

**Nonthermal leptogenesis with strongly hierarchical right-handed neutrinos**

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(Received 1 May 2007; published 25 July 2007)

Assuming the Dirac-type neutrino masses  $m_D$  are related to quark or charged lepton masses, neutrino oscillation data indicate that right-handed neutrino masses are in general strongly hierarchical. In particular, if  $m_D$  is similar to the up-type quark masses, the mass of the lightest right-handed neutrino  $M_1 \lesssim 10^6$  GeV. We show that nonthermal leptogenesis by inflaton decay can yield sufficient baryon asymmetry despite this constraint, and discuss how the asymmetry is correlated with the low energy neutrino masses and  $CP$ -violating phases.

DOI: [10.1103/PhysRevD.76.013005](https://doi.org/10.1103/PhysRevD.76.013005)

PACS numbers: 98.80.Cq, 12.60.Jv, 14.60.Pq, 14.60.St

**I. INTRODUCTION**

An attractive mechanism for generating the observed baryon asymmetry of the universe (BAU) is baryogenesis via leptogenesis [1]. In the seesaw model [2], the out-of-equilibrium decays of right-handed (RH) neutrinos to lepton and Higgs fields create lepton asymmetry, which is partially converted to baryon asymmetry by electroweak sphaleron processes [3].

The RH neutrinos can be generated thermally after inflation, if their masses are comparable to or below the reheat temperature  $T_r$ . The thermal leptogenesis scenario has the nice feature that the final asymmetry is independent of initial conditions and inflaton couplings. However, it requires  $T_r \gtrsim 10^9$  GeV to generate the BAU [4,5], which is problematic in supersymmetric (SUSY) models due to the gravitino constraint [6]. Nonthermal leptogenesis by

inflaton decay is an alternative scenario that can work with lower values of  $T_r$  ( $\gtrsim 10^6$  GeV) [7–9]. These bounds can be saturated with  $M_1 \sim T_r$  and  $M_1 \gtrsim T_r$  for the thermal and nonthermal scenarios, respectively, where  $M_1$  is the lightest RH neutrino mass.

The seesaw relation

$$m = m_D M^{-1} m_D^T, \quad (1)$$

where  $m_D$  is the Dirac-type neutrino mass matrix, relates the RH neutrino mass matrix  $M$  to the low energy neutrino mass matrix  $m$ , given in the basis where the charged lepton mass matrix and gauge interactions are diagonal by

$$m = U_{\text{PMNS}}^* d_\nu U_{\text{PMNS}}^\dagger. \quad (2)$$

Here  $d_\nu \equiv \text{diag}(m_1, m_2, m_3)$ , and  $U_{\text{PMNS}}$  [10] is the leptonic mixing matrix

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot K_0, \quad (3)$$

$c_{ij} \equiv \cos\theta_{ij}$ ,  $s_{ij} \equiv \sin\theta_{ij}$ ,  $\delta$  is the  $CP$ -violating Dirac phase, and  $K_0 = \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$  contains the two  $CP$ -violating Majorana phases.

In Refs. [11–15], thermal leptogenesis was analyzed with the assumption that  $m_D$  is related to the mass matrices of quarks or charged leptons, as typically realized in grand unified theories. In this case the Dirac masses are hierarchical, and the Dirac left-handed rotation in the basis where the charged lepton mass matrix is diagonal (the leptonic analogue of  $U_{\text{CKM}}$ ) is expected to be nearly diagonal or similar to  $U_{\text{CKM}}$ . We will hereafter refer to these two assumptions as quark-lepton symmetry.

Hierarchical Dirac masses indicate strongly hierarchical RH neutrino masses [13,16], and the resulting BAU is suppressed due to the low value of  $M_1$ . In particular,  $M_1 \lesssim$

$10^6$  GeV if  $m_D$  is similar to the up-type quark masses. In this paper, we point out that sufficient asymmetry can nevertheless be generated through nonthermal leptogenesis by inflaton decay. The inflaton is assumed to decay predominantly to the next-to-lightest RH neutrino. The asymmetry resulting from decays of this neutrino is partially washed out since  $M_1 < T_r$ . The final asymmetry depends on the asymmetry per neutrino decay as well as how strong the washout is.

The plan of the paper is as follows: We first review the structure of seesaw parameters and estimate the asymmetry and the washout assuming quark-lepton symmetry. Numerical examples are provided in separate sections for normal and inverted hierarchical (or quasidegenerate) light neutrino masses. We discuss how the BAU is correlated with the  $CP$ -violating phases and conclude with a summary of results and some brief remarks on thermal leptogenesis.

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## II. SEESAW PARAMETERS AND LEPTOGENESIS

In the basis where the RH neutrino mass matrix is diagonal, the Dirac mass matrix can be written as

$$m_D = U_L^\dagger d_D U_R, \quad (4)$$

$d_D \equiv \text{diag}(m_{D1}, m_{D2}, m_{D3})$ . Equation (1) then takes the form

$$m = U_L^\dagger d_D W d_D U_L^*, \quad (5)$$

where

$$W \equiv U_R d_R^{-1} U_R^T \quad (6)$$

is the inverse mass matrix of the RH neutrinos in the basis where  $m_D = U_L^\dagger d_D$ , and  $d_R \equiv \text{diag}(M_1, M_2, M_3)$ . From Eq. (5) one obtains

$$W = \begin{pmatrix} \frac{\hat{m}_{ee}}{m_{D1}^2} & \frac{\hat{m}_{e\mu}}{m_{D1}m_{D2}} & \frac{\hat{m}_{e\tau}}{m_{D1}m_{D3}} \\ \cdots & \frac{\hat{m}_{\mu\mu}}{m_{D2}^2} & \frac{\hat{m}_{\mu\tau}}{m_{D2}m_{D3}} \\ \cdots & \cdots & \frac{\hat{m}_{\tau\tau}}{m_{D3}^2} \end{pmatrix}, \quad (7)$$

$$\hat{m} \equiv U_L m U_L^T. \quad (8)$$

As mentioned in the Introduction, we are assuming  $m_{D1} \ll m_{D2} \ll m_{D3}$ , and the Dirac left-handed rotation  $U_L \approx U_{\text{CKM}} \approx I$ . Elements of  $\hat{m} \approx m$  generally have a much milder hierarchy compared to the Dirac masses. The matrix  $W$  then has a simple hierarchical structure, and is diagonalized by [13]

$$U_R \approx \begin{pmatrix} 1 & -\left(\frac{\hat{m}_{e\mu}}{\hat{m}_{ee}}\right)^* \frac{m_{D1}}{m_{D2}} & \left(\frac{d_{23}}{d_{12}}\right)^* \frac{m_{D1}}{m_{D3}} \\ \left(\frac{\hat{m}_{e\mu}}{\hat{m}_{ee}}\right) \frac{m_{D1}}{m_{D2}} & 1 & -\left(\frac{d_{13}}{d_{12}}\right)^* \frac{m_{D2}}{m_{D3}} \\ \left(\frac{\hat{m}_{e\mu}}{\hat{m}_{ee}}\right) \frac{m_{D1}}{m_{D3}} & \left(\frac{d_{13}}{d_{12}}\right) \frac{m_{D2}}{m_{D3}} & 1 \end{pmatrix} \cdot K, \quad (9)$$

$$d_{23} \equiv \hat{m}_{e\mu} \hat{m}_{\mu\tau} - \hat{m}_{\mu\mu} \hat{m}_{e\tau},$$

$$d_{13} \equiv \hat{m}_{ee} \hat{m}_{\mu\tau} - \hat{m}_{e\mu} \hat{m}_{e\tau}, \quad d_{12} \equiv \hat{m}_{ee} \hat{m}_{\mu\mu} - \hat{m}_{e\mu}^2,$$

$$K = \text{diag}(e^{-i\phi_1/2}, e^{-i\phi_2/2}, e^{-i\phi_3/2}), \quad \phi_i \equiv \arg M_i.$$

Here the phases of RH neutrinos  $\phi_i$  are included in  $U_R$  to keep  $M_i$  real. The mass eigenvalues are

$$M_1 \approx \left| \frac{m_{D1}^2}{\hat{m}_{ee}} \right|, \quad M_2 \approx \left| \frac{m_{D2}^2 \hat{m}_{ee}}{d_{12}} \right|, \quad (10)$$

$$M_3 \approx \left| \frac{m_{D3}^2 d_{12}}{m_1 m_2 m_3} \right|.$$

The large neutrino mixings can originate from the seesaw, despite both  $U_L$  and  $U_R$  being nearly diagonal [16].

To estimate the BAU, suppose the inflaton predominantly decays into the  $i$ th family RH neutrino  $N_i$ . The comoving number density  $Y_N$  is given by

$$Y_N \equiv \frac{n_N}{s} = \frac{n_N}{n_\phi} \frac{n_\phi}{\rho_\phi} \frac{\rho_\phi}{s} = 2Br \frac{1}{m_\phi} \frac{3T_r}{4}, \quad (11)$$

$Br \leq 1$  is the branching ratio of the inflaton  $\phi$  to  $N_i$ , the factor 2 assumes  $\phi \rightarrow 2N_i$ ,  $m_\phi$  is the inflaton mass, and we have used the instantaneous decay approximation. A more accurate calculation shows  $Y_N$  to be  $\approx 25\%$  larger [17]. The asymmetry resulting from the decays of  $N_i$  (assuming it decays promptly [8]) is then

$$Y_\Delta \leq \frac{2T_r |\epsilon_i| \eta}{m_\phi}, \quad (12)$$

where  $\Delta \equiv (1/3)B - L$ ,  $\epsilon_i$  is the lepton asymmetry produced per decay of  $N_i$ , and  $\eta$  is a washout factor. In the simplest scenario,  $M_1 \gg T_r$  and there is no washout ( $\eta = 1$ ). On the other hand, if  $M_1 \lesssim T_r$ , part of the asymmetry will be washed out due to  $N_1$  mediated inverse decays and  $\Delta L = 1$  scatterings. For  $M_2 \lesssim T_r$ ,  $N_2$  mediated processes contribute to the washout as well. The  $\Delta$  asymmetry is multiplied by a conversion factor ( $C \approx 12/37$  for standard model (SM) and  $C \approx 10/31$  for minimal supersymmetric standard model (MSSM) to obtain the BAU resulting from sphaleron processes at equilibrium above the electroweak scale [18].

For hierarchical RH neutrino masses as in Eq. (10),

$$|\epsilon_i| \leq \frac{3a M_1 m_{\text{atm}}}{16\pi v^2}, \quad (13)$$

where the parameter  $a = 1$  for non-SUSY and  $a = 2/\sin^2\beta$  for SUSY ( $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$ ) and  $v = 174$  GeV [9,19,20]. Equations (12) and (13) imply that if the inflaton decays into  $N_1$ , the WMAP best fit  $Y_{B0} = 8.7 \times 10^{-11}$  [21] for the BAU requires

$$M_1 \gtrsim \left(\frac{1}{a\eta}\right) \left(\frac{1/3}{C}\right) \left(\frac{0.05 \text{ eV}}{m_{\text{atm}}}\right) 1.3 \times 10^6 \text{ GeV}, \quad (14)$$

since  $m_\phi \gtrsim T_r$  (in effect [22]).

If the Dirac masses are related to the up-type quark masses, Eq. (10) indicates that  $M_1$  is too light to generate the BAU. We will therefore assume  $m_\phi > 2M_2$  so that  $\phi$  predominantly decays into  $N_2$  instead of  $N_1$ . Using Eqs. (4) and (9),  $(m_D^\dagger m_D)_{ij} \sim m_{Di} m_{Dj}$  with coefficients involving elements of  $\hat{m}$ . It follows that for hierarchical Dirac masses the dominant contribution to the asymmetry from the decays of  $N_2$  involves  $N_3$  in the loop [23]:

$$\epsilon_2 \equiv \sum_\alpha \epsilon_{2,\alpha}$$

$$\approx -\frac{3a}{16\pi v^2} \frac{\sum \text{Im}[(m_D^\dagger)_{2\alpha} (m_D^\dagger m_D)_{23} (m_D^T)_{3\alpha}]}{(m_D^\dagger m_D)_{22}} \frac{M_2}{M_3}. \quad (15)$$

In the expression  $\epsilon_{2,\alpha}$ , the label  $i = 1, 2, 3$  refers to the RH neutrino, and  $\alpha = e, \mu, \tau$  to the lepton flavor that it decays

into. For quark-lepton symmetry (that is, also assuming  $U_L \approx I$ ), the Dirac mass matrix has the form

$$m_D \sim \begin{pmatrix} \mathcal{O}(m_{D1}) & \ll m_{D2} & \ll m_{D3} \\ \mathcal{O}(m_{D1}) & \mathcal{O}(m_{D2}) & \ll m_{D3} \\ \mathcal{O}(m_{D1}) & \mathcal{O}(m_{D2}) & \mathcal{O}(m_{D3}) \end{pmatrix}, \quad (16)$$

with coefficients involving elements of  $\hat{m}$ , and the terms above the main diagonal proportional to nondiagonal elements of  $U_L$ . It follows that the dominant term in Eq. (15) is

$$|\epsilon_{2,\tau}| \approx \frac{3a\varphi}{16\pi v^2} \frac{|m_{D32}^* m_{D33}|^2 M_2}{(m_D^\dagger m_D)_{22} M_3}, \quad (17)$$

$$\approx \frac{3a\varphi M_2}{16\pi v^2} \frac{|d_{13}|^2 m_1 m_2 m_3}{|d_{12}|(|d_{12}|^2 + |d_{13}|^2)}, \quad (18)$$

where  $\varphi \leq 1$  is an effective phase that depends on  $d_\nu$ ,  $U_{\text{PMNS}}$ ,  $d_D$ , and  $U_L$ . [The phases  $\phi_i$  in  $U_R$  can be calculated using Eqs. (5) and (6) given the above masses, mixings, and phases.]

To estimate  $\eta_i$  (the washout involving  $N_i$ ), we define the washout parameters

$$K_{i,\alpha} \equiv \frac{\tilde{m}_{i,\alpha}}{m_*}, \quad \tilde{m}_{i,\alpha} \equiv \frac{|m_{D\alpha i}|^2}{M_i}, \quad (19)$$

$m_* \approx 1.08 \times 10^{-3}$  eV for non-SUSY and  $m_* \approx (\sin^2 \beta) 1.58 \times 10^{-3}$  eV for SUSY. Note that lepton flavors should be treated separately for an accurate calculation of the washout [14,24–26]. Setting  $\alpha = \tau$ , the washout is given in the instantaneous decay approximation by [5,26]

$$\eta_{i,\tau} \approx \exp \left[ \int_{z_0}^{\infty} -\frac{1}{4} z^3 \mathcal{K}_1(z) j(z) K_{i,\tau} A_{\tau\tau} dz \right], \quad (20)$$

$z \equiv M_i/T$ ,  $z_0 \equiv M_i/T_r$ ,  $\mathcal{K}_1$  is a modified Bessel function of the second kind, and

$$Y_{\ell\alpha} = -A_{\alpha\beta} Y_{\Delta\beta}, \quad (21)$$

with  $\ell$  denoting the lepton doublet. The value of  $A_{\tau\tau}$  depends on which interactions are in thermal equilibrium [24]. For MSSM,  $A_{\tau\tau} = 19/30$  between  $(1 + \tan^2 \beta) \times 10^5$  GeV and  $(1 + \tan^2 \beta) \times 10^9$  GeV [27]. For SM,  $A_{\tau\tau} = 344/537$  and  $390/589$  below and above  $10^9$  GeV, respectively [26].<sup>1</sup>

The function  $j(z)$  takes  $\Delta L = 1$  scatterings into account. We will not attempt a detailed calculation which involves finite temperature effects. Instead, we will use  $j(z) = 1$  to define  $\eta_{\text{max}}$  which underestimates the washout, and

$$j(z) = \frac{\mathcal{K}_2(z)}{\mathcal{K}_1(z)} \left( \frac{9m_i^2}{8\pi^2 v^2 z} + 1 \right) \quad (22)$$

to define  $\eta_{\text{min}}$  which overestimates the washout [5].

<sup>1</sup>A more accurate analysis around this temperature should take quantum oscillations into account [25].

It is also required that  $\Delta L = 2$  processes mediated by RH neutrinos are out of equilibrium. As discussed in Ref. [8], it is sufficient to have  $T_r \lesssim (m_{\text{atm}}/m_i)^2 \times 10^{13.5}$  GeV provided

$$\frac{\Gamma_{N_2}}{\Gamma_\phi} = \frac{(a/8\pi v^2) \sum |m_{D\alpha 2}|^2 M_2}{\sqrt{2\pi^2 g_*/45} T_r^2 / m_p} > 1, \quad (23)$$

where  $m_p \approx 2.4 \times 10^{18}$  GeV is the reduced Planck scale, and the relativistic degrees of freedom  $g_* = 106.75$  (228.75) for SM (MSSM). Using  $\sum_\alpha \tilde{m}_{2,\alpha} \sim m_{\text{atm}}$ , this condition corresponds to  $M_2 \gtrsim T_r/5$ .

For  $M_2 \gtrsim T_r$ ,  $\Gamma_{N_2} \gg \Gamma_\phi$  and we can use the following simplified equations [4,27,28]:

$$Z \frac{d\rho_\phi}{dz} = -\frac{3}{z} \rho_\phi - \frac{\Gamma_\phi}{Hz} \rho_\phi, \quad (24)$$

$$\begin{aligned} ZX \frac{dY_{\Delta\tau}}{dz} &= \frac{3}{z} (Z-1) XY_{\Delta\tau} + \frac{2\Gamma_\phi \rho_\phi X \epsilon_{2,\tau}}{sHzm_\phi} \\ &\quad - \frac{1}{4} z^3 \mathcal{K}_1(z) j(z) K_{2,\tau} A_{\tau\tau} Y_{\Delta\tau} \\ &\quad - \frac{1}{4} \gamma(\gamma z)^3 \mathcal{K}_1(\gamma z) j(\gamma z) K_{1,\tau} A_{\tau\tau} Y_{\Delta\tau}. \end{aligned} \quad (25)$$

Here  $z \equiv M_2/T$ ,  $\gamma \equiv M_1/M_2$ , and

$$Z \equiv 1 - \frac{\Gamma_\phi \rho_\phi}{4H\rho_r}, \quad X \equiv \left( \frac{\rho_r + \rho_\phi}{\rho_r} \right)^{1/2}, \quad (26)$$

with  $\rho_r = (M_2/z)^4 g_* \pi^2/30$ . The equations are solved from  $z_i = M_2/T_{\text{max}}$  to  $z_f \gg \gamma^{-1}$ , and  $Y_B \approx CY_{\Delta\tau}(z_f)$ .<sup>2</sup>

### III. RESULTS FOR NH SPECTRUM

In this section we assume a normal hierarchical (NH) spectrum of light neutrino masses ( $m_3 \approx m_{\text{atm}}$ ,  $m_2 \approx m_\odot$ ,  $m_1 \ll m_2$ ). To simplify the discussion we also set  $U_L = I$  and  $s_{13} = 0$ . In this limit the RH neutrino masses are given by [11,13]

$$M_1 \approx \frac{m_{D1}^2}{s_{12}^2 m_2}, \quad M_2 \approx \frac{2m_{D2}^2}{m_3}, \quad M_3 \approx \frac{m_{D3}^2 s_{12}^2}{2m_1}, \quad (27)$$

and with  $|d_{12}| = |d_{13}| = s_{12}^2 m_2 m_3 / 2$  we obtain

$$|\epsilon_{2,\tau}| \approx \frac{3a\varphi m_1 M_2}{16\pi v^2 s_{12}^2}. \quad (28)$$

Using Eqs. (12) and (28),

<sup>2</sup> $T_{\text{max}} \sim (H_I m_p)^{1/4} T_r^{1/2}$  is the maximum temperature attained just after the inflaton starts oscillating, at time  $H_I^{-1}$ . We took  $H_I = m_\phi$  for the numerical calculation, but the results are not sensitive to  $H_I$  as long as  $T_{\text{max}}$  is at least a few times larger than  $T_r$  ( $H_I \gg T_r^2/m_p$ ).

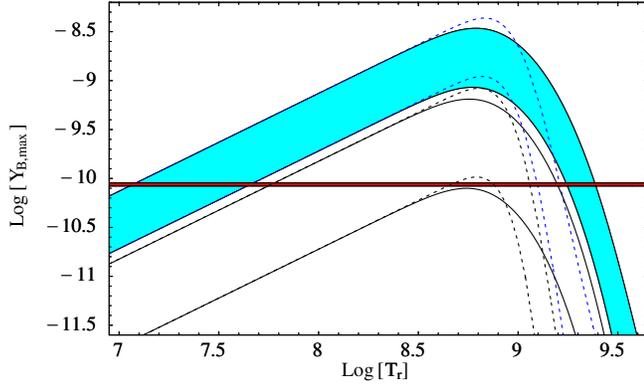


FIG. 1 (color online).  $Y_{B,\max}$  versus the reheat temperature  $T_r$ , for  $d_D = d_u$ ,  $U_L = I$ , and  $s_{13} = 0$ . The horizontal band corresponds to the WMAP range  $(8.7 \pm 0.3) \times 10^{-11}$  [21], the solid and the dotted curves are calculated using Eqs. (24)–(26) and Eq. (20), respectively. Filled: SUSY; unfilled: non-SUSY; upper bounds:  $j(z) = 1$ ; lower bounds: Eq. (22).

$$Y_B \approx \left(\frac{2M_2}{m_\phi}\right)\left(\frac{m_1}{m_2}\right)\varphi Y_{B,\max}, \quad (29)$$

$$Y_{B,\max} \approx \frac{3aCT_r m_2 \eta_1 \eta_2}{16\pi v^2 s_{12}^2}. \quad (30)$$

For  $M_1 \ll T_r$  we can take  $z_0 = 0$  in Eq. (20) to obtain

$$\eta_{1,\max} = \exp\left[-\frac{3\pi}{8}K_{1,\tau}A_{\tau\tau}\right], \quad (31)$$

$$\eta_{1,\min} = \exp\left[-\left(2 + \frac{3\pi}{8}\frac{9m_\tau^2}{8\pi^2 v^2}\right)K_{1,\tau}A_{\tau\tau}\right].$$

$\eta_2$  can be estimated by using Eq. (20), and becomes significant for  $z_0 \lesssim 10$ . It follows from Eqs. (29) and (30) that the maximum asymmetry is obtained for  $T_r \approx M_2/10$  and  $m_\phi \approx 20T_r$ . Taking the reheating phase into account by solving the Boltzmann equations gives similar results.

A numerical example is shown in Fig. 1, where we have used Eq. (30) and set  $d_D = d_u \equiv \text{diag}(m_u, m_c, m_t)$  with the values  $m_u = 1.5$  MeV,  $m_c = 0.43$  GeV,  $m_t = 150$  GeV (taken from Ref. [29], for a renormalization scale of  $10^9$  GeV), for which  $M_1 \approx 6 \times 10^5$  GeV and  $M_2 \approx 6 \times 10^9$  GeV.<sup>3</sup> Assuming  $m_1 \ll m_2$ , we can ignore the contributions to  $\tilde{m}_{i,\alpha}$  that involve  $m_1$ , and it follows from Eq. (29) that  $Y_B \propto m_1$ .

While the washout due to  $N_2$  is severe when  $M_2 \lesssim T_r$ , the washout due to  $N_1$  is rather mild, of order 0.1, for  $U_L = I$  and  $s_{13} = 0$ . This follows from Eq. (20), with  $\tilde{m}_{2,\tau} \approx m_3/2$  compared to  $\tilde{m}_{1,\tau} \approx c_{12}^2 m_2/2$  (in the limit  $m_1 \ll$

<sup>3</sup>We take  $s_{12} = 1/\sqrt{3}$ ,  $s_{23} = 1/\sqrt{2}$ , and  $\sin\beta \approx 1$  in the numerical calculations. We also take  $m_3 = 0.06$  eV and  $m_2 = 0.011$  eV, roughly approximating renormalization group effects by increasing the neutrino mass scale 20%.

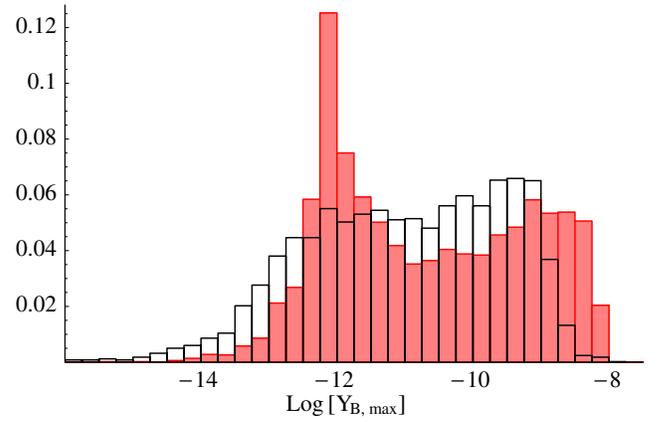


FIG. 2 (color online). Histograms for  $Y_{B,\max}$  with  $d_D = d_u$ ,  $m_1 = 0.2m_2$ , and  $U_L = U_{\text{CKM}}$ , calculated for SUSY with  $j(z) = 1$ . Filled:  $s_{13} = 0$ ; unfilled:  $s_{13} = 0.2$ .

$m_{2,3}$ ). For  $U_L \approx U_{\text{CKM}}$ , there are additional, order  $\theta_C^2 m_3^2/m_2$  contributions to  $\tilde{m}_{1,\tau}$ , where  $\theta_C$  is the leptonic analogue of the Cabibbo angle. The result then depends on the  $CP$ -violating phases of  $U_{\text{PMNS}}$ , but on average the washout gets stronger.

We also take into account an effect due to off-diagonal elements of  $A_{\alpha\beta}$  [30]. Namely, in case of a strong washout related to a large  $\tilde{m}_{1,\tau}$ , part of the asymmetry can still survive if  $\tilde{m}_{1,\mu}$  or  $\tilde{m}_{1,e} \sim m_*$ .<sup>4</sup> Typically  $\tilde{m}_{1,e}$  is the smallest washout parameter. For an estimate we can ignore  $\tilde{m}_{1,\mu}$  which is of order  $\tilde{m}_{1,\tau}$ , and modify the Boltzmann equations by adding

$$-\frac{1}{4}\gamma(\gamma z)^3 \mathcal{K}_1(\gamma z)j(\gamma z)K_{1,\tau}A_{\tau e}Y_{\Delta_e} \quad (32)$$

to Eq. (25), and including an analogous equation for  $Y_{\Delta_e}$  (with  $\tau \leftrightarrow e$  and  $\epsilon_{2,e} \approx 0$ ). The final asymmetry is then  $Y_B \approx C(Y_{\Delta_\tau}(z_f) + Y_{\Delta_e}(z_f))$ .

To estimate the probability distribution of  $Y_{B,\max}$  for  $U_L = U_{\text{CKM}}$ , we numerically solved the Boltzmann equations 5000 times with uniformly distributed random phases of  $U_{\text{PMNS}}$ . We define  $Y_{B,\max}$  by taking  $m_\phi = 2M_2$  as in Eq. (29), but here we take  $m_1 = 0.2m_2$  to be specific and include  $\varphi$ . In addition, the asymmetry is maximized by varying  $T_r$  for each run. Figure 2 shows the results for  $s_{13} = 0$  and  $s_{13} = 0.2$ . The percentage of runs yielding  $Y_{B,\max} > Y_{B0}$  was 38% and 32% for  $s_{13} = 0$  and  $s_{13} = 0.2$  respectively. Including the  $A_{\tau e}$  term significantly alters the low end of the probability distribution for  $Y_{B,\max}$ , but the effect on these percentages is only a few points.<sup>5</sup>

<sup>4</sup>This is also true for the washout due to  $N_2$ , and part of the asymmetry can survive for  $M_2 \lesssim T_r$ . However, the maximum BAU is still obtained for  $M_2 \approx 10T_r$ .

<sup>5</sup>Note that in Eq. (4), there are generally Majorana phases on the left side of  $U_R$  as well. These phases enter  $\tilde{m}_{i,\alpha}$ , leading to  $\mathcal{O}(\theta_C)$  corrections for  $\alpha = e, \mu$ . (They can be rotated away in the limit  $U_L = I$ .) This does not affect the results appreciably.

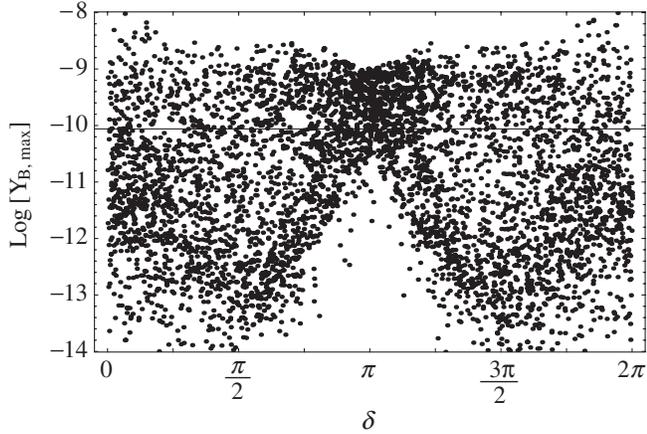


FIG. 3.  $Y_{B,\max}$  versus the Dirac phase  $\delta$ , with  $d_D = d_u$ ,  $m_1 = 0.2m_2$ ,  $U_L = U_{\text{CKM}}$ , and  $s_{13} = 0.2$ , calculated for SUSY with  $j(z) = 1$ .

For  $s_{13} = 0$ , the peak at  $Y_B \approx 10^{-12}$  results from  $\tilde{m}_{1,e}$  having a relatively small deviation [ $\tilde{m}_{1,e} \approx [1 + \mathcal{O}(\theta_C) + \mathcal{O}(\theta_C^2 m_3/m_2)]s_{12}^2 m_2$ ]. For  $s_{13} \neq 0$ , there are additional contributions to  $\tilde{m}_{1,e}$  (as well as  $\tilde{m}_{1,\tau}$ ), and the probability distribution becomes more dispersed. Using Eqs. (4), (9), (10), and (19),  $\tilde{m}_{1,\tau} \approx |\hat{m}_{e\tau}|^2/|\hat{m}_{ee}|$ , which from Eqs. (2) and (8) is given by

$$\approx \frac{|-c_{12}s_{12}e^{i\alpha_2}m_2 + (s_{13}e^{-i\delta} + \frac{\theta_C}{\sqrt{2}})m_3|^2}{2|s_{12}^2e^{i\alpha_2}m_2 + (s_{13}e^{-i\delta} + \frac{\theta_C}{\sqrt{2}})m_3|^2}. \quad (33)$$

The terms including  $m_1$  and  $\alpha_1$  are subdominant. Assuming  $\theta_C$  is similar to the Cabibbo angle,  $\tilde{m}_{1,\tau}$  is minimized for  $\delta \approx \pi$ .  $Y_{B,\max}$  also depends on  $\epsilon_{2,\tau}$ , which from Eq. (18) is maximized for  $\delta \approx 0$ . For random Majorana phases,  $Y_{B,\max} > Y_{B0}$  is most likely at  $\delta \approx \pi$  due to the exponential dependence on  $\tilde{m}_{1,\tau}$ , however it remains possible for all values of  $\delta$  (Fig. 3).

The values of  $s_{13}$  and  $\delta$  will be probed by neutrino beam experiments within a decade for  $s_{13} \gtrsim 0.05$  [31]. The value of  $\alpha_2$  can in principle be probed by neutrinoless double beta decay experiments. However, for normal hierarchy the effective Majorana mass  $|\langle m_{\beta\beta} \rangle| = |m_{ee}| \approx |s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 e^{-2i\delta} m_3|$  is too small to detect using current techniques.

#### IV. RESULTS FOR IH AND QD SPECTRA

For inverted hierarchical (IH) spectrum of neutrino masses,  $m_3 \ll m_1 < m_2 \approx m_{\text{atm}}$  and in the limit  $m_3 \rightarrow 0$ ,  $U_L \rightarrow I$ ,

$$\tilde{m}_{1,\tau} = \frac{c_{12}^2 s_{12}^2 |e^{i\alpha_1} m_1 - e^{i\alpha_2} m_2|^2}{2|c_{12}^2 e^{i\alpha_1} m_1 + s_{12}^2 e^{i\alpha_2} m_2|^2}. \quad (34)$$

Taking  $s_{12} = 1/\sqrt{3}$ ,  $\tilde{m}_{1,\tau}$  ranges from  $4m_{\text{atm}}/3$  for  $\alpha_2 = \alpha_1 + \pi$  to  $m_{\text{atm}}^4/36m_{\text{atm}}^3 \approx 0$  for  $\alpha_2 = \alpha_1$ . (Including the Cabibbo mixing,  $\tilde{m}_{1,\tau}$  for  $\alpha_2 = \alpha_1$  becomes  $m_{\text{atm}}\theta_C^2/4$ ,

which is still  $\lesssim m_*$ .) As a result, the asymmetry is suppressed by a factor  $\sim 10^8$  for  $\alpha_2 = \alpha_1 + \pi$ , but  $Y_{B,\max} > Y_{B0}$  is possible if  $\alpha_1 \approx \alpha_2$ .

The RH neutrino masses are given in this limit by

$$M_1 \approx \frac{m_{D1}^2}{m_{\text{atm}}}, \quad M_2 \approx \frac{2m_{D2}^2}{m_{\text{atm}}}, \quad M_3 \approx \frac{m_{D3}^2}{2m_3} \quad (35)$$

for  $\alpha_1 \approx \alpha_2$ . With  $|d_{12}| = |d_{13}| = m_1 m_2 / 2$  we obtain

$$|\epsilon_{2,\tau}| \approx \frac{3a\varphi m_3 M_2}{16\pi v^2}, \quad (36)$$

similar to Eq. (28).

The asymmetry can only survive if  $\alpha_1 \approx \alpha_2$  for quasi-degenerate (QD) spectra of neutrino masses as well, since terms involving  $m_3$  in  $\tilde{m}_{1,\tau}$  are suppressed either by  $s_{13}$  or  $\theta_C$ . The RH neutrino masses  $M_i \sim m_{D_i}^2/\bar{m}$ , where  $\bar{m}$  is the QD neutrino mass scale. Assuming  $\alpha_1 \approx \alpha_2$ ,  $|\epsilon_{2,\tau}| \sim (3a/16\pi v^2)m_{D2}^2$  is maximized for  $\alpha_2 \approx \pi$ . On the other hand,  $\tilde{m}_{1,\tau} \approx \theta_C^2 \bar{m}$  for  $\alpha_2 \approx \pi$  and it is minimized for  $\alpha_2 \approx 0$ . The maximum asymmetry is determined by the interplay of these two factors.

In the numerical examples we used the following neutrino masses: IH spectrum:  $m_1 = 0.059$  eV,  $m_2 = 0.06$  eV,  $m_3 = m_2/5$ ; QD spectrum:  $m_1 = 0.1$  eV,  $m_2 = 0.1006$  eV,  $m_3 = 0.117$  eV. (Similar results are obtained for inverted hierarchical QD masses.) The resulting probability distribution of  $Y_{B,\max}$  is displayed in Fig. 4. The percentage of runs yielding  $Y_{B,\max} > Y_{B0}$  was 31% and 18% for IH and QD spectra, respectively, for  $U_L = U_{\text{CKM}}$  and  $s_{13} = 0$ . Since  $\tilde{m}_{1,\tau} \propto \bar{m}$ ,  $Y_{B,\max}$  decreases as  $\bar{m}$  is increased, with the percentage of runs yielding  $Y_{B,\max} > Y_{B0}$  decreasing to 7% (3%) for  $m_1 = 0.2(0.3)$  eV. These percentages increase a few points if  $U_L \approx I$ , and decrease a few points if  $s_{13} \approx 0.2$ .

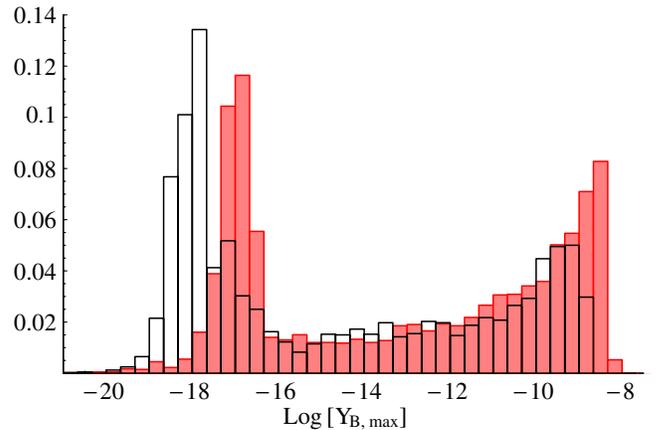


FIG. 4 (color online). Histograms for  $Y_{B,\max}$  with  $d_D = d_u$ ,  $U_L = U_{\text{CKM}}$ , and  $s_{13} = 0$ , calculated for SUSY with  $j(z) = 1$ . Filled: IH spectrum; unfilled: QD spectrum.

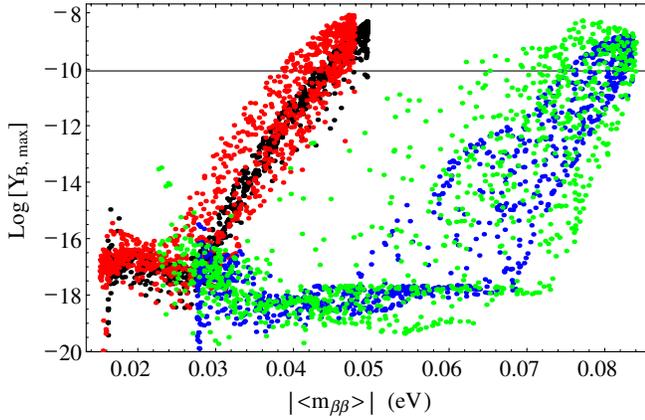


FIG. 5 (color online).  $Y_{B,\max}$  versus the effective Majorana mass  $|\langle m_{\beta\beta} \rangle|$ , with  $d_D = d_u$  and  $U_L = U_{\text{CKM}}$ , calculated for SUSY with  $j(z) = 1$ . Black (red): IH spectrum with  $s_{13} = 0$  ( $s_{13} = 0.2$ ). Blue (green): QD spectrum with  $s_{13} = 0$  ( $s_{13} = 0.2$ ).

The effective Majorana mass, given by

$$|\langle m_{\beta\beta} \rangle| \approx |c_{12}^2 e^{i\alpha_1} m_1 + s_{12}^2 e^{i\alpha_2} m_2| \quad (37)$$

for both IH and QD spectra, is maximized by the condition  $\alpha_2 \approx \alpha_1$ . As shown in Fig. 5,  $Y_{B,\max} > Y_{B0}$  requires  $|\langle m_{\beta\beta} \rangle| \gtrsim 0.04$  eV for IH.<sup>6</sup> This range of  $|\langle m_{\beta\beta} \rangle|$  can be probed within a decade [32].

## V. CONCLUSION

In this paper we considered nonthermal leptogenesis by inflaton decay under the assumption that the Dirac-type neutrino mass matrix  $m_D$  is related to the up-type quark (or charged lepton) mass matrix. Following the approach of Ref. [13], we did not make any specific assumptions on the textures of these matrices, but rather considered the general structure that follows from fitting to the low energy data. In this approach, the RH neutrino masses are almost always strongly hierarchical ( $\propto d_D^2$ ).

This strong hierarchy and the rest of our analysis follows from the light neutrino mass matrix having a less hierarchical structure compared to  $d_D$ , except for specific values of  $s_{13}$  and the  $U_{\text{PMNS}}$  phases which occur very rarely in a random scan.<sup>7</sup> On the other hand,  $m_D$  can be constrained further in particular  $SO(10)$  or other GUT models with flavor symmetries. It can then have structures different from Eq. (16), and fitting to the low energy data then yields those specific values as predictions. In such cases which

<sup>6</sup>Note that since we took the neutrino mass scale 20% larger at the leptogenesis scale, we scaled  $|\langle m_{\beta\beta} \rangle|$  down 20% in the figure to correspond to low energy values.

<sup>7</sup>Since nothing is currently known about the values of these phases, we assumed a flat probability distribution for numerical analysis. Such a distribution also follows from ‘‘anarchy’’ [33].

are beyond the scope of this paper, the RH neutrinos can be less hierarchical and nonthermal leptogenesis with  $M_1 \gg T_r$  also becomes possible.

Assuming strongly hierarchical RH neutrinos with  $d_D \approx d_u$ , the matter asymmetry created by the decays of  $N_2$  is partially washed out since  $M_1 < T_r$ , but can still account for the BAU. For either NH or IH spectra of light neutrino masses,  $Y_B = Y_{B0}$  requires

$$\left(\frac{10T_r}{M_2}\right)\left(\frac{2M_2}{m_\phi}\right)\left(\frac{\min(m_i)}{m_2}\right) \gtrsim \left(\frac{m_c}{m_{D2}}\right)^2 10^{-2}, \quad (38)$$

with each term on the left  $< 1$  [ $\min(m_i) = m_1$  for NH,  $\min(m_i) = m_3$  for IH]. The case  $d_D = d_u$  corresponds to  $M_2 \sim 6 \times 10^9$  GeV.<sup>8</sup> Equation (38) can then be satisfied with  $m_\phi = 10^{10}$ – $10^{11}$  GeV and  $T_r \sim 10^8$  GeV. This value of  $T_r$  can be consistent with the gravitino constraint, while  $m_\phi$  in the above range is possible in small field or hybrid inflation models. On the other hand, for simplest large field inflation models  $m_\phi \gtrsim 2 \times 10^{13}$  GeV. The upcoming Planck satellite can discriminate these classes of models [34].

For the NH spectrum of light neutrino masses and  $s_{13} \gtrsim 0.1$ , sufficient asymmetry is most likely to be obtained if the  $U_{\text{PMNS}}$  Dirac phase  $\delta \approx \pi$  (assuming  $U_L \approx U_{\text{CKM}}$ ). For IH or QD spectra of light neutrino masses, sufficient asymmetry can only be obtained if the  $U_{\text{PMNS}}$  Majorana phases are approximately equal to each other, implying  $|\langle m_{\beta\beta} \rangle| \approx m_{\text{atm}}$  for IH spectrum and larger for QD spectrum. The asymmetry decreases as the QD neutrino mass scale is increased, and if  $|\langle m_{\beta\beta} \rangle| \gtrsim 0.2$  eV, the leptogenesis scenario discussed here is strongly disfavored assuming  $d_D \approx d_u$ .

If we relate the Dirac masses to masses of the charged leptons  $d_\ell \equiv \text{diag}(m_e, m_\mu, m_\tau)$  instead,  $m_{D2} \approx m_\mu \tan\beta$  and Eq. (27) yields  $M_2 \sim (\tan^2\beta)2 \times 10^8$  GeV. Provided  $\tan\beta$  is large, it is then easier to satisfy Eq. (38), especially for non-SUSY where there is no gravitino constraint on  $T_r$ . For large  $\tan\beta$ , it also becomes possible to generate the BAU with the inflaton decaying to  $N_1$ , as  $M_1 \sim (\tan^2\beta)5 \times 10^4$  GeV can satisfy Eq. (14).

Thermal leptogenesis where the asymmetry is created by the decays of  $N_2$  was discussed in Refs. [20,30,35] as well as Refs. [14,15] which also relate  $m_D$  to the up-type quark masses. It is difficult to obtain sufficient asymmetry in this case. For  $\tilde{m}_{2,\tau} \approx m_{\text{atm}}/2$ , the bounds are  $T_r \gtrsim 10^{10}$  GeV and  $M_2 \gtrsim 5 \times 10^{10}$  GeV assuming that  $\epsilon_2$  is given by Eq. (13) with  $M_1$  replaced by  $M_2$ , and that there is negligible washout from  $N_1$  [5,20]. However, for quark-lepton symmetry  $\epsilon_2$  is suppressed by the lightest neutrino mass,

<sup>8</sup>More precisely,  $M_2$  varies depending on the  $U_{\text{PMNS}}$  phases and is in the range  $10^{9.7}$ – $10^{10.2}$  GeV for NH with  $s_{13} = 0$ . For NH with  $s_{13} = 0.2$  or IH, it varies within an order (two orders) of magnitude for about 90% (99%) of the runs.

and the phase values that maximize it do not coincide with those that suppress the washout. We therefore expect these bounds to be at least a few times larger. Similar conclusions are reached in Ref. [15]. Notwithstanding the high  $T_r$ , the value of  $M_2$  would then not be compatible with the assumption  $d_D \approx d_u$ , although it may be compatible with  $d_D \approx d_\ell \tan\beta$  for large  $\tan\beta$ .

## ACKNOWLEDGMENTS

I thank Azar Mustafayev and Danny Marfatia for useful discussions. I also thank L. Velasco-Sevilla and W. Rodejohann for helpful comments and pointing out relevant papers. This research was supported by the U.S. Department of Energy under Grant No. DE-FG02-04ER41308.

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