F and *D* values with explicit flavor symmetry breaking and Δs contents of nucleons

T. Yamanishi

Department of Management Information Science, Fukui Institute of Technology, Gakuen, Fukui 910–8505, Japan

(Received 1 April 2007; published 18 July 2007)

We propose a new model for describing baryon semileptonic decays for estimating F and D values with explicit breaking effects of both SU(3) and SU(2) flavor symmetry, where all possible SU(3) and SU(2) breaking effects are induced from an effective interaction. An overall fit including the weak magnetism form factor yields $F = 0.477 \pm 0.001$, $D = 0.835 \pm 0.001$, $V_{ud} = 0.975 \pm 0.002$, and $V_{us} = 0.221 \pm 0.002$ with $\chi^2 = 4.43/5$ d.o.f.. The spin content of strange quarks Δs is estimated from the obtained values F and D, and the nucleon spin problem is reexamined. Furthermore, the unmeasured values of (g_1/f_1) and (g_1) for other hyperon semileptonic decays are predicted from this new formula.

DOI: 10.1103/PhysRevD.76.014006 PACS numbers: 13.30.Ce, 11.30.Hv, 12.15.-y, 14.20.Dh

I. INTRODUCTION

Triggered by the measurement of the polarized structure function of proton $g_1^p(x)$ by EMC in 1987[1], the internal structure of a nucleon remains a challenging subject in nuclear and particle physics. Surprisingly, the measured $g_1^p(x)$ implies that only a small part of the nucleon spin is carried by quarks, and the polarization of the strange quark is negative and quite large. At the leading order of QCD, the amount of the strange quark carrying the nucleon spin $\Delta s(Q^2)(\equiv \Delta s_s + \Delta \bar{s}_s)$ is given by

$$\Delta s(Q^2) = 3\Gamma_1^p(Q^2) - \frac{1}{4}g_A - \frac{5}{12}(3F - D), \quad (1)$$

where F and D are Cabibbo parameters [2] and $\Gamma_1^p(Q^2)$ is the first moment of the proton structure function $g_1^p(x)$. g_A is the ratio of axial-vector to vector coupling constants (equivalent to the axial-vector to vector form factor g_1/f_1) of the neutron β decay. The values of F and D can be uniquely determined by the ratio of the axial-vector to vector form factor g_1/f_1 of various baryon semileptonic decays. The EMC value of $g_1^p(x)$ leads to

$$\Delta s(Q^2 = 10.7\ \text{GeV}^2) = -0.190 \pm 0.032 \pm 0.046, \quad (2)$$

using F and D values with the assumption of SU(3) flavor symmetry. This result is not anticipated in conventional quark models, and is often referred to as *the proton spin crisis* [3]. However, note that the SU(3) flavor symmetry is not a good description because of the rather large mass difference between strange and nonstrange quarks. Furthermore, recent data on the longitudinally polarized semi-inclusive deep inelastic scattering [4] suggest the asymmetry between $\Delta\bar{u}(x)$ and $\Delta\bar{d}(x)$; that is, even SU(2) flavor is broken.

Now, significant deviations between high precision data on various baryon semileptonic decays and g_1/f_1 expressed from F and D under SU(3) flavor symmetry are seen. Accordingly, it is important to reconsider the result of Eq. (2) using F and D values without SU(3) and SU(2) flavor symmetry.

There have been several attempts to estimate F and D values by taking the SU(3) or SU(2) flavor breaking into account [5]. For the SU(3) flavor, the breaking effects have been considered from the effective Hamiltonian formalism [6,7], possible SU(3) breaking with hypercharge matrix λ_8 [8], the $1/N_c$ expansion [9,10], the baryon mass differences [11], and the center-of-mass corrections [12]. In the case of SU(2), F and D values have been estimated by considering the Λ^0-Σ^0 mixing induced from the isospin violation [13]. Although these analyses are interesting in themselves, they are not yet general and are insufficient to constrain the parameter of polarized quark distribution functions describing quark spin contents. In this paper, to estimate more reliable F and D values, we propose a new model of baryon semileptonic decays that incorporates both SU(3) and SU(2) flavor symmetry breaking generally, and we attempt to derive the polarized strange-quark content in the proton from those F and D values.

In the next section, we derive the most general formulas for F and D values without SU(3) and SU(2) flavor symmetry. The χ^2 analysis is carried out in Sec. III. Using the obtained F and D values, we estimate the first moment of polarized strange quarks. Section IV provides the summary and discussion.

II. FORM FACTORS WITH BOTH SU(3) AND SU(2) FLAVOR SYMMETRY BREAKING

The matrix element for baryon semileptonic decay $A \rightarrow B + \ell + \bar{\nu}$ is given by

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\langle B|J_h^\mu|A\rangle \bar{u}_\ell(p_\ell)\gamma_\mu(1+\gamma_5)u_\nu(p_\nu), \quad (3)$$

where G_F is the universal weak coupling constant, and p_ℓ and p_ν the four-momenta of the lepton and antineutrino, respectively. The hadronic current is written as [14]

$$\langle B|J_h^\mu|A\rangle = C\bar{u}_B(p_B)\bigg\{f_1(q^2)\gamma^\mu + i\frac{f_2(q^2)}{M}\sigma^{\mu\nu}q_\nu + \frac{f_3(q^2)}{M}q^\mu + g_1(q^2)\gamma^\mu\gamma_5 + i\frac{g_2(q^2)}{M}\sigma^{\mu\nu}q_\nu\gamma_5 + \frac{g_3(q^2)}{M}q^\mu\gamma_5\bigg\}u_A(p_A), \quad (4)$$

where C is the mixing parameter V_{ud} or V_{us} with the Cabibbo-Kobayashi-Maskawa (CKM) matrix element for $|\Delta S| = 0$ or 1 transitions, respectively. p_A and p_B are the four-momenta of the initial and final baryons, and M the mass of the initial baryon. q is the momentum transfer given by $q = p_A - p_B$. The functions $f_i(q^2)$ and $g_i(q^2)$ with $i = 1$, 2, and 3 are the vector and axial-vector current form factors, respectively, and include all information on hadron dynamics. In the literature, f_1, f_2, and f_3 are called the vector, the induced tensor or weak magnetism, and the induced scalar form factors, respectively, and g_1, g_2, and g_3 are the axial-vector, the induced pseudotensor or weak-electricity, and the induced pseudoscalar form factors, respectively. In Weinberg's classification, f_3 and g_2 are associated with second-class currents, whereas the others are first-class currents [15].

Now, we consider the Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + h', \quad (5)$$

describing mass-splitting interactions, where $\mathcal{H}_0$ is SU(3) symmetric, and h' is responsible for the mass differences among particles in baryon or meson SU(3) multiplets [16,17]. Assuming that $\mathcal{H}$ is invariant under charge conjugation, and that the breaking term h' is originated from the third and eighth components of an octet, we can expand the weak current to the first order in h', as [18]

$$a_0\mathrm{Tr}(\bar{B}B\lambda_i) + b_0\mathrm{Tr}(\bar{B}\lambda_i B) + a\mathrm{Tr}(\bar{B}B\{\lambda_i, (\alpha\lambda_3 + \beta\lambda_8)\}) + b\mathrm{Tr}(\bar{B}\{\lambda_i, (\alpha\lambda_3 + \beta\lambda_8)\}B) + c[\mathrm{Tr}(\bar{B}\lambda_i B(\alpha\lambda_3 + \beta\lambda_8)) - \mathrm{Tr}(\bar{B}(\alpha\lambda_3 + \beta\lambda_8)B\lambda_i)] + g\mathrm{Tr}(\bar{B}B)\mathrm{Tr}(\lambda_i(\alpha\lambda_3 + \beta\lambda_8)) + h[\mathrm{Tr}(\bar{B}\lambda_i)\mathrm{Tr}(B(\alpha\lambda_3 + \beta\lambda_8)) + \mathrm{Tr}(\bar{B}(\alpha\lambda_3 + \beta\lambda_8)) \times\mathrm{Tr}(B\lambda_i)], \quad (6)$$

for the first-class covariants of γ_μ, $\sigma_{\mu\nu}q_\nu$, $\gamma_\mu\gamma_5$, and $\gamma_5 q_\mu$. Here, i gives the ith component of the weak current, and α and β are taken as parameters for SU(2) and SU(3) symmetry breaking effects, respectively. $a_0, b_0, \cdots, h$ are the first-class amplitudes. In particular, at zero, four-momentum transfer $q^2 \to 0$, Eq. (6) is reduced to its first two terms, where $a_0 = D - F$ and $b_0 = D + F$ with F and D in the SU(3) symmetry limit. Note that the SU(2) symmetry is realized only when $\alpha = 0$. B in Eq. (6) is the matrix representing the baryon octet:

$$B = \begin{bmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{bmatrix}. \quad (7)$$

A. The vector and axial-vector form factors f_1 and g_1

The Ademollo-Gatto theorem guarantees that the vector form factor f_1 is not modified in the first order in the symmetry breaking, but occurs in the second order [18]. In contrast, the axial-vector form factor g_1 is affected by the first-order symmetry breaking. In the present analysis, we only consider the corrections from the first-order symmetry breaking of SU(2) and SU(3). After some algebraic calculation with Eq. (6), we obtain the ratios of the axial-vector to vector form factors g_1/f_1 as

$$(g_1/f_1)_{n\to p} = F + D + 2\beta(b - c), \quad (8a)$$

$$(g_1/f_1)_{\Lambda^0\to p} = F + D/3 - (\alpha - \beta)(a - 2b)/3 + (\alpha - 3\beta)c/3 - 2\beta h/3, \quad (8b)$$

$$(g_1/f_1)_{\Sigma^-\to n} = F - D - (\alpha - \beta)(a + c), \quad (8c)$$

$$(g_1/f_1)_{\Xi^-\to\Lambda^0} = F - D/3 - (\alpha - \beta)(2a - b)/3 + (\alpha - 3\beta)c/3 + 2\beta h/3, \quad (8d)$$

$$(g_1/f_1)_{\Xi^0\to\Sigma^+} = F + D + (\alpha - \beta)(b - c), \quad (8e)$$

$$(g_1)_{\Sigma^-\to\Lambda^0} = \sqrt{\tfrac{2}{3}}\{D + \beta(a + b) + \alpha c + 3\beta h\}, \quad (8f)$$

$$(g_1/f_1)_{\Xi^-\to\Xi^0} = F - D - 2\beta(a + c), \quad (8g)$$

$$(g_1/f_1)_{\Xi^-\to\Sigma^0} = F + D + (\alpha - \beta)b + (\alpha + \beta)c + 2\alpha h, \quad (8h)$$

$$(g_1)_{\Sigma^+\to\Lambda^0} = \sqrt{\tfrac{2}{3}}\{D + \beta(a + b) - \alpha c + 3\beta h\}, \quad (8i)$$

with the first-class amplitudes a, b, c, h of the correction terms and α, β describing the contributions of SU(2) and SU(3) symmetry breaking. Since the vector form factor is absent in $\Sigma^- \to \Lambda^0$ and $\Sigma^+ \to \Lambda^0$, we present the expression for the axial-vector form factor. In the absence of the breaking parameters α and β for SU(2) and SU(3) symmetries, i.e., the SU(3) limit, we see that Eqs. (8a)–(8i) are reduced to the formulas given by the Cabibbo model.

B. The weak magnetism form factor f_2 and others including f_3, g_2, and g_3

The weak magnetism form factor f_2 at $q^2 = 0$ for each baryon semileptonic decay is obtained in terms of the proton and neutron anomalous magnetic moments in the Cabibbo model with the SU(3) limit [14]. When the SU(3)/SU(2) symmetry is broken, f_2 would be corrected by the symmetry breaking effects. Although the electron spectrum in baryon semileptonic decays is sensitive to the magnitude of f_2/f_1 [19,20], the measurement of f_2 has not yet been carried out experimentally with sufficient preci-

TABLE I. f_2/f_1 values for three cases, SU(3) symmetry, SU(3) breaking, and both SU(3) and SU(2) breaking, are listed. For $\Sigma^- \to \Lambda^0 \ell \bar{\nu}$ and $\Sigma^+ \to \Lambda^0 \bar{\ell} \nu$ processes, the values are given for f_2 since $f_1 = 0$, even though the first-order symmetry breaking is taken into account. The μ_{Σ^0} of SU(3) breaking and both SU(3) and SU(2) breaking for $\Xi^- \to \Sigma^0 \ell \bar{\nu}$ is taken as $\mu_{\Sigma^0} = (\mu_{\Sigma^+} + \mu_{\Sigma^-})/2$.

	SU(3) symmetry case	SU(3) breaking case	SU(3) and SU(2) breaking case
$n \to p \ell \bar{\nu}$	$\frac{1}{2}(\mu_p - \mu_n) = 1.853$	$\frac{1}{2}\frac{M_N}{M_N}(\mu_p - \mu_n) = 1.853$	$\frac{1}{2}\frac{M_n}{M_p}(\mu_p - \mu_n) = 1.855$
$\Lambda^0 \to p \ell \bar{\nu}$	$\frac{1}{2}\mu_p = 0.896$	$\frac{1}{2}\frac{M_\Lambda}{M_N}\mu_p = 1.065$	$\frac{1}{2}\frac{M_\Lambda}{M_p}\mu_p = 1.066$
$\Sigma^- \to n \ell \bar{\nu}$	$\frac{1}{2}(\mu_p + 2\mu_n) = -1.017$	$\frac{1}{2}(\mu_n - \mu_{\Sigma^-}) = -0.877$	$\frac{1}{2}(\mu_n - \mu_{\Sigma^-}) = -0.877$
$\Xi^- \to \Lambda^0 \ell \bar{\nu}$	$-\frac{1}{2}(\mu_p + \mu_n) = 0.060$	$-\frac{1}{2}\mu_{\Xi^-} = 0.175$	$-\frac{1}{2}\mu_{\Xi^-} = 0.175$
$\Xi^0 \to \Sigma^+ \ell \bar{\nu}$	$\frac{1}{2}(\mu_p - \mu_n) = 1.853$	$\frac{1}{2}(\mu_{\Sigma^+} - \mu_{\Xi^0}) = 1.354$	$\frac{1}{2}(\mu_{\Sigma^+} - \mu_{\Xi^0}) = 1.354$
$\Xi^- \to \Sigma^0 \ell \bar{\nu}$	$\frac{1}{2}(\mu_p - \mu_n) = 1.853$	$\frac{1}{2}(4\mu_{\Sigma^0} - \mu_{\Xi^-}) = 1.123$	$\frac{1}{2}(4\mu_{\Sigma^0} - \mu_{\Xi^-}) = 1.123$
$\Sigma^- \to \Lambda^0 \ell \bar{\nu}$	$-\sqrt{\frac{3}{2}}\frac{1}{2}\mu_n = 1.172$	$-\frac{\sqrt{6}}{2}\frac{M_\Sigma}{M_\Lambda}\mu_\Lambda = 0.803$	$-\frac{\sqrt{6}}{2}\frac{M_{\Sigma^-}}{M_\Lambda}\mu_\Lambda = 0.806$
$\Sigma^+ \to \Lambda^0 \bar{\ell} \nu$	$-\sqrt{\frac{3}{2}}\frac{1}{2}\mu_n = 1.172$	$-\frac{\sqrt{6}}{2}\frac{M_\Sigma}{M_\Lambda}\mu_\Lambda = 0.803$	$-\frac{\sqrt{6}}{2}\frac{M_{\Sigma^+}}{M_\Lambda}\mu_\Lambda = 0.800$

sion. Traditionally, the symmetry effect is expressed by multiplying the M/M_p factor by the SU(3) symmetry parameters represented by the Cabibbo model at $q^2 = 0$, where M and M_p are the mass of the decaying baryon A and the proton, respectively. However, we find a significant deviation in the model prediction for symmetry breaking $f_2/f_1 = \frac{1}{2}\frac{M_{\Sigma^-}}{M_p}(\mu_p + 2\mu_n) = -1.297$ from the experimental data -0.96 ± 0.15 for $\Sigma^- \to n \ell \bar{\nu}_\ell$ [21]. Here, we express f_2 for various decay processes in the symmetry breaking case in terms of the anomalous magnetic moments of relevant baryons instead of the proton μ_p and neutron μ_n anomalous magnetic moments. The expressions of f_2 are given in Table I for three cases: SU(3) symmetry, SU(3) symmetry breaking, and both SU(3) and SU(2) symmetry breaking. The f_2/f_1 for $\Sigma^- \to n \ell \bar{\nu}_\ell$ in the symmetry breaking case becomes -0.8765, and is consistent within the range of error with the experimental data.

The weak-electricity form factor g_2 vanishes at the SU(3) limit and V-spin invariance in the absence of second-class current. The induced scalar form factor f_3 also goes to zero in the SU(3) limit. In the real world the SU(3) symmetry is broken and therefore these form factors are expected to have some significant value. However, we ignore the g_2 terms because the corrections to g_2 in first-order symmetry breaking contribute to the decay amplitude in the second order [22,23]. Furthermore, since the contribution of f_3 to baryon semileptonic decay is suppressed by the mass of the emitted lepton, we can safely omit the term including f_3. Since the induced pseudoscalar form factor g_3 is also proportional to m_ℓ, this contribution is negligible as well.

III. NUMERICAL ANALYSES AND RESULTS

We determine the parameters of g_1/f_1 given in Eqs. (8a)–(8i) and the CKM matrix elements V_{ud} and V_{us} from χ^2 analysis with experimental data for ratios of the axial-vector to vector form factors and for differential semileptonic decay rates of baryons. Then, we obtain the values of the parameters for each case of symmetry and symmetry breaking—SU(3) symmetry, SU(3) symmetry breaking, and both SU(3) and SU(2) symmetry breaking. Table II shows the data used here [24]. Note that the KTeV group implied two values for g_1/f_1 of $\Xi^0 \to \Sigma^+ \ell \bar{\nu}$ [25]. One is the case of SU(3) symmetry, and its value is $1.32 \pm^{0.22}_{0.18}$. The other is the SU(3) breaking case: $1.17 \pm 0.28 \pm 0.05$. We adopt the former for analysis of the SU(3) symmetry case and the latter for symmetry breaking cases.

In fitting our expression for the decay rate to experimental data, we include not only the form factors f_1 and g_1 but

TABLE II. The baryon semileptonic decay data used in our analysis.

Decay	Rate (10^6 s^{-1}) $\ell = e$	$\ell = \mu$	g_1/f_1
$n \to p \ell \bar{\nu}$	1.129 ± 0.001[a]		1.2695 ± 0.0029
$\Lambda^0 \to p \ell \bar{\nu}$	3.16 ± 0.06	0.597 ± 0.133	0.718 ± 0.015
$\Sigma^- \to n \ell \bar{\nu}$	6.876 ± 0.236	3.0 ± 0.2	-0.340 ± 0.017
$\Xi^- \to \Lambda^0 \ell \bar{\nu}$	3.35 ± 0.37	$2.1 \pm^{2.1}_{1.3}$	0.25 ± 0.05
$\Xi^0 \to \Sigma^+ \ell \bar{\nu}$	0.93 ± 0.14		$1.32 \pm^{0.22}_{0.18}$ or $1.17 \pm 0.28 \pm 0.05$
$\Xi^- \to \Sigma^0 \ell \bar{\nu}$	0.53 ± 0.10		
$\Sigma^- \to \Lambda^0 \ell \bar{\nu}$	0.387 ± 0.018		
$\Sigma^+ \to \Lambda^0 \ell^+ \nu$	0.25 ± 0.06		

[a]Rate in 10^{-3} s^{-1}.

TABLE III. The radiative corrections to semileptonic decay rates of baryons in %. For $\Xi^0 \to \Sigma^+ e\bar{\nu}$ and $\Xi^- \to \Sigma^0 e\bar{\nu}$ processes, they are made the same value of $\Xi^- \to \Lambda^0 e\bar{\nu}$.

$n \to pe\bar{\nu}$	+7.3
$\Lambda^0 \to pe\bar{\nu}$	+3.9
$\Sigma^- \to ne\bar{\nu}$	+1.1
$\Xi^- \to \Lambda^0 e\bar{\nu}$	+1.3
$\Xi^0 \to \Sigma^+ e\bar{\nu}$	+1.3
$\Xi^- \to \Sigma^0 e\bar{\nu}$	+1.3
$\Sigma^- \to \Lambda^0 e\bar{\nu}$	+1.6
$\Sigma^+ \to \Lambda^0 \bar{e}\nu$	+1.6

also f_2. We also take into account the radiative corrections and the q^2 dependence of these form factors.

Radiative corrections play an important role in the precise determination of the CKM matrix, and hence should not be neglected in estimating the effects of SU(3) symmetry breaking [26]. There are various sets of radiative corrections to the semileptonic decay emitting an electron, but the differences in the differential cross section obtained using them varied only by a few percent [26]. Here, we adopt a set of radiative corrections for the hyperon semileptonic decays calculated by Tóth *et al.* [27] shown in Table III. The radiative corrections for $\Xi^- \to \Sigma^0 e\bar{\nu}$ and $\Xi^0 \to \Sigma^+ e\bar{\nu}$, which are not estimated by Ref. [27], are simply assumed to be the same as that for $\Xi^- \to \Lambda e\bar{\nu}$.

For the q^2 dependence we use the dipole form

$$f_i(q^2) = \frac{f_i(0)}{(1 - q^2/M_V^2)^2}, \qquad g_i(q^2) = \frac{g_i(0)}{(1 - q^2/M_A^2)^2}, \quad (9)$$

for $i = 1,\ 2$ with $M_V = 0.84(0.97)$ GeV and $M_A = 1.08(1.25)$ GeV at $|\Delta S| = 0(1)$ decays, respectively [20]. As the physical range of q^2 is given in $m_\ell^2 \le q^2 \le (M - M')^2$, and its value is smaller than $M_{V,A}^2$, we approximate the products of the form factors as follows:

$$f_i(q^2)f_j(q^2) = f_i(0)f_j(0)(1 + 4q^2/M_{V,A}^2). \quad (10)$$

The contributions of the term $4q^2/M_{V,A}^2$ in Eq. (10) to the differential cross section for f_2^2 and $f_1 f_2$ are an order of 10^{-2}–10^{-3} less than those for f_1^2 and g_1^2. Then, we ignore the term $4q^2/M_{V,A}^2$ of f_2^2 and $f_1 f_2$ in our analysis.

The χ^2 values for fitting are shown in Table IV, where we see that the χ^2 values of SU(3) symmetry for the decay rates of $\Xi^- \to \Lambda^0 e\bar{\nu}$ and $\Xi^- \to \Sigma^0 e\bar{\nu}$ are significantly improved by considering the symmetry breaking effects. Furthermore, we could say that not only SU(3) but also SU(2) symmetry breaking effects should be considered to reproduce the experimental data beyond the Cabibbo model with SU(3) symmetry since the χ^2/d.o.f. value for both SU(3) and SU(2) breaking is smaller than for SU(3) breaking alone.

Table V lists the optimal parameters for each case. In the present analysis, the parameters a, b, c, and h are smaller than F and D; thus a perturbative expansion with our effective interaction is reliable. From these results, we obtain $F/D = 0.599 \pm 0.006$ for the SU(3) symmetry, 0.648 ± 0.004 for the SU(3) breaking, and 0.572 ± 0.010 for both SU(3) and SU(2) breaking, respectively. The effects of both SU(3) and SU(2) symmetry breaking make F/D rather small. Figure 1 shows the F and D values calculated for SU(3) symmetry and for both SU(3) and SU(2) breaking, where the bands of allowed F and D

TABLE IV. The χ^2 values for three cases are listed.

		SU(3) symmetry case	SU(3) breaking case	SU(2) and SU(3) breaking case
g_1/f_1	$n \to p$	0.08	0.01	5.3×10^{-7}
	$\Lambda^0 \to p$	2.10	0.41	0.18
	$\Sigma^- \to n$	1.65	0.19	0.38
	$\Xi^- \to \Lambda^0$	0.62	0.04	0.21
	$\Xi^0 \to \Sigma^+$	0.08	0.21	0.38
Decay rate	$n \to pe\bar{\nu}$	5.9×10^{-3}	2.5×10^{-3}	2.4×10^{-7}
	$\Lambda^0 \to pe\bar{\nu}$	1.33	0.07	0.09
	$\Lambda^0 \to p\mu\bar{\nu}$	0.37	0.51	0.51
	$\Sigma^- \to ne\bar{\nu}$	1.01	0.92	0.92
	$\Sigma^- \to n\mu\bar{\nu}$	0.03	0.20	0.22
	$\Xi^- \to \Lambda^0 e\bar{\nu}$	9.22	1.24	0.55
	$\Xi^- \to \Lambda^0 \mu\bar{\nu}$	1.00	0.97	0.91
	$\Xi^0 \to \Sigma^+ e\bar{\nu}$	0.11	0.14	1.6×10^{-3}
	$\Xi^- \to \Sigma^0 e\bar{\nu}$	12.70	0.24	4.8×10^{-4}
	$\Sigma^- \to \Lambda^0 e\bar{\nu}$	0.15	0.84	0.01
	$\Sigma^+ \to \Lambda^0 e^+\nu$	0.08	0.12	0.05
	total χ^2	30.51	6.09	4.43
	χ^2/d.o.f	2.77	1.02	0.89

TABLE V. The parameter values for all three cases.

Parameters	SU(3) symmetry case	SU(3) breaking case	SU(3) and SU(2) breaking case
F	0.475 ± 0.004	0.499 ± 0.001	0.477 ± 0.007
D	0.793 ± 0.005	0.770 ± 0.004	0.835 ± 0.007
a	...	0.454 ± 0.213	0.099 ± 0.049
b	...	0.067 ± 0.049	0.072 ± 0.005
c	...	0.065 ± 0.055	0.043 ± 0.005
h	...	-0.099 ± 0.050	-0.031 ± 0.010
α	0 (putting)	0 (putting)	-0.949 ± 0.200
β	0 (putting)	-0.205 ± 0.105	-1.301 ± 0.211
V_{ud}	0.976 ± 0.002	0.976 ± 0.002	0.975 ± 0.002
V_{us}	0.222 ± 0.001	0.221 ± 0.002	0.221 ± 0.002

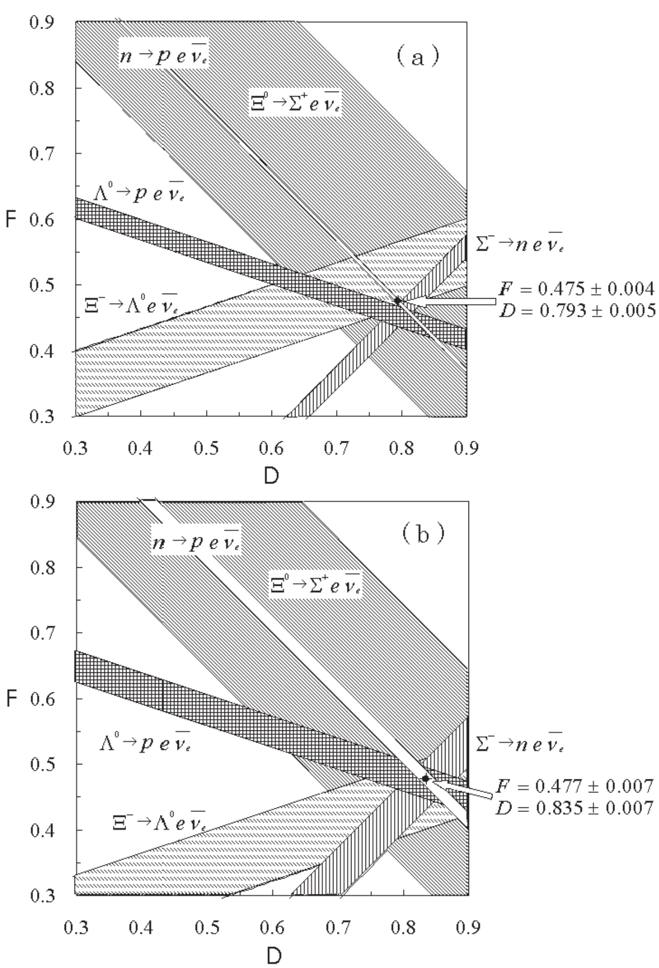

FIG. 1. The allowed area of F and D obtained from the χ^2 fit to the measurements of (g_1/f_1) for various baryon semileptonic decays is shown for two cases: (a) SU(3) symmetry and (b) both SU(3) and SU(2) breaking. The black circle in each figure indicates the optimal value of F and D for the fit. Each band indicates the experimental errors for SU(3) symmetry in (a). On the other hand, it is slightly modified by the breaking parameters in Eqs. (8a)–(8i) for both SU(3) and SU(2) breaking in (b).

values are deduced from (g_1/f_1) measurements. For the SU(3) symmetry case [Fig. 1(a)], the bands that present the experimental errors do not cross at a point. However, Fig. 1(b) shows that the bands completely corrected by the effects of both SU(3) and SU(2) symmetry breaking share a common overlapping region, and one can determine the F and D values in this overlapping region.

These results for χ^2 values (Fig. 1) imply the contribution of SU(2) flavor symmetry breaking on baryon semileptonic decays. However, should the SU(2) breaking effect for these processes actually be considered? We consider the $\Sigma^- \to \Lambda^0 \ell \bar{\nu}$ process because in that process only $|\Delta S| = 0$ interactions work, and one can easily test the presence or absence of SU(2) symmetry breaking effects for baryon semileptonic decays. Note that our Eqs. (8f) and (8i), only in the case of SU(3) symmetry breaking but SU(2) symmetry is kept, lead to a natural result that $(g_1)_{\Sigma^- \to \Lambda^0}$ and $(g_1)_{\Sigma^+ \to \Lambda^0}$ degenerate. Our prediction for the axial-vector form factor (g_1) at $\Sigma^- \to \Lambda^0 \ell \bar{\nu}$ becomes

$$\begin{aligned} g_1 \cos\theta_c &= 0.632 \pm 0.004 \quad \text{for the SU(3) symmetry case,} \\ &= 0.593 \pm 0.038 \quad \text{for the SU(3) breaking case,} \\ &= 0.586 \pm 0.040 \quad \text{for the SU(3)} \\ &\quad \text{and SU(2) breaking case,} \end{aligned}$$

with the Cabibbo angle $\cos\theta_c$ using each value of $V_{ud}(\equiv \cos\theta_c)$. This $(g_1)_{\Sigma^- \to \Lambda^0}$ was already measured in the WA2 experiment [28][1]:

$$g_1 \cos\theta_c = 0.572 \pm 0.016. \tag{11}$$

The case of both SU(3) and SU(2) breaking is in agreement with the data of Eq. (11) within the error boundary. Experimental data indicate the importance of corrections due to SU(2) breaking.

Using our results, one can predict the unmeasured ratios of axial-vector to vector form factors $(g_1/f_1)_{\Xi^- \to \Xi^0}$,

[1]The experimental result is not used to determine our parameters by χ^2 fitting, because it is not listed in the present particle data table.

$(g_1/f_1)_{\Xi^-\to\Sigma^0}$ and the axial-vector form factor $(g_1)_{\Sigma^+\to\Lambda^0}$ of the hyperon semileptonic decay. The $\Sigma^+ \to \Lambda^0\ell^+\nu$ process is interesting as well, because in that process only $|\Delta S| = 0$ interactions work the same as when $\Sigma^- \to \Lambda^0\ell\bar{\nu}$, and we can easily test whether the presence or absence of SU(2) symmetry breaking is effective. In the case of both SU(3) and SU(2) flavor symmetry breaking, we get

$$(g_1/f_1)_{\Xi^-\to\Xi^0} = -0.144 \pm 0.082,$$
$$(g_1/f_1)_{\Xi^-\to\Sigma^0} = 1.283 \pm 0.033,$$
$$(g_1)_{\Sigma^+\to\Lambda^0} = 0.668 \pm 0.041,$$

by using Eqs. (8g)–(8i), respectively. The measurement of unmeasured (g_1/f_1) and (g_1) is important to determine the magnitude of the symmetry breaking effect and test the validity of our model, which hopefully will be carried out in the near future.

Now, we attempt to estimate the amount of strange-quark content carrying the proton spin using our F and D values.

From the recent HERMES result $\Gamma_1^d(Q^2 = 5\ \mathrm{GeV}^2) = 0.0437 \pm 0.0035$ [29] and our F and D values, we have

$$\Delta s = \Delta s_s + \Delta\bar{s}_s = \frac{3\Gamma_1^d}{1 - \frac{3}{2}\omega_D} - \frac{5}{12}(3F - D) = -0.107 \pm 0.015, \quad (12)$$

for both SU(3) and SU(2) symmetry breaking with the D-state admixture to the deuteron wave function, $\omega_D = 0.05 \pm 0.01$ at the leading order of QCD, while

$$\Delta s = -0.122 \pm 0.013 \quad (13)$$

for SU(3) symmetry. Thus, the central value of Δs for symmetry breaking increases by about 12% compared with the symmetry case. Therefore, the flavor breaking effect contributes significantly to quark spin contents. This suggests that reanalysis of the proton spin structure using F and D values with full breaking effects is necessary for solving the proton spin crisis.

IV. SUMMARY AND DISCUSSION

We have proposed a new model of baryon semileptonic decays to derive F and D with both SU(3) and SU(2) flavor symmetry breaking effects. Numerical analysis using present experimental data determined that F and D values with both SU(3) and SU(2) symmetry breaking effects described the experimental data well, rather than those with the exact SU(3) symmetry case. The χ^2 fit leads to values of F/D in the case of both SU(3) and SU(2) breaking that becomes smaller by about 5% than that of the SU(3) symmetry case.

The central value for the amount of the strange quarks carrying the nucleon spin Δs for both the SU(3) and SU(2) breaking cases increases by about 12% compared with the one for the SU(3) symmetry case. Therefore, it is very important to reanalyze the polarized parton distribution functions using F and D values with the SU(3)/SU(2) symmetry breaking effect. The Japan Proton Accelerator Research Complex (J-PARC) is now under construction. It provides high intensity neutrino beams via charged pion decays from 50 GeV high intensity proton and antiproton beams. One expects that direct measurements of Δs for the nucleon scattering off the neutrino will be conducted at J-PARC with high precisions.

After completion of this work, flavor SU(3) breaking effects in quenched lattice QCD simulations by Sasaki *et al.* [30] were brought to my attention. They reached a similar result for the V_{us} value.

ACKNOWLEDGMENTS

I thank T. Morii and K. Ohkuma for enlightening comments and for careful reading of the manuscript. This work was supported by FUT Research Promotion Fund.

[1] J. Ashman *et al.* (EM Collaboration), Phys. Lett. B **206**, 364 (1988); Nucl. Phys. **B328**, 1 (1989).
[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
[3] For a review on the nucleon spin structure, see B. Lampe and E. Reya, Phys. Rep. **332**, 1 (2000).
[4] A. Airapetian *et al.* (HERMES Collaboration), Phys. Rev. Lett. **92**, 012005 (2004).
[5] For a recent review, see P. G. Ratcliffe, Czech. J. Phys. **54**, B11 (2004).
[6] P. G. Ratcliffe, in *Deep Inelastic Scattering off Polarized Targets: Theory Meets Experiment*, edited by J. Blümlein *et al.* (Zeuthen, Germany, 1997).
[7] P. G. Ratcliffe, Phys. Rev. D **59**, 014038 (1999).
[8] X. Song, P. K. Kabir, and J. S. McCarthy, Phys. Rev. D **54**, 2108 (1996).
[9] J. Dai, R. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D **53**, 273 (1996).
[10] R. Flores-Mendieta, E. Jenkins, and A. V. Manohar, Phys. Rev. D **58**, 094028 (1998).
[11] B. Ehrnsperger and A. Schäfer, Phys. Lett. B **348**, 619 (1995).
[12] J. F. Donoghue, B. R. Holstein, and S. W. Klimt, Phys. Rev. D **35**, 934 (1987).
[13] G. Karl, Phys. Lett. B **328**, 149 (1994); **341**, 449(E) (1995).
[14] P. Renton, *Electroweak Interactions—An Introduction to*

the Physics of Quarks & Leptons (Cambridge University Press, Cambridge, England, 1990).

[15] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

[16] N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. **149**, 1288 (1966).

[17] A. Sirlin, Nucl. Phys. **B161**, 301 (1979).

[18] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).

[19] A. García and P. Kielanowski, Lect. Notes Phys. **222**, 1 (1985).

[20] N. Cabibbo, E. C. Swallow, and R. Winston, Annu. Rev. Nucl. Part. Sci. **53**, 39 (2003).

[21] S. Y. Hsueh *et al.*, Phys. Rev. D **38**, 2056 (1988).

[22] A. García, R. Huerta, and P. Kielanowski, Phys. Rev. D **45**, 879 (1992).

[23] V. Mateu and A. Pich, J. High Energy Phys. 10 (2005) 41.

[24] A. B. Balantekin *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).

[25] A. Alavi-Harati *et al.* (KTeV Collaboration), Phys. Rev. Lett. **87**, 132001 (2001).

[26] M. Bourquin *et al.*, Z. Phys. C **21**, 27 (1983).

[27] K. Tóth, T. Margaritisz, and K. Szegõ, Acta Physiol. Acad. Sci. Hung. **55**, 481 (1984); T. Margaritisz, K. Szegõ, and K. Tóth, Report No. KFKI-1982-52.

[28] M. Bourquin *et al.*, Z. Phys. C **12**, 307 (1982).

[29] A. Airapetian *et al.* (HERMES Collaboration), arXiv:hep-ex/0609039.

[30] S. Sasaki and T. Yamazaki, arXiv:hep-lat/0610082.