

Coupled variations of fundamental couplings and primordial nucleosynthesis

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(Received 1 November 2006; revised manuscript received 1 June 2007; published 20 July 2007)

The effect of variations of the fundamental nuclear parameters on big-bang nucleosynthesis are modeled and discussed in detail taking into account the interrelations between the fundamental parameters arising in unified theories. Considering only ^4He , strong constraints on the variation of the neutron lifetime, neutron-proton mass difference are set. These constraints are then translated into constraints on the time variation of the Yukawa couplings and the fine structure constant. Furthermore, we show that a variation of the deuterium binding energy is able to reconcile the ^7Li abundance deduced from the WMAP analysis with its spectroscopically determined value while maintaining concordance with D and ^4He . The analysis is strongly based on the potential model of Flambaum and Shuryak that relates the binding energy of the deuteron with the nucleon and σ and ω mesons masses; however, we show that an alternative approach that consists of a pion mass dependence necessarily leads to equivalent conclusions.

DOI: [10.1103/PhysRevD.76.023511](https://doi.org/10.1103/PhysRevD.76.023511)

PACS numbers: 98.80.Cq

I. INTRODUCTION

Primordial nucleosynthesis (BBN) is one of the most successful predictions of the standard hot big-bang model. Its success rests on the concordance between the observational determinations of the light element abundances of D and ^4He specifically, and their theoretically predicted abundances [1,2]. Furthermore, measurements of the cosmic microwave background anisotropies by WMAP [3] have led to precision determinations of the baryon density or equivalently the baryon-to-photon ratio, η . As η is the sole parameter of the standard model of BBN, it is possible to make very accurate predictions [4–7] and hence infer the expected theoretical abundances of all of the light elements (including ^7Li , ^6Li , ^9Be , ^{10}Be , and ^{11}B).

At present, a discrepancy between the predicted abundance of ^7Li and its spectroscopically determined abundance persists. The ^7Li abundance based on the WMAP baryon density is predicted to be [5]

$$^7\text{Li}/\text{H} = 4.15_{-0.45}^{+0.49} \times 10^{-10}. \quad (1)$$

The systems best suited for Li observations are metal-poor halo stars in our Galaxy. Analyses of the abundances in these stars yield [8] $\text{Li}/\text{H} = (1.23_{-0.16}^{+0.34}) \times 10^{-10}$ and more recently [9] $\text{Li}/\text{H} = (1.26 \pm 0.26) \times 10^{-10}$. This value is in clear contradiction with most estimates of the primordial Li abundance, as also shown by [7] who find

$$^7\text{Li}/\text{H} = 4.26_{-0.60}^{+0.73} \times 10^{-10}. \quad (2)$$

In both cases, the ^7Li abundance is a factor of ~ 3 higher than the value observed in most halo stars.

There have been several attempts to account for the discrepancy between the BBN/WMAP predicted value of $^7\text{Li}/\text{H}$ and its observational determination. These include depletion mechanisms due to rotationally induced mixing and/or diffusion. Current estimates for possible depletion factors are in the range ~ 0.2 – 0.4 dex [10]. However, the negligible intrinsic spread in Li [11] leads to the conclusion that depletion in halo stars is as low as 0.1 dex. It is also possible that the stellar parameters used to determine the Li abundance from the spectroscopic measurements may be systematically off. Most important among these is the effective temperature assumed for stellar atmospheres. These can differ by up to 150–200 K, with higher temperatures resulting in estimated Li abundances which are higher by ~ 0.08 dex per 100 K. Thus, accounting for a difference of 0.5 dex between BBN and the observations would require a serious offset of the stellar parameters. We note that there has been a recent analysis [12] which does support higher temperatures, and brings the discrepancy between theory and observations to 2σ .

Another potential source for systematic uncertainty lies in the BBN calculation of the ^7Li abundance. The predictions for ^7Li carry the largest uncertainty of the 4 light elements which stem from uncertainties in the nuclear rates. The effect of changing the yields of certain BBN reactions was recently considered in Ref. [5]. In particular, they concentrated on the set of cross sections which affect ^7Li and are poorly determined both experimentally and

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theoretically. It was found, for example, that an increase of the ${}^7\text{Be}(d, p){}^2{}^4\text{He}$ reaction by a factor of 100 would reduce the ${}^7\text{Li}$ abundance by a factor of about 3 in the WMAP η range. This reaction has since been remeasured and precludes this solution [13]. The possibility of systematic errors in the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction, which is the only important ${}^7\text{Li}$ production channel in BBN, was considered in detail in [14]. However, the agreement between the standard solar model and solar neutrino data provides constraints on variations in this cross section. Using the standard solar model of Bahcall [15], and recent solar neutrino data [16], one can exclude systematic variations of the magnitude needed to resolve the BBN ${}^7\text{Li}$ problem at the $\approx 95\%$ C.L. [14]. The “nuclear fix” to the ${}^7\text{Li}$ BBN problem is unlikely.

On the other hand, various theoretical explanations involving physics beyond the standard model have been proposed [17]. One possible extension of the standard BBN scenario allows for inhomogeneous nucleosynthesis [18] but this seems to overproduce ${}^7\text{Li}$. It has also been argued that particle decay after BBN could lower the ${}^7\text{Li}$ abundance and produce some ${}^6\text{Li}$ as well [19]. This has been investigated in the framework of the constrained minimal supersymmetric standard model if the lightest supersymmetric particle is assumed to be the gravitino [20]. Some models have been found which accomplish these goals [21]. Another route is to assume that gravity is not described by general relativity but is attracted toward general relativity during the cosmic evolution [22]. BBN has been extensively studied in that scenario (see e.g. Ref. [23]). The effect of the modification of gravity is mainly to induce a time variation of the equivalent speedup that can be tuned to happen during BBN but it can have other signatures both on cosmological and local scales [24].

In this article, we want to investigate the possible variations of fundamental constants. It is well known that variations in the fundamental coupling constants such as the fine structure constant, α , can affect the light element abundance during BBN. Most analyses have concentrated on the effect of such variations on the abundance of ${}^4\text{He}$ [17,25–33]. Changes in α directly induce changes in the nucleon mass, Δm_N , which affects the neutron-to-proton ratio.

Much of the recent excitement over the possibility of a time variation in the fine structure constant stems from a series of recent observations of quasar absorption systems and a detailed fit of positions of the absorption lines for several heavy elements using the “many-multiplet” method [34,35]. When this method is applied to a set of Keck/Hires data, a statistically significant trend for a variation in α was reported: $\Delta\alpha/\alpha = (-0.54 \pm 0.12) \times 10^{-5}$ over a redshift range $0.5 \lesssim z \lesssim 3.0$. The minus sign indicates a smaller value of α in the past. In Ref. [36], a set of high signal-to-noise systems yielded the result $\Delta\alpha/\alpha =$

$(-0.06 \pm 0.06) \times 10^{-5}$. One should note that both results are sensitive to assumptions regarding the isotopic abundances of elements used in the analysis which further complicates the interpretation of any positive signal from these analyses [37].

In addition to the possible variation in α , it is reasonable to search for other time-varying quantities such as the ratio of the proton-to-electron mass, μ [38]. Indeed recent analyses [39] claim to observe a variation at the level

$$\frac{\Delta\mu}{\mu} = (2.4 \pm 0.6) \times 10^{-5} \quad (3)$$

using Lyman bands of H_2 spectra in two quasars. In the following, we investigate the effect on BBN of changes in fundamental couplings which could also account for a variation in μ . As we will see, the largest effect can be traced to a variation of the Higgs vacuum expectation value leading to a variation in the binding energy of deuterium. We find that, for a suitably large variation (of order a few hundredths of a percent) in μ at the time of BBN, the ${}^7\text{Li}$ abundance can be decreased by the requisite factor without overly affecting the agreement between theory and observations for D and ${}^4\text{He}$.

Although we do not directly tie our calculations to the observation with the result in Eq. (3), we do take, as our starting point, the possibility that $\mu \equiv m_p/m_e$ could have differed from its present value at the time of BBN. As a result we will be interested in variations of Yukawa couplings, h (we can assume, or not, that all Yukawas vary identically); the Higgs vacuum expectation value (vev), v , and $\Lambda \equiv \Lambda_{\text{QCD}}$. Some effects of the variations of v [30,31] and Λ [32] on BBN have been considered in the past. Here we will be primarily interested in the interdependence of these variations (some of which are *not* model dependent) and their effects on quantities of direct importance to BBN, such as the binding energy of the deuteron, the nucleon mass difference, and the neutron lifetime.

In complete generality, the effect of varying constants on BBN predictions is difficult to model because of the intricate structure of QCD and its role in low energy nuclear reactions (see Refs. [40,41]). The abundances of light nuclei produced during BBN mainly depend on the value of a series of fundamental constants which include the gravitational constant G , the three gauge couplings, and the Yukawa couplings of the electron and quarks. One needs to relate the nuclear parameters (cross sections, binding energies, and the masses of the light nuclei) to these fundamental constants. This explains why most studies are restricted to a subset of constants, such as e.g. the gravitational constant [17,23,42], the fine structure constant [17,25–27], or v [30,31].

The approach we adopt here recognizes that many variations of parameters we deem fundamental are interrelated in model dependent ways [27]. For example, if one assumes gauge coupling unification at some high energy

(grand unified) scale, there is a direct relation between variations in the fine structure constant and Λ . In string theories, the variations of gauge couplings will be related to variations in Yukawa couplings, and in models where the weak scale is determined by dimensional transmutation [43], there is a relation between variations in the Yukawa couplings and variations in the Higgs vev. Variations in the latter will also trigger variations in Λ . While the exact relation between these variations is model dependent, the fact that they are interrelated is not. Therefore it is inconsistent for example to consider a variation in v without simultaneously varying Λ . We will make use of these dependencies to study variations in several (tractable) quantities which affect BBN. Coupled variations of this type were used to strengthen existing bounds on the fine structure constant based on Oklo and meteoritic data [44]. As noted above, we cannot fully evolve the variations in all nuclear reactions, because their dependence is unknown. Here, we will be primarily interested in the induced variations of the nucleon mass difference, the neutron lifetime, and the binding energy of deuterium. We recognize that this represents a limitation on our results.

In Sec. II, we relate the variation of the BBN parameters, mainly Q , τ_n , and B_D , to the variation of the fundamental parameters such as the Yukawa couplings, h , the QCD energy scale, Λ , and the fine structure constant, α . Section III focuses on the relations that can be drawn between the variations of the fundamental parameters, taking into account successively grand unification, dimensional transmutation, and the possibility that the variation is driven by a dilaton. In order to deal with some of the theoretical uncertainties, we introduce two phenomenological parameters and we then make the connection with the variation of the proton-to-electron mass ratio at low redshift. Section IV focuses on the BBN computation and first describes the implementation of the variations in our BBN code. Assuming that the fine structure constant does not vary, we show that deuterium and ${}^4\text{He}$ data set strong constraints on the variation of the Yukawa couplings [see Eq. (39)] but that inside this bound there exists a range reconciling the ${}^7\text{Li}$ abundance with spectroscopic observations. We then allow the fine structure constant to vary and set a sharp constraint on its variation in the dilaton scenario [see Eqs. (40) and (41)].

II. FROM FUNDAMENTAL PARAMETERS TO BBN QUANTITIES

As discussed above, we focus our attention on three physical quantities which have direct bearing on the resulting abundances from BBN, the nucleon mass difference $Q = m_n - m_p = 1.29$ MeV, the neutron lifetime τ_n , and the binding energy of deuterium B_D .

The neutron-proton mass difference is expressed in terms of α , Λ , v , and the u and d quark Yukawa couplings as

$$Q \equiv m_n - m_p = a\alpha\Lambda + (h_d - h_u)v, \quad (4)$$

where the electromagnetic contribution at present is $a\alpha_0\Lambda_0 = -0.76$, and therefore the weak contribution is $(h_{d0} - h_{u0})v_0 = 2.05$ [45]. The variation of Q will then scale as

$$\frac{\Delta Q}{Q} = -0.6 \left[\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda} \right] + 1.6 \left[\frac{\Delta(h_d - h_u)}{h_d - h_u} + \frac{\Delta v}{v} \right]. \quad (5)$$

The neutron lifetime can be well approximated by

$$\tau_n^{-1} = \frac{1}{60} \frac{1 + 3g_A^2}{2\pi^3} G_F^2 m_e^5 \left[\sqrt{q^2 - 1} (2q^4 - 9q^2 - 8) + 15 \ln(q + \sqrt{q^2 - 1}) \right], \quad (6)$$

where $q = Q/m_e$. Since $G_F = 1/\sqrt{2}v^2$ and $m_e = h_e v$, we have for the relative variation of the neutron lifetime,

$$\frac{\Delta\tau_n}{\tau_n} = -4.8 \frac{\Delta v}{v} + 1.5 \frac{\Delta h_e}{h_e} - 10.4 \frac{\Delta(h_d - h_u)}{h_d - h_u} + 3.8 \left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda} \right). \quad (7)$$

In addition to Q and τ_n , which have been well studied in the context of BBN, we consider the variation of B_D . This is one of the better known quantities in the nuclear domain: it is experimentally measured to a precision better than 10^{-6} [46] so that allowing a change of its value by a few percent at BBN can only be reconciled with laboratory measurements if its value is varying with time.

Recently, in a series of works [28,29,47] Flambaum and collaborators have considered the dependence of hadronic properties on quark masses and have set constraints on the deuterium binding energy from BBN [29] following Refs. [30–33]. The importance of B_D can be understood by the fact that the equilibrium abundance of deuterium and the reaction rate $p(n, \gamma)\text{D}$ depend exponentially on B_D and on the fact that the deuterium is in a shallow bound state.

Here, we follow Refs. [29,47] to compute the quark-mass dependence of the deuterium binding energy. Using a potential model, the dependence of B_D on the nucleon mass and σ and ω meson masses have been determined [47],

$$\frac{\Delta B_D}{B_D} = -48 \frac{\Delta m_\sigma}{m_\sigma} + 50 \frac{\Delta m_\omega}{m_\omega} + 6 \frac{\Delta m_N}{m_N}, \quad (8)$$

for constant Λ . One can see that the coefficients of the quantities on the right-hand side (rhs) of (8) do not add up to unity as it is required on dimensional grounds. Clearly there is a variation of a dimensional quantity that has not been taken into account, which at low energy can be expressed in terms of an *a priori* unknown combination of Λ and v . For definiteness we will write the missing term in

terms of Λ only, keeping in mind that this somewhat artificial method of fixing units is accounted for in the uncertainty in the relations between the variations of the fundamental parameters that we will discuss in the next section. Hence, when the nucleon and meson masses are kept constant, we write $\Delta B_D/B_D = -7\Delta\Lambda/\Lambda$. On the other hand, fixing Λ , when varying quark masses (the largest contribution comes from m_s), their result is $\Delta B_D/B_D = -17\Delta m_s/m_s$.

The importance of the strange quark in the nucleon and meson masses can be traced to the π -nucleon Σ term, which is given by

$$\sigma_{\pi N} \equiv \Sigma = \frac{1}{2}(m_u + m_d)(B_u + B_d). \quad (9)$$

where

$$B_q \equiv \langle p | \bar{q}q | p \rangle. \quad (10)$$

Defining $y = 2B_s/(B_u + B_d)$, the combination $\Sigma(1 - y)$ is the change in the nucleon mass due to the nonzero u , d quark masses, which is estimated on the basis of octet baryon mass differences to be $\sigma_0 = 36 \pm 7$ MeV [48]. Following Ref. [49], we have $(B_u - B_s)/(B_d - B_s) = 1.49$ and, given a value of Σ , one can determine B_q . In Ref. [47], the value $B_s = 1.5$ was adopted and corresponds to $\Sigma \simeq 51$ MeV, which is a reasonable value. This corresponds to

$$\frac{\Delta m_N}{m_N} = \left(\frac{m_s B_s}{m_N} \right) \frac{\Delta m_s}{m_s} \simeq 0.19 \frac{\Delta m_s}{m_s}.$$

For these values we find a similar value (though slightly larger than the one found in Ref. [47]) for the light quark (u and d) contributions which give

$$\frac{\Delta m_N}{m_N} \simeq 0.052 \frac{\Delta m_q}{m_q}.$$

This implies that

$$\frac{\Delta m_p}{m_p} \simeq 0.76 \frac{\Delta\Lambda}{\Lambda} + 0.24 \left(\frac{\Delta h}{h} + \frac{\Delta v}{v} \right). \quad (11)$$

The value of Σ however has substantial uncertainties which were recently discussed in Ref. [50]. An often used value is $\Sigma = 45$ MeV which was already somewhat larger than naive quark model estimates, and corresponded to $y \simeq 0.2$. However, recent determinations of the π -nucleon Σ term have found higher values [51], $\Sigma = 64$ MeV. Still higher values can be ascertained for the observation of exotic baryons [52]. For $\Sigma = 45(64)$ MeV, $B_s = 0.9(2.8)$ and $\Delta m_N/m_N = 0.12(0.36)\Delta m_s/m_s$. The contribution from u and d quarks is 0.046 and 0.066 for $\Sigma = 45, 64$ MeV, respectively. A similar calculation for the ω meson leads to $\Delta m_\omega/m_\omega = (0.09, 0.15, 0.29)\Delta m_s/m_s$ for $\Sigma = 45, 51, 64$ MeV, respectively. For the σ meson, three contributions were identified in [47], only one of which is related to Σ , yielding $\Delta m_\sigma/m_\sigma =$

$(0.44, 0.54, 0.75)\Delta m_s/m_s$. Combining these sensitivities using Eq. (8), we would arrive at $\Delta B_D/B_D = (-16, -17, -19)\Delta m_s/m_s$ (when the u and d contributions are neglected). Thus, despite the large uncertainties in the individual sensitivities, the dependence of B_D on the strange quark mass is relatively stable. Because of the cancellations in Eq. (8), the u and d quark contributions are indeed small: $\Delta B_D/B_D = (0.08, -0.009, -0.20)\Delta m_q/m_q$ and can safely be neglected.

Choosing the central value $\Sigma = 51$ MeV and since $m_s = h_s v$, we immediately have the relation between B_D , h , and v . Adding these two contributions and using $\Delta B_D/B_D = -17\Delta m_s/m_s$ we have, in general,

$$\frac{\Delta B_D}{B_D} = 18 \frac{\Delta\Lambda}{\Lambda} - 17 \left(\frac{\Delta v}{v} + \frac{\Delta h_s}{h_s} \right), \quad (12)$$

where once again we have repaired the mass dimension by adding the appropriate powers of $\Delta\Lambda/\Lambda$.

Equations (5), (7), and (12), form the initial basis for our computation.

III. RELATIONS BETWEEN FUNDAMENTAL PARAMETERS

A. General relations in a GUT

We note that several relations among our fundamental parameters can be found. First, changes in either h or v trigger changes in Λ [53]. This is evident from the low energy expression for Λ when mass thresholds are included:

$$\Lambda = \mu \left(\frac{m_c m_b m_t}{\mu^3} \right)^{2/27} \exp \left(-\frac{2\pi}{9\alpha_s(\mu)} \right), \quad (13)$$

for $\mu > m_t$ up to some unification scale in the standard model¹

$$\frac{\Delta\Lambda}{\Lambda} = R \frac{\Delta\alpha}{\alpha} + \frac{2}{27} \left(3 \frac{\Delta v}{v} + \frac{\Delta h_c}{h_c} + \frac{\Delta h_b}{h_b} + \frac{\Delta h_t}{h_t} \right). \quad (14)$$

The value of R is determined by the particular grand unified theory and particle content which control both the value of $\alpha(M_{\text{GUT}}) = \alpha_s(M_{\text{GUT}})$ and the low energy relation between α and α_s , leading to significant model dependence in R [54,55]. Here we will assume a value of $R = 36$ corresponding to a set of minimal assumptions [27,56]. However, in most BBN computations, we will neglect the variation in α , and therefore the precise value of R chosen will not affect our conclusions. Nevertheless, the relation between h , v , and Λ is quite robust and has been neglected in most studies discussing the effect of varying v (or varying G_F) [30,31].

¹In supersymmetric models, additional thresholds related to squark and gluino masses would affect this relation [54].

For the quantities we are interested in, we now have

$$\frac{\Delta B_D}{B_D} = -13\left(\frac{\Delta v}{v} + \frac{\Delta h}{h}\right) + 18R\frac{\Delta\alpha}{\alpha}, \quad (15)$$

$$\frac{\Delta Q}{Q} = 1.5\left(\frac{\Delta v}{v} + \frac{\Delta h}{h}\right) - 0.6(1+R)\frac{\Delta\alpha}{\alpha}, \quad (16)$$

$$\frac{\Delta\tau_n}{\tau_n} = -4\frac{\Delta v}{v} - 8\frac{\Delta h}{h} + 3.8(1+R)\frac{\Delta\alpha}{\alpha}, \quad (17)$$

where we have assumed that all Yukawa couplings vary identically, $\Delta h_i/h_i = \Delta h/h$. For clarity, we have written only rounded values of the coefficients, however, the numerical computation of the light element abundances uses the more precise values. We also recall that $\Delta G_F/G_F = -2\Delta v/v$ and $\Delta m_e/m_e = \Delta h/h + \Delta v/v$.

B. Interrelations between fundamental parameters

Second, in all models in which the weak scale is determined by dimensional transmutation, changes in the largest Yukawa coupling, h_t , will trigger changes in v [43]. In such cases, the Higgs vev is derived from some unified mass scale (or the Planck scale) and can be written as (see Ref. [27])

$$v = M_P \exp\left(-\frac{8\pi^2 c}{h_t^2}\right), \quad (18)$$

where c is a constant of order unity. Indeed, in supersymmetric models with unification conditions such as the constrained minimal supersymmetric standard model [57], there is in general a significant amount of sensitivity to the Yukawa couplings and the top quark Yukawa in particular. This sensitivity can be quantified by a fine-tuning measure defined by [58]

$$\Delta_i \equiv \frac{\partial \ln m_W}{\partial \ln a_i}, \quad (19)$$

where m_W is the mass of the W boson and can be substituted with v . The a_i are the input parameters of the supersymmetric model and include h_t . In regions of the parameter space which provide a suitable dark matter candidate [59], the total sensitivity $\Delta = \sqrt{\sum_i \Delta_i^2}$ typically ranges from 100–400 for which the top quark contribution is in the range $\Delta_t = 80$ –250. In models where the neutralino is more massive, Δ may surpass 1000 and Δ_t may be as large as ~ 500 .

Clearly, there is a considerable model dependence in the relation between Δv and Δh_t . Here we assume a relatively central value obtained from Eq. (18) with $c \approx h_0 \approx 1$. In this case we have

$$\frac{\Delta v}{v} = 16\pi^2 c \frac{\Delta h}{h^3} \approx 160 \frac{\Delta h}{h}, \quad (20)$$

but in light of the model dependence, we will set

$$\frac{\Delta v}{v} \equiv S \frac{\Delta h}{h}, \quad (21)$$

hence defining $S \equiv d \ln v / d \ln h \sim \Delta_t$ and keeping in mind that $S \approx 160$. It follows that the variations of B_D , Q , and τ_n are expressed in the following way:

$$\frac{\Delta B_D}{B_D} = -17(S+1)\frac{\Delta h}{h} + 18\frac{\Delta\Lambda}{\Lambda}, \quad (22)$$

$$\frac{\Delta Q}{Q} = 1.6(S+1)\frac{\Delta h}{h} - 0.6\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda}\right), \quad (23)$$

$$\frac{\Delta\tau_n}{\tau_n} = -(8.8 + 4.8S)\frac{\Delta h}{h} + 3.8\left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda}\right), \quad (24)$$

where we have again assumed common variations in all of the Yukawa couplings. It also follows that $\Delta G_F/G_F = -2S\Delta h/h$ and $\Delta m_e/m_e = (1+S)\Delta h/h$.

Now, using the relation (14) we arrive at

$$\frac{\Delta B_D}{B_D} = -13(1+S)\frac{\Delta h}{h} + 18R\frac{\Delta\alpha}{\alpha} \quad (25)$$

$$\frac{\Delta Q}{Q} = 1.5(1+S)\frac{\Delta h}{h} - 0.6(1+R)\frac{\Delta\alpha}{\alpha}, \quad (26)$$

$$\frac{\Delta\tau_n}{\tau_n} = -(8+4S)\frac{\Delta h}{h} + 3.8(1+R)\frac{\Delta\alpha}{\alpha}. \quad (27)$$

Finally we can take into account the possibility that the variation of the constants is induced by an evolving dilaton [27]. In this scenario, it was shown that $\Delta h/h = (1/2)\Delta\alpha/\alpha$, therefore the expressions above can be simplified to

$$\frac{\Delta B_D}{B_D} = -[6.5(1+S) - 18R]\frac{\Delta\alpha}{\alpha} \quad (28)$$

$$\frac{\Delta Q}{Q} = (0.1 + 0.7S - 0.6R)\frac{\Delta\alpha}{\alpha} \quad (29)$$

$$\frac{\Delta\tau_n}{\tau_n} = -[0.2 + 2S - 3.8R]\frac{\Delta\alpha}{\alpha}, \quad (30)$$

though these relations will also be affected by model dependent threshold corrections.

C. Sensitivity of B_D to the pion mass

An independent calculation suggests a large dependence of the binding energy of the deuteron to the pion mass [60] parametrized in Ref. [31], for constant Λ , by

$$\frac{\Delta B_D}{B_D} = -r \frac{\Delta m_\pi}{m_\pi}, \quad (31)$$

where r is a fitting parameter found to be between 6 and 10. The mass of the pion is given by $f_\pi^2 m_\pi^2 = (m_u + m_d)\langle \bar{q}q \rangle$,

where $f_\pi \propto \Lambda$ is a coupling and $\langle \bar{q}q \rangle \propto \Lambda^3$ is the quark condensate. Hence, the sensitivity of the binding energy to the fundamental parameters is

$$\frac{\Delta B_D}{B_D} = \left(1 + \frac{r}{2}\right) \frac{\Delta \Lambda}{\Lambda} - \frac{r}{2} \left(\frac{\Delta v}{v} + \frac{\Delta h}{h}\right), \quad (32)$$

which must be compared with Eq. (12). The coefficients are different, at most, by a factor of 4. Substituting relations (14) and (21) into Eq. (32), we obtain

$$\frac{\Delta B_D}{B_D} = (0.2 - 0.4r)(1 + S) \frac{\Delta h}{h} + (1 + 0.5r)R \frac{\Delta \alpha}{\alpha}, \quad (33)$$

which is to be compared with Eq. (25).

For the dilaton model considered, $\Delta h/h = (1/2)\Delta\alpha/\alpha$, and we find

$$\frac{\Delta B_D}{B_D} = [(0.1 - 0.2r)(1 + S) + (1 + 0.5r)R] \frac{\Delta \alpha}{\alpha}. \quad (34)$$

Again, the ratio of the coefficients in Eq. (28) to these is at the most of order 4. We therefore conclude that taking a scaled dependence with the pion mass (31) leads to the same constraints on the variation of the fundamental parameters (up to a factor of a few) as the nucleon/meson mass dependence (8).

D. Links to the variation of m_p/m_e

Before we use the above relations in our BBN code, it is interesting to first compare these relations with the observed variation in μ . Using Eq. (11), and then Eqs. (14) and (21), the proton-to-electron mass ratio $\mu = m_p/m_e$ varies according to

$$\frac{\Delta \mu}{\mu} = 0.8R \frac{\Delta \alpha}{\alpha} - 0.6(S + 1) \frac{\Delta h}{h}. \quad (35)$$

Using the current value on the observational variation of μ at redshift $z \sim 3$ [39], i.e. $\Delta \mu/\mu \approx 3 \times 10^{-5}$, we obtain, assuming α constant,

$$\frac{\Delta h}{h} \approx -3.2 \times 10^{-7} \left(\frac{161}{1 + S}\right). \quad (36)$$

Interestingly we deduce from Eq. (25) that when α is constant

$$\frac{\Delta B_D}{B_D} \approx 22 \frac{\Delta \mu}{\mu} \approx 6.6 \times 10^{-4}, \quad (37)$$

at $z \sim 3$, independent of the value of S .

In the case where the variation is driven by a dilaton, we can link the observational variation in μ to a variation in α to get

$$\frac{\Delta \alpha}{\alpha} = -1.5 \times 10^{-6} \left[\frac{-20.2}{0.8R - 0.3(S + 1)} \right], \quad (38)$$

which is compatible with the measurement of the time variation of the fine structure constant in Refs. [34,35] but higher than the stronger bound found in Ref. [36], for the considered value $(R, S) = (36, 160)$. Note that this corresponds to $\Delta B_D/B_D \approx 6 \times 10^{-4}$, by applying Eq. (28), which is comparable to the value found in Eq. (37) where α was taken to be constant.

IV. NUCLEOSYNTHESIS

A. Implementation in a BBN code

We incorporate the relations derived above in a BBN network. We use the reaction rates provided by the compilation and analysis of experimental data of Ref. [61] covering ten of the twelve nuclear reactions involved in the BBN. The two remaining reactions of importance, $n \leftrightarrow p$ and $p(n, \gamma)D$, come from theory and are numerically evaluated, taking into account the variations discussed above.

The $p(n, \gamma)D$ reaction rate is calculated according to the Chen and Savage [62] derivation of the cross section in the framework of effective field theory. The weak reaction rates $n \leftrightarrow p$ are calculated by numerical integration of the electron, positron, and neutrino Fermi distributions and phase space for the six weak interaction reactions. Zero temperature radiative corrections [63,64] to the weak rates are also evaluated numerically. The neutrino versus photon temperature used in these rate calculations is a by-product of the numerical integration of the Friedmann equation where we follow the electron-positron annihilation exactly by the numerical integration of their Fermi distributions.

In principle, one could have introduced a variation in B_D in the effective field theory cross section provided the scattering length in the 1S_0 channel follows this variation. We found it simpler to use the Dmitriev *et al.* [29] prescription for the reaction rate change that takes into account these effects. The binding energy of the deuteron is also directly involved in the calculation of the reverse rate.

Variations in the binding energy of the dineutron also have little effect on the primordial abundances provided its absolute value remains smaller than the deuteron's binding energy [65]. Considering that, in this work the variations on the binding energy of the deuteron are only of a few percent, we do not expect any important role played by the binding energy of the dineutron in the calculations.

By varying B_D , one also changes the size of the deuteron which consequently modifies the $D(D, n)^3\text{He}$, $D(D, p)T$, and $D(p, \gamma)^3\text{He}$ reaction rates [66]. We have, however, not included this effect in our calculation.

The $n \leftrightarrow p$ weak rates depend on both Q and τ_n . Equation (6) gives the decay rate of free neutrons; that is, the zero temperature limit of the weak $n \rightarrow p$ rate. This equation is used, in conjunction with the experimental

value of the neutron lifetime ($\tau_n = 885.7$ s; see Ref. [67]) to fix the normalization of the finite temperature weak rates. Hence, variations in τ_n directly affect those rates.² More precisely, we use Eq. (6) to fix the *present day* normalization, then scale it according to the values of m_e and G_F at BBN and use the BBN values of Q and m_e to evaluate the integrals in the finite temperature weak rate calculations involving Q/m_e , T/m_e and T_ν/m_e . Nevertheless, the approximation of scaling the weak rates with τ_n is very good.

B. Results under the hypothesis $\Delta\alpha/\alpha = 0$

We proceed with the calculation of the light element abundances during nucleosynthesis using Eqs. (25)–(27), *first* neglecting the contribution from $\Delta\alpha/\alpha$. From the discussions of the two previous sections, we conclude that variations in the Yukawa couplings lead directly to the changes in τ_n , Q , and B_D . In addition, the nucleon mass also changes [$\Delta m_N/m_N \simeq 0.76\Delta\Lambda/\Lambda + 0.24(\Delta v/v + \Delta h/h) \simeq 66\Delta h/h$] as does the electron mass ($\Delta m_e/m_e \simeq 161\Delta h/h$).

In Fig. 1, we show the time evolution of the light elements for the standard BBN model (solid curves) and when a variation $\Delta h/h = 1.5 \times 10^{-5}$ (dashed curves) is considered, assuming the WMAP value for η and using the central value $S = 160$. We see that the largest effect is indeed a decrease in the ${}^7\text{Be}$ abundance (which contributes to the final ${}^7\text{Li}$ abundance after decay) correlated to an increase of n/p and a slight increase in the D/H abundance.

In Fig. 2, we show the final abundances of D/H, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}/\text{H}$ as a function of the variation in the Yukawa coupling h , for three assumed values of the parameter S . The horizontal cross-hatched regions indicate the current observational spectroscopic determinations. For ${}^4\text{He}$, we use $Y = 0.232\text{--}0.258$ [70]. For D/H, we use the 2σ range based the latest average of six quasar absorption systems, $\text{D}/\text{H} = (2.83 \pm 0.52) \times 10^{-5}$ [71]. For ${}^7\text{Li}/\text{H}$, we show two ranges: the first given by ${}^7\text{Li}/\text{H} = 1.23^{+0.68}_{-0.32} \times 10^{-10}$ [8] and the second given by ${}^7\text{Li}/\text{H} = (2.33 \pm 0.64) \times 10^{-10}$ when higher surface temperatures are assumed [12] and is represented with dashed lines. The dotted vertical line indicates the standard BBN results (i.e. $\Delta h/h = 0$) for $\eta = 6.12 \times 10^{-10}$.

We recall that there is significant model dependence in several of the assumed relations between the fundamental parameters. For example, in Eq. (21), we adopted $c/h^2 = 1$ (that is $S \simeq 160$). However, the origin of the dependence between v and h depends on physics beyond the standard

²While a more recent determination of neutron lifetime has been published [68], its value has not been adopted by the Review of Particle Physics [67]. Consequences of this measurement on BBN were considered in [69].

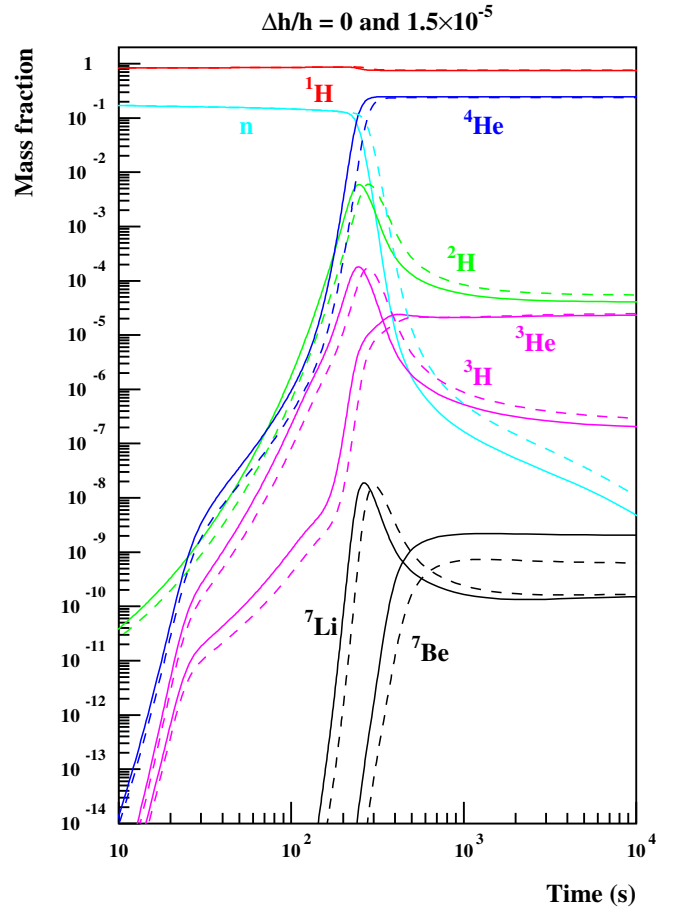


FIG. 1 (color online). The time evolution of the light element abundances for standard BBN (solid curves) and when variations $\Delta h/h = 1.5 \times 10^{-5}$ are included (dashed), assuming α remains constant. The WMAP value of $\eta = 6.12 \times 10^{-10}$ was assumed and we have used $S = 160$.

model, and c/h^2 could be significantly larger or smaller than unity.

As one can see from Fig. 2, each of the light elements, D, ${}^4\text{He}$, and ${}^7\text{Li}$ show strong dependences on $\Delta h/h$. In fact, D/H provides us with the strongest constraint (under the hypothesis that α is constant) which for $S = 160$ is,

$$-1.5 \times 10^{-5} < \frac{\Delta h}{h} < 1.9 \times 10^{-5}. \quad (39)$$

Using Eq. (25), this bound translates to $-4 \times 10^{-2} < \Delta B_D/B_D < 3.1 \times 10^{-2}$. Note that we have not used the ${}^7\text{Li}$ abundance to set the lower bound on $\Delta h/h$. However, we also observe that, for values of $\Delta h/h \gtrsim 1.8 \times 10^{-5}$ ($\Delta h/h \gtrsim 0.9 \times 10^{-5}$ for the second range of observational ${}^7\text{Li}$), the ${}^7\text{Li}$ abundance is sufficiently small so as to come into agreement with the observational data. So long as we do not exceed the upper bound given in Eq. (39), all of the light elements can be brought into agreement with data. Thus, we must saturate the limit, but recall that this con-

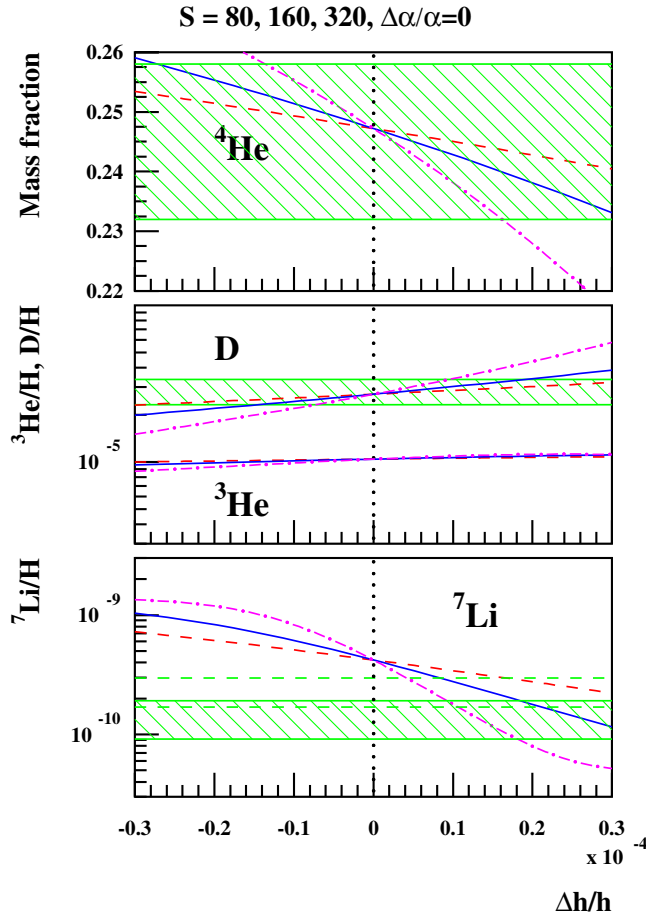


FIG. 2 (color online). Primordial abundances of ${}^4\text{He}$, D, ${}^3\text{He}$, and ${}^7\text{Li}$ when allowing a variation of the Yukawa couplings. The horizontal cross-hatched regions depict the 2σ spectroscopic data. We have assumed 3 values for S : $S = 80$ (dashed lines), $S = 160$ (solid lines), and $S = 320$ (dot-dashed lines) for $\eta = 6.12 \times 10^{-10}$.

clusion is based under the restrictive assumption that α is constant. On the other hand, it also means that our hypothesis can be falsified by decreasing the error bars of either ${}^7\text{Li}$ or D.

In Fig. 3, we show the individual contributions of the varying BBN quantities to the light element abundances. When varying Q and m_e individually, τ_n is held constant, i.e. Eqs. (6) and (7) have not been used. The effects of the variations of τ_n , Q , B_D , and m_e can be seen explicitly. The curves for m_e are due to the effects of the electron mass on the expansion of the universe: m_e effectively enters in the rhs of the Friedmann equation, affecting the timing and magnitude of the photon bath reheating following electron-positron annihilations. This effect is however very small as seen in Fig. 3. The effect of varying m_e in the weak rates is accounted for in the overall variation of τ_n . The electron mass does not affect the abundance of any of the isotopes, however, τ_n and Q have a significant effect on ${}^4\text{He}$ leaving deuterium and ${}^7\text{Li}$ almost unchanged.

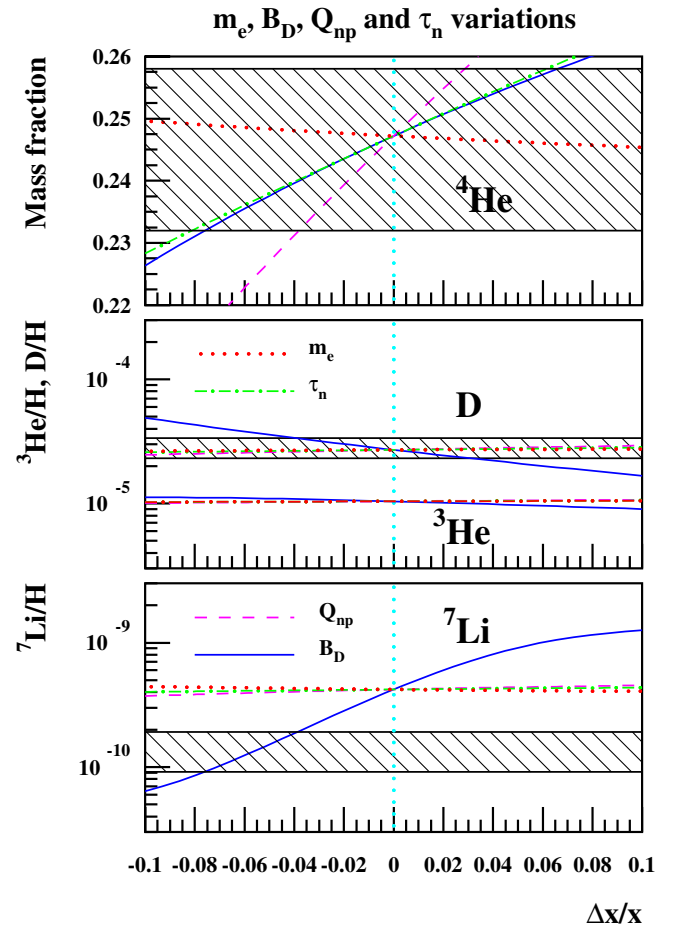


FIG. 3 (color online). Primordial abundances of the light nuclei as a function of the relative variation of m_e (dotted lines), τ_n (dot-dashed lines), Q (dashed lines), and B_D (solid lines) with the same conventions as in Fig. 2.

From the ${}^4\text{He}$ data, we deduce the bounds, $-7.5 \times 10^{-2} \lesssim \Delta B_D/B_D \lesssim 6.5 \times 10^{-2}$, $-8.2 \times 10^{-2} \lesssim \Delta \tau_n/\tau_n \lesssim 6 \times 10^{-2}$, and $-4 \times 10^{-2} \lesssim \Delta Q/Q \lesssim 2.7 \times 10^{-2}$ at 2σ . A variation of the deuterium binding energy affects all the abundances, in particular, the deuterium data sets the tighter constraint $-4 \times 10^{-2} \lesssim \Delta B_D/B_D \lesssim 3 \times 10^{-2}$. Interestingly, these bounds are equivalent to the ones obtained from the constraint (39) considering the interrelations between the fundamental parameters. The ${}^7\text{Li}$ abundance is brought in concordance with spectroscopic observations provided its change falls within the interval $-7.5 \times 10^{-2} \lesssim \Delta B_D/B_D \lesssim -4 \times 10^{-2}$. We thus conclude that B_D is the most important parameter connected to the discrepancy of the ${}^7\text{Li}$ abundance, and again, we see that there exists a window allowing for consistent ${}^7\text{Li}$ and deuterium abundances with data.

One may also consider the effect of the variation of the nucleon mass. The proton and neutron reduced mass enters as a factor $(m_p^{-1} + m_n^{-1})^{1/2}$ in the $p(n, \gamma)\text{D}$ rate. For var-

iations of the order we are considering, this effect is negligible.

C. Allowing for $\Delta\alpha/\alpha \neq 0$

We now allow the fine structure constant to vary and we further assume that it is tied to the variation of the Yukawa couplings according to $\Delta h/h = (1/2)\Delta\alpha/\alpha$, using Eqs. (28)–(30). The results are shown in Fig. 4 where the abundances are depicted for three values of the parameter R . Comparison of this figure with Fig. 2 shows the effect of including the variation in α . Not considering ${}^7\text{Li}$, the tighter bounds on $\Delta h/h$ are again given by the deuterium abundance and are comparable in order of magnitude to the ones found in Eq. (39):

$$-1.6 \times 10^{-5} < \frac{\Delta h}{h} < 2.1 \times 10^{-5}, \quad (40)$$

for $R = 36$ and

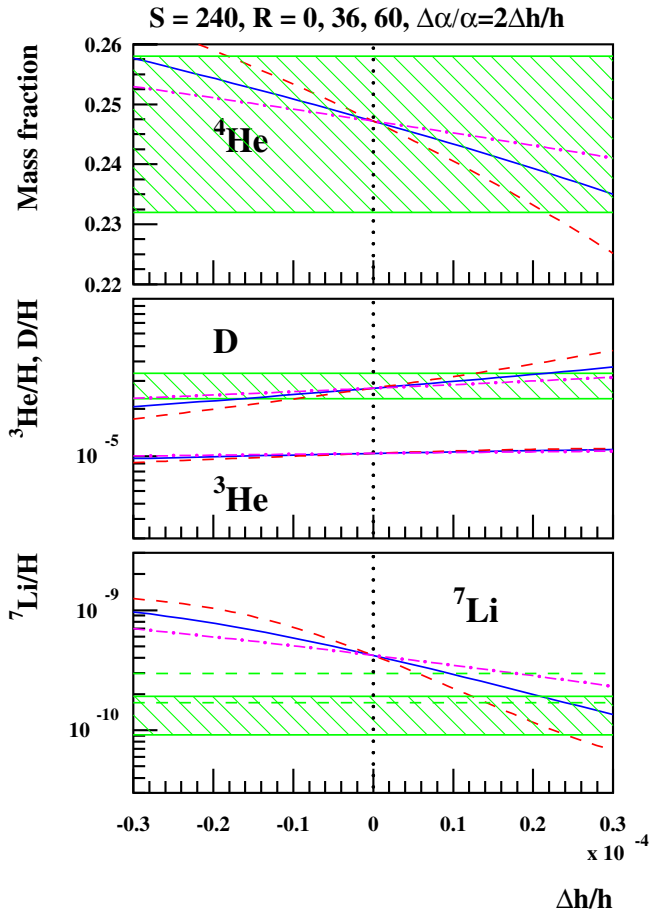


FIG. 4 (color online). Primordial abundances of ${}^4\text{He}$, D , ${}^3\text{He}$, and ${}^7\text{Li}$ as a function of $\Delta h/h = (1/2)\Delta\alpha/\alpha$ when allowing a variation of the fine structure constant for three values of the R parameter: $R = 0$ (dashed lines), $R = 36$ (solid lines), and $R = 60$ (dot-dashed lines).

$$-3 \times 10^{-5} < \frac{\Delta h}{h} < 4 \times 10^{-5}, \quad (41)$$

for $S = 240$ and $R = 60$.

While these limits are far more stringent than the one found in Ref. [25], it is consistent with those derived in Refs. [26,27] where coupled variations were considered. Once again, for a variation near the upper end of the range (40) and (41), we can simultaneously fit all of the observed abundances.

As noted above, a variation of α induces a multitude of changes in nuclear cross sections that have not been included here. We have checked, however, that a variation of $\Delta\alpha/\alpha \approx 4 \times 10^{-5}$ leads to variations in the reaction rates (numerically fit), mainly through the Coulomb barrier, of the most important α -dependent reactions in BBN [25] that never exceed one tenth of a percent in magnitude.

Before concluding, we return once more to the question of model dependence. We have parametrized the uncertainty between Δv and Δh with the quantity S and the uncertainty between $\Delta\Lambda$ and $\Delta\alpha$ through R . In full gen-

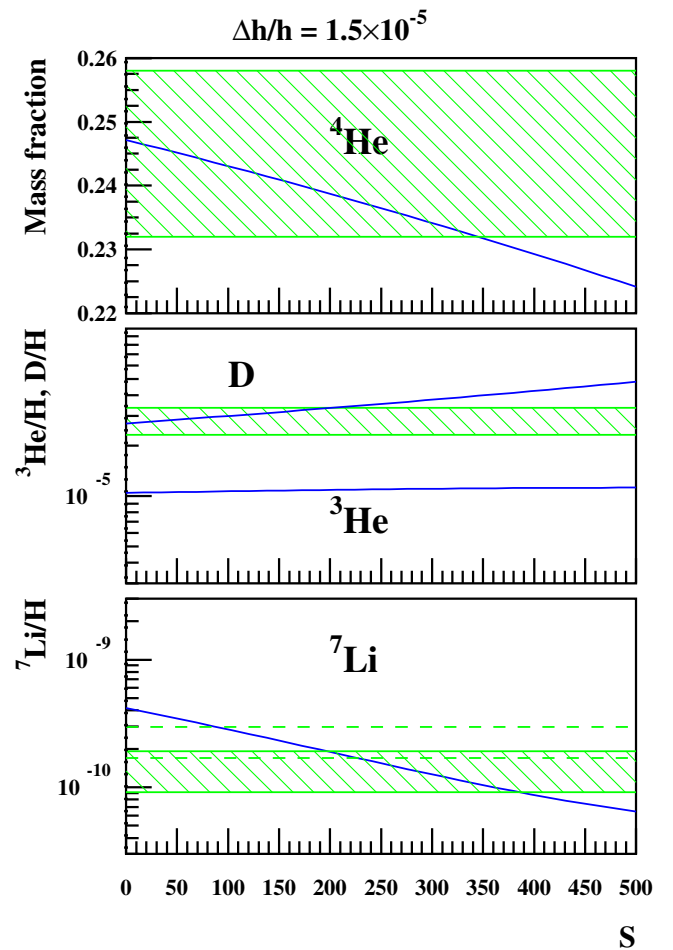


FIG. 5 (color online). Primordial abundances of the light nuclei as a function of the parameter S assuming a change in the Yukawa couplings $\Delta h/h = 1.5 \times 10^{-5}$ and $\Delta\alpha/\alpha = 0$.

erality, we ought to include one more unknown, say T , that parametrizes the relation between $\Delta\alpha$ and Δh , $T \equiv d \ln h / d \ln \alpha$ [56]. In this work, however, we focused our investigation in the dilaton model where $T = 1/2$. It is now important to evaluate more precisely how sensitive our results are to the value these parameters may take. In Fig. 5 we illustrate the evolution of the primordial abundances of the light nuclei with S for a fixed value of the change in the Yukawa couplings assuming $\Delta\alpha/\alpha = 0$. We clearly see that, in this case, the theoretical ${}^7\text{Li}$ abundance is compatible with its observational measurement provided $200 \lesssim S \lesssim 370$ (for the lower range of observational ${}^7\text{Li}$ abundances).

We can also evaluate the impact of changing R in the dilaton model, when we allow a variation in α . To this end, we show in Fig. 6 the evolution of the primordial abundances for two different values of $\Delta h/h$. We observe that when $\Delta h/h = 1.5 \times 10^{-5}$, we require $R = 16$. On the

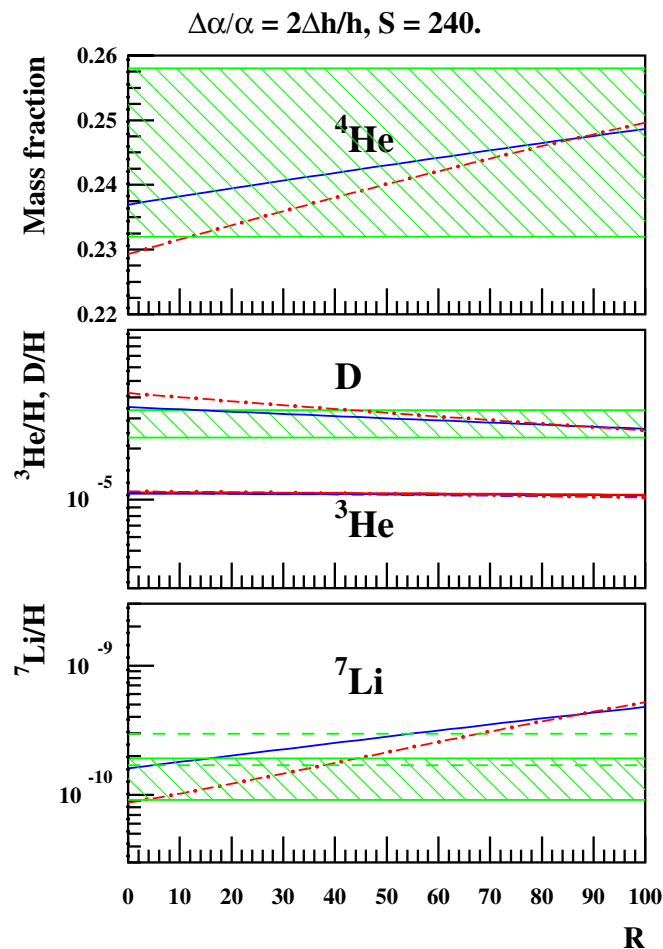


FIG. 6 (color online). Primordial abundances of the light nuclei as a function of the parameter R assuming a change in the Yukawa couplings: $\Delta h/h = 1.5 \times 10^{-5}$ (solid lines) and $\Delta h/h = 2.5 \times 10^{-5}$ (dot-dashed lines), for $S = 240$.

other hand, if we take $\Delta h/h = 2.5 \times 10^{-5}$, the abundances are more sensitive to the value of R as the slope of the corresponding curves are steeper, but there is also a narrow window around $R = 45$ where all the light nuclei abundances are compatible with the full observational data.

V. SUMMARY

In this article, we have considered the influence of a possible variation of the fundamental constants on the abundances of the light elements synthesized during BBN. We have focused our attention on three fundamental quantities central to BBN, namely Q , τ_n , and B_D , the variation of which was related to the one of the fundamental constants. Specifying our theoretical framework, we have reduced the fundamental constants to two independent ones, the Yukawa coupling and the fine structure constant.

We have shown that these constants have a strong effect on ${}^4\text{He}$ allowing us to set strong constraints on the variation of m_e , B_D , Q , and τ_n . Interestingly, the deuterium and ${}^7\text{Li}$ abundances are mainly sensitive to B_D and we have shown that there is a window in which ${}^7\text{Li}$ is compatible with spectroscopic data (see Fig. 3). The existence of such a narrow window also implies that our mechanism can be falsified by an increase of the precision of deuterium and/or ${}^7\text{Li}$ data. Our analysis also enables one to set sharper constraints on the variation of the fundamental constants.

Assuming that the fine structure constant does not vary, we have shown that deuterium and ${}^4\text{He}$ data set strong constraints on the variation of the Yukawa couplings [see Eq. (39)] but that inside this bound there exists a range reconciling the ${}^7\text{Li}$ abundance with spectroscopic observations. We then allow the fine structure constant to vary and set a sharp constraint on its variation in the dilaton scenario [see Eqs. (40) and (41)]. The theoretical limitations have also been discussed in detail. More specifically, we have parametrized the relations between $\Delta\nu$ and Δh and between $\Delta\Lambda$ and $\Delta\alpha$ with two free quantities, S and R , respectively. We found that the specific value of these quantities plays an important role alongside the change in the fundamental parameters in solving the ${}^7\text{Li}$ abundance problem. We conclude therefore, that a better understanding of the values of these parameters from the theoretical standpoint can help us to better constrain the variation of the fundamental parameters at the time of BBN.

ACKNOWLEDGMENTS

We would like to thank M. Voloshin for discussions. The work of K. A. O. and N. J. N. was supported in part by DOE Grant No. DE-FG02-94ER-40823. The work is also supported by the project “INSU-CNRS/USA.”

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