Signatures of axionlike particles in the spectra of TeV gamma-ray sources

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One interpretation of the unexplained signature observed in the PVLAS experiment invokes a new axionlike particle (ALP) with a two-photon vertex, allowing for photon-ALP oscillations in the presence of magnetic fields. In the range of masses and couplings suggested by PVLAS, the same effect would lead to a peculiar dimming of high-energy photon sources. For typical parameters of the turbulent magnetic field in the galaxy, the effect sets in at $E_\gamma \gtrsim 10$ TeV, providing an ALP signature in the spectra of TeV gamma sources that can be probed with Cherenkov telescopes. A dedicated search will be strongly motivated if the ongoing photon regeneration experiments confirm the PVLAS particle interpretation.

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I. INTRODUCTION

One of the phenomenologically most important properties of the hypothetical axions is their two-photon vertex that allows for axion-photon conversions in external electric or magnetic fields [1,2]. In particular, this coupling is used by the ADMX experiment to search for axion dark matter [3,4] and by the CAST experiment to search for solar axions [5,6]. Generically such particles affect the propagation of photons in magnetic fields. For a linearly polarized laser beam propagating in a transverse B-field, signatures are a rotation of the plane of polarization and the development of an elliptical polarization component [7–9]. The latter effect is also caused by the effective four-photon interaction predicted by QED [10,11].

Recently the laser experiment PVLAS has reported such results with an amplitude about 10^4 times larger than expected from QED [12]. If one interprets this signal in terms of photon-axion conversions, these measurements imply an axion mass $m_a \approx 1.3$ meV and a coupling with photons $g_{a\gamma} \approx 3 \times 10^{-6}$ GeV^{-1}, where the coupling constant is defined in Eq. (1) below. This combination of m_a and $g_{a\gamma}$ is incompatible with axions in the usual sense. Therefore, the new states require a different interpretation and are generically termed ''axionlike particles'' (ALPs), meaning bosons with a two-photon vertex where the mass and coupling strength are taken as independent parameters.

The main problem with the PVLAS signature is that it violates simple astrophysical limits by a huge margin. ALPs are produced in the Sun and other stars by the Primakoff process where thermal photons convert in the fluctuating electric fields of the stellar plasma [1,13,14]. Assuming the PVLAS-inspired parameters, a standard solar model leads to an ALP luminosity so large that the Sun would burn out in 1000 years. Circumventing this vast discrepancy is the main theoretical challenge for the PVLAS particle interpretation [15–20].

It is conceivable that the presence of the hot stellar plasma modifies the effective couplings or that these couplings are different at the momentum transfers relevant in stars. Therefore, it has been stressed that the PVLAS particle interpretation should be tested with experiments where the transition takes place in vacuum and where the momentum transfer is small [21]. Photon regeneration experiments (''shining light through a wall'') are of particular interest because it will be fairly easy to confirm PVLAS if the particle interpretation is indeed correct. Several such efforts are now being discussed or are already under way [22–24], notably ALPs at DESY, BMV at LULI, GammeV at Fermilab, LIPSS at Jefferson Laboratory, OSQAR at CERN, and PVLAS-regeneration at INFN Laboratory in Legnaro.

If the PVLAS particle interpretation is confirmed, some radical new low-energy physics must be at work that prevents ALP emission from stars. However, other astrophysical settings provide conditions similar to the laboratory experiments, i.e., a vacuum environment and near-vanishing momentum transfers. One example is ''shining light through the Sun'' where a high-energy photon source would become visible through the Sun by photon-ALP conversion in the solar magnetic field on the far side of the Sun, and their regeneration on our side [25]. Another example is the double pulsar J0737-3039, where gamma rays emitted by one pulsar periodically pass through the magnetosphere of the other on their way to us [26].

We here consider another example, the photon-ALP conversion in the turbulent magnetic field of our galaxy. Beyond energies of order 10 TeV, the gamma-ray flux would be depleted, leaving a distinct signature in the spectrum of TeV gamma-ray sources. Current data from Imaging Atmospheric Cherenkov Telescopes (IACTs) do not allow for a stringent constraint on this effect. However, if the laboratory experiments confirm the existence of ALPs with the properties suggested by PVLAS, this depletion must be included in the analysis of TeV gamma-ray sources by IACTs. Given the strong motivation that would be provided by a positive laboratory ALP confirmation, dedicated efforts by present and future instruments would be mandatory that could provide an independent astrophysical signature of these novel particles and/or allow one to study or constrain the turbulent galactic B field.

We begin in Sec. II with a summary of the formalism to describe photon-ALP conversions and turn in Sec. III to phenomenological consequences on the propagation of TeV photons in our galaxy. In Sec. IV we briefly touch on the possible effect of millicharged particles on photon propagation. We conclude in Sec. V.

II. PHOTON-AXION CONVERSION

Axionlike particles by definition have a two-photon coupling. For pseudoscalars, it is of the form

$$\mathcal{L}_{a\gamma} = -\tfrac{1}{4} g_{a\gamma} F_{\mu\nu}\tilde{F}^{\mu\nu} a = g_{a\gamma}\mathbf{E}\cdot\mathbf{B} a, \tag{1}$$

where a is the axionlike field with mass m_a, $F_{\mu\nu}$ the electromagnetic field-strength tensor, $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ its dual, and $g_{a\gamma}$ the ALP-photon coupling with dimension of inverse energy. For a scalar particle, the coupling is proportional to $F_{\mu\nu}F^{\mu\nu}a$. To be definite we limit our discussion to the pseudoscalar case, but similar consequences apply to scalars.

As a consequence of this coupling, ALPs and photons oscillate into each other in an external magnetic field. Under quite general assumptions, the probability for an unpolarized photon beam to convert to ALPs after traversing a magnetic field $\mathbf{B} = (B_x, B_y, B_z)$ from 0 to z is (Appendix A)

$$P_{\gamma\to a}(z) = \frac{g_{a\gamma}^2}{8}\left(\left|\int_0^z dz' e^{-i2\pi z'/l_0} B_x(x, y, z')\right|^2 + \left|\int_0^z dz' e^{-i2\pi z'/l_0} B_y(x, y, z')\right|^2\right), \tag{2}$$

where for simplicity we have chosen the z-axis along the propagation direction. Further, $l_0 = 4\pi E/m_a^2$ is the oscillation length with m_a the axion mass and E the photon energy. The meV range of ALP masses, relevant for the PVLAS particle interpretation, is so large that the photon plasma mass is completely negligible by comparison, in contrast to the case of cosmic microwave conversion into intergalactic magnetic fields, studied in [27,28].

We consider a simplified case where the field is of constant magnitude and random direction in each patchy domain, each with typical size $s \ll z$, so that a large number N of domains is crossed. The previous expression then further simplifies to (Appendix A)

$$P_{\gamma\to a}(z) = NP_0, \tag{3}$$

where the probability per single domain is

$$P_0 \approx \frac{g_{a\gamma}^2\langle|\mathbf{B}|^2\rangle s^2}{4}\frac{\sin^2(\pi s/l_0)}{(\pi s/l_0)^2}. \tag{4}$$

Equation (3) only holds in the perturbative regime where $NP_0 \ll 1$. For N sufficiently large, this result violates unitarity. It can be shown (Appendix of Ref. [28]) that the correct continuum limit after traveling over $z \gg s$ is

$$P_{\gamma\to a}(z) = \frac{1}{3}\left[1 - \exp\left(-\frac{3P_0 z}{2s}\right)\right]. \tag{5}$$

As physically expected, Eq. (5) implies for $z/s \to \infty$ that the conversion probability saturates so that on average one third of all photons converts to axions.

III. CONVERSIONS IN THE TURBULENT GALACTIC MAGNETIC FIELD

For the PVLAS-inspired parameters $m_a = 1.3$ meV and $g_{a\gamma} = 3\times 10^{-6}$ GeV^{-1}, it is useful to write P_0 in suitable numerical units,

$$P_0 = (1.5 g_6 B_{\mu G} s_{pc})^2 \frac{\sin^2(3.8\times 10^3 m_{meV}^2 s_{pc}/E_{10})}{(3.8\times 10^3 m_{meV}^2 s_{pc}/E_{10})^2} \simeq 0.8\times 10^{-7}\left(\frac{g_6 B_{\mu G} E_{10}}{m_{meV}^2}\right)^2. \tag{6}$$

Here, we have introduced $g_6 = g_{a\gamma}/10^{-6}$ GeV^{-1}, $B_{\mu G}$ is the root mean square (rms) magnetic field strength in micro-Gauss, s_{pc} is the domain size in pc, m_{meV} is the ALP mass in meV, and E_{10} the photon energy in units of 10 TeV. In the second line we have replaced $\sin^2$ with its average value $\frac{1}{2}$ because its argument is large and oscillates rapidly for any realistic energy resolution.

Although the galactic B field has a regular component with several kpc coherence length, on small scales a turbulent component dominates (Ref. [29] and references therein). The power spectrum follows a Kolmogorov power law, with a lower cutoff at very small dissipative scales, perhaps as small as 6×10^{-4} pc [30], but in any case at most comparable to 0.01 pc, with an rms intensity of order μG on pc scales. For typical galactic distances of 10 kpc, there are approximately 10^6 domains with $s \approx 0.01$ pc towards a typical TeV gamma source such as the one at the galactic center [31–34]. For nominal values of the parameters in Eq. (6), $P_0 \simeq 10^{-7}$–10^{-6} at energies of order 10 TeV, implying $NP_0 \simeq 0.1$–1. Therefore, observable effects must be expected.

In Fig. 1 we show the spectral modification of a TeV source at the galactic center (distance 8.5 kpc) superimposed with H.E.S.S. data [35]. For illustration we have used Eq. (5) with $g_6 = 3$, $B_{\mu G} = 0.7$, $s_{pc} = 0.01$, and $m_{meV} = 1$ and we have assumed that the power-law spectrum does not break before 60 TeV. Photon-ALP oscillations (dashed curve) cause a downward shift of the spectrum at high energies, i.e., a change of normalization of the typical power-law spectrum (continuous curve) between low and high energies ($E \gtrsim 10$ TeV). The maximum shift is 33% when the conversion saturates.

Evidently current data do not allow for a serious constraint on this depletion effect. Note, however, that the

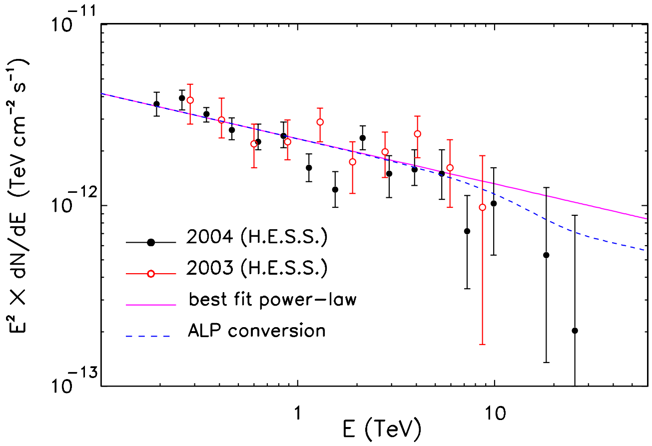

FIG. 1 (color online). Spectral energy density $E^2 \times \mathrm{d}N/\mathrm{d}E$ of photons from the galactic center source, for the 2004 data (full points) and 2003 data (open points) of H.E.S.S. [35]. Error bars represent 95% C.L. The continuous line shows the best-fit power law $\mathrm{d}N/\mathrm{d}E \sim E^{-\Gamma}$ with $\Gamma = 2.25$ [35]. The dashed line shows the effect of photon-ALP conversion with coupling and mass suggested by PVLAS.

large error bars at high energy are only due to a lack of statistics. The points reported in Fig. 1 are based on 17 hours of data in 2003 with two telescopes and 48.7 hours in 2004 in the four-telescope array mode. Already the current generation of IACTs (CANGAROO-III, H.E.S.S., MAGIC, VERITAS) may have a sufficient aperture to probe this scenario, if dedicated campaigns were motivated by a positive laboratory detection.

To be more quantitative, one would model the turbulent field as a Gaussian random field with zero mean and an rms value B_{rms} [29]. Each of its components can thus be written in terms of its Fourier transform as

$$B_i(\mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \tilde{B}_i(\mathbf{k}) \mathrm{e}^{\mathrm{i}[\mathbf{x}\cdot\mathbf{k}+\phi_i(\mathbf{k})]}, \tag{7}$$

where the phases $\phi_i(\mathbf{k})$ are random. For an isotropic and homogeneous turbulence, the Fourier modes satisfy

$$\langle \tilde{B}_i(\mathbf{k}) \tilde{B}_j^*(\mathbf{k}') \rangle = \frac{\mathcal{B}^2(k)}{8\pi k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)(2\pi)^6 \delta^3(\mathbf{k} - \mathbf{k}'), \tag{8}$$

where the tensor in brackets implements the condition $\nabla \cdot \mathbf{B} = 0$. In the generic case of a power-law spectrum with index α between the scales s_{min} and s_{max}, i.e., between wave numbers $k_{\mathrm{L}} \equiv 2\pi/s_{\mathrm{max}}$ and $k_{\mathrm{H}} \equiv 2\pi/s_{\mathrm{min}}$, one has

$$\mathcal{B}^2(k) = B_{\mathrm{rms}}^2(\alpha - 1)k^{-\alpha}(k_{\mathrm{L}}^{1-\alpha} - k_{\mathrm{H}}^{1-\alpha})^{-1}, \tag{9}$$

which is already normalized such that $\langle |\mathbf{B}(\mathbf{x})|^2 \rangle = B_{\mathrm{rms}}^2$. In the limit $k_{\mathrm{L}} \ll k_{\mathrm{H}}$, and if $\alpha > 1$, one finds

$$\mathcal{B}^2(k) \simeq B_{\mathrm{rms}}^2(\alpha - 1)k^{-\alpha}k_{\mathrm{L}}^{\alpha-1}. \tag{10}$$

Therefore, the field averaged over scales less than s is

$$\langle |\mathbf{B}(\mathbf{x})|^2 \rangle_s = B_{\mathrm{rms}}^2(s/s_{\mathrm{max}})^{\alpha-1}. \tag{11}$$

For the Kolmogorov spectrum $\alpha = 5/3$ suggested by the data, this means that the rms intensity of the field varies as $s^{1/3}$. The intensity below 0.001 pc is then only a factor 10 weaker than the μG level at the pc scale. Below 0.01 pc it is only a factor $\sim$4 lower than at the pc scale. Therefore, our simple estimate of P_0 may be too optimistic by an order of magnitude. However, the effect would still be observable simply by looking at a factor $\sim$3 larger energies. Additionally, the true field configuration may be more complicated, and recently a more intense turbulence than previously estimated has been suggested [36].

In a more detailed treatment, one would consider stochastic realizations of the realistic power spectrum of the turbulent B field. However, for our purpose simple estimates are probably more instructive and show that: (i) Possible effects may start manifesting themselves around 10 TeV, and are more and more likely to show up at 20–30 TeV. (ii) The smaller the characteristic scale of turbulence of a given intensity, the larger the number of domains available, and the lower the energy at which the effect appears. (iii) The conversion probability depends on E^2. Therefore, on the scale of the typical energy resolution of a Cherenkov telescope, the depletion rapidly drops from negligible to the saturation value of 1/3. (iv) The phenomenology described here would be universal, affecting both galactic and extragalactic sources. Yet, the exact energy at which the shift manifests depends on the properties of the field along that line of sight. Although for all sources the light must cross the galactic B field to reach us, one may not exclude an additional role of a small-scale field close to the sources. Our estimate for the onset of the effect is conservative, especially for extragalactic sources. As a general rule, for sources in similar directions, the more distant ones may manifest the signature at lower energies.

IV. MILLICHARGED PARTICLES

Another particle-physics explanation of the PVLAS anomaly postulates the existence of low-mass millicharged particles [37,38]. We briefly check if this hypothesis would also affect the propagation of photons in the astrophysical context.

At TeV energies, the extragalactic medium becomes opaque due to the onset of $e^{\pm}$ pair production on the diffuse low-energy photon backgrounds. At a few PeV, the mean-free path of photons reaches a minimum of $\lambda_e \lesssim$ 10 kpc due to pair production on the cosmic microwave background (CMB) [39]. The threshold energy $E_{\mathrm{th}} \sim 3 \times 10^{14}$ eV scales as m_e^2 and the cross section as e^4/m_e^2. Scaling these quantities to millicharged particles with charge $q \ll e$ and mass $m_q \ll m_e$, one finds

$$\lambda_q^{-1} \simeq \lambda_e^{-1}\left(\frac{q}{e}\right)^4\left(\frac{m_e}{m_q}\right)^2, \tag{12}$$

and

$$E_{\mathrm{th}}^{q} \simeq E_{\mathrm{th}}^{e}\left(\frac{m_q}{m_e}\right)^2. \tag{13}$$

The preferred mass range of the millicharged candidate is 0.01–0.1 eV, i.e., $m_q/m_e \sim 2\times 10^{-8}$–$2\times 10^{-7}$. The peak of the cross section is very close to the threshold and would fall in the $(10^{-16}$–$10^{-14})\times 10^{15}$ eV range, i.e., ranging from infrared to ultraviolet. Sources at cosmological distances do not show such a universal dimming. The conservative requirement $\lambda_q \gtrsim 1$ Gpc implies $q \lesssim 10^{-5}e$.

A much more constraining limit of $q \lesssim 10^{-7}e$ arises from spectral distortion effects of the CMB that may already rule out the millicharged particle explanation of PVLAS [40]. In any event, it appears safe to assume that millicharged particles with the relevant properties would not affect TeV photon observations.

V. CONCLUSIONS

The unexpected optical properties of the vacuum suggested by the PVLAS experiment has inspired various interpretations in terms of axionlike particles. The severe conflict with stellar structure arguments implies that this interpretation requires radical new physics at low energies. If the new particles interact differently in a stellar plasma or at vanishing momentum transfers, they may still show up in the upcoming photon regeneration experiments. In this case, one necessarily expects signatures also in other settings that are characterized by a vacuum environment and/or small momentum transfers.

We have discussed possible signatures of PVLAS particles in the spectra of TeV gamma-ray sources in our galaxy. If the PVLAS signal can be attributed to photon-ALP conversion in the laboratory, the same effect must occur in the astrophysical context. For an ALP mass around 1 meV, as suggested by PVLAS, one would observe a peculiar distortion in the photon spectra at $E_\gamma \gtrsim 10$ TeV due to conversions in the turbulent galactic B-field. This process would take place under better vacuum conditions than are achievable in the laboratory and the momentum transfer would be extremely small.

Present data from TeV gamma-ray telescopes do not allow for a stringent constraint on this effect. However, a positive ALP detection would strongly motivate a dedicated search, perhaps allowing one to find signatures for ALPs in current or future instruments and to investigate or constrain the properties of the turbulent magnetic field in the galaxy and beyond.

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APPENDIX A: PHOTON-ALP CONVERSION IN A RANDOM MAGNETIC FIELD

We here derive the photon-ALP conversion probabilities [Eqs. (2) and (3)] in a random magnetic field distribution. These detailed results are not used for the simple estimates derived in our paper, but would be necessary for a detailed treatment involving the numerical study of different realizations of the turbulent galactic B-field.

For relativistic ALPs, the equations of motion following from Eq. (1) reduce to the linearized system [9]

$$(\omega - \mathrm{i}\partial_z + \mathcal{M})\begin{pmatrix} A_x \\ A_y \\ a \end{pmatrix} = 0, \tag{A1}$$

where z is the direction of propagation, A_x and A_y are orthogonal components of the photon field in a fixed frame perpendicular to z, and ω is the photon energy. The mixing matrix is

$$\mathcal{M} = \begin{pmatrix} \Delta_{xx} & \Delta_{xy} & \frac{1}{2}g_{a\gamma}B_x \\ \Delta_{yx} & \Delta_{yy} & \frac{1}{2}g_{a\gamma}B_y \\ \frac{1}{2}g_{a\gamma}B_x & \frac{1}{2}g_{a\gamma}B_y & \Delta_a \end{pmatrix}, \tag{A2}$$

where $\Delta_a = -m_a^2/2\omega$. Notice that the component of $\mathbf{B}$ parallel to the direction of motion does not induce photon-axion mixing, since only B_x and B_y enter the third row/column of $\mathcal{M}$. The entries Δ_{ij} $(i, j = x, y)$ that mix the photon polarization states are energy-dependent terms determined by the properties of the medium and the QED vacuum polarization effect. We will neglect the latter because it is subdominant here.

The Δ_{ij} terms have a simple interpretation when the x or y direction coincides with the transverse field direction $\mathbf{B}_\mathrm{T} = \mathbf{B} - (\mathbf{B}\cdot\hat{\mathbf{z}})\hat{\mathbf{z}}$. We can then specify the previous equations for the case of a single domain with uniform magnetic field $\mathbf{B}_\mathrm{T}$, whose modulus will be denoted by $B_T = |\mathbf{B}_\mathrm{T}|$. Equation (A1) is in the new basis

$$(\omega - \mathrm{i}\partial_z + \mathcal{M})\begin{pmatrix} A_\perp \\ A_\parallel \\ a \end{pmatrix} = 0, \tag{A3}$$

where $i = \perp$ or $\parallel$ refer to the $\mathbf{B}_\mathrm{T}$ direction. The mixing matrix is now

$$\mathcal{M} \equiv \begin{pmatrix} \Delta_\perp & \Delta_R & 0 \\ \Delta_R & \Delta_\parallel & \Delta_{a\gamma} \\ 0 & \Delta_{a\gamma} & \Delta_a \end{pmatrix}, \tag{A4}$$

where

$$\begin{aligned} \Delta_\perp &= \Delta_{\rm pl} + \Delta_\perp^{\rm CM}, \qquad \Delta_{a\gamma} = \tfrac{1}{2} g_{a\gamma} B_{\rm T}, \\ \Delta_\parallel &= \Delta_{\rm pl} + \Delta_\parallel^{\rm CM}, \qquad \Delta_{\rm pl} = -\omega_{\rm pl}^2/2\omega. \end{aligned} \tag{A5}$$

Here, $\omega_{\rm pl}^2 = 4\pi\alpha n_e/m_e$ is the plasma frequency with m_e the electron mass and α the fine-structure constant. The Faraday rotation term $\Delta_{\rm R}$, which depends on the energy and the longitudinal component B_z, would couple the modes $A_\parallel$ and $A_\perp$. While it is important when analyzing polarized photon sources, it plays a negligible role here. The $\Delta_{\rm CM}$ terms describe the Cotton-Mouton effect, i.e., the birefringence of fluids in the presence of a longitudinal magnetic field, with $|\Delta_\parallel^{\rm CM} - \Delta_\perp^{\rm CM}| \propto B_{\rm T}^2$. These terms are of little importance for the following arguments and will be neglected hereafter.

Therefore, we finally concentrate on the simple two-level mixing problem,

$$\left[\omega - i\partial_z + \begin{pmatrix} \Delta_{\rm pl} & \Delta_{a\gamma} \\ \Delta_{a\gamma} & \Delta_a \end{pmatrix}\right]\begin{pmatrix} A_\parallel \\ a \end{pmatrix} = 0. \tag{A6}$$

The solution of this system follows from a diagonalization of the mixing matrix by a rotation with an angle,

$$\vartheta = \frac{1}{2}\arctan\left(\frac{2\Delta_{a\gamma}}{\Delta_{\rm pl} - \Delta_a}\right). \tag{A7}$$

In analogy to the neutrino case, the probability for a photon emitted in the state $A_\parallel$ to convert to an ALP after traveling a distance s in a constant transverse magnetic field $\mathbf{B}_{\rm T}$ is

$$P_0(\gamma \to a) = |\langle A_\parallel(0) \mid a(s)\rangle|^2 \tag{A8}$$

$$= \sin^2(2\vartheta)\sin^2(\Delta_{\rm osc} s/2) \tag{A9}$$

$$= (\Delta_{a\gamma} s)^2 \frac{\sin^2(\Delta_{\rm osc} s/2)}{(\Delta_{\rm osc} s/2)^2}, \tag{A10}$$

where the oscillation wave number is given by

$$\Delta_{\rm osc}^2 = (\Delta_{\rm pl} - \Delta_a)^2 + 4\Delta_{a\gamma}^2. \tag{A11}$$

The conversion probability is energy independent when $2|\Delta_{a\gamma}| \gg |\Delta_{\rm pl} - \Delta_a|$ or in any case when the oscillatory term $\sin^2 x/x^2 \approx 1$ in Eq. (A10), corresponding to $\Delta_{\rm osc} s/2 \ll 1$.

We now return to the 3×3 formalism to derive a perturbative solution. In a fixed x-y-z frame with z the direction of motion, the propagation equations are

$$\left[\omega - i\partial_z + \begin{pmatrix} \Delta_{xx} & \Delta_{xy} & \Delta_{a\gamma} s_\gamma \\ \Delta_{yx} & \Delta_{yy} & \Delta_{a\gamma} c_\gamma \\ \Delta_{a\gamma} s_\gamma & \Delta_{a\gamma} c_\gamma & \Delta_a \end{pmatrix}\right]\begin{pmatrix} A_x \\ A_y \\ a \end{pmatrix} = 0, \tag{A12}$$

where $c_\gamma = \cos\gamma$ and $s_\gamma = \sin\gamma$ with γ the angle between $\mathbf{B}_T$ and the y axes (measured clockwise). Further, from Eq. (A5) one can write

$$\Delta_{xx} \simeq \Delta_{\rm pl}, \qquad \Delta_{xy} \simeq 0, \qquad \Delta_{yy} \simeq \Delta_{\rm pl}. \tag{A13}$$

The field strength entering $\Delta_{a\gamma}$ is $B_{\rm T} = |\mathbf{B}_{\rm T}| = |\mathbf{B}||\sin\psi|$, where ψ is the angle between the field and the photon propagation direction. Thus we have $B_x = B_{\rm T} c_\gamma$, $B_y = B_{\rm T} s_\gamma$ that are all z-dependent quantities. All of the Δ_{ij} are z-dependent as well because this applies to γ, n_e, and B_T, entering the quantities in Eq. (A5).

Since the ALP is weakly coupled, the 3rd row/column off-diagonal terms are much smaller than ω, and it makes sense to write

$$i\,\partial_z \mathbf{A} = (\mathcal{H}_0 + \mathcal{H}_1)\mathbf{A} \tag{A14}$$

where $\mathbf{A} = (A_x, A_y, a)$,

$$\mathcal{H}_0 = \omega \mathbf{I} + \begin{pmatrix} \Delta_{\rm pl} & 0 & 0 \\ 0 & \Delta_{\rm pl} & 0 \\ 0 & 0 & \Delta_a \end{pmatrix}, \tag{A15}$$

and

$$\mathcal{H}_1 = \begin{pmatrix} 0 & 0 & \Delta_{a\gamma} s_\gamma \\ 0 & 0 & \Delta_{a\gamma} c_\gamma \\ \Delta_{a\gamma} s_\gamma & \Delta_{a\gamma} c_\gamma & 0 \end{pmatrix}. \tag{A16}$$

For $g_{a\gamma} \to 0$ this equation is solved exactly by $\mathbf{A}^{(0)}(z) = \mathcal{U}_0(z)\mathbf{A}(0)$, where

$$\mathcal{U}_0(z) = \exp\left[-i\int_0^z dz' \mathcal{H}_0(z')\right]. \tag{A17}$$

If we now include the perturbation, the complete solution can be written perturbatively in the interaction representation. In particular, to first order we have $\mathbf{A}_{\rm int} = \mathcal{U}_0^\dagger \mathbf{A}$, $\mathcal{H}_{\rm int} = \mathcal{U}_0^\dagger \mathcal{H}_1 \mathcal{U}_0$, and

$$\mathbf{A}_{\rm int}^{(1)}(z) = -i\int_0^z dz' \mathcal{H}_{\rm int}(z')\mathbf{A}_{\rm int}^{(0)}(0), \tag{A18}$$

and $\mathbf{A}_{\rm int}^{(0)}(z) = \mathbf{A}(0)$ because

$$\mathbf{A}_{\rm int}^{(0)}(z) = \mathcal{U}_0^\dagger \mathbf{A}^{(0)}(z) = \mathcal{U}_0^\dagger \mathcal{U}_0 \mathbf{A}(0). \tag{A19}$$

Since $\mathcal{H}_0$ is diagonal, $\mathcal{U}_0$ has the general form $\mathcal{U}_0(z) = {\rm diag}[e^{-ia(z)}, e^{-ib(z)}, e^{-ic(z)}]$ so that

$$\mathcal{H}_{\rm int} = \begin{pmatrix} 0 & 0 & e^{-i(c-a)}\Delta_{a\gamma} s_\gamma \\ 0 & 0 & e^{-i(c-b)}\Delta_{a\gamma} c_\gamma \\ e^{i(c-a)}\Delta_{a\gamma} s_\gamma & e^{i(c-b)}\Delta_{a\gamma} c_\gamma & 0 \end{pmatrix}. \tag{A20}$$

The ALP amplitude developed at distance z is then

$$\begin{aligned} a^{(1)}(z) = -i\frac{g_{a\gamma}}{2}\int_0^z dz' \{&A_x(0)B_x(z')e^{i[c(z')-a(z')]} \\ &+ A_y(0)B_y(z')e^{i[c(z')-b(z')]}\}. \end{aligned} \tag{A21}$$

This result is a straightforward generalization of the one derived in Ref. [9]. The probability for photon-ALP conversion is then schematically

$$P_{\gamma\to a}(z) = |A_x(0)|^2|\mathcal{I}_1|^2 + |A_y(0)|^2|\mathcal{I}_2|^2 + 2\,\mathrm{Re}[A_x(0)A_y(0)\mathcal{I}_1\mathcal{I}_2]. \tag{A22}$$

For an unpolarized source, an average over the initial state has to be performed. The interference term averages to zero, and $\langle|A_x(0)|^2\rangle = \langle|A_y(0)|^2\rangle = 1/2$. Then

$$P_{\gamma\to a}(z) = \frac{g_{a\gamma}^2}{8}\left(\left|\int_0^z dz' e^{i(\Delta_a-\Delta_{\rm pl})z'}B_x(z')\right|^2 + \left|\int_0^z dz' e^{i(\Delta_a-\Delta_{\rm pl})z'}B_y(z')\right|^2\right) \tag{A23}$$

or equivalently

$$P_{\gamma\to a}(z) = \frac{g_{a\gamma}^2|\mathbf{B}|^2}{8}\left(\left|\int_0^z dz' \sin\psi(z')e^{i(\Delta_a-\Delta_{\rm pl})z'}c_\gamma(z')\right|^2 + \left|\int_0^z dz' \sin\psi(z')e^{i(\Delta_a-\Delta_{\rm pl})z'}s_\gamma(z')\right|^2\right). \tag{A24}$$

Here we have assumed $\Delta_{\rm pl}$ to be independent of z.

We next consider a "patchy" pattern of domains of equal size s and constant field in each of them. We will show that, when evaluated after a distance $z \approx Ns$, with $N \gg 1$, the conversion probability is roughly the product of the conversion probability in a single domain times the number of domains. Except for the replacement $s_\gamma \to c_\gamma$, each one of the two integrals in Eq. (A24) can be evaluated as follows, where $l_0 = 2\pi/(\Delta_{\rm pl} - \Delta_a)$:

$$\begin{aligned}\mathcal{I} &= \left|\int_0^z dz' \sin\psi(z')e^{-2\pi i z'/l_0}s_\gamma(z')\right|^2 \\ &= \left|\sum_{k=1}^N \mu_k\int_{z_k}^{z_{k+1}} dz' e^{-2\pi i z'/l_0}\right|^2 \\ &= \frac{l_0^2}{\pi^2}\sin^2\left(\frac{\pi s}{l_0}\right)\left|\sum_{k=1}^N \mu_k e^{-i\pi(2z_k+s)/l_0}\right|^2 \\ &= \frac{l_0^2}{\pi^2}\sin^2\left(\frac{\pi s}{l_0}\right)\left(\sum_{k=1}^N \mu_k^2 + \sum \text{interference terms}\right).\end{aligned} \tag{A25}$$

Here, $N = z/s \gg 1$ and $\mu_k = |\sin\psi_k|s_\gamma(k)$ or $\mu_k = |\sin\psi_k|c_\gamma(k)$ is a random variable in the interval $[-1, 1]$. The random nature of the field directions implies that the interference term vanishes on average. For the geometrical factor we have $\langle\mu_k^2\rangle = \langle\sin^2\psi\sin^2\gamma\rangle = 1/3$. Then we find

$$\begin{aligned}P_{\gamma\to a}(z) &\approx \frac{g_{a\gamma}^2|\mathbf{B}|^2}{8}\frac{l_0^2}{\pi^2}\sin^2\left(\frac{\pi s}{l_0}\right)\times 2\times\frac{N}{3} \\ &= N(\langle\Delta_{a\gamma}\rangle s)^2\frac{\sin^2(|\Delta_{\rm pl}-\Delta_a|s/2)}{(|\Delta_{\rm pl}-\Delta_a|s/2)^2} = NP_0,\end{aligned} \tag{A26}$$

having the structure of a probability per single domain P_0 times the number of domains N. We stress that Eq. (A26) only holds perturbatively, i.e., $\langle\Delta_{a\gamma}\rangle s \ll 1$ is a necessary condition.

In the limit $|\Delta_{\rm pl} - \Delta_a| \gg \langle\Delta_{a\gamma}\rangle$, we have in Eq. (A10) that $\Delta_{\rm osc} = |\Delta_{\rm pl} - \Delta_a|$ and Eq. (A10) coincides with Eq. (A26), provided that $B_T \to \langle|\mathbf{B}|\rangle = |\mathbf{B}|/\sqrt{3}$ because of the projection effect. In the opposite limit $|\Delta_{\rm pl} - \Delta_a| \ll \langle\Delta_{a\gamma}\rangle$, Eq. (A26) reduces to

$$P_{\gamma\to a}(z) \approx N(\langle\Delta_{a\gamma}\rangle s)^2, \tag{A27}$$

again in agreement with the corresponding limit of Eq. (A10).

This exercise shows explicitly how the classical rule of "adding the probabilities" instead of amplitudes arises from the randomness of the polarization and of the field configuration over scales much larger than s. However, since we used first-order perturbation theory, the validity of these results breaks down when $P_{\gamma\to a}(z)$ becomes large. This is always the case for z large enough, since we are not including the backreaction $a \to \gamma$, that are second order in $g_{a\gamma}$ and that prevent the violation of unitarity. In the saturation regime, the correct generalization of Eq. (A10) is provided by Eq. (5) as discussed in the text.

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