Azimuthal correlation between the $(\vec{p}_l, \vec{p}_{X_b})$ and $(\vec{p}_l, \vec{P}_t)$ planes in the semileptonic rest frame decay of a polarized top quark: An $O(\alpha_s)$ effect

S. Groote,[1,2] W. S. Huo,[1,3] A. Kadeer,[1] and J. G. Körner[1]

[1]*Institut für Physik der Johannes-Gutenberg-Universität, Staudinger Weg 7, D-55099 Mainz, Germany*
[2]*Tartu Ülikooli Teoreetilise Füüsika Instituut, Tähe 4, EE-51010 Tartu, Estonia*
[3]*Department of Physics, Xinjiang University, Shengli Road 14, 830046 Ürümqi, People's Republic of China*
(Received 14 March 2007; published 31 July 2007)

The azimuthal correlation between the planes formed by the vectors $(\vec{p}_\ell, \vec{p}_{X_b})$ and $(\vec{p}_\ell, \vec{P}_t)$ in the semileptonic rest frame decay of a polarized top quark $t(\uparrow) \rightarrow X_b + \ell^+ + \nu_\ell$ belongs to a class of polarization observables involving the top quark which vanish at the Born term level in the standard model. We determine the next-to-leading-order QCD corrections to the aforementioned azimuthal correlation and compare the result to the corresponding contribution of a non-standard-model right-chiral quark current.

DOI: 10.1103/PhysRevD.76.014012 PACS numbers: 12.38.Bx, 13.30.Ce, 13.88.+e, 14.65.Ha

I. INTRODUCTION

The azimuthal correlation between the $(\vec{p}_\ell, \vec{p}_{X_b})$ and $(\vec{p}_\ell, \vec{P}_t)$ planes in the semileptonic rest frame decay of a polarized top quark (see Fig. 1) belongs to a class of polarization observables involving the top quark in which the leading-order (LO) contribution gives a zero result in the standard model (SM). As we shall see later on, the vanishing of this azimuthal correlation is a consequence of the left-chiral $(V-A)(V-A)$ nature of the current-current interaction in the SM. Another example of a LO zero polarization observable is the decay of a top quark into a polarized transverse-plus W boson and a (massless) bottom quark where the rate into the transverse-plus W boson is zero at the Born term level due to the left-chiral $(V-A)$ coupling structure of the SM. Still another example is the production of longitudinally polarized top quarks in e^+-e^- annihilation produced from the longitudinal part of the intermediate gauge bosons (Z and/or γ). The corresponding rate is zero due to the absence of second–class currents in the SM.

For the latter two cases above, the next-to-leading-order (NLO) corrections have been computed in [1,2]. In [1] we determined the NLO QCD corrections to longitudinally polarized top quarks from the longitudinal part of the intermediate gauge bosons (Z and/or γ) in e^+-e^- annihilation. The NLO QCD and electroweak corrections to transverse-plus W bosons in top quark decays have been calculated in [2]. The purpose of this paper is to determine the NLO QCD corrections to the aforementioned azimuthal correlation in polarized top quark decay. We compare the results with the corresponding contribution of a non-SM right-chiral quark current.

Nonzero contributions to the aforementioned polarization observables can either arise from non-SM effects or from higher order SM radiative corrections. Clearly it is important to determine the size of the NLO corrections to the aforementioned polarization observables before non-SM effects can be claimed to be responsible for nonzero values of these polarization observables.

We mention that highly polarized top quarks will become available in singly produced top quarks at hadron colliders (see e.g. [3]) and in top quark pairs produced in future linear e^+-e^--colliders (see e.g. [4–10]). It will then be possible to experimentally measure the azimuthal correlation between the $(\vec{p}_\ell, \vec{p}_{X_b})$ and $(\vec{p}_\ell, \vec{P}_t)$ planes. To define the planes one needs to measure the momentum directions of the momenta $\vec{p}_\ell$ and $\vec{p}_{X_b}$ and the polarization direction of the top quark. The momentum direction of $\vec{p}_\ell$ can be directly measured, whereas the measurement of the momentum direction of $\vec{p}_{X_b}$ requires the use of a jet finding algorithm. The direction of the polarization of the top quark must be obtained from theoretical input. In e^+-e^- interactions the degree of polarization of the top quark can be tuned with the help of polarized beams [9]. For sufficiently high energies the polarization of the top quark will be longitudinal in both production processes, i.e. it

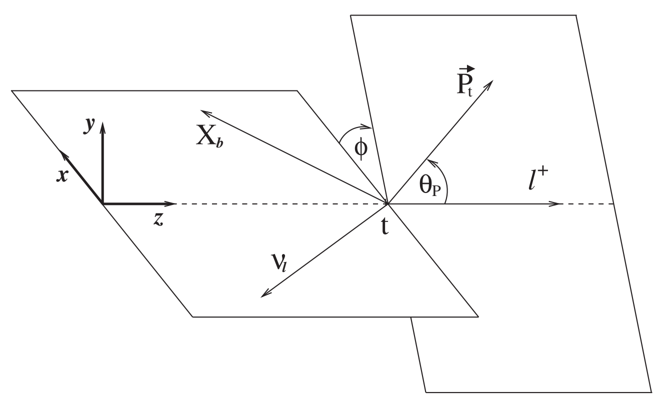

FIG. 1. The definition of the azimuthal angle θ_P in the rest frame decay of a polarized top quark. The event plane defines the (x, z) plane. The momenta of the top quark, bottom quark, charged lepton, and neutrino are denoted by p_t, p_b, p_ℓ, and p_ν. $\vec{P}_t$ is the polarization vector of the top quark.

will point in the direction of its motion. The measurement or a bound on the aforementioned azimuthal correlation in polarized top quark decays will be difficult, but may yet be feasible as the recent measurements of the helicity content of the W boson in semileptonic top quark decays by the CDF and D0 collaborations have shown [11,12].

II. ANGULAR RATE STRUCTURE

We shall closely follow the notation of [13] where D. Pirjol and one of us discussed the inclusive semileptonic rest frame decay of a polarized bottom baryon Λ_b. Of course one needs to take into account the necessary modifications when going from the $(b \rightarrow c)$ case to the $(t \rightarrow b)$ case. Reference [13] also contains a discussion of nonperturbative effects in the inclusive decay of the polarized Λ_b which were treated in next-to-leading order of heavy quark effective theory . This is not necessary in the present application since the top quark decays essentially as a free quark.

The general angular decay distribution of a polarized top quark decaying into a jet X_b with bottom quantum numbers and a charged lepton ℓ^+ and a neutrino ν_ℓ is given by [13]

$$\frac{d\Gamma}{dx d\cos\theta_P d\phi} = \frac{1}{4\pi}\left(\frac{d\Gamma_A}{dx} + \frac{d\Gamma_B}{dx} P\cos\theta_P + \frac{d\Gamma_C}{dx} P\sin\theta_P\cos\phi\right) \qquad (1)$$

where the polar and azimuthal angles θ_P and ϕ are defined in Fig. 1. In the classification of [13] this is the system 1b where the z axis is determined by the lepton's momentum and $\vec{p}_{X_b}$ has a positive x component. As usual we have defined a scaled lepton energy through $x = 2E_\ell/m_t$. P is the magnitude of the top quark polarization. $d\Gamma_A/dx$ corresponds to the unpolarized differential rate. $d\Gamma_B/dx$ and $d\Gamma_C/dx$ describe the polar and azimuthal correlation between the polarization of the top quark and its decay products, respectively.

The radiative corrections to the rate Γ_A [14] and the polar correlation function Γ_B [15–17] have been studied extensively before. We have repeated the calculations and have found agreement with the results in [14–17]. The radiative corrections to the azimuthal correlation function Γ_C have not been done before. As we shall explicitly see in the next section the LO Born term contribution to Γ_C vanishes as was mentioned. Technically this means that one does not have to introduce any IR regularization scheme such as a fictitious gluon mass or dimensional regularization when calculating the azimuthal correlation since at NLO the virtual one-loop and the tree-graph (real emission) contributions are separately infrared (IR) finite.

III. BORN TERM RESULTS

It is straightforward to calculate the Born term contribution to the decay $t(\uparrow) \rightarrow X_b + \ell^+ + \nu_\ell$. In the narrow resonance approximation for the W^+ boson the differential rates are given by ($x = 2E_\ell/m_t$)

$$\frac{d\Gamma_A^{(0)}}{dx} = \frac{d\Gamma_B^{(0)}}{dx} = \Gamma_F 2\pi\frac{m_W}{\Gamma_W} 6x(1-x)y^2, \qquad (2)$$

$$\frac{d\Gamma_C^{(0)}}{dx} = 0, \qquad (3)$$

where

$$\Gamma_F = \frac{G_F^2 m_t^5}{192\pi^3}|V_{tb}|^2 \qquad (4)$$

is a reference rate corresponding to a (hypothetical) pointlike four-Fermion interaction and $y^2 = m_W^2/m_t^2$. Note that we put the bottom quark mass to zero throughout the paper except for Sec. V where we discuss non-SM effects.

For the integrated rates we obtain ($y^2 \leq x \leq 1$)

$$\Gamma_A^{(0)} = \Gamma_B^{(0)} = \Gamma_F 2\pi\frac{m_W}{\Gamma_W} y^2(1-y^2)^2(1+2y^2), \qquad (5)$$

$$\Gamma_C^{(0)} = 0. \qquad (6)$$

One can read off from (4) that the width of the top quark is enhanced by a factor of $2\pi m_W/\Gamma_W \cdot y^2(1-y^2)^2 \times (1+2y^2) = 44.09$ compared to a pointlike four-Fermion interaction due to the presence of the W-pole ($\Gamma_W = 2.141$ GeV, $m_W = 80.403$ GeV).

Let us return to Eq. (2). The fact that $\Gamma_A = \Gamma_B$ means that the proposed polar correlation measurement has 100% analyzing power to analyze the polarization of the top quark whereas the azimuthal correlation measurement has zero analyzing power. In the following we shall present some simple arguments to show that $\Gamma_A = \Gamma_B$ can be directly traced to the fact that we are dealing with a $(V-A)(V-A)$ current-current structure in this transition. Once this is established we then present a physics argument that $\Gamma_C = 0$ necessarily follows.

Let us rewrite the original $(V-A)(V-A)$ SM form into a more convenient form using the Fierz transformation of the second kind which transforms the $(V-A)(V-A)$ form into a $(S+P)(S-P)$ form (see e.g. [18]):

$$M = \bar{u}(b)\gamma^\mu(1-\gamma_5)u(t)\bar{u}(\nu)\gamma_\mu(1-\gamma_5)v(\ell) \qquad (7)$$

$$= 2\bar{u}(b)(1+\gamma_5)C\bar{u}^T(\nu)v^T(\ell)C^{-1}(1-\gamma_5)u(t) \qquad (8)$$

$$= 2\bar{u}(b)(1+\gamma_5)v(\nu)\bar{u}(\ell)(1-\gamma_5)u(t) \qquad (9)$$

where we have used $C\bar{u}^T(\nu) = v(\nu)$ and $v^T(\ell)C^{-1} = \bar{u}(\ell)$. The advantage of the form of Eq. (9) is that the spinors of the top quark and the lepton are now connected by one Dirac string. In particular, this means that there is no correlation between the top quark spin and the momenta of the b-quark jet or the neutrino, i.e. there will be no azimuthal correlation term. Returning to the spinor amplitude $\bar{u}(\ell)(1-\gamma_5)u(t)$ one notes that the combination $(1-\gamma_5)$ acts to project out the positive helicity spinor of the (massless) lepton. One can evaluate the amplitude $\bar{u}(\ell)\times(1-\gamma_5)u(t)$ for a top quark polarized in the (θ_P, ϕ) direction (see Fig. 1) using $u_+(t)^T = \sqrt{2m_t}(\cos\theta_P/2, e^{i\phi}\sin\theta_P/2, 0, 0)$ and $\bar{u}_+(\ell) = \sqrt{E_\ell}(1, 0, -1, 0)$ for a positive helicity lepton moving in the z direction. One obtains

$$\bar{u}_+(\ell)(1-\gamma_5)u_+(t) = 2\sqrt{2E_\ell m_t}\cos\frac{\theta_P}{2}. \tag{10}$$

On squaring the amplitude in Eq. (10) one finally obtains

$$|\bar{u}_+(\ell)(1-\gamma_5)u_+(t)|^2 = 4E_\ell m_t(1+\cos\theta_P). \tag{11}$$

An identical result is of course obtained by evaluating the trace

$$\sum_{s_b,s_\ell,s_\nu}|M|^2 = 4\,\mathrm{Tr}(\not{p}_b(1+\gamma_5)\not{p}_\nu(1-\gamma_5))\mathrm{Tr}(\not{p}_\ell(1-\gamma_5)\times(\not{p}_t - +\mathrm{m}_t)\frac{1}{2}(1+\gamma_5\not{s}_t)(1+\gamma_5)) = 128(p_b\cdot p_\nu)(\bar{p}_t\cdot p_\ell), \tag{12}$$

where we have used the abbreviation

$$\bar{p}_t^\mu = p_t^\mu - m_t s_t^\mu, \tag{13}$$

with s_t denoting the polarization four-vector of the top quark. The scalar products in Eq. (11) can be evaluated using explicit representations of the pertinent four-vectors in the rest frame of the top quark. From Fig. 1 one has ($x = 2E_\ell/m_t; y^2 = m_W^2/m_t^2$)

$$\begin{aligned} p_t &= m_t(1;0,0,0),\\ p_\ell &= \frac{m_t}{2}x(1;0,0,1),\\ p_\nu &= \frac{m_t}{2}(1-x+y^2)(1;-\sin\theta_\nu,0,\cos\theta_\nu),\\ p_b &= \frac{m_t}{2}(1-y^2)(1;\sin\theta_b,0,\cos\theta_b),\\ s_t &= (0;\vec{P}_t) = (0;\sin\theta_P\cos\phi,\sin\theta_P\sin\phi,\cos\theta_P),\end{aligned} \tag{14}$$

where

$$\cos\theta_\nu = \frac{x(1-x+y^2)-2y^2}{x(1-x+y^2)}, \qquad \cos\theta_b = \frac{2y^2-x(1+y^2)}{x(1-y^2)}. \tag{15}$$

For the spin summed squared matrix element we then obtain

$$\sum_{s_b,s_\ell,s_\nu}|M|^2 = 32m_t^4 x(1-x)(1+\cos\theta_P), \tag{16}$$

which, in the narrow width approximation for the W, leads to the partial rate formulas in Eq. (2).

The above derivation shows that the LO result $\Gamma\sim(1+\cos\theta_P)$ does not depend on the mass of the bottom quark. It does, however, depend on the mass of the lepton. The lepton mass effect can be easily calculated using the trace formula in Eq. (12). One obtains $|M|^2\sim 1+(1-\frac{1}{2}m_\ell^2/E_\ell^2+\ldots)\cos\theta_P$. The lepton mass correction is thus negligibly small since, in the narrow resonance approximation for the W^+, the minimal lepton energy is given by $E_\ell^{\min} = (m_W^4 + m_t^2 m_\ell^2)/(2m_t m_W^2)$ and is thus very much larger than the lepton mass appearing in the lepton mass correction.

Returning to the original current-current form (7) and its Fierz-transformed form (9) it is clear that there will be no azimuthal correlation, i.e. one has $\Gamma_C = 0$ at the Born term level. It is nevertheless instructive and interesting to go through the exercise to show that $\Gamma_C = 0$ directly follows from $\Gamma_A = \Gamma_B$ if the rate is to remain positive definite over all of phase space. We use a shorthand notation and write A for $d\Gamma_A/dx$ and B for $d\Gamma_B/dx$, etc. With $A = B$ the angular decay distribution is given by (we set $P = 1$)

$$\Gamma\sim A\Big(1+\cos\theta_P+\frac{C}{A}\sin\theta_P\cos\phi\Big). \tag{17}$$

From the structure of Eq. (17) one can immediately conclude that the ratio C/A necessarily has to vanish if the rate is to remain positive definite over all of angular phase space. This can be seen in the following way. Assume first that C/A is positive. Set $\cos\phi = -1$ and expand the resulting decay distribution around $\theta_P = \pi$ ($\theta_P \le \pi$). One obtains

$$\Gamma\sim A(\pi-\theta_P)\Big(\frac{\pi-\theta_P}{2}-\frac{C}{A}\Big). \tag{18}$$

For any given value of C/A the piece $(\pi-\theta_P)/2$ can always be chosen small enough to render the rate to become negative. If C/A is assumed to be negative one chooses $\cos\phi = +1$ and goes through the same steps of arguments as before. The upshot is that C has to be zero if one has $A = B$ in order for the rate to be positive definite everywhere. As mentioned before the explicit calculation using the form (7) or more directly (9) of course confirms this conclusion.

IV. QCD NLO CONTRIBUTION TO THE AZIMUTHAL CORRELATION FUNCTION Γ_C

The ingredients for the NLO calculation are the virtual one-loop contributions on the one hand, and the tree-graph (real emission) contributions on the other hand. Both of these have been calculated before (as e.g. in [19]) and we can make use of the previous results.

The virtual one-loop amplitudes are defined by covariant expansions [$J_\mu^V = \bar{\psi}(b)\gamma_\mu\psi(t)$, $J_\mu^A = \bar{\psi}(b)\gamma_\mu\gamma_5\psi(t)$]:

$$\langle b(p_b)|J_\mu^V|t(p_t)\rangle = \bar{u}(b)(\gamma_\mu F_1^V + p_{t,\mu}F_2^V + p_{b,\mu}F_3^V)u(t), \tag{19}$$

$$\langle b(p_b)|J^A_\mu|t(p_t)\rangle = \bar{u}(b)(\gamma_\mu F^A_1 + p_{t,\mu}F^A_2 + p_{b,\mu}F^A_3)\gamma_5 u(t). \tag{20}$$

The standard model current combination is given by $J^V_\mu - J^A_\mu$. At the one-loop level the form factors are [19] ($C_F = 4/3$)

$$F^V_1 = F^A_1 = 1 - \frac{\alpha_s}{4\pi}C_F\bigg[4 + \frac{1}{y^2}\ln(1-y^2) + 2\ln\bigg(\frac{\Lambda^2}{\epsilon}\frac{1}{1-y^2}\bigg) \times \ln\bigg(\frac{\epsilon}{1-y^2}\bigg) + \ln\bigg(\frac{\epsilon}{1-y^2}\frac{\Lambda^4}{(1-y^2)^2}\bigg) + 2\mathrm{Li}_2(y^2)\bigg], \tag{21}$$

$$F^V_2 = -F^A_2 = \frac{1}{m_t}\frac{\alpha_s}{4\pi}C_F\frac{2}{y^2}\bigg[+1 + \frac{1-y^2}{y^2}\ln(1-y^2)\bigg], \tag{22}$$

$$F^V_3 = -F^A_3 = \frac{1}{m_t}\frac{\alpha_s}{4\pi}C_F\frac{2}{y^2}\bigg[-1 + \frac{2y^2-1}{y^2}\ln(1-y^2)\bigg], \tag{23}$$

where a gluon mass regulator was used to regularize the gluon IR singularity. The scaled gluon mass and the scaled bottom quark mass are denoted by $\Lambda = m_g/m_t$ and $\epsilon = m_b/m_t$. As mentioned earlier on, the logarithmic terms in the gluon mass will not contribute to the azimuthal correlation function and can therefore be dropped. The dilog function $\mathrm{Li}_2(x)$ is defined by

$$\mathrm{Li}_2(x) := -\int_0^x \frac{\ln(1-z)}{z}dz. \tag{24}$$

The tree-graph contribution results from the square of the real gluon emission graphs. For the corresponding hadron tensor one obtains [19]

$$\begin{aligned}\mathcal{H}^{\mu\nu} = 4\pi\alpha_s C_F\frac{4}{(k\cdot p_t)(k\cdot p_b)}\bigg\{&-i\frac{k\cdot p_t}{k\cdot p_b}(\epsilon^{\alpha\beta\mu\nu}(p_b-k)\cdot\bar{p}_t - \epsilon^{\alpha\beta\gamma\nu}(p_b-k)^\mu\bar{p}_{t,\gamma} + \epsilon^{\alpha\beta\gamma\mu}(p_b-k)^\nu\bar{p}_{t,\gamma})k_\alpha p_{b,\beta}\\ &+\frac{k\cdot p_b}{k\cdot p_t}[(\bar{p}_t\cdot p_t)(k^\mu p^\nu_b + k^\nu p^\mu_b - k\cdot p_b g^{\mu\nu} - i\epsilon^{\alpha\beta\mu\nu}k_\alpha p_{b,\beta}) - (\bar{p}_t\cdot k)((p_t-k)^\mu p^\nu_b + (p_t-k)^\nu p^\mu_b\\ &-(p_t-k)\cdot p_b g^{\mu\nu} - i\epsilon^{\alpha\beta\mu\nu}(p_t-k)_\alpha p_{b,\beta})] - (\bar{p}_t\cdot p_b)(k^\mu p^\nu_b + k^\nu p^\mu_b - k\cdot p_b g^{\mu\nu} - i\epsilon^{\alpha\beta\mu\nu}k_\alpha p_{b,\beta})\\ &+(p_t\cdot p_b)(k^\mu\bar{p}^\nu_t + k^\nu\bar{p}^\mu_t - k\cdot\bar{p}_t g^{\mu\nu}) - (k\cdot p_b)(p^\mu_t\bar{p}^\nu_t + p^\nu_t\bar{p}^\mu_t - p_t\cdot\bar{p}_t g^{\mu\nu}) + (k\cdot p_t)((p_b+k)^\mu\bar{p}^\nu_t\\ &+(p_b+k)^\nu\bar{p}^\mu_t - (p_b+k)\cdot\bar{p}_t g^{\mu\nu}) + 2(k\cdot\bar{p}_t)p^\mu_b p^\nu_b + i(\epsilon^{\alpha\beta\mu\nu}(k\cdot\bar{p}_t) + \epsilon^{\alpha\beta\gamma\mu}k^\nu\bar{p}_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu}k^\mu\bar{p}_{t,\gamma})p_{b,\alpha}p_{t,\beta}\\ &+i(\epsilon^{\alpha\beta\mu\nu}(p_t\cdot\bar{p}_t) + \epsilon^{\alpha\beta\gamma\mu}p^\nu_t\bar{p}_{t,\gamma} - \epsilon^{\alpha\beta\gamma\nu}p^\mu_t\bar{p}_{t,\gamma})k_\alpha p_{b,\beta}\bigg\} + B^{\mu\nu}\cdot\Delta_{\mathrm{SGF}},\end{aligned} \tag{25}$$

where k is the gluon momentum. The abbreviation $\bar{p}^\mu_t$ is defined in (13). The hadron tensor has been written such that the IR singular part in the hadronic tensor has been isolated in the term $B^{\mu\nu}\cdot\Delta_{\mathrm{SGF}}$ where $B^{\mu\nu}$ is the Born term hadron tensor. The remaining pieces are IR finite. Again, when calculating the azimuthal correlation, the IR divergent term will not contribute and can thus be dropped. Its explicit form does not need to concern us here. It can be found in [19].

In the following we will concentrate on the azimuthal correlation function. For the fully differential azimuthal correlation function $d\Gamma_C/dxdz$ we find ($z = \frac{(p_b+k)^2}{m_t^2}$)

$$\frac{d\Gamma_C}{dxdz} = \Gamma_F 2\pi\frac{m_W}{\Gamma_W}C_F\bigg(-\frac{\alpha_s}{2\pi}\bigg)6y^2(M^C_t(x,z) + M^C_\ell(x)\delta(z)), \tag{26}$$

where $M^C_t(x,z)$ and $M^C_\ell(x)$ denote the tree-graph and the virtual one-loop contribution, respectively. The virtual one-loop contribution is multiplied by $\delta(z)$ since there is no gluon emission in the one-loop contribution and hence one has $z = 0$.

For the virtual one-loop contribution one finds

$$M^C_\ell(x) = -\sqrt{y^2(1-x)(x-y^2)}\bigg(\frac{1-x}{y^2}\bigg)\ln(1-y^2). \tag{27}$$

Integrating the one-loop contribution over the scaled lepton energy one obtains

$$\int_{y^2}^1 dx M^C_\ell(x) = -\frac{\pi}{16}\frac{(1-y^2)^3}{y}\ln(1-y^2). \tag{28}$$

The tree-graph contribution is rather more involved. One finds

$$M^C_t(x,z) = -\sqrt{y^2(1-x)(x-y^2) - xy^2z} \times\bigg[\frac{y^2}{x\lambda^3}j_1 + \frac{1}{\lambda^3}j_2 + \frac{x}{\lambda^3}j_3 + 4Y_p\bigg(\frac{6y^2z}{x\lambda^{7/2}}j_4 + \frac{1}{\lambda^{7/2}}j_5 + \frac{x}{\lambda^{7/2}}j_6\bigg)\bigg], \tag{29}$$

where

$$j_1 = (1-y^2)^4 + (1-y^2)^2(25+2y^2-y^4)z - 4(1-y^2)(11+y^2+y^4)z^2 + 2(4-8y^2-3y^4)z^3 + (11+4y^2)z^4 - z^5, \tag{30}$$

$$j_2 = -(1-y^2)^4(11+2y^2) + 2(1-y^2)^3(13-2y^2)z - 4(3+2y^2+3y^4)z^2 - 2(5-23y^2+2y^4)z^3 + (7+2y^2)z^4, \tag{31}$$

$$j_3 = 12(1-y^2)^4 - 2(1-y^2)^2(6+7y^2)z - 4(3-13y^2+2y^4)z^2 + 2(6+5y^2)z^3, \tag{32}$$

$$j_4 = -(1-y^2)^3 - y^2(1-y^2)z + 3z^2 - 2z^3, \tag{33}$$

$$j_5 = 2(1-y^2)^5 + 5y^2(1-y^2)^3z - (1-y^2)(11-23y^2 + 4y^4)z^2 + (13-19y^2+14y^4)z^3 - (3+10y^2)z^4 - z^5, \tag{34}$$

$$j_6 = -2(1-y^2)^5 - (1-y^2)^3(4-5y^2)z + (1-y^2) \times (12-11y^2-3y^4)z^2 - (4+15y^2-y^4)z^3 - (2+y^2)z^4, \tag{35}$$

with $\lambda := \lambda(1, y^2, z) = 1 + y^4 + z^2 - 2(y^2 + z + y^2 z)$ and $Y_p = \frac{1}{2}\ln\frac{1-y^2+z+\sqrt{\lambda}}{1-y^2+z-\sqrt{\lambda}}$.

In order to obtain the lepton energy spectrum $d\Gamma_C/dx$ the tree-graph contribution has to be integrated over z in the interval $0 \le z \le (1-x)(1-\frac{y^2}{x})$. We have not been able to do this integration in closed form so the integration was done numerically.

We show the x spectrum $d\Gamma_C/dx$ ($y^2 \le x \le 1$) in Fig. 2 by adding the tree-graph and virtual one-loop contributions $M_t^C(x)$ and $M_\ell^C(x)$. The azimuthal correlation function is small and negative over the whole spectrum and peaks at the lower end of the spectrum. The smallness of the azimuthal correlation function can be assessed by comparing the integrated azimuthal correlation function with the integrated unpolarized Born term rate as done in Eq. (39). Figure 2 also shows the spectrum of a possible right-chiral contribution to the $t \to b$ transition which will be discussed in the next section. Note, though, that the spectrum of the right-chiral contribution is positive.

As a last point we calculate the fully integrated NLO azimuthal correlation function Γ_C. It turns out that the full analytical integration of the tree-graph contribution can be done by reversing the order of integrations, i.e. by first integrating over x in the limits $w_- \le x \le w_+$ where

$$w_\pm = \frac{1}{2}\left(1 + y^2 - z \pm \sqrt{\lambda(1, y^2, z)}\right) \tag{36}$$

and then over z $[0 \le z \le (1-y)^2]$. We then obtain

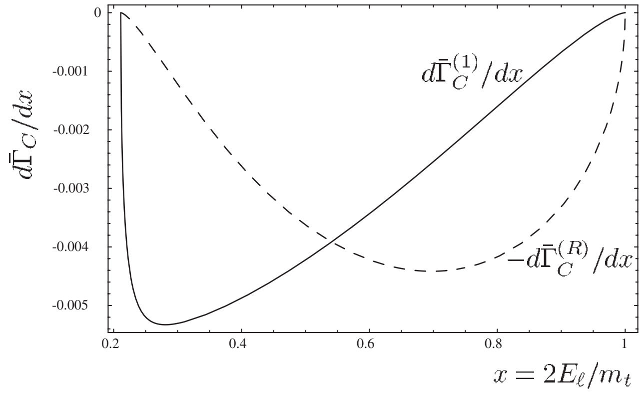

FIG. 2. Lepton energy dependence of the azimuthal correlation functions: the SM $O(\alpha_s)$ contribution $d\Gamma_C^{(1)}/dx$ (solid line) and the right-chiral contribution $d\Gamma_C^{(R)}/dx$ (dashed line). With the bar we denote that they are scaled to the LO unpolarized total rate $\Gamma_A^{(0)}$, i.e. $d\bar{\Gamma}_C/dx = (\Gamma_A^{(0)})^{-1} d\Gamma_C/dx$.

$$\int_0^{(1-y)^2} dz \int_{w_-}^{w_+} dx M_t^C(x,z) = \frac{\pi}{16}\Big\{4y(4+3y^2-3y^4)\mathrm{Li}_2(-y) - 2(1-y^2)(8-7y+8y^2-5y^3)\ln(1+y) + \frac{1}{3}y[6(1-y)^2(1-y-2y^2) + \pi^2(4+3y^2-3y^4)]\Big\}. \tag{37}$$

Finally we add up (28) and (37) to obtain the NLO fully integrated azimuthal correlation function Γ_C. We find

$$\Gamma_C^{(1)} = \Gamma_F 2\pi \frac{m_W}{\Gamma_W} C_F\left(-\frac{\alpha_s}{2\pi}\right)\frac{3}{8}\pi y^2\Big\{4y(4+3y^2-3y^4) \times \mathrm{Li}_2(-y) - 2(1-y^2)(8-7y+8y^2-5y^3) \times \ln(1+y) - \frac{(1-y^2)^3}{y}\ln(1-y^2) + \frac{1}{3}y[6(1-y)^2(1-y-2y^2) + \pi^2(4+3y^2-3y^4)]\Big\}. \tag{38}$$

In the last step we combine our results for the azimuthal correlation rate with the results for the unpolarized rate and the polar correlation rate from [17]. Numerically we obtain $[\Gamma^{\rm NLO} = \Gamma^{(0)} + \Gamma^{(1)};\ \alpha_s(m_t^2) = 0.107,\ y^2 = 0.211]$

$$\frac{d\Gamma^{\rm NLO}}{d\cos\theta_P d\phi} = \frac{\Gamma_A^{(0)}}{4\pi}[(1-8.54\%) + (1-8.71\%)P\cos\theta_P - 0.24\% P\sin\theta_P\cos\phi] \tag{39}$$

$$= \frac{\Gamma_A^{\rm NLO}}{4\pi}[1 + 0.998P\cos\theta_P - 0.0026P\sin\theta_P\cos\phi]. \tag{40}$$

The radiative corrections to the rate Γ_A and the polar correlation function Γ_B go in the same direction and are very close in magnitude. The polar analyzing power therefore remains largely unchanged by the radiative corrections as (40) shows (100% → 99.8%). The azimuthal correlation generated by the radiative corrections is quite small. It is safe to say that, if top quark decays reveal a violation of the SM $(V - A)$ current structure in the azimuthal correlation function which exceeds the 1% level, the violation must have a non-SM origin.

As discussed in Sec. III for the Born term contribution, the positivity of the rate is an issue. We find that the NLO numerical rate values also satisfy positivity. Note that the positivity is not automatic in NLO calculations. Although the NLO tree-graph contribution is positive definite the one-loop contribution is not necessarily positive since it involves an interference with the Born term amplitude. To prove positivity we use a standard trigonometric identity to rewrite the NLO result (39) as (we set $P = 1$)

$$\frac{\Gamma_A^{(0)}}{4\pi}\left[\left(1 + \frac{\Gamma_A^{(1)}}{\Gamma_A^{(0)}}\right) + \left(1 + \frac{\Gamma_B^{(1)}}{\Gamma_A^{(0)}}\right)\cos\theta_P + \frac{\Gamma_C^{(1)}}{\Gamma_A^{(0)}}\sin\theta_P\cos\phi\right]$$
$$= \frac{\Gamma_A^{(0)}}{4\pi}\left[\left(1 + \frac{\Gamma_A^{(1)}}{\Gamma_A^{(0)}}\right) + \sqrt{\left(1 + \frac{\Gamma_B^{(1)}}{\Gamma_A^{(0)}}\right)^2 + \left(\frac{\Gamma_C^{(1)}}{\Gamma_A^{(0)}}\cos\phi\right)^2} \times \sin(\theta_P + \delta)\right], \tag{41}$$

where

$$\tan\delta = \frac{\Gamma_A^{(0)} + \Gamma_B^{(1)}}{\Gamma_C^{(1)}\cos\phi}. \tag{42}$$

For $\sin(\theta_P + \delta) = -1$ and $\cos\phi = \pm 1$ the rate becomes minimal. With the numbers in Eq. (39) one can check that the minimal value of the rate is positive.

V. NON-SM RIGHT-CHIRAL QUARK CURRENT

In order to be able to assess the size of the NLO contribution to the azimuthal correlation we add a right-chiral piece to the quark current

$$\bar{\psi}(b)[\gamma^\mu(1 - \gamma_5)]\psi(t) \to \bar{\psi}(b)[\gamma^\mu(1 - \gamma_5) + \delta_R\gamma^\mu(1 + \gamma_5)]\psi(t) \tag{43}$$

where δ_R parametrizes the strength of the right-chiral contribution. From the discussion in Sec. III we anticipate that the right-chiral quark current will generate a nonvanishing azimuthal correlation. The current-current matrix element involving the new right-chiral quark current will then read

$$M = \delta_R\bar{u}(b)\gamma^\mu(1 + \gamma_5)u(t)\bar{u}(\nu)\gamma_\mu(1 - \gamma_5)v(\ell) \tag{44}$$

$$= 2\delta_R\bar{u}(b)(1 - \gamma_5)v(\ell)\bar{u}(\nu)(1 + \gamma_5)u(t) \tag{45}$$

where we have used a Fierz identity of the first kind to simplify the matrix element (44).

There are some indirect model dependent constraints on the strength of the right-chiral quark $\delta_R \leq 0.004$ from an analysis of the rare decay $b \to s\gamma$ [20–22]. In this paper we take a phenomenological point of view and leave the size of δ_R unconstrained.

To start with we assume that $m_b = 0$ or more generally $\delta_R \gg m_b/m_t$. For $m_b = 0$ there will be no interference contribution from the interference of the left- and right-chiral quark currents when squaring the full matrix element. The case $\delta_R \simeq m_b/m_t$ will be discussed at the end of this section. Using the form (44) it is not difficult to obtain the square of the right-chiral matrix element. One has

$$\sum_{s_b,s_\ell,s_\nu}|M|^2 = 4\delta_R^2\,{\rm Tr}\Big(\not{p}_\nu(1 + \gamma_5)(\not{p}_t + {\rm m}_t)\frac{1}{2}(1 + \gamma_5\not{s}_t) \times (1 - \gamma_5)\Big){\rm Tr}(\not{p}_b(1 - \gamma_5)\not{p}_\ell(1 + \gamma_5))$$
$$= 128\delta_R^2(p_\nu\cdot p_t + m_t p_\nu\cdot s_t)(p_b\cdot p_\ell). \tag{46}$$

The scalar products in Eq. (46) can again be evaluated using the explicit representations of the pertinent four-vectors given in Eq. (14). Using again the narrow resonance approximation one has ($x = 2E_\ell/m_t$; $y^2 = m_W^2/m_t^2$)

$$\frac{d\Gamma_A^{(R)}}{dx} = \delta_R^2\Gamma_F 2\pi\frac{m_W}{\Gamma_W}6y^2(x - y^2)(1 - x + y^2), \tag{47}$$

$$\frac{d\Gamma_B^{(R)}}{dx} = \delta_R^2\Gamma_F 2\pi\frac{m_W}{\Gamma_W}\frac{6y^2(x - y^2)}{x}(2y^2 - x(1 + y^2 - x)), \tag{48}$$

$$\frac{d\Gamma_C^{(R)}}{dx} = \delta_R^2\Gamma_F 2\pi\frac{m_W}{\Gamma_W}\frac{12y^2(x - y^2)}{x}\sqrt{y^2(1 - x)(x - y^2)}. \tag{49}$$

In Fig. 2 we show a plot of the spectrum of the azimuthal part of the right-chiral contribution where we have fixed $\delta_R = 0.051$ from arbitrarily setting $|\Gamma_C^{(R)}| = |\Gamma_C^{(1)}|$, i.e. the two spectra in Fig. 2 have the same area. One notes that, besides having a different sign, the x dependence of the right-chiral contribution is harder than that of the standard model. If the x dependence can be measured it should not be difficult to differentiate between the two cases.

For the integrated rates we obtain ($y^2 \leq x \leq 1$)

$$\Gamma_A^{(R)} = \delta_R^2\Gamma_F 2\pi\frac{m_W}{\Gamma_W}y^2(1 - 3y^4 + 2y^6), \tag{50}$$

$$\Gamma_B^{(R)} = \delta_R^2 \Gamma_F 2\pi \frac{m_W}{\Gamma_W} y^2(-1 + 12y^2 - 9y^4 - 2y^6 + 12y^4 \ln y^2), \tag{51}$$

$$\Gamma_C^{(R)} = \delta_R^2 \Gamma_F 2\pi \frac{m_W}{\Gamma_W} \frac{3}{2} \pi y^3(1 - 6y^2 + 8y^3 - 3y^4). \tag{52}$$

Of course, for small values of δ_R, i.e. when $\delta_R \simeq m_b/m_t$, the interference between SM and non-SM-type contributions cannot be neglected. If one takes a (one-loop) running b-quark mass of $m_b(m_t) = 1.79$ GeV and $m_t = 175$ GeV this would correspond to $\delta_R \simeq m_b/m_t = 0.0102$. If one takes a pole mass of $m_b = 4.8$ GeV this would correspond to $\delta_R \simeq m_b/m_t = 0.027$. The contribution of the interference term to the differential rate is

$$\frac{d\Gamma^{(\rm int)}}{dx} = -\delta_R \frac{m_b}{m_t} \Gamma_F 2\pi \frac{m_W}{\Gamma_W} 12y^2 \Big[y^2(1 + P\cos\theta_P) + \sqrt{y^2(1-x)(x-y^2)} P \sin\theta_P \cos\phi \Big]. \tag{53}$$

Finally, integrating over x one obtains

$$\Gamma^{(\rm int)} = -\delta_R \frac{m_b}{m_t} \Gamma_F 2\pi \frac{m_W}{\Gamma_W} \frac{3}{2} y^3(1 - y^2)[8y(1 + P\cos\theta_P) + \pi(1 - y^2) P \sin\theta_P \cos\phi]. \tag{54}$$

It is curious to note that for $\delta_R \simeq m_b/m_t$ the integrated interference and right-chiral contributions to Γ_C tend to cancel each other, cf.

$$\Gamma_C^{(R)} + \Gamma_C^{(\rm int)} = \Gamma_A^{(0)} \frac{3}{2} \pi \delta_R \Big(0.20\delta_R - 0.32 \frac{m_b}{m_t} \Big). \tag{55}$$

If one takes $\delta_R \leq 0.004$ as suggested by the analysis of the rare decay $b \to s\gamma$ [20–22] one finds $|\Gamma_C^{(R)} + \Gamma_C^{(\rm int)}| \leq 4.7 \times 10^{-5} \Gamma_A^{(0)}$ using $m_b/m_t = 0.0102$. This is far below the SM value $|\Gamma_C^{(1)}| = 2.4 \times 10^{-3} \Gamma_A^{(0)}$ that we have obtained in Eq. (39).

VI. SUMMARY AND CONCLUSIONS

We have calculated the $O(\alpha_s)$ corrections to an azimuthal correlation observable in polarized top quark decay which vanishes at the Born term level. We have found that the $O(\alpha_s)$ corrections to this particular azimuthal correlation are quite small. If top quark decays reveal a violation of the SM $(V - A)$ current structure in the azimuthal correlation function which exceeds the 1% level, the violation must have a non-SM origin.

We have used the helicity system for our analysis where the event plane lies in the (x, z) plane and the lepton momentum is along the z axis. Other helicity systems, where the z axis is defined by the neutrino or the bottom quark jet, provide independent probes of the polarized top quark decay dynamics. The Born term angular correlations in these two additional helicity systems were studied in [13]. The $O(\alpha_s)$ radiative corrections to the angular correlations in these helicity systems will be the subject of a future publication.

[1] S. Groote, J. G. Körner, and M. M. Tung, Z. Phys. C **70**, 281 (1996).
[2] H. S. Do, S. Groote, J. G. Körner, and M. C. Mauser, Phys. Rev. D **67**, 091501(R) (2003).
[3] G. Mahlon and S. J. Parke, Phys. Rev. D **55**, 7249 (1997).
[4] J. H. Kühn, Nucl. Phys. **B237**, 77 (1984).
[5] J. H. Kühn, A. Reiter, and P. M. Zerwas, Nucl. Phys. **B272**, 560 (1986).
[6] J. G. Körner, A. Pilaftsis, and M. M. Tung, Z. Phys. C **63**, 575 (1994).
[7] S. Groote and J. G. Körner, Z. Phys. C **72**, 255 (1996).
[8] S. Groote, J. G. Körner, and M. M. Tung, Z. Phys. C **74**, 615 (1997).
[9] S. J. Parke and Y. Shadmi, Phys. Lett. B **387**, 199 (1996).
[10] A. Brandenburg, M. Flesch, and P. Uwer, Phys. Rev. D **59**, 014001 (1998).
[11] A. Abulencia *et al.* (CDF II Collaboration), Phys. Rev. D **75**, 052001 (2007).
[12] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. D **75**, 031102 (2007).
[13] J. G. Körner and D. Pirjol, Phys. Rev. D **60**, 014021 (1999).
[14] M. Jezabek and J. H. Kühn, Nucl. Phys. **B314**, 1 (1989).
[15] A. Czarnecki, M. Jezabek, and J. H. Kühn, Nucl. Phys. **B351**, 70 (1991).
[16] A. Czarnecki, M. Jezabek, J. G. Körner, and J. H. Kühn, Phys. Rev. Lett. **73**, 384 (1994).
[17] A. Czarnecki and M. Jezabek, Nucl. Phys. **B427**, 3 (1994).
[18] H. Pietschmann, *Weak Interactions—Formulae, Results and Derivations* (Springer-Verlag, Berlin, 1983).
[19] M. Fischer, S. Groote, J. G. Körner, and M. C. Mauser, Phys. Rev. D **65**, 054036 (2002).
[20] K. Fujikawa and A. Yamada, Phys. Rev. D **49**, 5890 (1994).
[21] F. Larios, M. A. Perez, and C. P. Yuan, Phys. Lett. B **457**, 334 (1999).
[22] G. Burdman, M. C. Gonzalez-Garcia, and S. F. Novaes, Phys. Rev. D **61**, 114016 (2000).