggesting that our calculation is self-consistent. A second danger is that the coordinate system may provide an incomplete description of the gravitational collapse space-time. This remains a possibility. However, we find that Schwarzschild coordinates are sufficient to answer the very specific set of questions we ask from the asymptotic observer's viewpoint. Namely, does the asymptotic observer see objects disappear into a black hole in the time that he sees the collapsing body evaporate? And, is the spectrum of the radiation received ever truly thermal (even in the semiclassical approximation)?

In Sec. III we verify the standard result that the formation of an event horizon takes an infinite (Schwarzschild) time if we consider classical collapse. This is not surprising and is often viewed as a limitation of the Schwarzschild coordinate system. To see if this result changes when quantum effects are taken into account, we address the problem of quantum collapse using a minisuperspace version of the functional Schrödinger equation [2] in Sec. IV. We find that even in this case the black hole takes an infinite time to form, contrary to some speculations in the literature [3].

In Sec. V we consider the possible radiation associated with the collapsing shell by considering the interaction of a quantum scalar field and the classical background of a

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collapsing domain wall. We treat the problem using the functional Schrödinger picture, which we relate to the standard Bogoliubov treatment carried out in Sec. VI. Here we find that the shell, even as it collapses, radiates away its energy in a finite amount of time. With some assumptions about the metric close to the incipient horizon, we conclude that the evaporation time is shorter than what would be taken by objects to fall through a black hole horizon. This leads us to the conclusion that the asymptotic observer will see the evaporation of the collapsing shell before he can see any objects disappear.

We discuss our results from the point of view of an infalling observer in Sec. VII, where we attempt to reconcile the fact that such an observer will not see substantial radiation with the observations made by an asymptotic observer. Our conclusions are summarized in Sec. VIII, where we elucidate a possible resolution of the information loss problem suggested by our results, together with a discussion of possible loopholes and future directions.

II. SETUP AND FORMALISM

To study a concrete realization of black hole formation we consider a spherical Nambu-Goto domain wall that is collapsing. To include the possibility of (spherically symmetric) radiation we consider a massless scalar field, Φ, that is coupled to the gravitational field but not directly to the domain wall. The action for the system is

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G}\mathcal{R} + \frac{1}{2}(\partial_\mu \Phi)^2\right] - \sigma\int d^3\xi\sqrt{-\gamma} + S_{\rm obs}, \tag{1}$$

where the first term is the Einstein-Hilbert action for the gravitational field, the second is the scalar field action, the third is the domain wall action in terms of the wall world-volume coordinates, ξ^a ($a = 0, 1, 2$), the wall tension, σ, and the induced world-volume metric

$$\gamma_{ab} = g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu. \tag{2}$$

The coordinates $X^\mu(\xi^a)$ describe the location of the wall and Roman indices go over internal domain wall world-volume coordinates ζ^a, while Greek indices go over space-time coordinates. The term $S_{\rm obs}$ in Eq. (1) denotes the action for the observer.

We will begin first with the Wheeler-de Witt equation in order to explore and contrast quantum vs classical collapse of the domain wall, but we will eventually utilize the functional Schrödinger formalism to study both collapse and radiation in this system.

The Wheeler-de Witt equation for a closed universe is

$$H\Psi = 0, \tag{3}$$

where H is the Hamiltonian and $\Psi[X^\alpha, g_{\mu\nu}, \Phi, \mathcal{O}]$ is the wave functional for all the ingredients of the system, including the observer's degrees of freedom denoted by $\mathcal{O}$. Note that the wave functional Ψ is a functional of the fields but not of the space-time coordinates. We will separate the Hamiltonian into two parts, one for the system and the other for the observer

$$H = H_{\rm sys} + H_{\rm obs}. \tag{4}$$

Any (weak) interaction terms between the observer and the wall-metric-scalar system are included in $H_{\rm sys}$. The observer is assumed not to significantly affect the evolution of the system and similarly for the system *vis-à-vis* the observer. The total wave functional can be written as a sum over eigenstates

$$\Psi = \sum_k c_k \Psi^k_{\rm sys}({\rm sys}, t)\Psi^k_{\rm obs}(\mathcal{O}, t), \tag{5}$$

where k labels the eigenstates, c_k are complex coefficients, and we have introduced the observer time t via

$$i\frac{\partial \Psi^k_{\rm obs}}{\partial t} \equiv H_{\rm obs}\Psi^k_{\rm obs}. \tag{6}$$

With the help of an integration by parts, and the fact that the total wave functional is independent of t, the Wheeler-de Witt equation implies the Schrödinger equation

$$H_{\rm sys}\Psi^k_{\rm sys} = i\frac{\partial \Psi^k_{\rm sys}}{\partial t}. \tag{7}$$

For convenience, from now on we will denote the system wave function simply by Ψ and drop the superscript k and the subscript "sys." Similarly H will now denote $H_{\rm sys}$, and the Schrödinger equation reads

$$H\Psi = i\frac{\partial \Psi}{\partial t}. \tag{8}$$

A general treatment of the full Wheeler-de Witt equation is very difficult and we shall utilize the frequently employed strategy of truncating the field degrees of freedom to a finite subset. In other words, we will consider a minisuperspace version of the Wheeler-de Witt equation. As long as we keep all the degrees of freedom that are of interest to us, this is a useful truncation. With this in mind, we only consider spherical domain walls and assume spherical symmetry for all the fields. So the wall is described by only the radial degree of freedom $R(t)$. The metric is taken to be the solution of Einstein equations for a spherical domain wall. The metric is Schwarzschild outside the wall, as follows from spherical symmetry [4]

$$ds^2 = -\left(1 - \frac{R_S}{r}\right)dt^2 + \left(1 - \frac{R_S}{r}\right)^{-1}dr^2 + r^2 d\Omega^2, \quad r > R(t), \tag{9}$$

where $R_S = 2GM$ is the Schwarzschild radius in terms of the mass M of the wall, and

$$d\Omega^2 = d\theta^2 + r^2\sin^2\theta d\phi^2. \tag{10}$$

In the interior of the spherical domain wall, the line element is flat, as expected by Birkhoff's theorem,

$$ds^2 = -dT^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \quad r < R(t). \tag{11}$$

The interior time coordinate T is related to the observer time coordinate t via the proper time τ of the domain wall.

$$\frac{dT}{d\tau} = \left[1 + \left(\frac{dR}{d\tau}\right)^2\right]^{1/2} \tag{12}$$

and

$$\frac{dt}{d\tau} = \frac{1}{B}\left[B + \left(\frac{dR}{d\tau}\right)^2\right]^{1/2}, \tag{13}$$

where

$$B \equiv 1 - \frac{R_S}{R}. \tag{14}$$

The ratio of these equations gives

$$\frac{dT}{dt} = \frac{(1 + R_\tau^2)^{1/2} B}{(B + R_\tau^2)^{1/2}} = \left[B - \frac{(1 - B)}{B}\dot{R}^2\right]^{1/2}, \tag{15}$$

where $R_\tau = dR/d\tau$ and $\dot{R} = dR/dt$. Integrating Eq. (15) still requires knowing $R(\tau)$ (or $R(t)$) as a function of τ (or t).

Since we are restricting our minisuperspace to fields with spherical symmetry, we need not include any other metric degrees of freedom. The scalar field can also be truncated to the spherically symmetric modes ($\Phi = \Phi(t, r)$).

By integrating the equations of motion for the spherical domain wall, Ipser and Sikivie [4] found that the mass is a constant of motion and is given by

$$M = \tfrac{1}{2}[\sqrt{1 + R_\tau^2} + \sqrt{B + R_\tau^2}] 4\pi\sigma R^2, \tag{16}$$

where it is assumed that $\max(R) > 1/4\pi G\sigma$ [4]. This expression for M is implicit since $R_S = 2GM$ occurs in B. Solving for M explicitly in terms of R_τ gives

$$M = 4\pi\sigma R^2[\sqrt{1 + R_\tau^2} - 2\pi G\sigma R] \tag{17}$$

and with the relations between T and τ given above we get

$$M = 4\pi\sigma R^2\left[\frac{1}{\sqrt{1 - R_T^2}} - 2\pi G\sigma R\right], \tag{18}$$

where R_T denotes dR/dT.

We now discuss the classical collapse of the domain wall.

III. CLASSICAL TREATMENT OF DOMAIN WALL COLLAPSE

A naive approach to obtaining the dynamics for the spherical domain wall is to insert the spherical ansatz for both the wall and the metric in the original action. This would lead to an effective action for the radial coordinate $R(t)$. However, it is known that this approach does not give the correct dynamics for gravitating systems. We find that this approach does not straightforwardly lead to mass conservation as given in Eq. (16). So we take the alternative approach of finding an action that leads to the correct mass conservation law. The form of the action can be deduced from Eq. (18) quite easily

$$S_{\rm eff} = -4\pi\sigma \int dT R^2[\sqrt{1 - R_T^2} - 2\pi G\sigma R] \tag{19}$$

which can now be written in terms of the external time t

$$S_{\rm eff} = -4\pi\sigma \int dt R^2\left[\sqrt{B - \frac{\dot{R}^2}{B}} - 2\pi G\sigma R\sqrt{B - \frac{1 - B}{B}\dot{R}^2}\right] \tag{20}$$

and the effective Lagrangian is

$$L_{\rm eff} = -4\pi\sigma R^2\left[\sqrt{B - \frac{\dot{R}^2}{B}} - 2\pi G\sigma R\sqrt{B - \frac{1 - B}{B}\dot{R}^2}\right]. \tag{21}$$

The generalized momentum Π can be derived from Eq. (21)

$$\Pi = \frac{4\pi\sigma R^2 \dot{R}}{\sqrt{B}}\left[\frac{1}{\sqrt{B^2 - \dot{R}^2}} - \frac{2\pi G\sigma R(1 - B)}{\sqrt{B^2 - (1 - B)\dot{R}^2}}\right]. \tag{22}$$

The Hamiltonian (in terms of $\dot{R}$) is

$$H = 4\pi\sigma B^{3/2} R^2\left[\frac{1}{\sqrt{B^2 - \dot{R}^2}} - \frac{2\pi G\sigma R}{\sqrt{B^2 - (1 - B)\dot{R}^2}}\right]. \tag{23}$$

To obtain H as a function of (R, Π), we need to eliminate $\dot{R}$ in favor of Π using Eq. (22). This can be done but is messy, requiring solutions of a quartic polynomial. Instead we consider the limit when R is close to R_S and hence $B \to 0$. In this limit the denominators of the two terms in Eqs. (22) (also in (23)) are equal and

$$\Pi \approx \frac{4\pi\mu R^2 \dot{R}}{\sqrt{B}\sqrt{B^2 - \dot{R}^2}}, \tag{24}$$

where

$$\mu \equiv \sigma(1 - 2\pi G\sigma R_S) \tag{25}$$

and

$$H \approx \frac{4\pi\mu B^{3/2} R^2}{\sqrt{B^2 - \dot{R}^2}} \tag{26}$$

$$= [(B\Pi)^2 + B(4\pi\mu R^2)^2]^{1/2}. \tag{27}$$

The Hamiltonian has the form of the energy of a relativistic particle, $\sqrt{p^2 + m^2}$, with a position dependent mass.

The Hamiltonian is a conserved quantity and so, from Eq. (26),

$$\frac{B^{3/2} R^2}{\sqrt{B^2 - \dot{R}^2}} = h, \tag{28}$$

where $h = H/4\pi\mu$ is a constant. (Up to the approximation used to obtain the simpler form of the Hamiltonian in Eq. (26), the Hamiltonian is the mass.)

Solving Eq. (28) for $\dot{R}$ we get

$$\dot{R} = \pm B\left(1 - \frac{BR^4}{h^2}\right)^{1/2}, \tag{29}$$

which, near the horizon, takes the form

$$\dot{R} \approx \pm B\left(1 - \frac{1}{2}\frac{BR^4}{h^2}\right) \tag{30}$$

since $B \to 0$ as $R \to R_S$.

The dynamics for $R \sim R_S$ can be obtained by solving the equation $\dot{R} = \pm B$. To leading order in $R - R_S$, the solution is

$$R(t) \approx R_S + (R_0 - R_S)e^{\pm t/R_S}, \tag{31}$$

where R_0 is the radius of the shell at $t = 0$. As we are interested in the collapsing solution, we choose the negative sign in the exponent. This solution implies that, from the classical point of view, the asymptotic observer never sees the formation of the horizon of the black hole, since $R(t) = R_S$ only as $t \to \infty$. This result is similar to the well-known result (for example, see [5]) that it takes an infinite time for objects to fall into a preexisting black hole as viewed by an asymptotic observer [6]. In our case there is no preexisting horizon, which is itself taking an infinite amount of time to form during collapse. To see if this conclusion will change when quantum effects are taken into account (e.g. Sec. 10.1.5 of [3]) we now explore the quantum dynamics of the collapsing domain wall.

IV. QUANTUM TREATMENT OF DOMAIN WALL COLLAPSE

The classical Hamiltonian in Eq. (27) has a square root and so we first consider the squared Hamiltonian

$$H^2 = B\Pi B\Pi + B(4\pi\mu R^2)^2, \tag{32}$$

where we have made a choice for ordering B and Π in the first term. In general, we should add terms that depend on the commutator $[B, \Pi]$. However, in the limit $R \to R_S$, we find

$$[B, \Pi] \sim \frac{1}{R_S}.$$

Estimating H by the mass M of the domain wall, the terms due to the operating order ambiguity will be negligible provided

$$M \gg \frac{1}{R_S} \sim \frac{m_P^2}{M},$$

where m_P is the Planck mass. Hence the operator ordering ambiguity can be ignored for domain walls that are much more massive than the Planck mass.

Now we apply the standard quantization procedure. We substitute

$$\Pi = -i\frac{\partial}{\partial R} \tag{33}$$

in the squared Schrödinger equation,

$$H^2\Psi = -\frac{\partial^2\Psi}{\partial t^2}. \tag{34}$$

Then

$$-B\frac{\partial}{\partial R}\left(B\frac{\partial\Psi}{\partial R}\right) + B(4\pi\mu R^2)^2\Psi = -\frac{\partial^2\Psi}{\partial t^2}. \tag{35}$$

To solve this equation, define

$$u = R + R_S \ln\left|\frac{R}{R_S} - 1\right| \tag{36}$$

which gives

$$B\Pi = -i\frac{\partial}{\partial u}. \tag{37}$$

Equation (34) then gives

$$\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial u^2} + B(4\pi\mu R^2)^2\Psi = 0. \tag{38}$$

This is just the massive wave equation in a Minkowski background with a mass that depends on the position. Note that R needs to be written in terms of the coordinate u and this can be done (in principle) by inverting Eq. (36). However, care needs to be taken to choose the correct branch since the region $R \in (R_S, \infty)$ maps onto $u \in (-\infty, +\infty)$ and $R \in (0, R_S)$ onto $u \in (0, -\infty)$.

We are interested in the situation of a collapsing wall. In the region $R \sim R_S$, the logarithm in Eq. (36) dominates and

$$R \sim R_S + R_S e^{u/R_S}.$$

We look for wave packet solutions propagating toward R_S, that is, toward $u \to -\infty$. In this limit

$$B \sim e^{u/R_S} \to 0$$

and the last term in Eq. (38) can be ignored. Wave packet dynamics in this region is simply given by the free wave equation and any function of light-cone coordinates $(u \pm t)$

is a solution. In particular, we can write a Gaussian wave packet solution that is propagating toward the Schwarzschild radius

$$\Psi = \frac{1}{\sqrt{2\pi}s} e^{-(u+t)^2/2s^2}, \tag{39}$$

where s is some chosen width of the wave packet in the u coordinate. The width of the Gaussian wave packet remains fixed in the u coordinate while it shrinks in the R coordinate via the relation $dR = Bdu$ which follows from Eq. (36). This fact is of great importance, since if the wave packet remained of constant size in R coordinates, it might cross the event horizon in finite time.

The wave packet travels at the speed of light in the u coordinate—as can be seen directly from the wave equation (38) or from the solution, Eq. (39). However, to get to the horizon, it must travel out to $u = -\infty$, and this takes an infinite time. So we conclude that the quantum domain wall does not collapse to R_S in a finite time, as far as the asymptotic observer is concerned, so that quantum effects do not alter the classical result that an asymptotic observer does not observe the formation of an event horizon.

The above analysis shows that the collapsing wall takes an infinite time to reach $R = R_S$. The analysis leaves room for processes by which the wave packet can jump from the (R_S, ∞) region to the $(0, R_S)$ region, without ever going through R_S. Note that this is different from tunneling through a barrier. In that case, the wave function is nonzero within the barrier, and a small part of it leaks over to the other side of the barrier. In the present case, R_S occurs at $u = -\infty$ and so, if there is any barrier, it is infinitely far away. If there is to be a jump from outside to inside R_S, it does not show up in the present description using the Wheeler-de Witt equation.

We have obtained the massive wave equation (38), by first squaring the classical Hamiltonian, Eq. (27). This procedure eliminated the square root occurring in the Hamiltonian. It is possible that some other quantization procedure will yield different conclusions. In this context, we note, in fact, that we need not square the Hamiltonian to get rid of the square root provided we work in the near horizon limit. In that case

$$H = [(B\Pi)^2 + B(4\pi\mu R^2)^2]^{1/2} \approx \pm B\Pi, \tag{40}$$

where the sign is chosen to make H non-negative. Then the Schrödinger equation again yields wave packets propagating at the speed of light in the (t, u) coordinate system and with the horizon located at $u = -\infty$.

V. RADIATION-SEMICLASSICAL TREATMENT

If an external observer never sees the formation of an event horizon, we need to explore what radiation might be observed that characterizes gravitational collapse. To do so we consider a quantum scalar field in the background of the collapsing domain wall. We do not consider gravitational radiation since this is excluded by our restriction to spherically symmetric modes in minisuperspace. In this section, we approach the problem using the functional Schrödinger equation since (i) we have already set up this approach and used it in the previous section, (ii) we believe the approach is more suited to treating the backreaction problem, and (iii) it allows us to calculate the total radiation of which Hawking radiation may only be a subset. To connect with earlier work, we discuss the problem of Hawking radiation using the conventional Bogoliubov transformations in Sec. VI.

The action for the scalar field is

$$S = \int d^4x \sqrt{-g} \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi. \tag{41}$$

We decompose the (spherically symmetric) scalar field into a complete set of real basis functions denoted by $\{f_k(r)\}$

$$\Phi = \sum_k a_k(t) f_k(r). \tag{42}$$

The exact form of the functions $f_k(r)$ will not be important for us. We will be interested in the wave function for the mode coefficients $\{a_k\}$.

In the functional Schrödinger picture, we wish to find the wave functional $\Psi[\Phi; t]$ by solving a functional Schrödinger equation. This is equivalent to finding the wave function of an infinite set of variables, $\psi(\{a_k\}, t)$, by solving an ordinary Schrödinger equation in an infinite dimensional space. The procedure (detailed below) is to find independent eigenmodes, $\{b_k\}$, for the system for which the Hamiltonian is a sum of terms, one for each independent eigenmode. Then the wave function factorizes and can be found by solving a time-dependent Schrödinger equation of just one variable.

Since the metric inside and outside the shell have different forms, we split the action into two parts

$$S = S_{\rm in} + S_{\rm out}, \tag{43}$$

where

$$S_{\rm in} = 2\pi \int dt \int_0^{R(t)} drr^2 \left[-\frac{(\partial_t \Phi)^2}{\dot{T}} + \dot{T}(\partial_r \Phi)^2 \right], \tag{44}$$

$$S_{\rm out} = 2\pi \int dt \int_{R(t)}^{\infty} drr^2 \left[-\frac{(\partial_t \Phi)^2}{1 - R_S/r} + \left(1 - \frac{R_S}{r}\right)(\partial_r \Phi)^2 \right], \tag{45}$$

$\dot{T}$ is given by Eq. (15), which with Eq. (29), gives

$$\dot{T} = \frac{dT}{dt} = B\left[1 + (1 - B)\frac{R^4}{h^2}\right]^{1/2}. \tag{46}$$

As $R \to R_S$, $\dot{T} \sim B \to 0$. Therefore the kinetic term in $S_{\rm in}$ diverges as $(R - R_S)^{-1}$ in this limit and dominates over the softer logarithmically divergent contribution to the kinetic term from $S_{\rm out}$. Similarly the gradient term in $S_{\rm in}$ vanishes in this limit and is subdominant compared to the contribution coming from $S_{\rm out}$. Hence,

$$S \sim 2\pi \int dt\Big[-\frac{1}{B}\int_0^{R_S} drr^2(\partial_t\Phi)^2 + \int_{R_S}^{\infty} drr^2\Big(1-\frac{R_S}{r}\Big)(\partial_r\Phi)^2\Big], \quad (47)$$

where we have changed the limits of the integrations to R_S since we are working in the regime $R(t) \sim R_S$. This approximation is valid provided the contribution from $r \in (R_S, R(t))$ to the integrals remains subdominant, and also the time variation introduced by the true integration limit ($R(t)$) can be ignored. These requirements are not arduous.

Now, we use the expansion in modes in Eq. (42) to write

$$S = \int dt\Big[-\frac{1}{2B}\dot a_k \mathbf{M}_{kk'}\dot a_{k'} + \frac{1}{2}a_k\mathbf{N}_{kk'}a_{k'}\Big], \quad (48)$$

where $\mathbf{M}$ and $\mathbf{N}$ are matrices that are independent of $R(t)$ and are given by

$$\mathbf{M}_{kk'} = 4\pi\int_0^{R_S} drr^2 f_k(r)f_{k'}(r), \quad (49)$$

$$\mathbf{N}_{kk'} = 4\pi\int_{R_S}^{\infty} drr^2\Big(1-\frac{R_S}{r}\Big)f'_k(r)f'_{k'}(r). \quad (50)$$

Using the standard quantization procedure, the wave function $\psi(a_k, t)$ satisfies

$$\Big[\Big(1-\frac{R_S}{R}\Big)\frac{1}{2}\Pi_k(\mathbf{M}^{-1})_{kk'}\Pi_{k'} + \frac{1}{2}a_k\mathbf{N}_{kk'}a_{k'}\Big]\psi = i\frac{\partial\psi}{\partial t}, \quad (51)$$

where

$$\Pi_k = -i\frac{\partial}{\partial a_k} \quad (52)$$

is the momentum operator conjugate to a_k.

So the problem of radiation from the collapsing domain wall is equivalent to the problem of an infinite set of coupled harmonic oscillators whose masses go to infinity with time. Since the matrices $\mathbf{M}$ and $\mathbf{N}$ are symmetric and real (i.e. Hermitian), it is possible to do a principal axis transformation to simultaneously diagonalize $\mathbf{M}$ and $\mathbf{N}$ (see Sec. 6-2 of Ref. [7] for example). Then for a single eigenmode, the Schrödinger equation takes the form

$$\Big[-\Big(1-\frac{R_S}{R}\Big)\frac{1}{2m}\frac{\partial^2}{\partial b^2} + \frac{1}{2}Kb^2\Big]\psi(b,t) = i\frac{\partial\psi(b,t)}{\partial t}, \quad (53)$$

where m and K denote eigenvalues of $\mathbf{M}$ and $\mathbf{N}$, and b is the eigenmode.

We rewrite Eq. (53) in the standard form

$$\Big[-\frac{1}{2m}\frac{\partial^2}{\partial b^2} + \frac{m}{2}\omega^2(\eta)b^2\Big]\psi(b,\eta) = i\frac{\partial}{\partial\eta}\psi(b,\eta), \quad (54)$$

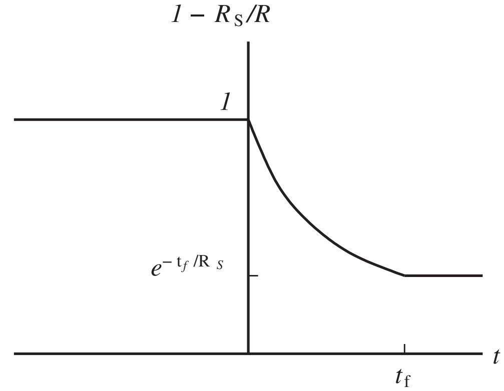

FIG. 1. Model for $R(t)$.

where

$$\eta = \int_0^t dt\Big(1-\frac{R_S}{R}\Big) \quad (55)$$

and

$$\omega^2(\eta) = \frac{K}{m}\frac{1}{1-R_S/R} \equiv \frac{\omega_0^2}{1-R_S/R}. \quad (56)$$

We have chosen to set $\eta(t=0) = 0$.

To proceed further, we need to choose the background space-time, i.e. the behavior of $R(t)$. The classical solution in Eq. (31), tells us that $1 - R_S/R \sim \exp(-t/R_S)$ at late times. We are mostly interested in the particle production during this period. At early times, the behavior depends on how the spherical domain wall was created and we are free to choose a behavior for $R(t)$ that is convenient for calculations and interpretation. To be able to interpret particle production at very late times it is easiest to have a static situation. This can be obtained if we artificially take the collapse to stop at some time, t_f. Eventually we can take $t_f \to \infty$ to go over to the eternal collapse case. So our choice for R will be

$$1-\frac{R_S}{R} = \begin{cases} 1 & t\in(-\infty,0) \\ e^{-t/R_S} & t\in(0,t_f) \\ e^{-t_f/R_S} & t\in(t_f,\infty) \end{cases} \quad (57)$$

as depicted in Fig. 1. This choice does have the issue that the derivative of R has discontinuities at $t=0$ and $t=t_f$. However, we shall show below that these discontinuities do not affect particle production.

With the chosen behavior of R, the space-time is static at early times and the initial vacuum state for the modes is the simple harmonic oscillator ground state,

$$\psi(b,\eta=0) = \Big(\frac{m\omega_0}{\pi}\Big)^{1/4} e^{-m\omega_0 b^2/2}. \quad (58)$$

Then the exact solution to Eq. (54) at later times is [8]

$$\psi(b,\eta) = e^{i\alpha(\eta)}\left(\frac{m}{\pi\rho^2}\right)^{1/4} \exp\left[i\frac{m}{2}\left(\frac{\rho_\eta}{\rho} + \frac{i}{\rho^2}\right)b^2\right], \quad (59)$$

where ρ_η denotes the derivative of $\rho(\eta)$ with respect to η, and ρ is given by the real solution of the ordinary (though nonlinear) differential equation

$$\rho_{\eta\eta} + \omega^2(\eta)\rho = \frac{1}{\rho^3} \quad (60)$$

with initial conditions

$$\rho(0) = \frac{1}{\sqrt{\omega_0}}, \qquad \rho_\eta(0) = 0. \quad (61)$$

The phase α is given by

$$\alpha(\eta) = -\frac{1}{2}\int_0^\eta \frac{d\eta'}{\rho^2(\eta')}. \quad (62)$$

In Appendix A we discuss the behavior of ρ as $\eta \to R_S$ $(t \to \infty)$. Also note that the solution for ρ and ρ_η is continuous.

Consider an observer with detectors that are designed to register particles of different frequencies for the free field ϕ at early times. Such an observer will interpret the wave function of a given mode b at late times in terms of simple harmonic oscillator states, $\{\varphi_n\}$, at the *final* frequency,

$$\bar{\omega} = \omega_0 e^{t_f/2R_S}. \quad (63)$$

The number of quanta in eigenmode b can be evaluated by decomposing the wave function (Eq. (59)) in terms of the states, $\{\varphi_n\}$, and by evaluating the occupation number of that mode. To implement this evaluation, we start by writing the wave function for a given mode at time $t > t_f$ in terms of the simple harmonic oscillator basis at $t = 0$.

$$\psi(b,t) = \sum_n c_n(t)\varphi_n(b), \quad (64)$$

where

$$c_n = \int db\,\varphi_n^*(b)\psi(b,t) \quad (65)$$

which is an overlap of a Gaussian with the simple harmonic oscillator basis functions. The occupation number at eigenfrequency $\bar{\omega}$ (i.e. in the eigenmode b) by the time $t > t_f$, is given by the expectation value

$$N(t,\bar{\omega}) = \sum_n n|c_n|^2. \quad (66)$$

In Appendix B we evaluate the occupation number in the eigenmode b and the result is given in Eq. (B12)

$$N(t,\bar{\omega}) = \frac{\bar{\omega}\rho^2}{\sqrt{2}}\left[\left(1 - \frac{1}{\bar{\omega}\rho^2}\right)^2 + \left(\frac{\rho_\eta}{\bar{\omega}\rho}\right)^2\right] \quad (67)$$

for $t > t_f$.

By calculating $\dot{N}$, it can be checked that N remains constant for $t < 0$ and also $t > t_f$. Hence all the particle production occurs for $0 \le t \le t_f$. There is a possibility that the particle production is due to discontinuities in the derivative of R at $t = 0, t_f$. However, as we shall see below, the particle number grows with increasing t_f, while the discontinuity at $t = 0$ is fixed, and that at $t = t_f$ gets weaker. This indicates that particle production occurs only during $0 < t < t_f$ and is a consequence of the gravitational collapse.

Now we can take the $t_f \to \infty$ limit. In Appendix A we have shown that ρ remains finite but $\rho_\eta \to -\infty$ as $t > t_f \to \infty$, provided $\omega_0 \neq 0$. However, we are interested in the behavior of N for fixed frequency, $\bar{\omega}$. Since $\bar{\omega} = \omega_0 e^{+t_f/2R_S}$, $t_f \to \infty$ also implies $\omega_0 \to 0$. From the discussion in Appendix A, we also know that $\rho \to \infty$ as $\omega_0 \to 0$. Hence we find

$$N(t,\bar{\omega}) \sim \frac{\bar{\omega}\rho^2}{\sqrt{2}} \sim \frac{e^{t/(2R_S)}}{\sqrt{2}}, \qquad t > t_f \to \infty. \quad (68)$$

This is confirmed by our numerical evaluation of N as a function of time $t > t_f$ for several different values of ω (see Fig. 2).

Therefore the occupation number at any frequency diverges in the infinite time limit when backreaction is not taken into account. This implies that backreaction due to particle creation will have important consequences for gravitational collapse.

We have also numerically evaluated the spectrum of mode occupation numbers at any finite time and show the results in Fig. 3 for several different values of t. The similar shapes of the different curves suggest that there might be a simple relationship between them. By rescaling both axes we find that the curves roughly (though not completely) collapse into a single curve as shown in Fig. 4. Hence, knowing the spectrum at time t_i approximately gives us the spectrum at all times via

$$\lambda^{-1}(t)N(t,\bar{\omega}/\lambda'(t)) = \lambda^{-1}(t_i)N(t_i,\bar{\omega}/\lambda'(t_i)), \quad (69)$$

where we can determine the function $\lambda(t)$ by considering the time variation of $N(t,0)$, and λ' by Eq. (63). The result is

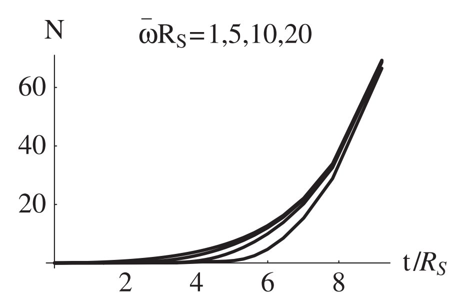

FIG. 2. N versus t/R_S for various fixed values of $\bar{\omega}R_S$. The curves are lower for higher $\bar{\omega}R_S$. At late times the behavior is given by Eq. (68).

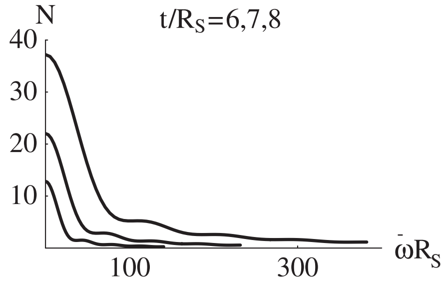

FIG. 3. N versus $\bar{\omega}R_S$ for various fixed values of t/R_S. The occupation number at any frequency grows as t/R_S increases.

$$\lambda(t) = \frac{1}{\sqrt{2}}[e^{t/2R_S} + e^{-t/2R_S} - 2], \tag{70}$$

$$\lambda'(t) = e^{t/2R_S}. \tag{71}$$

We can compare the curve in Fig. 4 with the occupation numbers for the Planck distribution

$$N_P(\omega) = \frac{1}{e^{\beta\omega} - 1}, \tag{72}$$

where β is the inverse temperature. It is clear that the spectrum of occupation numbers is nonthermal. In particular, there is no singularity in N at $\omega = 0$ at finite time, there are oscillations in N, and the rescaling law of Eq. (69) is not applicable to a thermal distribution. As $t \to \infty$, the peak at $\omega = 0$ does diverge and the distribution becomes more and more thermal. Even at finite times, at small frequencies

$$N_P(\omega \ll \beta^{-1}) \approx \frac{1}{\beta\omega} \tag{73}$$

and the rescaling law amounts to rescaling the temperature by a factor $\lambda\lambda'$.

Now, from Eq. (54), since the time derivative of the wave function on the right-hand side is with respect to η, ω is the mode frequency with respect to η and not with respect to time t. Equation (55) tells us that the frequency in t is $(1 - R_S/R)$ times the frequency in η, and at time t_f, this implies

$$\omega^{(t)} = e^{-t_f/R_s}\bar{\omega}, \tag{74}$$

where the superscript (t) on ω refers to the fact that this frequency is with respect to time t. This rescaling of the frequency implies that the temperature for the asymptotic observer (with time coordinate t) can be obtained by finding the "best fit temperature" β^{-1} and then rescaling by $(1 - R_S/R)$. So the temperature seen by the asymptotic observer is

$$T = e^{-t_f/R_S}\beta^{-1}(t_f). \tag{75}$$

(The temperature T is not to be confused with the time coordinate within the spherical domain wall, also denoted by T in Sec. II.) By using the scaling in Eq. (71), it is easy to see that β^{-1} grows as $e^{+t_f/2R_S}$ at late times and so T is constant. We can fit a thermal spectrum to the collapsed spectrum of Fig. 4, as shown in Fig. 5 to obtain

$$T \approx \frac{0.19}{R_S} = \frac{2.4}{4\pi R_S} = 2.4T_H, \tag{76}$$

where $T_H = 1/4\pi R_S \sim .08/R_S$ is the Hawking temperature. Since there is ambiguity in fitting the nonthermal spectrum by a thermal distribution, we can only say that the constant temperature T and the Hawking temperature are of comparable magnitude.

The occupation number $N(t, \omega)$ can be related to the asymptotic flux of radiation following standard procedures (e.g. Chapter 8 of Ref. [1]) and will result in the usual greybody factors.

We thus see that in the context of the Schrödinger formalism there is evidence of Hawking-like, but nonthermal radiation emitted during gravitational collapse before any event horizon is formed. There are several possible sources that one can imagine for this radiation, including radiation due to a time-dependent metric, and also Hawking emission [9]. Since the Schrödinger method in principle accounts for all such sources of radiation, it is worth reexamining the original Hawking calculation, done using the Heisenberg picture and Bogoliubov machinery, in the context of our above results.

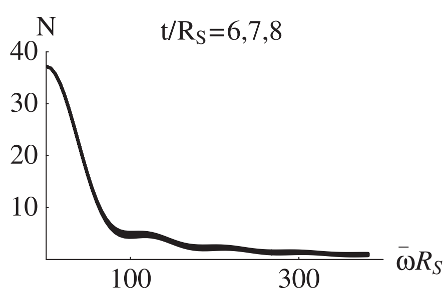

FIG. 4. The same as Fig. 3 but with the axes rescaled as in Eq. (69). This graph shows that the spectrum at different times is approximately self-similar.

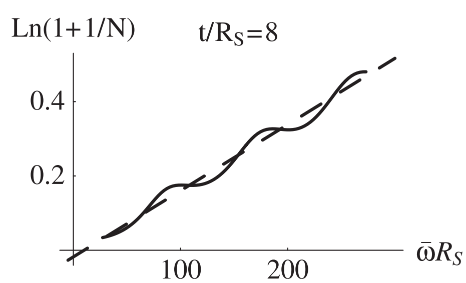

FIG. 5. $\ln(1 + 1/N)$ versus $\bar{\omega}R_S$ for $t = 8R_S$. The dashed line shows $\ln(1 + 1/N_P)$ versus $\bar{\omega}R_S$ where N_P is a Planck distribution. The slope gives β^{-1} and the temperature in Eq. (76).

VI. HAWKING'S CALCULATION

In Hawking's pioneering paper [9], he considered a collapsing body. By matching asymptotic field operators, he could find the Bogoliubov coefficients, and then the particle emission rate. The result is the famous Hawking thermal radiation at temperature

$$T_H = \frac{\kappa}{2\pi}, \tag{77}$$

where $\kappa = 1/2R_S$ is the surface gravity.

Since Hawking radiation is calculated in the $t \to \infty$ limit (asymptotic field operators), the result does not provide an answer to our original question: what will an experimentalist observe at a finite time? So we must recalculate the radiation from a collapsing domain wall which is close to, but still larger than, the Schwarzschild radius. Stated in a slightly different way—does the experimentalist see Hawking radiation before the event horizon is formed?

As Hawking showed, the mode functions of a massless scalar field in the black hole space-time have a "phase pileup" near the event horizon [9]. In other words, if we retrace the mode functions from $\mathcal{I}^+$ back in time up to $\mathcal{I}^-$, the phase of the mode function diverges on $\mathcal{I}^-$ at the point v_0 in Fig. 6, where the coordinate v is defined by

$$v = t + r + R_S \ln\left|\frac{r}{R_S} - 1\right|. \tag{78}$$

The radial part of the ingoing mode functions on $\mathcal{I}^-$ are (Eq. (2.11) of [9])

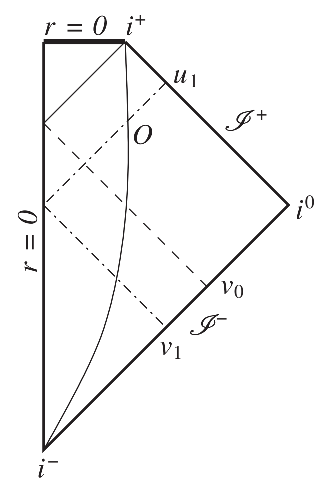

FIG. 6. Space-time of a blackhole with null rays originating at $\mathcal{I}^-$ and going to $\mathcal{I}^+$. The last ray that makes it to $\mathcal{I}^+$ is emitted at v_0. An observer, O, far from the collapsing wall will attempt to detect a flux of radiation over a finite but large time interval. The last ray to get to the observer originates at $\mathcal{I}^-$ at $v_1 < v_0$ and arrives at $\mathcal{I}^+$ at $u = u_1$. We are interested in finding the particle flux in the section of $\mathcal{I}^+$ between the points marked i^0 and u_1.

$$f_{\omega'} = \frac{F_{\omega'}(r)}{\sqrt{2\pi\omega'}r} e^{i\omega' v}. \tag{79}$$

The relevant part of the outgoing mode function at frequency ω when extended back to $\mathcal{I}^-$ is given in Eq. (2.18) of [9]

$$p_\omega^{(2)} \sim \frac{P_\omega^-}{\sqrt{2\pi\omega}r} \exp\left(-i\frac{\omega}{\kappa}\ln\left(\frac{v_0 - v}{CD}\right)\right), \qquad v < v_0, \tag{80}$$

and zero for $v > v_0$, where P_ω^-, C, D are constants, and $\kappa = 1/2R_S$. The expression in Eq. (80) is only valid for small $v_0 - v$, and for large ω' (geometrical optics limit).

The overlaps of $p_\omega^{(2)}$ with $f_{\omega'}$ and $\bar{f}_{\omega'}$ determine the Bogoliubov coefficients. This is equivalent to taking the Fourier transform of $p_\omega^{(2)}$. Following Hawking's calculation, the Bogoliubov coefficients for large ω' are (see Eq. (2.19), (2.20) of [9]; also see [5])

$$\alpha_{\omega\omega'}^{(2)} \approx \frac{P_\omega^-}{2\pi}(CD)^{i\omega/\kappa} e^{i(\omega-\omega')v_0}\sqrt{\frac{\omega'}{\omega}}\Gamma\left(1 - \frac{i\omega}{\kappa}\right) \times (-i\omega')^{-1+i\omega/\kappa}, \tag{81}$$

$$\beta_{\omega\omega'}^{(2)} \approx -i\alpha_{\omega(-\omega')}^{(2)}. \tag{82}$$

Even though the expression for $\alpha_{\omega\omega'}^{(2)}$ is only valid for large ω', Hawking argues on analyticity grounds that the singularity at $\omega' = 0$ should be present. So to obtain $\beta_{\omega\omega'}^{(2)}$ it becomes necessary to go around the pole at $\omega' = 0$ to negative values of ω'. The choice of deformation of the contour around the pole is determined on the grounds of analyticity, and the result is

$$|\beta_{\omega\omega'}^{(2)}| = |\alpha_{\omega(-\omega')}^{(2)}| = \exp\left(-\frac{\pi\omega}{\kappa}\right)|\alpha_{\omega\omega'}^{(2)}|. \tag{83}$$

From here, the calculation of the thermal flux of Hawking radiation follows.

Now consider an observer who only sees the collapsing object for a finite time (see Fig. 6). The last ray detected by such an observer emerges from $\mathcal{I}^-$ at $v = v_1 < v_0$. For this observer, the phase of the mode functions have a tendency to pile up but there is no divergence as in Eq. (80) because $v \le v_1 < v_0$. As far as this observer is concerned, the behavior in Eq. (80) holds for $v \le v_1$, while for $v > v_1$ the backtracked mode functions vanish. The Fourier transform of $p_\omega^{(2)}$ now gives the Bogoliubov coefficients the following ω' dependence

$$\alpha_{\omega\omega'}^{(2)} \approx \int^{v_1} dv \exp\left[-i\omega' v - i\frac{\omega}{\kappa}\ln(v_0 - v)\right]. \tag{84}$$

Following Ref. [5], for $\omega' > 0$ we rotate the integration contour to the negative imaginary axis ($v \to -iy$) and for $\omega' < 0$ to the positive imaginary axis (see Fig. 7). Simple manipulations for $\omega' > 0$ then give

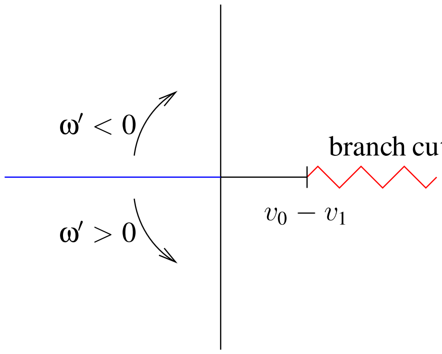

FIG. 7 (color online). The integration contour in the calculation of the Bogoliubov coefficients runs from $v = -\infty$ to $v = 0$ in the complex v plane. In Hawking's calculation $v_0 - v_1 = 0$, and the branch cut starts at the origin. For $\omega' < 0$, the integration contour is rotated to the upper imaginary axis, and for $\omega' > 0$ to the negative imaginary axis. In Hawking's case, this relates the Bogoliubov coefficients $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ as pedagogically described in Ref. [5]. (As pointed out to us by F. Dowker, care is required in comparing the calculations in [5,9] since Hawking considers modes $e^{+i\omega' v}$ while Townsend considers $e^{-i\omega' v}$. We are following Hawking's calculation.) In our case, these very rotations can also be done. However, the branch cut starts at $v_0 - v_1 > 0$ and the simple relation between the Bogoliubov coefficients needed for thermal emission is not obtained.

$$|\alpha^{(2)}_{\omega\omega'}| = e^{\pi\omega/2\kappa} \left| \int_0^\infty dy e^{-\omega' y - \omega\theta/\kappa} (y^2 + \delta^2)^{i\omega/2\kappa} \right|,$$

where $\delta = (v_0 - v_1)$ and $\theta = \tan^{-1}(\delta/y)$. Similarly, for $\omega' < 0$ we get

$$|\alpha^{(2)}_{\omega\omega'}| \approx e^{-\pi\omega/2\kappa} \left| \int_0^\infty dy e^{-|\omega'|y + \omega\theta/\kappa} (y^2 + \delta^2)^{i\omega/2\kappa} \right|.$$

Since $\beta^{(2)}_{\omega\omega'} = \alpha^{(2)}_{\omega(-\omega')}$ the above expressions yield both Bogoliubov coefficients.

The crucial difference between Hawking's asymptotic result and the finite time result is the factor $\exp(\pm\omega\theta/\kappa)$ within the integral. Because of this factor, the relation in Eq. (83) does not hold and thermality is lost. However, if this extra factor is nearly unity, we can expect the spectrum to be nearly thermal. The integral is cut off exponentially for $y > 1/\omega'$ and hence we estimate that the spectrum will be nearly thermal provided $\omega\omega'\delta/\kappa \ll 1$. Hence the spectrum is thermal at low frequencies and gets closer to being thermal as time goes on ($\delta \to 0$), both of which seem plausible on physical grounds.

It is difficult however, to go beyond these qualitative statements in attempting to compare our results with what one might derive in the Hawking approach, in particular, to determine possibly how much of the effect we obtain might be due to Hawking radiation, as opposed to particle creation by a changing metric. This is because the spectrum depends on a sum over all ω', while the Hawking analysis is done in the geometrical optics limit, at large frequencies. Hence to find the spectrum in this approach, we need a more complete solution to the equations of motion for all the modes of the scalar field in the domain wall background. Such solutions are more difficult to obtain (as described in [1] for example).

VII. INFALLING OBSERVER

So far we have considered the wall collapse from the point of view of an asymptotic observer. From the point of view of an infalling observer, the time coordinate is τ of Sec. II and the collapse appears to proceed differently. For example, if we ignore radiation, the classical equation of motion can be written from the conservation of M in Eq. (17). Then, as the wall approaches the Schwarzschild radius,

$$R_\tau^2 \approx \left[\frac{M}{4\pi\sigma R_S^2} + 2\pi G\sigma R_S\right]^2 - 1. \qquad (85)$$

The right-hand side is a nonzero constant, implying that the wall is collapsing with constant velocity in the τ coordinate. This shows that the collapse into a black hole occurs in a finite time interval for the infalling observer. Further, Hawking has argued [9] that the infalling observer does not detect significant Hawking radiation since the emission is dominantly at low frequencies compared to $1/R_S$, while the infalling observer can only have local detectors of size less than R_S. Thus the infalling observer would appear to see event horizon formation in a finite time, with no significant radiation emanating from the black hole.

These paradoxical views of the asymptotic and infalling observers need to be reconciled, and the conventional way to reconcile them is summarized in the space-time diagram of an evaporating black hole shown in Fig. 8. The diagram is drawn so that the asymptotic observer sees evaporation in a finite time and the infalling observer falls into the black hole in a finite time also.

We have to note that the diagram in Fig. 8 does not follow from a rigorous solution to the problem of radiation from a collapsing object with backreaction included. There are some analyses of this problem in (1 + 1)-dimensional models [10,11] whose connection with the (3 + 1)-dimensional problem is unclear (e.g. [12]). Thus, the diagram in Fig. 8 is a conjectured diagram that is widely used in the literature. While this diagram may well be the correct one once the full problem of gravitational collapse with backreaction is solved, we have to emphasize that it also has some puzzling features that indicate that it may not be the best conjecture to make in the absence of a backreaction analysis.

The conventionally drawn space-time of an evaporating black hole has features that are not consistent with our findings. Since the asymptotic observer sees Hawking-like radiation from the collapsing wall *prior* to event horizon formation, the mass of the collapsing wall must be decreasing, and at the point denoted by F in Fig. 8 the entire

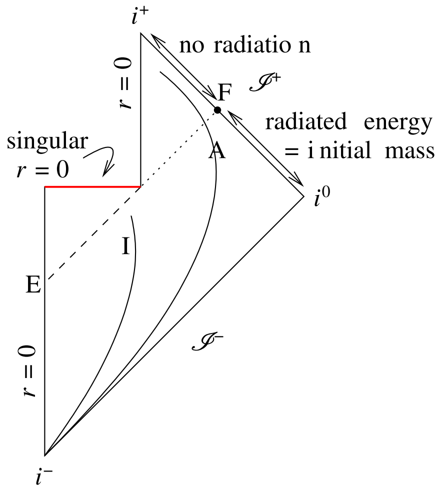

FIG. 8 (color online). The conventional space-time diagram for an evaporating black hole. The observer A will register a flux of quantum radiation even during collapse, and will be able to account for the entire energy of the shell by the time he/she gets to the line EF. From this point on, A will conclude that there is no energy left in the region of the collapsing shell. Yet A will see objects and other observers (such as I) disappear into what can at most be a Planck scale object and this is a puzzling feature of this picture.

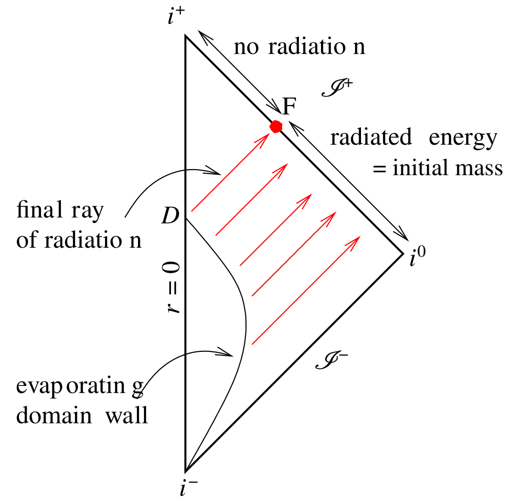

FIG. 9 (color online). The space-time of a collapsing domain wall. During collapse the wall emits nonthermal (quasi-Hawking) radiation as depicted by the arrows. Our calculations indicate that the total energy flux between the point i^0 to some point indicated by F is equal to the energy of the initial domain wall. Hence we conjecture that the domain wall evaporates completely at point D. Between F and i^+, there is no radiation flux arriving at $\mathcal{I}^+$. The event horizon and singularity present in the customary treatment are not formed and the space-time structure is the same as that of Minkowski space-time.

energy of the wall has been radiated to $\mathcal{I}^+$. However, in the space-time of Fig. 8, it is at precisely this instant that the asymptotic observer sees infalling objects disappear into the event horizon, even though there is nothing left of the collapsing wall to form the singularity. A space-time region such as the triangular region behind the event horizon only seems reasonable if not all of the collapsing shell energy has been lost to $\mathcal{I}^+$ up to the point F, and there is some energy-momentum source left behind to crunch up in the singularity. Also, if the space-time near the event horizon is described by the Schwarzschild metric, there is infinite gravitational redshift of signals escaping to infinity, while the diagram shows that signals escape to infinity in a finite time. Finally, as is well known, the diagram in Fig. 8 also gives rise to the information loss paradox. While these features of the diagram in Fig. 8 are not inconceivable, they are sufficiently strange as to cast doubt on the validity of the picture.

Instead it may happen that the true event horizon never forms in a gravitational collapse. We saw that an outside observer never sees formation of a horizon in finite time, not even in the full quantum treatment. What about an infalling observer? As in Hawking's case, the infalling observer does not see radiation, but this is due to size limitations of his detectors. The mode occupation numbers we have calculated will also be the mode occupation numbers that the infalling observer will calculate, even if they be associated with frequency modes that he cannot personally detect. The infalling observer never crosses an event horizon, not because it takes an infinite time, but because there is no event horizon to cross. As the infalling observer gets closer to the collapsing wall, the wall shrinks due to radiation backreaction, evaporating before an event horizon can form. The evaporation appears mysterious to the infalling observer since his detectors do not register any emission from the collapsing wall. Yet he reconciles the absence of radiation with the evaporation as being due to a limitation of the frequency range of his detectors. Both he and the asymptotic observer would then agree that the space-time diagram for an evaporating black hole is as shown in Fig. 9. In this picture a global event horizon and singularity never form. A trapped surface (from within which light cannot escape) may exist temporarily, but after all of the mass is radiated, the trapped surface disappears and light gets released to infinity.

The space-time picture that we are advocating is similar to that described in Refs. [13,14] and, more recently, Refs. [15–17].

VIII. DISCUSSION

In this paper we have studied the collapse of a gravitating spherical domain wall using the functional Schrödinger equation. We would like to clearly delineate our analysis of the collapse and the emitted nonthermal quantum radiation from the interpretational issues about the formation of an event horizon.

First, we studied the collapse of a gravitating spherical domain in both classical and quantum theory, ignoring any evaporative processes. It has been suggested in the literature that quantum fluctuations can cause the collapse and formation of a black hole in a finite (Schwarzschild) time [3]. However, our results show that this is not the case and the horizon does not form in a finite time even in the full quantum treatment.

Then we studied radiation from the collapsing shell as seen by the asymptotic observer. In the process of gravitational collapse, there are two, perhaps related, sources of radiation: first is the radiation from particle creation in the changing gravitational field of the collapsing ball, and second may be Hawking-like radiation due to a mismatch of vacua at early and late times. The functional Schrödinger analysis takes all such sources into account and therefore gives the total particle production. We have found a nonthermal distribution of particle occupation numbers, with departures from thermality as illustrated in Fig. 3 and discussed toward the end of Sec. V. In a limited range of frequencies, the spectrum is approximately thermal and the temperature fitted in a restricted range of frequencies is constant and roughly equal to the Hawking temperature $1/4\pi R_S$. The radiation becomes thermal in the entire range of frequencies only in the limit $t \to \infty$, i.e. when the horizon is formed. Further, the mode occupation number diverges in the infinite time limit, if the backreaction is neglected (i.e. the background is held fixed). Since an outside observer never sees formation of a horizon in a finite time, radiation observed by him is never quite thermal. (Nonthermal features also get greatly amplified once the background is also treated quantum mechanically [18].) This nonthermal radiation has strong implications for the information loss paradox since it can carry information about the collapsing matter.

Without a rigorous calculation that includes backreaction, one cannot give a definite answer to the final fate of a collapsing object. It may happen that the diagram in Fig. 8 is correct and some radical and elaborate solutions to the problems we mentioned in Sec. VII are needed. However, one can imagine an alternative picture, different from the one in Fig. 8, which seems to have fewer problems, and that is that an event horizon never forms. Since the mass of the shell is decreasing during the collapse, the shell will be chasing its own Schwarzschild radius, and the question is whether the shell will catch up to its own Schwarzschild radius or completely evaporate before that happens [14].

With backreaction included, the radiation should lead to a continual reduction of the Schwarzschild radius, R_S, occurring in the Ipser-Sikivie metric (see Sec. II). Then, as seen by the asymptotic observer, one of two possibilities occurs: either the collapsing domain wall evaporates and $R_S \to 0$ in a finite time, or else backreaction causes the radiation rate to slow down and vanish in a finite time. This latter possibility is unlikely, as our estimates suggest that the rate of emission increases as R_S decreases [19]. We therefore conjecture that the backreaction due to particle production will cause the collapsing domain wall space-time to completely evaporate in a finite time. In this case, the space-time can either be as given in Fig. 8, or have the same global space-time structure as Minkowski space, as shown in Fig. 9. If the latter picture is correct, it also means that the infalling observer will not encounter an event horizon, because this feature is simply absent from the space-time. Another way to see this is to note that the causal relation between two events is the same for all observers. Hence if the asymptotic observer sees a signal from an infalling object after he sees the last radiation ray emitted by the evaporating wall, this will also be the sequence of signals seen by the infalling observer. As discussed in Sec. VII, the infalling observer would expect to see an intense burst of radiation as the wall approaches the Schwarzchild radius, but can fail to do so because his detectors are too small to detect the emitted range of frequencies.

In the absence of an exact backreaction calculation, we also have to allow for the possibility that a value of the critical mass exists above which Fig. 8 applies and below which Fig. 9 holds. Also, as discussed by Hawking [20], the question of "whether a black hole forms" is not sharp enough and may not make sense in the full quantum theory since all of the measurements are made by an asymptotic observer at infinity, while a collapsing object exists for a finite time and disappears by emitting radiation in the strong field region in the middle. An asymptotic observer can never be sure if a black hole formed because of underlying quantum uncertainty [20].

The broad picture we have obtained is consistent with that proposed in Refs. [13,14], though there are differences in the analysis and the conclusions. In particular, we find a nonthermal spectrum whereas Gerlach argues for thermality. Our picture also supports the interpretation of Hawking radiation given in Ref. [5] whereby particles are created during the process of gravitational collapse and are then radiated slowly to form what we call Hawking radiation. We have indeed found particle production during the collapse but the radiation is not quite thermal. It is only in the frequency range where the occupation number spectrum can be approximated by T/ω (Eq. (73)) that thermality holds at finite time. Also note that the nonthermality we find is in the mode occupation numbers. Propagation of the radiation in the background metric will cause further nonthermality due to greybody factors.

If we live in a world of low scale gravity, the collision of particles in high energy accelerators will lead to a situation where the particles are in a continual state of gravitational collapse from which nonthermal radiation is being emitted. The lifetime of such a state can be estimated once we know the details of the radiation more precisely from an analysis which includes backreaction. However, on dimensional grounds, Hawking's estimate for the lifetime of a black hole ($\sim R_S^3/G$) may well apply to the colliding particles as well.

In reality the collapse is further complicated by the fact that the collapsing object is not kept in isolation and there are external forces that can disrupt the collapse at any point in time. From the perspective of potential information loss, note that any infalling encyclopedias can be returned to the asymptotic observer if the collapse is disrupted at any time, as it could be, for example, by a bomb set to go off at some late, but finite time. Most importantly, since we calculate that the radiation emitted during the gravitational collapse is never truly thermal, the classic information loss issue in black holes should, in this case, be a nonproblem for the asymptotic observer [21].

Our primary result, that no event horizon forms in gravitational collapse as seen by an asymptotic observer is suggestive of the possibility of using the number of local event horizons to classify and divide Hilbert space into superselection sectors, labeled by the number of local event horizons. Our result suggests that no operator could increase the number of event horizons, but the possibility of reducing the number of preexisting primordial event horizons is not so clear and would require that Hawking radiation not cause any primordial black hole event horizons to evaporate completely.

Our conclusions have been derived on the basis of a number of assumptions which we now discuss. The first is the truncation of superspace to minisuperspace. We have only included spherically symmetric field configurations. Even then, the metric is restricted to be the classical solution sourced by a spherical domain wall. A more general analysis would include more metric degrees of freedom, though it is hard to see how this would make a difference to our conclusion. Similarly, we have restricted ourselves to a zero thickness domain wall. A more general analysis would allow for a thick wall. Finally, the Wheeler-de Witt formalism as we have used it, does not allow for the creation and annihilation of domain walls ("third quantization"). Perhaps third quantization could allow for the spontaneous creation of a black hole and the annihilation of the wall, effectively leading to black hole formation. A fourth possibility is that our Lagrangian breaks down near the Schwarzschild horizon and "quantum gravity" effects become important. This is usually thought not to be the case since the space-time curvature near the horizon is small for large black holes.

Perhaps the most serious drawback of our analysis is that it does not include backreaction on the gravitational collapse due to radiation. While we do not expect such inclusion to alter our conclusions regarding the nonexistence of event horizons for asymptotic observers, we are currently exploring ways to extend our treatments to include backreaction.

No theoretical idea is complete without the possibility of experimental verification and so it is important to ask if the picture we have developed in this paper can also be tested experimentally. We have already mentioned the relevance of our conclusions to black hole production in particle accelerators provided low scale gravity is correct. However, there is an even more accessible experimental system where these theoretical ideas can be put to the test. These are condensed matter systems in which sonic black holes (dumbholes) may exist [22]. It is very hard to realize a dumbhole in the laboratory for various experimental reasons; the closest known realization seems to be the propagating He-3 AB interface in the experiment of Ref. [23] as discussed in [24]. Yet the crucial aspect of our work in this paper is that *there is no need to produce a dumbhole in order to see acoustic "pre-Hawking" radiation*. The process of collapse toward a dumbhole will give off radiation. This is also the conclusion of Ref. [25] though the details of the analysis and conclusions are different—for example, we find nonthermal emission whereas these authors claim thermal emission with a modified temperature that is lower than the Hawking temperature. In any case, it should be much easier to do experiments in the laboratory that do not go all the way to forming a dumbhole, and this could be an ideal arena to test pre-Hawking radiation.

Our conclusions are important not only for the general issue of the breakdown of unitarity via information loss, but also for more general studies of black hole formation, whether they be in the context of astrophysics (e.g. galactic black holes) or in future accelerator experiments. In all these situations, we are asymptotic observers watching the gravitational collapse of matter, and we may never see effects associated with a black hole event horizon. Only effects occurring during the gravitational collapse itself appear to be visible.

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APPENDIX A: ρ EQUATION

In the range $t < 0$, ω is a constant and the solution to Eq. (60) is

$$\rho(\eta) = \frac{1}{\sqrt{\omega_0}}. \tag{A1}$$

In the range $0 < t < t_f$, we do not have an analytic solution but we can derive certain useful properties. First note that in terms of η

$$\omega^2 = \frac{\omega_0^2}{1 - \eta/R_S}. \tag{A2}$$

Then the equation for ρ after rescalings can be written as:

$$\frac{d^2 f}{d\eta'^2} = -(\omega_0 R_S)^2 \left[\frac{f}{1-\eta'} - \frac{1}{f^3} \right], \tag{A3}$$

where $\eta' = \eta/R_S$, $f = \sqrt{\omega_0}\rho$. The boundary conditions are

$$f(0) = 1, \qquad \frac{df(0)}{d\eta'} = 0. \tag{A4}$$

The last term with the $1/f^3$ becomes singular as $f \to 0$. Let us consider another equation with the $1/f^3$ replaced by something better behaved. For example,

$$\frac{d^2 g}{d\eta'^2} = -(\omega_0 R_S)^2 \left[\frac{g}{1-\eta'} - g \right] \tag{A5}$$

with boundary conditions

$$g(0) = 1, \qquad \frac{dg(0)}{d\eta'} = 0. \tag{A6}$$

Equation (A5) implies that $g(\eta')$ is monotonically decreasing as long as $g(\eta') > 0$. Furthermore, it is decreasing faster than the solution for f as long as $f < 1$, since the $1/f^3$ term in Eq. (A3) is a larger "repulsive" force than the g term in the Eq. (A5). So

$$g(\eta') \leq f(\eta') \tag{A7}$$

for all η' such that $g(\eta') > 0$.

Equations (A5) with initial conditions (A6) can be solved in terms of degenerate hypergeometric functions. For us, the important point is that the solution for g is positive for all η' and, in particular, $g(1) > 0$ for all the values of $\omega_0 R_S$ that we have checked. Therefore $f(\eta')$ is positive, at least for a wide range of $\omega_0 R_S$.

Let $f_1 = f(1) \neq 0$. Then the equation for f can be expanded near $\eta' = 1$,

$$\frac{d^2 f}{d\eta'^2} \sim -(\omega_0 R_S)^2 \left[\frac{f_1}{1-\eta'} - \frac{1}{f_1^3} \right]. \tag{A8}$$

This shows that

$$\frac{df}{d\eta'} \sim (\omega_0 R_S)^2 f_1 \ln(1-\eta') \to -\infty \tag{A9}$$

as $\eta' \to 1$.

Hence $\rho(\eta = R_S)$ is strictly positive and finite while $\rho_\eta(\eta = R_S) = -\infty$ for finite and nonzero ω_0. Since $f = \sqrt{\omega_0}\rho$, and $f \to 1$ for $\omega_0 \to 0$, we also see that $\rho \to \infty$ and $\rho_\eta \to 0$ as $\omega_0 \to 0$.

In the range $t_f < t$, ω is a constant. However, the solution for ρ is not a constant, unlike in the range $t < 0$, since the constant solution $1/\sqrt{\omega(t_f)}$ does not necessarily match up with $\rho(t_f-)$ to ensure a continuous solution. Yet it is easy to check that in this region $\dot{N} = 0$ and so there is no change in the occupation numbers. So we need only find $N(t_f-, \bar{\omega})$ to determine $N(t \to \infty, \bar{\omega})$.

APPENDIX B: NUMBER OF PARTICLES RADIATED AS A FUNCTION OF TIME

We use the simple harmonic oscillator basis states but at a frequency $\bar{\omega}$ to keep track of the different ω's in the calculation. To evaluate the occupation numbers at time $t > t_f$, we need only set $\bar{\omega} = \omega(t_f)$. So

$$\phi_n(b) = \left(\frac{m\bar{\omega}}{\pi}\right)^{1/4} \frac{e^{-m\bar{\omega}b^2/2}}{\sqrt{2^n n!}} H_n(\sqrt{m\bar{\omega}}b), \tag{B1}$$

where H_n are Hermite polynomials. Then Eq. (65) together with Eq. (59) gives

$$\begin{aligned} c_n &= \left(\frac{1}{\pi^2 \bar{\omega}\rho^2}\right)^{1/4} \frac{e^{i\alpha}}{\sqrt{2^n n!}} \int d\xi e^{-P\xi^2/2} H_n(\xi) \\ &\equiv \left(\frac{1}{\pi^2 \bar{\omega}\rho^2}\right)^{1/4} \frac{e^{i\alpha}}{\sqrt{2^n n!}} I_n, \end{aligned} \tag{B2}$$

where

$$P = 1 - \frac{i}{\bar{\omega}} \left(\frac{\rho_\eta}{\rho} + \frac{i}{\rho^2} \right). \tag{B3}$$

To find I_n consider the corresponding integral over the generating function for the Hermite polynomials

$$J(z) = \int d\xi e^{-P\xi^2/2} e^{-z^2 + 2z\xi} = \sqrt{\frac{2\pi}{P}} e^{-z^2(1-2/P)}. \tag{B4}$$

Since

$$e^{-z^2+2z\xi} = \sum_{n=0}^{\infty} \frac{z^n}{n!} H_n(\xi), \tag{B5}$$

$$\int d\xi e^{-P\xi^2/2} H_n(\xi) = \frac{d^n}{dz^n} J(z) \bigg|_{z=0}. \tag{B6}$$

Therefore

$$I_n = \sqrt{\frac{2\pi}{P}} \left(1 - \frac{2}{P}\right)^{n/2} H_n(0). \tag{B7}$$

Since

$$H_n(0) = (-1)^{n/2} \sqrt{2^n n!} \frac{(n-1)!!}{\sqrt{n!}}, \qquad n = \text{even} \tag{B8}$$

and $H_n(0) = 0$ for odd n, we find the coefficients c_n for even values of n,

$$c_n = \frac{(-1)^{n/2} e^{i\alpha}}{(\bar{\omega}\rho^2)^{1/4}} \sqrt{\frac{2}{P}} \left(1 - \frac{2}{P}\right)^{n/2} \frac{(n-1)!!}{\sqrt{n!}}. \tag{B9}$$

For odd n, $c_n = 0$.

Next we find the number of particles produced. Let

$$\chi = \left| 1 - \frac{2}{P} \right|. \tag{B10}$$

Then

$$\begin{aligned} N(t, \bar{\omega}) &= \sum_{n=\text{even}} n|c_n|^2 \\ &= \frac{2}{\sqrt{\bar{\omega}\rho^2}|P|} \chi \frac{d}{d\chi} \sum_{n=\text{even}} \frac{(n-1)!!}{n!!} \chi^n \\ &= \frac{2}{\sqrt{\bar{\omega}\rho^2}|P|} \chi \frac{d}{d\chi} \frac{1}{\sqrt{1-\chi^2}} \\ &= \frac{2}{\sqrt{\bar{\omega}\rho^2}|P|} \frac{\chi^2}{(1-\chi^2)^{3/2}}. \end{aligned} \tag{B11}$$

Inserting the expressions for χ and P, leads to

$$N(t, \bar{\omega}) = \frac{\bar{\omega}\rho^2}{\sqrt{2}} \left[\left(1 - \frac{1}{\bar{\omega}\rho^2} \right)^2 + \left(\frac{\rho_\eta}{\bar{\omega}\rho} \right)^2 \right]. \tag{B12}$$

In summary, we have found the occupation number of modes as a function of ρ which is a function of time as given by the nonlinear differential equation (60). The equation connecting ρ and time t has only been solved numerically but we have discussed the behavior of ρ and ρ_η as $\eta \to R_S$ ($t \to \infty$) in Appendix A.

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