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Semileptonic bc to cc baryon decay and heavy quark spin symmetry

Jonathan M. Flynn[1] and Juan Nieves[2]

[1]*School of Physics and Astronomy, University of Southampton, Highfield, Southampton SO17 1BJ, United Kingdom*
[2]*Departamento de Física Atómica, Molecular y Nuclear, Universidad de Granada, E-8071 Granada, Spain*
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We study the semileptonic decays of the lowest-lying bc baryons to the lowest-lying cc baryons ($\Xi_{bc}^{(\prime *)} \to \Xi_{cc}^{(*)}$ and $\Omega_{bc}^{(\prime *)} \to \Omega_{cc}^{(*)}$), in the limit $m_b, m_c \gg \Lambda_{\rm QCD}$ and close to the zero-recoil point. The separate heavy quark spin symmetries make it possible to describe all these decays using a single form factor. We recover results derived previously by White and Savage in a manner which we think is more straightforward and parallels the method applied later to study B_c semileptonic decays. We further discuss the resemblance between the bc baryon decays and those of B_c mesons to η_c and J/ψ mesons and comment on the relation between the slopes of the single functions describing each set of decays. Our results can straightforwardly be applied to the decays of bb baryons to bc baryons.

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I. INTRODUCTION

The static theory for a system with two heavy quarks has infrared divergences which can be regulated by the kinetic energy term $\bar{h}_Q(D^2/2m_Q)h_Q$. This term breaks the heavy quark flavor symmetry, but not the spin symmetry for each heavy quark flavor. The spin symmetry is sufficient to derive relations between form factors for decays of doubly heavy hadrons in the heavy quark limit, as was first shown in [1]. The consequences for semileptonic decays of B_c mesons were worked out in [2]. Here we extend the formalism to describe semileptonic decays of bc baryons to cc baryons. In Ref. [1], the two heavy quarks Q in a QQq baryon were treated as a pointlike color-triplet antiquark $\bar{Q}$ interacting with the light degrees of freedom. We will compare our results with those obtained using this diquark picture and make a link to the B_c to η_c and J/ψ decays. For recent developments using the diquark picture see [3–5].

We are interested in semileptonic decays of baryons containing two heavy quarks and a light quark. Specifically we study the decays of the cascade bc baryons Ξ_{bc}, Ξ'_{bc}, and Ξ^*_{bc} to cascade cc baryons Ξ_{cc} and Ξ^*_{cc}. The quantum numbers of these particles are listed in Table I. We find, in agreement with [1], that in the heavy quark limit a unique function describes the entire family of decays. This function satisfies a normalization condition (a consequence of vector current conservation) at zero recoil if the heavy quarks are degenerate. Our results can be straightforwardly applied to the corresponding decays involving Ω baryons and also to the decays of bb baryons to bc baryons. Some of these decays have also been studied in various quark model approaches [6–10].

II. SPIN SYMMETRY

The invariance of the effective Lagrangian under separate spin rotations of the b and c quarks leads to relations between the form factors for vector and axial-vector currents between the cascade bc baryons and cascade cc baryons. These decays are induced by the semileptonic weak decay of the b quark to a c quark. Near the zero-recoil point the velocities of the initial and final baryons are approximately the same. If the momenta of the initial bc and final cc baryons are $p_\mu = m_{bc}v_\mu$ and $p'_\mu = m_{cc}v'_\mu = m_{cc}v_\mu + k_\mu$, respectively, then k will be a small residual momentum near the zero-recoil point. Since the final baryon is on shell, $k \cdot v = \mathcal{O}(1/m_{cc})$. We will work near zero recoil and thus neglect $v \cdot k$ below.

Heavy quark spin symmetry implies that all baryons with the same flavor content listed in Table I are degenerate. The consequences of spin symmetry for weak matrix elements can be derived using the "trace formalism" [11,12]. To represent the lowest-lying S wave bcq baryons we will use wave functions comprising tensor products of Dirac matrices and spinors, namely:

$$B'_{bc} = -\left[\frac{(1+\not{v})}{2}\gamma_5\right]_{\alpha\beta} u_\gamma(v, r), \tag{1}$$

$$B_{bc} = \left[\frac{(1+\not{v})}{2}\gamma_\mu\right]_{\alpha\beta}\left[\frac{1}{\sqrt{3}}(v^\mu + \gamma^\mu)\gamma_5 u(v, r)\right]_\gamma, \tag{2}$$

TABLE I. Quantum numbers of double-heavy baryons. S and J^P are the strangeness and the spin parity of the baryon, I is the isospin, and $S^\pi_{hh'}$ is the spin parity of the heavy degrees of freedom, well defined in the infinite heavy mass limit. l denotes a light u or d quark.

	S	J^P	I	S^π_{hh}			S	J^P	I	S^π_{hh}	
Ξ_{cc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	ccl	Ω_{cc}	-1	$\frac{1}{2}^+$	0	1^+	ccs
Ξ^*_{cc}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	ccl	Ω^*_{cc}	-1	$\frac{3}{2}^+$	0	1^+	ccs
Ξ'_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	0^+	bcl	Ω'_{bc}	-1	$\frac{1}{2}^+$	0	0^+	bcs
Ξ_{bc}	0	$\frac{1}{2}^+$	$\frac{1}{2}$	1^+	bcl	Ω_{bc}	-1	$\frac{1}{2}^+$	0	1^+	bcs
Ξ^*_{bc}	0	$\frac{3}{2}^+$	$\frac{1}{2}$	1^+	bcl	Ω^*_{bc}	-1	$\frac{3}{2}^+$	0	1^+	bcs

$$B^*_{bc} = \Xi^*_{bc} = \left[\frac{(1+\not{v})}{2}\gamma_\mu\right]_{\alpha\beta} u^\mu_\gamma(v, r), \tag{3}$$

where we have indicated Dirac indices α, β, and γ explicitly on the right-hand sides and r is a helicity label for the baryon. For the B^*_{bc}, $u^\mu_\gamma(v, r)$ is a Rarita-Schwinger spinor. These wave functions can be considered as matrix elements of the form $\langle 0|c_\alpha \bar{q}^c{}_\beta b_\gamma|B^{(\prime *)}_{bc}\rangle$ where $\bar{q}^c = q^T C$ with C the charge-conjugation matrix. We couple the c quark and light quark to spin 0 for the B'_{bc} or 1 for the B_{bc} and B^*_{bc} states. Under a Lorentz transformation, Λ, and b and c quark spin transformations S_b and S_c, a wave function of the form $\Gamma_{\alpha\beta}u_\gamma$ transforms as

$$\Gamma u \to S(\Lambda)\Gamma S^{-1}(\Lambda)S(\Lambda)u, \qquad \Gamma u \to S_c\Gamma S_b u. \tag{4}$$

The states in Eqs. (1)–(3) have a common normalization $\bar{u}u\,\mathrm{Tr}(\Gamma\bar{\Gamma})$ and are mutually orthogonal.

To build states where the b and c quarks are coupled to definite spin, we need the linear combinations

$$|0; 1/2, M\rangle_{bc} = -\tfrac{1}{2}|0; 1/2, M\rangle_{cq} + \tfrac{\sqrt{3}}{2}|1; 1/2, M\rangle_{cq} \tag{5}$$

$$|1; 1/2, M\rangle_{bc} = \tfrac{\sqrt{3}}{2}|0; 1/2, M\rangle_{cq} + \tfrac{1}{2}|1; 1/2, M\rangle_{cq} \tag{6}$$

$$|1; 3/2, M\rangle_{bc} = |1; 3/2, M\rangle_{cq}, \tag{7}$$

where the second and third arguments are the total spin quantum numbers of the baryon and the first argument denotes the total spin of the bc or cq subsystem. We have chosen the relative phase of the states in Eqs. (5) and (6) to agree with that adopted above in Eqs. (1) and (2) (we will comment again on this when constructing the cc baryon states). We have not used definite spin combinations for the b and c quarks in Eqs. (1) and (2). This is to make both the spin transformations on the heavy quarks and the Lorentz transformation of the states convenient, making it straightforward to build spin-invariant and Lorentz covariant quantities.

Finally we observe that we could have combined the b quark with the light quark to a definite spin in Eqs. (1)–(3). This would clearly interchange the spin transformations in Eq. (4) (and alter the appearance of the matrix element expression in Eq. (11) below). Note also that when rewriting Eq. (5) with the roles of b and c exchanged, an extra minus sign arises from the antisymmetry of the $S_{bc} = 0$ state under $b \leftrightarrow c$ interchange. Physical results should be unaltered and we have checked that this is the case.

For the cc baryons there are some differences because we have two identical quarks. In this case the states are

$$B'_{cc} = -\sqrt{\frac{2}{3}}\left[\frac{(1+\not{v})}{2}\gamma_5\right]_{\alpha\beta} u_\gamma(v, r), \tag{8}$$

$$B_{cc} = \sqrt{2}\left[\frac{(1+\not{v})}{2}\gamma_\mu\right]_{\alpha\beta}\left[\frac{1}{\sqrt{3}}(v^\mu + \gamma^\mu)\gamma_5 u(v, r)\right]_\gamma, \tag{9}$$

$$B^*_{cc} = \Xi^*_{cc} = \sqrt{\frac{1}{2}}\left[\frac{(1+\not{v})}{2}\gamma_\mu\right]_{\alpha\beta} u^\mu_\gamma(v, r). \tag{10}$$

Two comments are in order here. First, the two charm quarks can only be in a symmetric spin-1 state and therefore B'_{cc} and B_{cc} correspond to the same baryon state Ξ_{cc} (or Ω_{cc} if the light quark is s). We can thus use either of them to build up spin invariants and we have confirmed that we obtain the same results from each. Second, in the normalization, there are two ways to contract the charm quark indices, leading to $\bar{u}u\,\mathrm{Tr}(\Gamma\bar{\Gamma}) + \bar{u}\Gamma\bar{\Gamma}u$. In order to have the same normalization as for the bc case, we have to include extra numerical factors as shown in Eqs. (8)–(10). Note that the equality between the B'_{cc} and B_{cc} states fixes the relative phase between them.

We can now construct amplitudes for semileptonic cascade bc to cascade cc baryon decays, determined by matrix elements of the weak current $J^\mu = \bar{c}\gamma^\mu(1-\gamma_5)b$. We first build transition amplitudes between the $B^{(\prime *)}_{bc}$ and $\Xi^{(*)}_{cc}$ states and subsequently take linear combinations to obtain transitions from $\Xi^{(\prime *)}_{bc}$ states. The most general form for the matrix element respecting the heavy quark spin symmetry is[1]

$$\begin{aligned}\langle \Xi^{(*)}_{cc}, v, k, M'|J^\mu(0)|B^{(\prime *)}_{bc}, v, M\rangle &= \bar{u}_{cc}(v, k, M')\gamma^\mu(1-\gamma_5)u_{bc}(v, M)\,\mathrm{Tr}[\Gamma_{bc}\Omega\bar{\Gamma}_{cc}] \\ &\quad + \bar{u}_{cc}(v, k, M')\Gamma_{bc}\Omega\bar{\Gamma}_{cc}\gamma^\mu(1-\gamma_5)u_{bc}(v, M),\end{aligned} \tag{11}$$

where M and M' are the helicities of the initial and final states and $\Omega = -\eta(\omega)/\sqrt{2}$, with $\omega = v \cdot v'$. We use the standard relativistic normalization for hadronic states and our spinors satisfy $\bar{u}u = 2m$, $\bar{u}^\mu u_\mu = -2m$ where m is the mass of the state. Terms with a factor of $\not{v}$ can be omitted because of the equations of motion ($\not{v}u = u$, $\not{v}\Gamma = \Gamma$, $\gamma_\mu u^\mu = 0$, $v_\mu u^\mu = 0$), while terms with $\not{k}$ will always lead to contributions proportional to $v \cdot k$ which is set to 0 at the order we are working. In performing the calculations, we make use of the relations $\bar{u}\gamma_\mu u = \bar{u}v_\mu u$, $\bar{u}\gamma_5 u = 0$, $\bar{u}\not{k}u = 0$ and, $\bar{u}\not{k}\gamma_\mu\gamma_5 u = -\bar{u}\not{k}v_\mu\gamma_5 u$. Our results for cascade bc to cascade cc transition matrix elements are

$$\Xi_{bc} \to \Xi_{cc} \qquad \eta\bar{u}_{cc}(2\gamma^\mu - \tfrac{4}{3}\gamma^\mu\gamma_5)u_{bc}, \tag{12}$$

$$\Xi'_{bc} \to \Xi_{cc} \qquad \tfrac{-2}{\sqrt{3}}\eta\bar{u}_{cc}(-\gamma^\mu\gamma_5)u_{bc}, \tag{13}$$

$$\Xi_{bc} \to \Xi^*_{cc} \qquad \tfrac{-2}{\sqrt{3}}\eta\bar{u}^\mu_{cc}u_{bc}, \tag{14}$$

$$\Xi'_{bc} \to \Xi^*_{cc} \qquad -2\eta\bar{u}^\mu_{cc}u_{bc}, \tag{15}$$

$$\Xi^*_{bc} \to \Xi_{cc} \qquad \tfrac{-2}{\sqrt{3}}\eta\bar{u}_{cc}u^\mu_{bc}, \tag{16}$$

$$\Xi^*_{bc} \to \Xi^*_{cc} \qquad -2\eta\bar{u}^\lambda_{cc}(\gamma^\mu - \gamma^\mu\gamma_5)u_{bc\lambda}. \tag{17}$$

[1]If the roles of the b and c quarks were interchanged, the matrix element would read $\bar{u}_{cc}u_{bc}\,\mathrm{Tr}[\Gamma_{bc}\Omega\bar{\Gamma}_{cc}\gamma^\mu(1-\gamma_5)] + \bar{u}_{cc}\gamma^\mu(1-\gamma_5)\Gamma_{bc}\Omega\bar{\Gamma}_{cc}u_{bc}$.

If the b and c quarks become degenerate, then vector current conservation ensures that $\eta(1) = 1$.

The consequences of taking the heavy quark limit for semileptonic decays of baryons with two heavy quarks were considered some time ago by Savage and White [1]. They adopted an approach where the two heavy quarks bind into a color antitriplet which appears as a pointlike color source to the light degrees of freedom. Applying the "superflavor" formalism of Georgi and Wise [13–15] allowed the matrix elements of the heavy-flavor-changing weak current to be evaluated between different baryon states. We find two differences to their results which cannot be eliminated by redefining the phases of the physical states. One difference, already pointed out in [16], is for the spin-3/2 to spin-1/2 transition in Eq. (16), where they find a vanishing weak transition matrix element, while ours is nonzero. The second difference is the relative sign of the vector and axial contributions in the $\Xi_{bc} \to \Xi_{cc}$ transition of Eq. (12). This does not affect the differential decay rate although it could change angular correlations between the outgoing charged lepton and baryon.

Spin symmetry for both the b and c quarks enormously simplifies the description of all of the above transitions in the heavy quark limit and near the zero-recoil point. All the weak transition matrix elements are given in terms of a single universal function. Lorentz covariance alone allows a large number of form factors (six form factors to describe $\Xi_{bc} \to \Xi_{cc}$, another six for $\Xi'_{bc} \to \Xi_{cc}$, eight each for $\Xi_{bc} \to \Xi^*_{cc}$, $\Xi'_{bc} \to \Xi^*_{cc}$, and $\Xi^*_{bc} \to \Xi_{cc}$, and even more for $\Xi^*_{bc} \to \Xi^*_{cc}$). The spin symmetry provides further simplifications beyond those coming from working at $v' = v$. For example, the transitions $\Xi^{(\prime)}_{bc} \to \Xi_{cc}$ are each described by six form factors in general, corresponding to the structures $v^\mu - \gamma^\mu$, $v'^\mu - \gamma^\mu$, γ^μ, $v^\mu\gamma_5$, $v'^\mu\gamma_5$, and $\gamma^\mu\gamma_5$. At the zero-recoil point only γ^μ and $\gamma^\mu\gamma_5$ survive, leaving four form factors to describe these two decays. Spin symmetry reduces this to a single function η, which also describes the rest of the transitions shown above.

III. DIQUARK PICTURE AND LINK TO B_c MESON DECAYS

Up to now we have used only the separate spin symmetries for the heavy charm and bottom quarks and our results are completely model independent. Now we will use constituent quark model ideas to estimate the scale of variation of the form factors and to make a link to B_c to η_c and J/ψ semileptonic decays.

The form factor η is calculable in terms of the overlap of the spatial wave functions of the bcq and ccq baryon states. Considering the $\Xi_{bc} \to \Xi_{cc}$ transition with the initial baryon at rest, we can find η using

$$\eta(\omega) = \int d^3r_1 d^3r_2 \exp[-i\mathbf{k}\cdot\mathbf{r}_{12}/2] \times [\Psi^{\Xi}_{cc}(r_1, r_2, r_{12})]^* \Psi^{\Xi}_{bc}(r_1, r_2, r_{12}), \tag{18}$$

where $r_{1,2}$ are the distances between each of the heavy quarks and the light quark, while r_{12} is the heavy quark separation. The wave functions depend on distances because we are assuming that the lowest-lying baryons are purely S wave and so the integral depends on $\mathbf{k}^2 = m^2_{\Xi_{cc}}[(\omega)^2 - 1]$ (see Eq. (34) in [10]).

If the distance between the two heavy quarks is much smaller than the distance of the light quark from either heavy quark, as expected in the heavy mass limit of a strong Coulomb binding potential where the radius of the QQ bound state should decrease as $1/m_Q$, then the baryon wave functions can be approximated by (see Appendix B of [10])

$$\Psi^{\Xi}_{Qc}(r_1, r_2, r_{12}) = \Phi_{Qc}(r_{12})\phi(r_Q), \tag{19}$$

where r_Q is the distance of the light quark from the center of mass of the two heavy quarks. We ignore all spin-dependent interactions which are suppressed by inverse powers of heavy quark masses, allowing us to drop the superscript Ξ from now on, and making all interquark potentials flavor independent. Φ_{Qc} is the ground-state wave function of the Qc diquark, while ϕ is the ground-state wave function for the relative motion of the light quark and a pointlike diquark of infinite mass with a potential which is twice the quark-quark potential. In these circumstances we have

$$\eta(\omega) = \int d^3r_{12} \exp[-i\mathbf{k}\cdot\mathbf{r}_{12}/2][\Phi_{cc}(r_{12})]^*\Phi_{bc}(r_{12}) \times \int d^3r \phi^*(r)\phi(r), \tag{20}$$

where $\mathbf{r} = \mathbf{r}_c$ and in the d^3r integral we have replaced $\phi(r_b)$ by $\phi(r)$ since $\mathbf{r}_b = \mathbf{r}_c + \mathcal{O}(\mathbf{r}_{12})$. This approximation leads to uncertainties of $\mathcal{O}(r^2_{12})$ after integration. The d^3r integration then gives 1 and thus

$$\eta(\omega) = \int d^3r_{12} \exp[-i\mathbf{k}\cdot\mathbf{r}_{12}/2][\Phi_{cc}(r_{12})]^*\Phi_{bc}(r_{12}), \tag{21}$$

which has an identical form to Eq. (4.11) in [2], where the unique form factor Δ describing the B_c to η_c and J/ψ semileptonic decays is given in terms of wave functions of the $\bar{b}c$ and $\bar{c}c$ bound states.[2] This does not mean that η and Δ are identical because the QQ and $Q\bar{Q}$ potentials used to compute the diquark and meson wave functions are not the same. For example a $\lambda_i\lambda_j$ color dependence (λ_i are the usual Gell-Mann matrices) would lead to $V^{QQ} = V^{Q\bar{Q}}/2$.

Assuming Coulomb wave functions, $\Phi_{Qc}(r) \propto e^{-r/a_Q}$, with the diquark radius $a_Q \propto 1/(\beta\mu_Q)$, where μ_Q is the Qc reduced mass and β is the strength of the $-1/r$ potential, we find

$$\eta(\omega) = 8\frac{a_b^{3/2}a_c^{3/2}}{(a_b + a_c)^3}\left[1 + \frac{\mathbf{k}^2 a_b^2 a_c^2}{4(a_b + a_c)^2}\right]^{-2}, \tag{22}$$

which agrees with the expression given in Eq. (3) of [1] and

[2]We believe that there should not be an explicit factor of 2 in (4.11) of [2].

clearly resembles Eq. (4.12) of [2]. Assuming $V^{QQ} = V^{Q\bar{Q}}/2$, we would expect the B_c and η_c radii a_0 and a_η introduced in [2] to be approximately one-half of a_b and a_c, respectively. The ω^2 slopes of the form factors Δ and η would then be in the ratio 1 to $4(m_{\Xi_{cc}}/m_{\eta_c})^2 \sim 6$.

To check the use of Coulomb wave functions and the slope prediction, we have calculated η and Δ using wave functions from a nonrelativistic quark model [10,17] and show the results in Fig. 1. The ω^2 slope of the Δ form factor is indeed smaller than that of η, but the ratio is around 1 to 3 rather than 1 to 6, so there are significant corrections to the Coulomb wave function description.

IV. CONCLUSION

We have studied the semileptonic decays of the lowest-lying bc baryons to lowest-lying cc baryons in the limit $m_b, m_c \gg \Lambda_{\rm QCD}$ and close to the zero-recoil point. The separate heavy quark spin symmetries make it possible to describe all these decays using a single form factor. We have discussed the resemblance of the bc baryon decays to those of B_c mesons to η_c and J/ψ mesons and commented on the relation between the slopes of the single functions describing each set of decays. Lattice QCD simulations work best near the zero-recoil point and thus are well-suited to check the validity of the results.

We studied specifically the semileptonic decays of cascade bc baryons to cascade cc baryons. Our results can be straightforwardly applied also to the corresponding decays involving Ω baryons as well as to the decays of bb baryons to bc baryons. It is also straightforward to extend the analysis to transitions involving the heavy-to-light weak current, using the bc baryon wave functions defined in Eqs. (1)–(3) together with the usual spinor wave function for a single heavy quark baryon. For example, to study

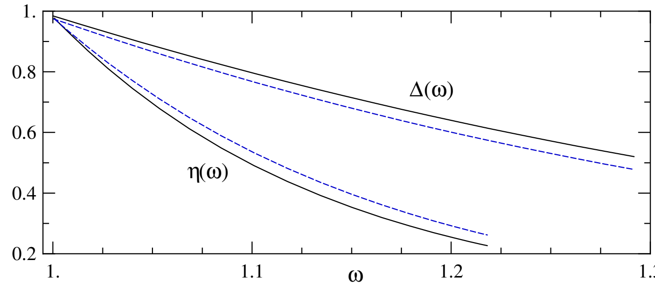

FIG. 1 (color online). Form factors in the heavy quark limit: $\eta(\omega)$ for cascade bc to cascade cc baryon decays and $\Delta(\omega)$ for B_c to η_c, J/ψ decays, calculated from a nonrelativistic quark model [10,17] (using the AL1 potential). The solid lines are calculated from the wave function overlaps, illustrated for $\eta(\omega)$ in Eq. (21), while the dashed lines are constructed from appropriate combinations of form factors: for η we consider $(F_1 + F_2 + F_3)/2$, where $F_{1,2,3}$ are defined in Eq. (23) of [10], while for Δ we use $\Sigma_1^{(0)}$ defined in Eq. (52) of [17]. The solid and dashed curves should agree close to zero recoil ($\omega \to 1$).

$\Xi_{bc}^{(\prime *)} \to \Lambda_b$ semileptonic decays, we would evaluate expressions like $\bar{u}_b u_{bc} {\rm Tr}[\gamma^\mu(1-\gamma_5)\Gamma_{bc}\Omega]$ where $\Omega = \Omega_1 + \not{v}\Omega_2$ and u_b is the spinor for the Λ_b.

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[1] M. J. White and M. J. Savage, Phys. Lett. B **271**, 410 (1991).
[2] E. Jenkins, M. E. Luke, A. V. Manohar, and M. J. Savage, Nucl. Phys. **B390**, 463 (1993).
[3] T. Mehen and B. C. Tiburzi, Phys. Rev. D **74**, 054505 (2006).
[4] J. Hu and T. Mehen, Phys. Rev. D **73**, 054003 (2006).
[5] S. Fleming and T. Mehen, Phys. Rev. D **73**, 034502 (2006).
[6] M. A. Sanchis-Lozano, Nucl. Phys. **B440**, 251 (1995).
[7] X. H. Guo, H. Y. Jin, and X. Q. Li, Phys. Rev. D **58**, 114007 (1998).
[8] A. Faessler, *et al.*, Phys. Lett. B **518**, 55 (2001).
[9] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, Phys. Rev. D **70**, 014018 (2004).
[10] C. Albertus, E. Hernandez, J. Nieves, and J. M. Verde-Velasco, Eur. Phys. J. A **32**, 183 (2007).
[11] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343**, 1 (1990).
[12] A. V. Manohar and M. B. Wise, *Heavy Quark Physics* (Cambridge University Press, Cambridge, England, 2000), ISBN 0 521 64241 8.
[13] H. Georgi and M. B. Wise, Phys. Lett. B **243**, 279 (1990).
[14] M. J. Savage and M. B. Wise, Phys. Lett. B **248**, 177 (1990).
[15] C. D. Carone, Phys. Lett. B **253**, 408 (1991).
[16] M. A. Sanchis-Lozano, Phys. Lett. B **321**, 407 (1994).
[17] E. Hernandez, J. Nieves, and J. M. Verde-Velasco, Phys. Rev. D **74**, 074008 (2006).