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Heavy-to-light form factors on the light cone

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The light-cone method provides a convenient nonperturbative tool to study the heavy-to-light form factors. We construct a light-cone quark model utilizing the soft collinear effective theory. In the leading order of effective theory, the form factors for B-to-light pseudoscalar and vector mesons are reduced to three universal form factors which can be calculated as overlaps of hadron light-cone wave functions. The numerical results show that the leading contribution is close to the results from other approaches. The q^2 dependence of the heavy-to-light form factors is also presented.

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I. INTRODUCTION

The hadronic matrix elements of weak B decays to a light pseudoscalar (P) and to a vector meson (V) are described by $B \to P$ and $B \to V$ transition form factors, respectively. These heavy-to-light form factors are essential to study the semileptonic and even nonleptonic B decays. Information on the form factors is crucial to test the mechanism of CP violation in the standard model and to extract the Cabibbo-Kobayashi-Maskawa (CKM) parameters [1]. For instance, the $B \to \pi(\rho)$ form factors are required to determine the CKM matrix element $|V_{ub}|$ precisely. In $B \to V\gamma$ and $B \to V l^+ l^-$ processes which are sensitive to new physics, the precise evaluation of $B \to V$ form factors is indispensable. Another interesting reason for the study of the heavy-to-light form factors is that they provide an ideal laboratory to explore the rich structures of QCD dynamics. At the large recoil region where the final state light meson moves fast, the heavy-to-light system contains internal information on both short and long distance QCD dynamics with the factorization theorem.

There are already many methods calculating the heavy-to-light transition form factors in the literature, such as the simple quark model [2], the light-cone quark model (LCQM) [3–6],[1] light-cone sum rules (LCSR) [7–9], the perturbative QCD (PQCD) approach based on k_T factorization [10], etc.

In Ref. [11], a model-independent way to look for relations between different form factors is suggested by analogy with the heavy-to-heavy transitions [12]. One important observation is that, in the heavy quark mass and large energy of light meson limits, the spin symmetry relates the form factors for $B \to P$ and $B \to V$ to three universal energy-dependent functions: ζ_P for the pseudoscalar meson; and $\zeta_\parallel$, $\zeta_\perp$ for longitudinally and transversely polarized vector mesons, respectively. The development of soft collinear effective theory (SCET) makes the analysis on a more rigorous foundation. The SCET is a powerful method to systematically separate the dynamics at different scales—the hard scale m_b (b quark mass), the hard intermediate scale $\mu_{hc} = \sqrt{m_b \Lambda_{\rm QCD}}$, the soft scale $\Lambda_{\rm QCD}$—and to sum large logs using the renormalization group techniques. After a series of studies [13–17], a factorization formula was established for the heavy-to-light form factors in the heavy quark mass and large energy limit as

$$F_i(q^2) = C_i(E, \mu_{\rm I})\zeta_j(\mu_{\rm I}, E) + \phi_B(\omega, \mu_{\rm II}) \otimes T_i(E, u, \omega, \mu_{\rm II}) \otimes \phi_M(u, \mu_{\rm II}), \qquad (1)$$

where the indices j represent $(P, \parallel, \perp)$ and $\otimes$ denotes the convolutions over light-cone momentum fractions. $\phi_B(\omega)$ and $\phi_M(u)$ are light-cone distribution amplitudes for $\bar{B}$ and light mesons. The coefficients C_i and T_i are perturbatively calculable functions which include hard gluon corrections. The functions ζ_j denote the universal functions that satisfy the spin symmetry.

Although soft collinear effective theory is really powerful and rigorous, the form factors ζ_j cannot be directly calculated. These functions are nonperturbative in principle and the evaluation of them relies on nonperturbative methods. Lattice simulation on heavy-to-light form factors is usually restricted to the region with final meson energy $E < 1$ GeV and cannot be applied to our case directly where the light meson carries the energy of order $M_B/2$.[2] The construction of LCSR within SCET has been explored recently in [20–22]. In these studies, only the pseudoscalar meson form factor ζ_P is calculated at present.

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[1]In some references, the authors prefer to use the term “light front.” We will use the term “light cone” which is widely adopted in SCET, LCSR, and other approaches.

[2]This may be changed by applying “moving” nonrelativistic QCD in lattice QCD [18]. For a recent development, please see [19] and references therein.

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The light-cone field theory provides another natural language to study these processes. As pointed out in [23], light-cone QCD has some unique features which are particularly suitable to describe a hadronic bound state. For instance, the vacuum state in this approach is much simpler than that in other approaches. The light-cone wave functions, which describe the hadron in terms of their fundamental quark and gluon degrees of freedom, are independent of the hadron momentum and thus are explicitly Lorentz invariant. The light-cone Fock space expansion provides a complete relativistic many-particle basis for a hadron. For hard exclusive processes with large momentum transfer, the factorization theorem in the perturbative light-cone QCD makes first-principle predictions [24]. For nonperturbative QCD, an approach which combines the advantage of the light-cone method with the low energy constituent quark model is more appealing. This approach, which we will call the LCQM, has been successfully applied to the calculation of the meson decay constants and hadronic form factors [3–6,25,26].

As far as the form factors are concerned, they can be generally represented by the convolution of B and light meson wave functions in the light-cone approach as

$$F(q^2) = \sum_{n_1,n_2} \int \left\{ \prod_i \frac{dx_i d^2k_{\perp i}}{16\pi^3} \right\} \left\{ \prod_j \frac{dx_j d^2k_{\perp j}}{16\pi^3} \right\} \times \Psi_M^{(n_1)*}(x_i, k_{\perp i}) \Psi_B^{(n_2)}(x_j, k_{\perp j}), \tag{2}$$

where the sum is over all Fock states with n_1, n_2 the particle numbers; i, j denote the ith and jth constituents of the light meson and $\bar{B}$ meson, respectively. The product is performed over the longitudinal momentum fractions $x_{i,j}$ and the transverse momenta $k_{\perp i,j}$. The light-cone wave function $\Psi(x, k_\perp)$ is the generalization of the distribution amplitude $\phi(x)$ by including the transverse momentum distributions. This formulation contains both hard and soft interactions.

The main purpose of this paper is to develop a nonperturbative light-cone approach within the soft collinear effective theory and to evaluate the three universal heavy-to-light form factors directly. The close relation between the light-cone QCD and soft collinear effective theory was noted in [15]. The SCET has the advantage that a systematic power expansion with small parameter $\Lambda_{\rm QCD}/m_b$ (or $\sqrt{\Lambda_{\rm QCD}/m_b}$) can be performed to improve the calculation accuracy order by order. The combination of the two methods can reduce the model dependence of nonperturbative methods. In the conventional light-cone approach, all the quarks are on shell. Now, in the new approach, it is convenient to choose the light energetic quark as the collinear mode in the soft collinear effective theory and the heavy quark field as that in the heavy quark effective theory. The spectator antiquark remains as the soft mode. By this way, the light-cone quark model within the soft collinear effective theory is established. Then we can calculate the $B \to P$ and $B \to V$ form factors order by order.

The paper is organized as follows. In Sec. II, we first present the definition of the three universal form factors from the spin symmetry relations. We then discuss a light-cone quark model within soft collinear effective theory. The numerical results for the form factors and discussions are presented in Sec. III. The final part contains our conclusion.

II. THE HEAVY-TO-LIGHT FORM FACTORS IN THE LIGHT-CONE APPROACH

A. Definitions of the heavy-to-light form factors

The $\bar{B} \to P$ and $\bar{B} \to V$ form factors are defined under the conventional form as follows:

$$\begin{aligned}
\langle P(P')|\bar{q}\gamma^\mu b|\bar{B}(P)\rangle &= f_+(q^2)\left[(P+P')^\mu - \frac{M_B^2 - M_P^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_B^2 - M_P^2}{q^2}q^\mu, \\
\langle P(P')|\bar{q}\sigma^{\mu\nu}q_\nu b|\bar{B}(P)\rangle &= i\frac{f_T(q^2)}{M_B + M_P}[q^2(P+P')^\mu - (M_B^2 - M_P^2)q^\mu], \\
\langle V(P', \epsilon^*)|\bar{q}\gamma^\mu b|\bar{B}(P)\rangle &= -\frac{2V(q^2)}{M_B + M_V}\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu P_\rho P'_\sigma, \\
\langle V(P', \epsilon^*)|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}(P)\rangle &= 2iM_V A_0(q^2)\frac{\epsilon^* \cdot q}{q^2}q^\mu + i(M_B + M_V)A_1(q^2)\left[\epsilon^*_\mu - \frac{\epsilon^* \cdot q}{q^2}q^\mu\right] \\
&\quad - iA_2(q^2)\frac{\epsilon^* \cdot q}{M_B + M_V}\left[(P+P')^\mu - \frac{M_B^2 - M_V^2}{q^2}q^\mu\right], \\
\langle V(P', \epsilon^*)|\bar{q}\sigma^{\mu\nu}q_\nu b|\bar{B}(P)\rangle &= -2iT_1(q^2)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu P_\rho P'_\sigma, \\
\langle V(P', \epsilon^*)|\bar{q}\sigma^{\mu\nu}\gamma_5 q_\nu b|\bar{B}(P)\rangle &= T_2(q^2)[(M_B^2 - M_V^2)\epsilon^{*\mu} - (\epsilon^* \cdot q)(P+P')^\mu] \\
&\quad + T_3(q^2)(\epsilon^* \cdot q)\left[q^\mu - \frac{q^2}{M_B^2 - M_V^2}(P+P')^\mu\right],
\end{aligned} \tag{3}$$

where $q = P - P'$ is the momentum transfer, M_B the $\bar{B}$ meson mass, $M_{P,V}$ the mass of the pseudoscalar and vector mesons, and ϵ the polarization vector of the vector meson. We have used the convention $\epsilon^{0123} = +1$. In the following, we choose the convention within which the vectors $n_\pm$ are $n_+^\mu = (1, 0, 0, -1)$, $n_-^\mu = (1, 0, 0, 1)$, and the light-cone momentum components are $p^+ = n_+ \cdot p = p^0 + p^3$, $p^- = n_- \cdot p = p^0 - p^3$, $p_b = m_b v$. Our convention for the vectors $n_\pm$ is different from that in most of the literature. In the above definitions, there are ten form factors in total: f_+, f_0, f_T for the pseudoscalar meson; $V, A_0, A_1, A_2, T_1, T_2, T_3$ for the vector meson. Note that the form factors are, in general, different for each hadron.

In SCET, the energetic light quark is described by its leading two-component spinor $\xi = \frac{\not{n}_- \not{n}_+}{4} q$, and the heavy quark is replaced by $h_v = e^{i m_b v \cdot x} \frac{(1+\not{v})}{2} b$. The weak current $\bar{q}\Gamma b$ in full QCD is matched onto the SCET current $\bar{\xi}\Gamma h_v$ at tree level where we have omitted the Wilson lines for simplicity. For an arbitrary matrix Γ, $\bar{\xi}\Gamma h_v$ has only three independent Dirac structures. One convenient choice is discussed in Refs. [13,27]: $\bar{\xi} h_v$, $\bar{\xi}\gamma_5 h_v$, and $\bar{\xi}\gamma_\perp^\mu h_v$. It can be seen from a trace technology by

$$\frac{\not{n}_+ \not{n}_-}{4} \Gamma \frac{(1+\not{v})}{2} = \frac{\not{n}_+ \not{n}_-}{4} [c_1 + c_2\gamma_5 + c_3\gamma_\perp^\mu] \frac{(1+\not{v})}{2}, \tag{4}$$

where c_is are defined as

$$c_1 = \tfrac{1}{4}\mathrm{Tr}[(1+\not{v})\not{n}_- \Gamma], \qquad c_2 = \tfrac{1}{4}\mathrm{Tr}[(1+\not{v})\not{n}_- \gamma_5 \Gamma], \qquad c_3 = \tfrac{1}{4}\mathrm{Tr}[(1+\not{v})\not{n}_- \gamma_{\perp\mu}\Gamma]. \tag{5}$$

The above spin symmetry leads to nontrivial relations for the heavy-to-light form factors: the ten form factors are reduced to three independent universal form factors. The B-to-light universal form factors ζ_P, $\zeta_{\parallel,\perp}$ are defined as

$$\begin{aligned} \langle P|\bar{\xi} h_v|\bar{B}\rangle &= 2E\zeta_P(E), \\ \langle V|\bar{\xi}\gamma_5 h_v|\bar{B}\rangle &= -2iM_V \zeta_\parallel(E) v\cdot\epsilon^*, \\ \langle V|\xi\gamma_\perp^\mu h_v|\bar{B}\rangle &= -2E\zeta_\perp(E)\epsilon^{\mu\nu\rho\sigma}\epsilon_\nu^* v_\rho n_{-\sigma}, \end{aligned} \tag{6}$$

where $E = (M_B^2 - q^2)/2M_B$ is the energy of the light meson (neglecting the small mass of the final state meson) and q is the momentum transfer. $\zeta_{i(i=P,\parallel,\zeta)}$ are functions of energy of the light meson. Up to leading order of α_s and leading power of $\Lambda_{\mathrm{QCD}}/m_b$, the total ten physical form factors are determined from the three independent factors to the leading order of α_s as

$$\begin{aligned} f_+(q^2) &= \frac{M_B}{2E} f_0(q^2) = \frac{M_B}{M_B + M_P} f_T(q^2) = \zeta_P(E), \\ \frac{M_B}{M_B + M_V} V(q^2) &= \frac{M_B + M_V}{2E} A_1(q^2) = \zeta_\perp(E), \\ A_0(q^2) &= \zeta_\parallel(E), \\ A_2(q^2) &= \frac{M_B}{M_B - M_V}\left[\zeta_\perp(E) - \frac{M_V}{E}\zeta_\parallel(E)\right], \\ T_1(q^2) &= \frac{M_B}{2E} T_2(q^2) = \zeta_\perp(E), \\ T_3(q^2) &= \zeta_\perp(E) - \frac{M_V}{E}\zeta_\parallel(E). \end{aligned} \tag{7}$$

As in [11,28], we keep the leading kinematic light meson mass correction and neglect the higher $M_{P,V}^2/M_B^2$ terms.

B. Light-cone quark model

We start with a discussion of hadron bound states on the light cone. The goal is to find a relativistic invariant description of the hadron in terms of its fundamental quark and gluon constituents. For a complete Fock state basis $|n\rangle$, the hadron is expanded by a series of wave functions: $|h\rangle = \sum_n |n\rangle\langle n|h\rangle = \sum_n |n\rangle\psi_{n/h}$. It is convenient to use a light-cone Fock state basis on which the hadron with momentum $\tilde{P} = (P^+, P_\perp)$ is described by [23]

$$|h{:}\tilde{P}\rangle = \sum_{n,\lambda_i}\int\left\{\bar{\prod_i}\frac{dx_i d^2k_{\perp i}}{\sqrt{x_i}16\pi^3}\right\}|n{:}x_iP^+, x_iP_{\perp i} + k_{\perp i}, \lambda_i\rangle\Psi_{n/h}(x_i, k_{\perp i}, \lambda_i), \tag{8}$$

where the sum is over all Fock states and helicities and the product is performed on the variables x_i and $k_{\perp i}$, not on the wave functions $\Psi_{n/h}(x_i, k_{\perp i}, \lambda_i)$,

$$\bar{\prod_i} dx_i d^2k_{\perp i} = \prod_i dx_i d^2k_{\perp i}\delta\Big(1 - \sum_j x_j\Big) \times 16\pi^3\delta^2\Big(\sum_j k_{\perp j}\Big). \tag{9}$$

The essential variables are boost-invariant light-cone momentum fractions $x_i = p_i^+/P^+$ with p_i momenta of quarks or gluons and the internal transverse momenta $k_{\perp i} = p_{\perp i} - x_i P_\perp$. The light-cone momentum fractions x_i and the internal transverse momenta $k_{\perp i}$ are relative variables which are independent of the hadron momentum. The wave functions in terms of these variables are explicitly Lorentz invariant and they are the probability amplitudes for finding n partons with momentum fractions x_i and relative momentum $k_{\perp i}$ in the hadron. The total probability is equal to 1 which implies a normalization condition

$$\sum_{n,\lambda_i}\int\left\{\bar{\prod_i}\frac{dx_i d^2k_{\perp i}}{16\pi^3}\right\}|\Psi_{n/h}(x_i, k_{\perp i}, \lambda_i)|^2 = 1. \tag{10}$$

The hadron state $|h\rangle$ is the eigenstate of the light-cone Hamiltonian $H_{LC}|h\rangle = M^2|h\rangle$ with the hadron mass M. Solving the eigenstate equation with the full Fock states is very difficult and is beyond our capability. We will meet an infinite number of coupled equations and the problems of some nonphysical singularities (endpoint singularities $x_i \to 0$ or ultraviolet singularities $k_\perp \to \infty$). What concerns us most is the wave function at the endpoint region. For the wave functions $\Psi_{n/h}(x_i, k_{\perp i}, \lambda_i)$, one general property is found [23]:

$$\Psi_{n/h}(x_i, k_{\perp i}, \lambda_i) \to 0 \quad \text{as } x_i \to 0. \tag{11}$$

This constraint means that the probability of finding partons with very small longitudinal momentum is little. In this mechanism, the $\bar{B}$ meson wave function is overlapped with the light meson wave function at the endpoint where the valence antiquark carries momentum of order of the hadron scale. In the infinite heavy quark mass limit, the light meson wave functions at the endpoint are suppressed. However, at the realistic m_b scale, the suppression is not so heavy that soft contribution still dominates the heavy-to-light form factors.

The solution of all wave functions from first principles is not obtainable at present. We will use the constituent quark model. The constituent quark masses are about several hundred MeV for light quarks which are much larger than the current quark mass obtained from the chiral perturbation theory. The appreciable mass absorbs dynamical effects from a complicated vacuum in the common instanton form [29]. A key approximation adopted in the light-cone quark model is the mock-hadron approximation [30] where the hadron is dominated by the lowest Fock state with free quarks. Under the valence quark assumption, we can write a meson state M constituting a quark q_1 and an antiquark $\bar{q}_2$ by

$$\begin{aligned}|M(P, S, S_z)\rangle = &\int \frac{dp_1^+ d^2p_{1\perp}}{16\pi^3}\frac{dp_2^+ d^2p_{2\perp}}{16\pi^3}16\pi^3 \\ &\times \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2)\sum_{\lambda_1,\lambda_2}\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) \\ &\times |q_1(p_1, \lambda)\bar{q}_2(p_2, \lambda)\rangle,\end{aligned} \tag{12}$$

with the meson denoted by its momentum P and spin S, S_z, the constituent quarks $q_1(\bar{q}_2)$ denoted by momenta $p_1(p_2)$, and the light-cone helicities $\lambda_1(\lambda_2)$. The 4-momentum p is defined as

$$\tilde{p} = (p^+, p_\perp), \qquad p_\perp = (p^1, p^2), \qquad p^- = \frac{m^2 + p_\perp^2}{p^+}. \tag{13}$$

From the momentum, we can see that the quarks in the meson are taken to be on the mass shell. In the following, we choose a frame where the transverse momentum of the meson is zero, i.e., $P_\perp = 0$. The light-front momenta p_1 and p_2 in terms of light-cone variables are

$$p_1^+ = x_1P^+, \qquad p_2^+ = x_2P^+, \qquad p_{1\perp} = -p_{2\perp} = k_\perp, \tag{14}$$

where x_i are the light-cone momentum fractions which satisfy $0 < x_1,\ x_2 < 1$, and $x_1 + x_2 = 1$. The invariant mass $M_0 = p_1 + p_2$ of the constituents and the relative momentum p_z in the z direction can be written as

$$\begin{aligned}M_0^2 &= \frac{m_1^2 + k_\perp^2}{x_1} + \frac{m_2^2 + k_\perp^2}{x_2}, \\ p_z &= \frac{x_2M_0}{2} - \frac{m_2^2 + k_\perp^2}{2x_2M_0}.\end{aligned} \tag{15}$$

Note that the invariant mass of the quark system is different from the meson total momentum, i.e. $p_1 + p_2 \neq P$.

The momentum-space wave function related to the meson bound state can be expressed as

$$\Psi^{SS_z}(p_1, p_2, \lambda_1, \lambda_2) = R^{SS_z}_{\lambda_1\lambda_2}(x, k_\perp)\phi(x, k_\perp), \tag{16}$$

where $\phi(x, k_\perp)$ describes the momentum distribution of the constituents in the bound state with $x \equiv x_2$, and $R^{SS_z}_{\lambda_1,\lambda_2}$ constructs a state of definite spin (S, S_z) out of the light-cone helicity (λ_1, λ_2) eigenstates. In practice, it is convenient to use the covariant form for $R^{SS_z}_{\lambda_1,\lambda_2}$ [3,25]:

$$R^{SS_z}_{\lambda_1,\lambda_2}(x, k_\perp) = \frac{\sqrt{p_1^+p_2^+}}{\sqrt{2}\tilde{M}_0}\bar{u}(p_1, \lambda_1)\Gamma v(p_2, \lambda_2), \tag{17}$$

where the parameter $\tilde{M}_0 \equiv \sqrt{M_0^2 - (m_1 - m_2)^2}$ and the Γ matrices for the mesons are defined as

$$\begin{aligned}\Gamma_P &= -i\frac{\gamma_5}{\sqrt{N_c}}, \quad \text{for the pseudoscalar meson,} \\ \Gamma_V &= \frac{-\hat{\not\epsilon}(S_z) + \frac{\hat{\epsilon}\cdot(p_1-p_2)}{M_0+m_1+m_2}}{\sqrt{N_c}}, \quad \text{for the vector meson}\end{aligned} \tag{18}$$

with $N_c = 3$. The transverse and longitudinal polarization vectors $\hat{\epsilon}$ are

$$\begin{aligned}\hat{\epsilon}^\mu(\pm 1) &= (0, 0, \vec{\epsilon}_\perp(\pm 1)), \\ \hat{\epsilon}^\mu(0) &= \frac{1}{M_0}\left(-\frac{M_0^2}{P^+}, P^+, 0\right),\end{aligned} \tag{19}$$

where $\vec{\epsilon}_\perp(\pm 1) = \mp(1, \pm i)/\sqrt{2}$. The Dirac spinors satisfy the relation

$$\begin{aligned}\sum_\lambda u(p, \lambda)\bar{u}(p, \lambda) &= \frac{(\not p + m)}{p^+}, \quad \text{for quarks,} \\ \sum_\lambda v(p, \lambda)\bar{v}(p, \lambda) &= \frac{(\not p - m)}{p^+}, \quad \text{for antiquarks.}\end{aligned} \tag{20}$$

The momentum distribution amplitude $\phi(x, k_\perp)$ is the generalization of the distribution amplitude $\phi(x)$ which is normalized as

$$\int \frac{dx d^2 k_\perp}{2(2\pi)^3} |\phi(x, k_\perp)|^2 = 1. \tag{21}$$

Before discussing the form factors, we will study the decay constants in the light-cone approach. The decay constants $f_{P,V}$ are defined by the matrix elements of the axial-vector current for the pseudoscalar meson and the vector current for the vector meson:

$$\langle 0|A^\mu|P(P)\rangle = i f_P P^\mu, \qquad \langle 0|V^\mu|V(P)\rangle = M_V f_V \epsilon^\mu, \tag{22}$$

where P is the meson momentum, M_V is the mass of the vector meson and ϵ^μ the polarization vector: $\epsilon^\mu(\pm 1) = (0, 0, \vec{\epsilon}_\perp)$, $\epsilon^\mu(0) = \frac{1}{M_V}(\frac{-M_V^2}{P^+}, P^+, 0)$. Note that the longitudinal polarization vector of the meson is not the same as that of the quark system due to $M_V \neq M_0$.

It is straightforward to show that the decay constant of a pseudoscalar meson and a vector meson can be represented by

$$f_P = 4\sqrt{\frac{3}{2}} \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \phi_P(x, k_\perp) \frac{\mathcal{A}}{\sqrt{\mathcal{A}^2 + k_\perp^2}},$$

$$f_V = 4\sqrt{\frac{3}{2}} \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \frac{\phi_V(x, k_\perp)}{\sqrt{\mathcal{A}^2 + k_\perp^2}} \frac{1}{M_0} \bigg\{ x(1-x) M_0^2 + m_1 m_2 + k_\perp^2 + \frac{\mathcal{B}}{2W_V}\bigg[\frac{m_1^2 + k_\perp^2}{1-x} - \frac{m_2^2 + k_\perp^2}{x} - (1-2x) M_0^2\bigg]\bigg\}, \tag{23}$$

where

$$\mathcal{A} = m_1 x + m_2(1-x), \quad \mathcal{B} = x m_1 - (1-x) m_2, \quad W_V = M_0 + m_1 + m_2. \tag{24}$$

In the above expression for the vector decay constant, we have used the plus component for the longitudinal polarization vector. When the decay constants are known from the experimental data, they can be used to constrain the parameters in the light-cone wave functions.

C. SCET light-cone quark model

Now, we discuss how to establish a light-cone quark model utilizing soft collinear effective theory. Since the $\bar{B}$ meson mass is dominated by the b quark mass, the momentum fraction for the spectator light antiquark x is of order $\Lambda_{\rm QCD}/m_b$. The variable $X \equiv x m_b$ is of order of $\Lambda_{\rm QCD}$ which is independent of m_b in the limit $m_b \to \infty$. The B meson wave function should have a scaling behavior in the heavy quark limit [31]

$$\phi_B(x, k_\perp) \to \sqrt{m_b}\Phi(X, k_\perp), \tag{25}$$

where the factor $\sqrt{m_b}$ subtracts out the m_b dependence of $\phi_B(x, k_\perp)$ and the function $\Phi(X, k_\perp)$ is normalized as $\int dX d^2 k_\perp |\Phi(X, k_\perp)|^2 = 1$. It is also found that $\Phi(X, k_\perp)$ is a function of $v \cdot p_q$: $\Phi(X, k_\perp) \to \Phi(v \cdot p_q)$ with p_q the momentum of the spectator antiquark. This observation is important in heavy-to-heavy transitions; however, because we work in the $\bar{B}$ meson rest frame, it does not help us to understand the heavy-to-light case. The light meson wave function $\phi_M(x, k_\perp)$ that appeared in the heavy-to-light form factors is the wave function at endpoint $x \sim \Lambda_{\rm QCD}/E \to 0$ in the large energy limit. The form of the light meson wave function at the endpoint is very important in determining the scaling behavior in m_b of the heavy-to-light form factors.

In the heavy quark limit, the heavy quark momentum is approximated as $p_b \cong m_b v$ with other components neglected. For the light energetic quark, $p^- \ll p_\perp \ll p^+$. Thus, the light quark momentum p is replaced by $p^\mu \cong (n_+ \cdot p)\frac{n_-^\mu}{2}$. As discussed before, in the soft collinear effective theory, the field describing the heavy quark is the two-component spinor h_v and the one describing the energetic quark is the spinor ξ. For our purpose, we need the expression for the helicity sums for Dirac spinors in the heavy quark limit. For the hheavy quark h_v, the leading-order contribution is

$$\sum_\lambda h_v(\lambda)\bar{h}_v(\lambda) = (1 + \not{v}). \tag{26}$$

For the light quark field ξ, the helicity sum gives

$$\sum_\lambda \xi(p, \lambda)\bar{\xi}(p, \lambda) = \frac{\not{n}_-}{2}. \tag{27}$$

The above two equations provide the spin symmetry relations for heavy-to-light form factors. While for the spectator which is a light antiquark, it satisfies the relation given in Eq. (20).

The momenta for $\bar{B}$ and light mesons are denoted by P and P', respectively. It is convenient to work in the $\bar{B}$ meson rest frame and set $P'_\perp = 0$. In this Lorentz frame, the momentum transfer q is purely longitudinal, i.e., $q_\perp = 0$ and $q^2 = q^+ q^- \geq 0$ cover the entire physical range.

The lowest order contribution to the form factor comes from the soft Feynman diagram where the spectator antiquark goes directly into the final light meson. The diagram is depicted in Fig. 1. The valence quark approximation guarantees that only the endpoint wave function of the light meson overlaps with the $\bar{B}$ meson. We use p_b, p_1, and p_q to denote the momenta of the b quark, the energetic quark, and the spectator:

$$p_b^+ = (1-x)P^+, \qquad p_{b\perp} = -k_\perp,$$
$$p_1^+ = (1-x')P'^+, \qquad p_{1\perp} = -k_\perp, \tag{28}$$
$$p_q^+ = xP^+ = x'P'^+, \qquad p_{q\perp} = k_\perp,$$

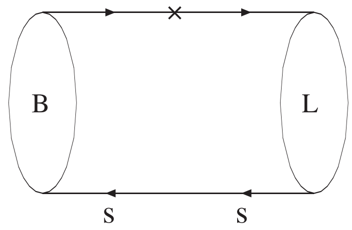

FIG. 1. The leading-order contribution to heavy-to-light form factors with "s" representing the soft momentum.

where $P^+ = M_B$ and $P'^+ = 2E$. x, x' are the momentum fractions of the spectator antiquark in the $\bar{B}$ meson and in the final state meson, respectively. x and x' are connected by $x = x'r$. It is useful to define a variable $r \equiv P'^+/P^+ = 1 - q^2/M_B^2$. Since x' varies from 0 to 1, x varies from 0 to r.

Now, we are able to present the derivation of form factors in the light-cone approach with some details. The $\bar{B}$-to-pseudoscalar meson matrix element can be expressed as

$$\langle P|\bar{\xi} h_v|\bar{B}\rangle = (-1)N_c \int_0^r dx \int \frac{d^2k_\perp}{2(2\pi)^3} P^+ \phi_P^*(x', k_\perp) \times \phi_B(x, k_\perp) \frac{P^+P'^+\sqrt{x(1-x)}\sqrt{x'(1-x')}}{\sqrt{2}\tilde{M}_0\sqrt{2}\tilde{M}'_0} \times \mathrm{Tr}\left[\frac{(\not{p}_q - m_q)}{p_q^+}\frac{(i\gamma_5)}{\sqrt{N_c}}\frac{\not{n}_-}{2}(1+\not{v})\frac{(-i\gamma_5)}{\sqrt{N_c}}\right], \tag{29}$$

where m_q is the mass of spectator antiquark. Since $x \sim x' \sim \Lambda_{\rm QCD}/m_b$, we will neglect x, x' compared to 1. The mass difference between the b quark mass and the $\bar{B}$ meson is neglected, i.e., $m_b \doteq M_B$. It is easy to obtain the relation $\sqrt{x(1-x)}\tilde{M}_0 = \sqrt{\mathcal{A}^2 + k_\perp^2}$. Expanding the momentum and keeping the leading power component, we get

$$\langle P|\bar{\xi} h_v|\bar{B}\rangle = \int_0^r dx \int \frac{d^2k_\perp}{2(2\pi)^3} \times \frac{\phi_P^*(x', k_\perp)\phi_B(x, k_\perp)}{\sqrt{\mathcal{A}_B^2 + k_\perp^2}\sqrt{\mathcal{A}_P^2 + k_\perp^2}} x m_b^2(p_q^- + m_q), \tag{30}$$

where $p_q^- = \frac{k_\perp^2 + m_q^2}{x m_b}$. From Eqs. (6) and (30), one obtains

$$\zeta_P = \frac{m_b}{2E}\int_0^r dx \int \frac{d^2k_\perp}{2(2\pi)^3} \frac{\phi_P^*(x', k_\perp)\phi_B(x, k_\perp)}{\sqrt{\mathcal{A}_B^2 + k_\perp^2}\sqrt{\mathcal{A}_P + k_\perp^2}} \times (x m_b m_q + m_q^2 + k_\perp^2). \tag{31}$$

We can see that the leading-order form factor ζ_P depends on the spectator quark mass m_q and the scaleless factor m_b/E, and nonperturbatively it depends on E through the light meson wave function $\phi_P(x', k_\perp)$ at $x' \sim \Lambda_{\rm QCD}/E$. The fact that m_b must be associated with x means that the form factor depends on the nonperturbative scale $X = x m_b$ rather than the hard scale m_b (except a normalization constant factor $\sqrt{m_b}$).

For $\bar{B}$ meson decays to a longitudinally polarized vector, substituting the polarization vector into the right-hand side of Eq. (6), we get

$$\langle V|\bar{\xi}\gamma_5 h_v|\bar{B}\rangle = -iM_V\zeta_\parallel\left(\frac{P'^+}{M_V} - \frac{M_V}{P'^+}\right) = -iP'^+\zeta_\parallel, \tag{32}$$

where we have dropped the subleading term. The expression in the light-cone approach gives

$$\begin{aligned}\langle V|\bar{\xi}\gamma_5 h_v|\bar{B}\rangle &= i\int_0^r dx \int \frac{d^2k_\perp}{2(2\pi)^3} \frac{\phi_V^*(x', k_\perp)\phi_B(x, k_\perp)}{2\sqrt{\mathcal{A}_B^2 + k_\perp^2}\sqrt{\mathcal{A}_V^2 + k_\perp^2}} x' m_b^2 \mathrm{Tr}\left[(\not{p}_q - m_q)\left(-\hat{\not{\epsilon}} + \frac{\hat{\epsilon}\cdot(p_1 - p_q)}{W_V}\right)\frac{\not{n}_-}{2}\gamma_5(1+\not{v})\gamma_5\right] \\ &= \frac{-im_b^2P'^+}{2}\int_0^r dx \int \frac{d^2k_\perp}{2(2\pi)^3} \frac{\phi_V^*(x', k_\perp)\phi_B(x, k_\perp)}{M_{0V}\sqrt{\mathcal{A}_B^2 + k_\perp^2}\sqrt{\mathcal{A}_V^2 + k_\perp^2}} x\left[2z^2(p_q^+ + m_q) + \frac{p_q^- + z^2p_1^+}{W_V}(p_q^- + m_q)\right],\end{aligned} \tag{33}$$

with $z \equiv M_{0V}/P'^+$. Although it seems that the first term is suppressed by $\lambda = \sqrt{\Lambda_{\rm QCD}/m_b}$, later we find that this term gives a relatively large contribution in the numerical calculation. We obtain the expression for the longitudinal leading-order form factor as

$$\zeta_\parallel = \frac{m_b^2}{2}\int \frac{dx d^2k_\perp}{2(2\pi)^3} \frac{\phi_V^*(x', k_\perp)\phi_B(x, k_\perp)}{M_{0V}\sqrt{\mathcal{A}_B^2 + k_\perp^2}\sqrt{\mathcal{A}_V^2 + k_\perp^2}} \times x\left[2z^2(p_q^+ + m_q) + \frac{p_q^- + z^2 r m_b}{W_V}(p_q^- + m_q)\right]. \tag{34}$$

Similarly, we can analyze the leading-order transverse form factor. When performing the calculation of $\zeta_\perp$, a formula for the transverse momentum integral is useful,

$$\int d^2k_\perp(\epsilon\cdot p_1)p_q^\alpha = \frac{1}{2}\int d^2k_\perp k_\perp^2\epsilon^\alpha. \tag{35}$$

The expression for $\bar{B}$ to a transversely polarized vector meson is

$$\langle V|\bar{\xi}\gamma_\perp^\mu h_v|\bar{B}\rangle = i\int_0^r dx \int \frac{d^2k_\perp}{2(2\pi)^3} \frac{\phi_V^*(x',k_\perp)\phi_B(x,k_\perp)}{2\sqrt{\mathcal{A}_B^2+k_\perp^2}\sqrt{\mathcal{A}_V^2+k_\perp^2}} x m_b^2 \,\mathrm{Tr}\Big[(\not{p}_q - m_q)\Big(-\hat{\not{\epsilon}} + \frac{\hat{\epsilon}\cdot(p_1-p_q)}{W_V}\Big)\frac{\not{n}_-}{2}\gamma_\perp^\mu(1+\not{v})\gamma_5\Big]$$
$$= -m_b^2 \int \frac{dx d^2k_\perp}{2(2\pi)^3} \frac{\phi_V^*(x',k_\perp)\phi_B(x,k_\perp)}{\sqrt{\mathcal{A}_B^2+k_\perp^2}\sqrt{\mathcal{A}_V^2+k_\perp^2}} \epsilon^{\mu\nu\rho\sigma}\epsilon_\nu^* v_\rho n_{-\sigma} x\Big(p_q^- + m_q + \frac{k_\perp^2}{W_V}\Big). \tag{36}$$

It is straightforward to get

$$\zeta_\perp = \frac{m_b^2}{2E}\int \frac{dx d^2k_\perp}{2(2\pi)^3} \frac{\phi_V^*(x',k_\perp)\phi_B(x,k_\perp)}{\sqrt{\mathcal{A}_B^2+k_\perp^2}\sqrt{\mathcal{A}_V^2+k_\perp^2}} \times x\Big(p_q^- + m_q + \frac{k_\perp^2}{W_V}\Big). \tag{37}$$

D. Higher order corrections to the heavy-to-light form factors in the light-cone perturbation theory

In this subsection, we will derive the higher order corrections for the heavy-to-light form factors in the light-cone perturbation theory of QCD. Besides the leading-order soft contributions to the universal form factors, the next-to-leading-order contribution is the kind of diagram shown in Fig. 2 with one hard gluon exchange (for the vertex corrections, please see [13,28]).

A four-component Dirac field ψ can be decomposed into two-component spinors ξ and η by

$$\psi = \xi + \eta, \qquad \xi \equiv P_-\psi = \frac{\not{n}_-\not{n}_+}{4}\psi, \qquad \eta \equiv P_+\psi = \frac{\not{n}_+\not{n}_-}{4}\psi, \tag{38}$$

where equations of motion for spinors ξ and η are

$$in_-\cdot D\frac{\not{n}_+}{2}\xi + (i\not{D}_\perp - m)\eta = 0, \tag{39}$$

$$in_+\cdot D\frac{\not{n}_-}{2}\eta + (i\not{D}_\perp - m)\xi = 0. \tag{40}$$

In light-cone quantization, the time variable is chosen to be different from the conventional one $t = x^3$. We adopt the light-cone time as $\tau = n_+\cdot x$ and then the timelike derivative is $n_-\cdot\partial$. In Eq. (40), there is no time derivative. Thus

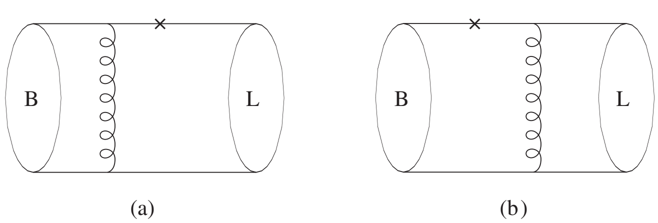

FIG. 2. The one-gluon exchange contributions to heavy-to-light form factors with the signs "×" representing the electroweak vertex.

η is a constrained field,[3] since it is determined by ξ at any time $n_+\cdot x$. From Eq. (40), the η field is obtained as

$$\eta = \frac{1}{in_+\cdot D}(i\not{D}_\perp + m)\frac{\not{n}_+}{2}\xi. \tag{41}$$

For the gluon field, it satisfies the color Maxwell equation $\partial_\mu F^{a\mu\nu} = gJ^{a\nu}$, where $J^{a\nu}$ is the quark current. By using the constraint $n_+\cdot A = 0$, we obtain one relation, $(n_+\cdot\partial)\partial_\mu A^{a\mu} = -g(n_+\cdot J^a)$. Thus, the field component $n_-\cdot A$ is not a dynamical variable but rather it is determined by $A_\perp$ through

$$n_-\cdot A = \frac{2}{n_+\cdot\partial}\partial_\perp\cdot A_\perp - \frac{2}{(n_+\cdot\partial)^2}g(n_+\cdot J^a). \tag{42}$$

The Feynman rules for ξ and $A_\perp$ have been derived, such as in [32], which are not useful for our purpose. We prefer to use another formulation given in [24]. In light-cone perturbation theory, the diagrams are $n_+\cdot x$ ordered and all particles are on mass shell. For the propagator of the quark, it contains an instantaneous part, in particular,

$$\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} = \frac{i(\not{p}_{\rm on}+m)}{p^2-m^2+i\epsilon} + \frac{i\not{n}_+}{2n_+\cdot p}, \tag{43}$$

where $p_{\rm on}$ is the on-shell momentum $p_{\rm on} = (n_-\cdot p, \frac{p_\perp^2+m^2}{n_-\cdot p}, p_\perp)$ and $p_{\rm on}^2 = m^2$. The second term in the quark propagator $\frac{\not{n}_+}{2n_+\cdot p}$ is the instantaneous part induced by integrating out the field η. For the gluon field, the polarization sum is written as

$$d_{\mu\nu}(k) \equiv \sum_{\lambda_g}\epsilon_\mu(k,\lambda_g)\epsilon_\nu^*(k,\lambda_g) = \sum_{i=1,2}\Big[-n_{+\mu}\frac{\epsilon^{(i)}\cdot k}{n_+\cdot k} + \epsilon_\mu^{(i)}\Big] \times\Big[-n_{+\nu}\frac{\epsilon^{(i)}\cdot k}{n_+\cdot k} + \epsilon_\nu^{(i)}\Big], \tag{44}$$

where $\epsilon^{(i)}$ are purely transverse vectors: $\epsilon^{(i)+} = \epsilon^{(i)-} = 0$ and $\epsilon_\perp^{(i)*}\cdot\epsilon_\perp^{(j)} = \delta^{ij}$. There are two terms in brackets in Eq. (44): the first term $n_{+\mu}\frac{\epsilon^{(i)}\cdot k}{n_+\cdot k}$ comes from the longitudinal component $n_+\cdot A$, and the second, $\epsilon_\mu^{(i)}$, comes from the transverse component $A_\perp$. If the gluon momentum is chosen to be in the longitudinal direction, then $\epsilon^{(i)}\cdot k = 0$ and only the transverse components $\epsilon^{(i)}$ remain. It reflects

[3]In some references, ξ is called a "good" component and η is called a "bad" component.

the fact that the physical gluon is transverse polarized. In the above rules, the choice of n_+ and n_- is arbitrary and there is a symmetry by exchanging them. In this way, we obtain the light-cone quantization rules for the light-cone time $n_- \cdot x$.

For the one-gluon exchange diagram given in Fig. 2, the amplitude at the quark level is given in the conventional covariant form as

$$A = \frac{g^2}{k^2} d_{\mu\nu}\left\{\bar{u}(p_1)\Gamma\frac{(\not{p}_{q1}+m_b)}{p_{q1}^2-m_b^2}T^A\gamma^\mu b(p_b)\bar{v}_s(p_q)T^A\gamma^\nu v(p_2) + \bar{u}(p_1)T^A\gamma^\mu\frac{\not{p}_{q2}}{p_{q2}^2}\Gamma b(p_b)\bar{v}_s(p_q)T^A\gamma^\nu v(p_2)\right\}, \tag{45}$$

where $u(v)$ are light quark (antiquark) spinors, $b(v_s)$ are b quark (spectator antiquark) spinors, $p_{q1,q2}$ are the internal quark momenta, k is the exchanged gluon momentum, and $k = p_2 - p_q$, $p_{q1} = p_b - p_2$, $p_{q2} = p_1 + p_2 - p_q$. The first term of the amplitude comes from the contribution of Fig. 2(a), and the second term comes from Fig. 2(b). We have neglected the light quark masses. For the second term in Eq. (45), we use the light-cone quantization rules of Eqs. (43) and (44), while for the first term in Eq. (45), the exchanged rules of Eqs. (43) and (44) by $n_- \leftrightarrow n_+$ are applied. Thus, the amplitude is rewritten in the light-cone form by

$$\begin{aligned} A = \frac{g^2}{k^2}\bigg\{&\bar{u}\Gamma\bigg[\frac{\not{n}_-}{2n_-\cdot p_{q1}} + \frac{(\not{p}_{q1})_{\rm on}+m_b}{p_{q1}^2-m_b^2}\bigg] \\ &\times T^A\bigg[\gamma_\perp^\mu - \frac{\not{n}_-}{n_-\cdot k}k_\perp^\mu\bigg]b\bar{v}_sT^A\bigg[\gamma_{\perp\mu} - \frac{\not{n}_-}{n_-\cdot k}k_{\perp\mu}\bigg]v \\ &+ \bar{u}T^A\bigg[\gamma_\perp^\mu - \frac{\not{n}_+}{n_+\cdot k}k_\perp^\mu\bigg]\bigg[\frac{\not{n}_+}{2n_+\cdot p_{q2}} + \frac{(\not{p}_{q2})_{\rm on}}{p_{q2}^2}\bigg] \\ &\times \Gamma b\bar{v}_sT^A\bigg[\gamma_{\perp\mu} - \frac{\not{n}_+}{n_+\cdot k}k_{\perp\mu}\bigg]v\bigg\}. \end{aligned} \tag{46}$$

Neglecting the contributions suppressed by $\Lambda_{\rm QCD}/m_b$, we find that the contribution from the instantaneous interaction part is

$$A^h = \frac{-g^2}{(n_+\cdot p_2)(n_-\cdot p_q)}\left\{\bar{\xi}_{n_-}\Gamma\frac{\not{n}_-}{2m_b}T^A\gamma_\perp^\mu h_v\bar{v}_sT^A\gamma_{\perp\mu}\xi_{n_-} + \bar{\xi}_{n_-}T^A\gamma_\perp^\mu\frac{\not{n}_+}{2n_+\cdot P'}\Gamma h_v\bar{v}_sT^A\gamma_{\perp\mu}\xi_{n_-}\right\}. \tag{47}$$

This contribution is not singular for the leading twist distribution amplitudes of B and light mesons. It is usually called a "hard" contribution which breaks the spin symmetry due to $\not{n}_-$ and $\not{n}_+$ matrices. In the light-cone language, the hard gluon exchange contributions come from the instantaneous quark interactions and the transversely polarized gluons. The hard one-gluon exchange contributions cannot be absorbed into the three universal form factors because this type of higher order contribution breaks the spin symmetry in the leading order.

III. NUMERICAL RESULTS AND DISCUSSIONS

The physical heavy-to-light form factors contain both hard and soft contributions. In this study, we concentrate on the leading-order soft form factors. The next-to-leading-order α_s corrections, which break the spin symmetry, will be calculated in a future work. In order to obtain the numerical results, we have to determine the wave functions of the hadrons which contain all information of the hadron state. The full solution takes great efforts, so we use the phenomenological Gaussian-type wave function:

$$\phi(x, k_\perp) = N\sqrt{\frac{dk_z}{dx}}\exp\left(-\frac{\vec{k}^2}{2\omega^2}\right), \tag{48}$$

where $N = 4(\pi/\omega^2)^{3/4}$ and k_z of the internal momentum $\vec{k} = (\vec{k}_\perp, k_z)$ is defined through

$$1 - x = \frac{e_1 - k_z}{e_1 + e_2}, \qquad x = \frac{e_2 + k_z}{e_1 + e_2}, \tag{49}$$

with $e_i = \sqrt{m_i^2 + \vec{k}_i^2} = \frac{x_iM_0}{2} + \frac{m_i^2+k_{\perp i}^2}{2x_iM_0}$. We then have

$$k_z = \frac{xM_0}{2} - \frac{m_2^2 + k_\perp^2}{2xM_0}, \qquad \frac{dk_z}{dx} = \frac{e_1e_2}{x(1-x)M_0}. \tag{50}$$

In this wave function, the distribution of the momentum is determined by the quark mass and the parameter ω. The quarks are constituent quarks and the quark masses are usually chosen as

$$m_{u,d} = 0.25 \text{ GeV}, \qquad m_s = 0.40 \text{ GeV}, \qquad m_b = 4.8 \text{ GeV}. \tag{51}$$

The parameter ω can be determined by the hadronic results, for example, the decay constants [33].

As for the decay constants of η and η', we should pay much more attention to the mixing of these two particles. Although the quark model has achieved great successes, we still do not have the definite answer on the exact components of these two mesons. The study of B to $\eta^{(\prime)}$ decays, especially the study of the $B \to \eta^{(\prime)}$ form factor, can help us to understand their intrinsic character (for a recent study, please see [34]). Here we view these two particles as the conventional two-quark states. As for the mixing, we use the quark flavor basis proposed by Feldmann and Kroll [35], i.e. these two mesons are made of $\eta_n = \bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $\eta_s = \bar{s}s$:

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = U(\phi)\begin{pmatrix} |\eta_n\rangle \\ |\eta_s\rangle \end{pmatrix}, \tag{52}$$

with the mixing matrix

$$U(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}, \tag{53}$$

where ϕ is the mixing angle. In this mixing scheme, only two decay constants f_n ($n = u, d$) and f_s are needed:

$$\langle 0|\bar{n}\gamma^{\mu}\gamma_5 n|\eta_n(P)\rangle = \frac{i}{\sqrt{2}} f_n P^{\mu}, \qquad \langle 0|\bar{s}\gamma^{\mu}\gamma_5 s|\eta_s(P)\rangle = i f_s P^{\mu}. \tag{54}$$

This is based on the assumption that the intrinsic $\bar{n}n(\bar{s}s)$ component is absent in the $\eta_s(\eta_n)$ meson, i.e., based on the Okubo-Zweig-Iizuka suppression rule. These decay constants have been determined from the related exclusive processes as [35]

$$f_n = (1.07 \pm 0.02) f_{\pi}, \qquad f_s = (1.34 \pm 0.06) f_{\pi}. \tag{55}$$

In the following we will calculate the form factors of $B \to \eta_n$ and $B_s \to \eta_s$. The gluonic contribution to $B \to \eta^{(\prime)}$ has also been studied in Ref. [34]. We will neglect it, as it is very small.

We use the following results for the decay constants as input in the light-front wave functions:

$$\begin{aligned} f_B &= 0.190\ \text{GeV}, & f_{B_s} &= 0.236\ \text{GeV}, \\ f_{\pi} &= 0.132\ \text{GeV}, & f_K &= 0.160\ \text{GeV}, \\ f_{\rho} &= 0.205\ \text{GeV}, & f_{\omega} &= 0.195\ \text{GeV}, \\ f_{K^*} &= 0.217\ \text{GeV}, & f_{\phi} &= 0.231\ \text{GeV}. \end{aligned} \tag{56}$$

Then the parameters ω in the light-front wave functions are determined from these decay constants as

$$\begin{aligned} \omega_B &= 0.55^{+0.05}_{-0.04}\ \text{GeV}, & \omega_{B_s} &= 0.64^{+0.05}_{-0.06}\ \text{GeV}, \\ \omega_{\pi} &= 0.33\ \text{GeV}, & \omega_K &= 0.38\ \text{GeV}, \\ \omega_n &= 0.38^{+0.09}_{-0.08}\ \text{GeV}, & \omega_s &= 0.39^{+0.06}_{-0.06}\ \text{GeV}, \\ \omega_{\rho} &= 0.31^{+0.03}_{-0.03}\ \text{GeV}, & \omega_{\omega} &= 0.29^{+0.03}_{-0.03}\ \text{GeV}, \\ \omega_{K^*} &= 0.33^{+0.03}_{-0.03}\ \text{GeV}, & \omega_{\phi} &= 0.35^{+0.03}_{-0.03}\ \text{GeV}, \end{aligned} \tag{57}$$

where the uncertainties come from varying the decay constants of the heavy and light mesons by 10%. Some light meson decay constants have been determined to a high accuracy, for example, f_{π}, f_K. We neglect the uncertainties for them.

A. Results for the $B \to P$ form factor ζ_P

Now we are ready to give the numerical results of the B-to-pseudoscalar soft form factors at $q^2 = 0$, i.e. $E = m_B/2$. Using the above parameters, we obtain the results as follows:

$$\begin{aligned} &\zeta_P^{B\to\pi}\left(\frac{m_B}{2}\right) = 0.247, \qquad \zeta_P^{B\to K}\left(\frac{m_B}{2}\right) = 0.297, \\ &\zeta_P^{B\to\eta_n}\left(\frac{m_B}{2}\right) = 0.287^{+0.059}_{-0.065}, \qquad \zeta_P^{B_s\to K}\left(\frac{m_B}{2}\right) = 0.290, \\ &\zeta_P^{B_s\to\eta_s}\left(\frac{m_B}{2}\right) = 0.288^{+0.047}_{-0.052}, \end{aligned} \tag{58}$$

where the uncertainties are from the decay constant of the light mesons. We also find that the uncertainties caused by B meson decay constants are rather small and thus we neglect these uncertainties. In Ref. [20], the SCET sum rule result is calculated as $\zeta_P^{B\to\pi} = 0.27$ which is consistent with our result within theoretical errors. The physical form factors can be obtained directly using the relation in Eq. (7). At maximal recoil $r = 1$, f_+ and f_0 are equal to each other, which are exactly the soft form factor ζ_P; f_T is slightly larger. Table I lists the $B \to P$ form factors at $q^2 = 0$.

These $B \to P$ form factors have also been studied systemically in the usual light-cone quark model [4–6], the light-cone sum rules [8], and the PQCD approach [10]. Although lattice QCD (LQCD) cannot give direct predic-

TABLE I. The physical $B_{(u,d,s)} \to P$ form factors at maximal recoil using the usual LCQM [6], LCSR [8], LQCD [36–38], and PQCD [10] approaches. The different values for $f_+(B \to \pi)$ in Ref. [37] correspond to different extrapolations.

		LCQM [6]	LCSR [8]	PQCD [10]	LQCD [36]	LQCD [37]	LQCD [38]	This work
$B \to \pi$	f_+	0.25	0.258	0.292	0.27	0.27(0.26)	0.23	0.247
	f_T		0.253	0.278				0.253
	f_0	0.25		0.292	0.27			0.247
$B \to K$	f_+	0.35	0.331	0.321				0.297
	f_T		0.358	0.311				0.325
	f_0	0.35		0.321				0.297
$B \to \eta_n$[a]	f_+		0.275					0.287
	f_T		0.285					
	f_0							0.287
$B_s \to K$	f_+							0.290
	f_T							0.317
	f_0							0.290
$B_s \to \eta_s$	f_+							0.288
	f_0							0.288

[a]The form factor of $B \to \eta$, rather than that of $B \to \eta_n$, is calculated in LCSR.

tions on the B-to-light form factors at large recoiling, there are some studies using the extrapolations from the results at large q^2: in quenched LQCD [36] and in unquenched LQCD [37,38]. We cite these results in Table I.

Comparing the results in Table I, we can find that our leading-order results agree with the results calculated using other approaches. The numerical results of higher order corrections which should be small in our approaches will be taken into account in future work.

We compare our approach with the previous light-cone quark models. As in the conventional form of [4] where the quarks are on shell, the calculation of form factors is performed in the physical momentum regions $q^2 \geq 0$. The difference between the approach in [4] and ours is that we make approximations in the heavy quark mass and large energy limits. The consistency of the numerical predictions in the two methods means that our result is the leading dominant contribution. In the covariant form in [25], the quarks are off shell. The evaluations are performed in the momentum regions $q^2 < 0$, and the analytic continuation is required to obtain the physical form factors. The advantage of this approach is that the zero-mode ($k^+ = 0$) contribution does not occur. In our method, the zero-mode contribution vanishes in the heavy quark mass and large energy limits.

Since our analysis is within the SCET framework, we should make sure that the final state meson is energetic. The energy of the light meson should be larger than $\sqrt{m_B \Lambda_{\rm QCD}} \sim 1.5$ GeV in order to ensure it is a collinear meson. From this constraint, we can get $q^2 = m_B^2 - 2m_B E < 10$ GeV^2. Thus we can directly calculate the form factors in the range of $0 < q^2 < 10$ GeV^2 and the results should be reliable. We plot the q^2 dependence of the $B_{(s)} \to P$ form factors in Fig. 3. In this figure, the form factors $f_+(q^2) = \zeta_P(E)$, $f_T(q^2) = \frac{M_B+M_P}{M_B}\zeta_P(E)$, and $f_0(q^2) = \frac{2E}{m_B}\zeta_P(E)$ are plotted. The q^2 dependence of f_+ and f_T is essentially the same, except for the difference of the form factor at $q^2 = 0$. The curve of $f_0(q^2)$ is more flat than the other two because of the compensation of the factor $r = \frac{2E}{m_B} = 1 - \frac{q^2}{m_B^2}$.

In order to study the analytic q^2 dependence of the results for the form factors, we fit the data by adopting the simple parametrization

$$f(q^2) = \frac{f(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}, \tag{59}$$

where $f(0)$ are the results at $q^2 = 0$ which have been discussed as above, while a and b are the parameters.

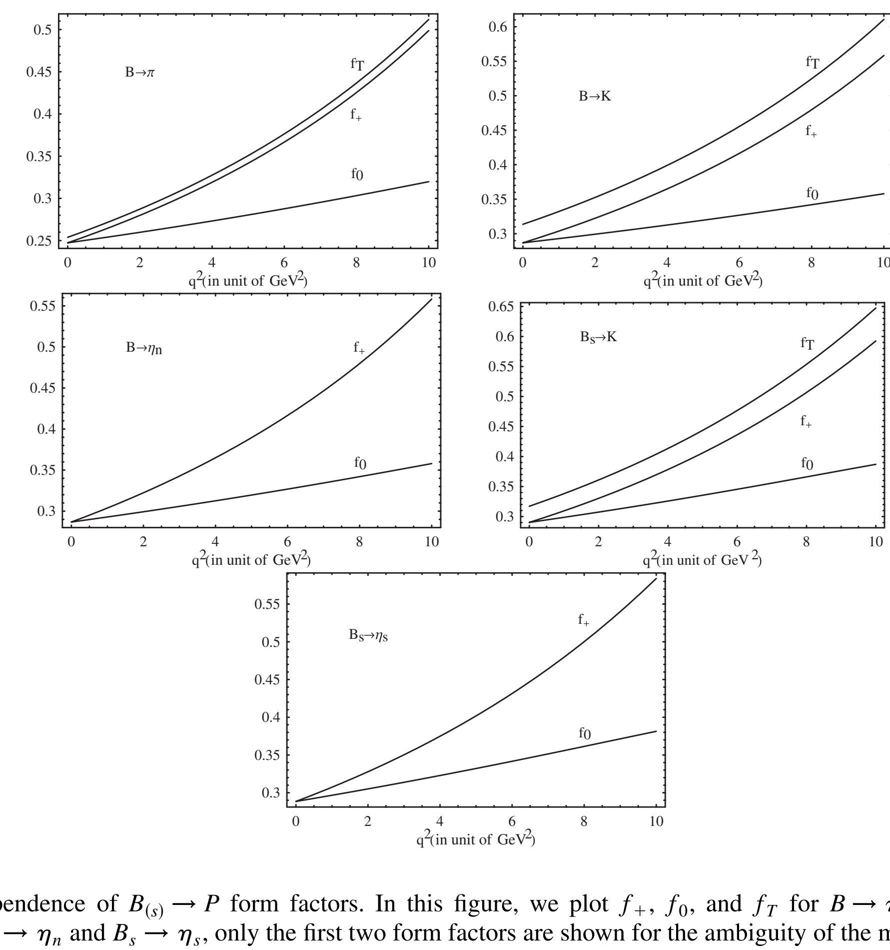

FIG. 3. The q^2 dependence of $B_{(s)} \to P$ form factors. In this figure, we plot f_+, f_0, and f_T for $B \to \pi$, $B \to K$, and $B_s \to K$ transitions. But for $B \to \eta_n$ and $B_s \to \eta_s$, only the first two form factors are shown for the ambiguity of the mass for η_n and η_s in f_T.

TABLE II. The parameters in the parametrization of $B_{(u,d,s)} \to P$ form factors. The fitted values of a and b for f_T are the same as the ones in f_+.

	$f_+^{B\to\pi}$	$f_+^{B\to K}$	$f_+^{B\to\eta_n}$	$f_+^{B_s\to K}$	$f_+^{B_s\to\eta_s}$
a	1.43	1.28	1.31	1.51	1.49
b	0.08	0.00	−0.00	0.23	0.22
	$f_0^{B\to\pi}$	$f_0^{B\to K}$	$f_0^{B\to\eta_n}$	$f_0^{B_s\to K}$	$f_0^{B_s\to\eta_s}$
a	0.56	0.46	0.48	0.66	0.64
b	−0.14	−0.08	−0.14	−0.00	0.00

The fitted results for these two parameters are summarized in Table II. From Fig. 3, we can see that all of the curves are close to being straight lines and the parameters b should be rather small. The results from the parametrization also verify this expectation. Our results for parameters a for different processes are also close to each other: around $a = 1.5$ for f_+ and f_T or $a = 0.5$ for f_0.

B. Results for $B \to V$ form factors

A similar analysis can also be applied to $B \to V$ form factors. At $q^2 = 0$, the results for the $B \to V$ soft form factors are

$$\zeta_\parallel^{B\to\rho}\left(\frac{m_B}{2}\right) = 0.260^{+0.028}_{-0.030}, \quad \zeta_\perp^{B\to\rho}\left(\frac{m_B}{2}\right) = 0.260^{+0.030}_{-0.031},$$
$$\zeta_\parallel^{B\to\omega}\left(\frac{m_B}{2}\right) = 0.240^{+0.029}_{-0.031}, \quad \zeta_\perp^{B\to\omega}\left(\frac{m_B}{2}\right) = 0.239^{+0.031}_{-0.031},$$
$$\zeta_\parallel^{B\to K^*}\left(\frac{m_B}{2}\right) = 0.284^{+0.025}_{-0.027}, \quad \zeta_\perp^{B\to K^*}\left(\frac{m_B}{2}\right) = 0.290^{+0.027}_{-0.029},$$
$$\zeta_\parallel^{B_s\to K^*}\left(\frac{m_B}{2}\right) = 0.279^{+0.030}_{-0.030}, \quad \zeta_\perp^{B_s\to K^*}\left(\frac{m_B}{2}\right) = 0.271^{+0.030}_{-0.030},$$
$$\zeta_\parallel^{B_s\to\phi}\left(\frac{m_B}{2}\right) = 0.279^{+0.029}_{-0.030}, \quad \zeta_\perp^{B_s\to\phi}\left(\frac{m_B}{2}\right) = 0.276^{+0.030}_{-0.030}, \tag{60}$$

where the uncertainties are from the decay constants of the light mesons. In order to make a comparison, we collect the results for the physical form factors in LCQM [4,6], LCSR [9], the PQCD [10] approach, LQCD [36,39], and our leading-order results in Table III. Our results are consistent with other approaches except for the smaller $T_{2,3}$ and larger T_1 in PQCD approaches.

The features of our results are as follows:

(i) Our results of $\zeta_\parallel$ and $\zeta_\perp$ for every meson are close to each other, which is mainly due to the similar wave function for the longitudinal and transverse polarizations.

(ii) The physical form factors can be directly calculated by using the soft form factors. The kinematic factor as in Eq. (7) makes the physical form factors different. V is the largest form factor which is enhanced by the factor $1 + M_V/M_B$, while T_3 is the smallest one because there is a minus term in Eq. (7).

TABLE III. The physical $B \to V$ form factors at maximal recoil, i.e. $q^2 = 0$.

		$B\to\rho$	$B\to K^*$	$B\to\omega$	$B_s\to K^*$	$B_s\to\phi$
LCQM [6]	V	0.27	0.31			
	A_0	0.28	0.31			
	A_1	0.22	0.26			
	A_2	0.20	0.24			
LCSR [9]	V	0.323	0.411	0.293	0.311	0.434
	A_0	0.303	0.374	0.281	0.360	0.474
	A_1	0.242	0.292	0.219	0.233	0.311
	A_2	0.221	0.259	0.198	0.181	0.234
	T_2	0.267	0.333	0.242	0.260	0.349
PQCD [10]	V	0.318	0.406	0.305		
	A_0	0.366	0.455	0.347		
	A_1	0.25	0.30	0.24		
	A_2	0.21	0.24	0.20		
LQCD [36]	V	0.35				
	A_0	0.30				
	A_1	0.27				
	A_2	0.26				
	T_1		0.24 [39]			
This work	V	0.298	0.339	0.275	0.323	0.329
	A_0	0.260	0.283	0.240	0.279	0.279
	A_1	0.227	0.248	0.209	0.228	0.232
	A_2	0.215	0.233	0.198	0.204	0.210
	T_1	0.260	0.290	0.239	0.271	0.276
	T_2	0.260	0.290	0.239	0.271	0.276
	T_3	0.184	0.194	0.168	0.165	0.170

(iii) The soft form factor of $B \to K^*$ is larger than that of $B \to \rho$ because the s quark in the K^* meson carries more momentum than the d quark in ρ, which can induce more overlap of the $\bar{B}$ meson wave function and the light K^* meson wave function. $\zeta_{\parallel,\perp}^{B\to\omega}$ is smaller than $\zeta_{\parallel,\perp}^{B\to\rho}$, which is a consequence of the fact that the decay constant of ω is smaller than that of ρ.

(iv) As we have discussed above, we keep the first term in $\zeta_\parallel$, although it is suppressed by $\lambda = \sqrt{\Lambda_{\rm QCD}/m_b}$. This term cannot be neglected in the numerics, as the suppression is not so effective: the $\zeta_\parallel^{B\to\rho}$ without this term becomes

$$\zeta_\parallel^{B\to\rho}\left(\frac{m_B}{2}\right) = 0.139, \tag{61}$$

which is quite smaller than the result with it. This small $\zeta_\parallel$ can lead to a small A_0 but a large A_2 and T_3.

The q^2 dependence ($0 < q^2 < 10\ \text{GeV}^2$) of the form factors is plotted in Fig. 4. The two form factors V and T_1 have the same q^2 dependence except for the different results at $q^2 = 0$, and both of them can be directly calcu-

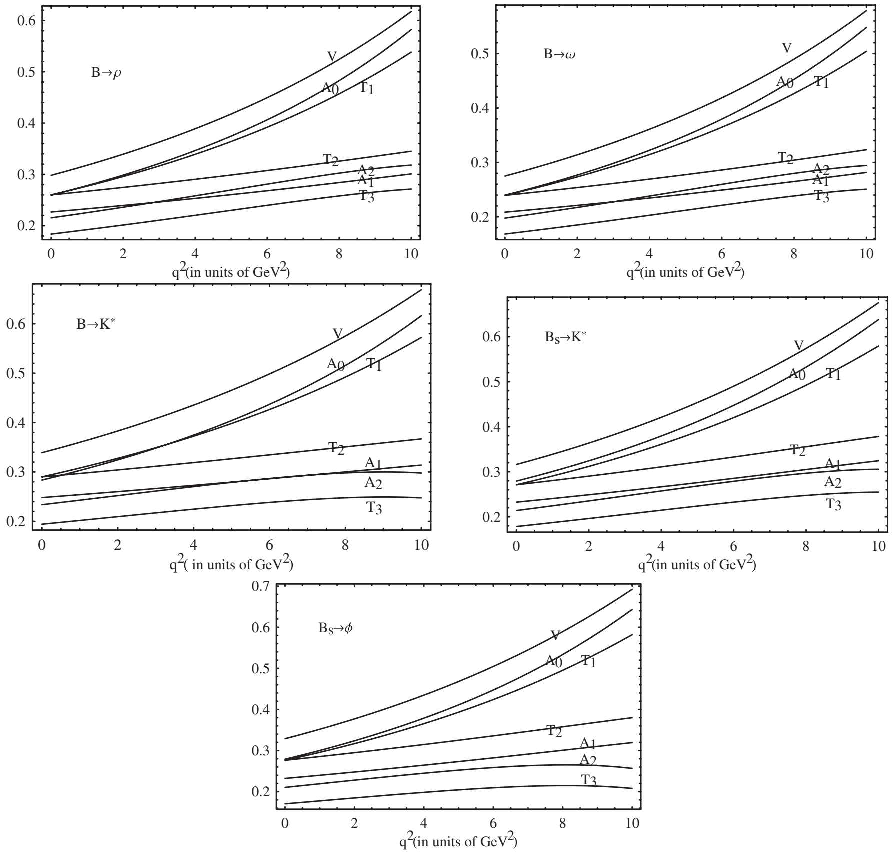

FIG. 4. The q^2 dependence of the $B_{(s)} \to V$ form factors.

lated by $\zeta_\perp(E)$. $A_0(q^2) = \zeta_\parallel(E)$ has similar q^2 dependence with $\zeta_\perp(E)$. When q^2 gets large, A_0 is a little sharper than V and T_1. The other four form factors are rather flat and are less sensitive to q^2. From the figure, we can see that A_2 and T_3 show a tendency to decrease at large q^2; these two form factors may not be described by the above parametrization and so we will not fit them as in B-to-pseudoscalar decays. We use the same parametrization to describe the q^2 dependence of the other form factors, and the results for the fitted parameters are given in Table IV. From the table, we can see that the parameters a for various channels are close to each other: around $a = 1.5$ for $\zeta_\parallel(A_0)$ and $\zeta_\perp(V, T_1)$ or $a = 0.5$ for $\frac{2E}{m_B}\zeta_\perp(A_1, T_2)$. Another interesting feature is that, for all form factors, the parameter b is not large and the form factor is dominated by the monopole term.

IV. CONCLUSIONS

A light-cone quark model within the soft collinear effective theory is constructed in this study. We calculated all the heavy-to-light $B_{(s)} \to P$ and $B_{(s)} \to V$ transition form factors at the large recoiling region. The three universal soft form factors are studied; in particular, the $B \to V$ form factors $\zeta_{\parallel,\perp}$ are given for the first time. Our numerical

TABLE IV. The parameters in the parametrization of $B \to V$ form factors.

	$\zeta_\parallel^{B\to\rho}(A_0)$	$\zeta_\parallel^{B\to\omega}(A_0)$	$\zeta_\parallel^{B\to K^*}(A_0)$	$\zeta_\parallel^{B_s\to K^*}(A_0)$	$\zeta_\parallel^{B_s\to\phi}(A_0)$
a	1.56	1.60	1.51	1.74	1.73
b	0.17	0.22	0.14	0.47	0.41
	$\zeta_\perp^{B\to\rho}(V, T_1)$	$\zeta_\perp^{B\to\omega}(V, T_1)$	$\zeta_\perp^{B\to K^*}(V, T_1)$	$\zeta_\perp^{B_s\to K^*}(V, T_1)$	$\zeta_\perp^{B_s\to\phi}(V, T_1)$
a	1.45	1.49	1.37	1.64	1.60
b	0.15	0.20	0.11	0.42	0.36
	$\frac{2E}{m_B}\zeta_\perp^{B\to\rho}(A_1, T_2)$	$\frac{2E}{m_B}\zeta_\perp^{B\to\omega}(A_1, T_2)$	$\frac{2E}{m_B}\zeta_\perp^{B\to K^*}(A_1, T_2)$	$\frac{2E}{m_B}\zeta_\perp^{B_s\to K^*}(A_1, T_2)$	$\frac{2E}{m_B}\zeta_\perp^{B_s\to\phi}(A_1, T_2)$
a	0.62	0.66	0.55	0.82	0.48
b	−0.11	−0.10	−0.05	0.08	0.04

results are, in general, consistent with other nonperturbative methods, such as light-cone sum rules and quark models within theoretical errors. Our numerical results are close to the results by other methods, which supports the fact that the leading-order soft contribution is dominant in the light-cone quark model. The theoretical uncertainties caused by the lesser known B meson decay constants are small. The q^2 dependence of the $B \to P$, V form factor is also studied in the range $0 < q^2 < 10$ GeV^2.

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