Lorentz conserving noncommutative standard model

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We consider Lorentz-conserving noncommutative field theory to construct the Lorentz-conserving noncommutative standard model based on the gauge group $SU(3) \times SU(2) \times U(1)$. We obtain the enveloping algebra-valued of Higgs field up to the second order of the noncommutativity parameter $\theta_{\mu\nu}$. We derive the action at the leading order and find new vertices which are absent in the ordinary standard model as well as the minimal noncommutative standard model. We briefly study the phenomenological aspects of the model.

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I. INTRODUCTION

In recent years, many authors have considered noncommutative (NC) field theories and their phenomenological aspects [1]. A strong motivation for investigating these field theories is their appearance in a definite limit of string theory [2]. On the other hand the standard model of electroweak and strong interactions has met the challenge of many high precision experiments. In the high energy limit the noncommutativity effects seem to be significant and therefore the new interactions in the noncommutative space and time can be potentially important to particle physics and cosmology. For example, in the minimal extension of the standard model in the noncommutative space, in contrast with the conventional theory, there is neutrino-photon vertex which leads to neutrino-photon interaction at the tree level [3]. In the canonical noncommutative space-time, the coordinates are operators and satisfy the following commutator relation:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{1}$$

where $\theta^{\mu\nu} = -\theta^{\nu\mu}$ is a real and constant Lorentz tensor. As $\theta^{\mu\nu}$ is constant, there is, obviously, a preferred direction in a given particle Lorentz frame which leads to the Lorentz symmetry violation. On the other hand, experimental inspections for Lorentz violation, including clock comparison tests, polarization measurements on the light from distant galaxies, analyses of the radiation emitted by energetic astrophysical sources, studies of matter-antimatter asymmetries for trapped charged particles and bound state systems [4] and so on, have thus far failed to produce any positive results. These experiments strictly bound the Lorentz-violating parameters, therefore, in the lower energy limit, the Lorentz symmetry is an almost exact symmetry of the nature. However, it is natural to explore the noncommutative field theories that are Lorentz invariant from the beginning. In this class of NC theories, the parameter of noncommutativity is not a constant but an operator which transforms as a Lorentz tensor. Of course in this way one needs to generalize the star product and operator trace for functions of both x^μ and $\theta^{\mu\nu}$, appropriately. However, in both cases experiment should confirm the theories. Using the enveloping algebra-valued method, introduced in [5,6], Carlson, Carone, and Zobin (CCZ) have constructed Lorentz-conserving noncommutative quantum electrodynamics based on a contracted Snyder algebra [7]. Afterward, the miscellaneous aspects of the theory had been considered by others [8–11]. In this paper we introduce Lorentz-conserving noncommutative standard model (LCNCSM) using the CCZ approach and consider differences between the LCNCSM and the Lorentz-violating noncommutative standard model. To construct the noncommutative field theory, according to the Weyl-Moyal correspondence, an ordinary function can be used instead of the corresponding noncommutative one by replacing the ordinary product with the star product as follows:

$$f * g(x) = f(x) \exp(i/2 \overleftarrow{\partial}_\mu \theta^{\mu\nu} \vec{\partial}_\nu) g(x). \tag{2}$$

Using this correspondence, however, there are two approaches to construct the gauge theories in the noncommutative space. In the first one the gauge group is restricted to $U(n)$ and the symmetry group of the standard model is achieved by reduction of $U(3) \times U(2) \times U(1)$ to $SU(3) \times SU(2) \times U(1)$ by an appropriate symmetry breaking [12]. Nevertheless, for $U(1)$ the charge of particles are allowed to be ± 1 and 0 [12,13]. In the second approach, the noncommutative gauge theory can be constructed for every charge and for $SU(n)$ gauge group via Seiberg-Witten map [5,6]. Meanwhile, to follow the second approach to construct LCNCSM one needs the Seiberg-Witten map of all fields in the standard model up to the second order of $\theta^{\mu\nu}$. Fortunately, the map for all of the fields except the Higgs field to this order has been obtained already [6].

In Sec. II we briefly review Lorentz-conserving noncommutative field theory. In Sec. III we study enveloping algebra and the Seiberg-Witten map for the fields of the standard model and obtain the corresponding expression for the Higgs field up to the second order of $\theta^{\mu\nu}$ which is not calculated elsewhere. In Sec. IV we construct the

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LCNCSM and discuss possible vertices in this model. Finally, we discuss the phenomenological aspects of this model in Sec. V, and give the concluding remarks in Sec. VI.

II. LORENTZ-CONSERVING NONCOMMUTATIVE FIELD THEORY

In 1947 Snyder considered the Lorentz symmetry in discrete space-time to avoid UV divergence [14]. For this purpose, he assumed that the space-time coordinates are noncommutative operators which led to a Lorentz-invariant discrete space-time. CCZ by contracting the proposed algebra found the following algebra:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\hat{\theta}^{\mu\nu}, \qquad [\hat{\theta}^{\mu\nu}, \hat{x}^\lambda] = 0, \qquad [\hat{\theta}^{\mu\nu}, \hat{\theta}^{\alpha\beta}] = 0, \tag{3}$$

which is similar to the canonical noncommutative algebra but $\hat{\theta}^{\mu\nu}$ is an antisymmetric operator that is not constant but transforms as a Lorentz tensor. The action for Lorentz-conserving field theories on noncommutative spaces are then obtained using the Weyl-Moyal correspondence. In fact, in order to find the noncommutative action, the usual product of fields should be replaced by the star product:

$$f * g(x, \theta) = f(x, \theta) \exp(i/2 \overleftarrow{\partial}_\mu \theta^{\mu\nu} \vec{\partial}_\nu) g(x, \theta). \tag{4}$$

It should be noted that here the mapping to c-number coordinates involves $\theta^{\mu\nu}$ as a c-number due to the presence of the operator $\hat{\theta}^{\mu\nu}$ in the Lorentz-conserving case. In this formulation, the operator trace that is a map from operator space to numbers is defined as

$$\mathrm{Tr}\, \hat{f} = \int d^4x d^6\theta W(\theta) f(x, \theta), \tag{5}$$

where $W(\theta)$ is a Lorentz-invariant weight function with the normalization $\int d^6\theta W(\theta) = 1$ and is assumed to be a positive and even function of $\theta_{\mu\nu}$. Therefore,

$$\int d^6\theta W(\theta) \theta^{\mu\nu} = 0. \tag{6}$$

Also for every even Lorentz-invariant weighting function $W(\theta)$ one has

$$\int d^6\theta W(\theta) \theta^{\mu\nu}\theta^{\kappa\lambda} = \frac{\langle\theta^2\rangle}{12}(g^{\mu\kappa}g^{\nu\lambda} - g^{\mu\lambda}g^{\nu\kappa}), \tag{7}$$

where

$$\langle\theta^2\rangle = \int d^6\theta W(\theta)\theta^{\mu\nu}\theta_{\mu\nu}. \tag{8}$$

Furthermore, the weight function is assumed to fall sufficiently fast so that all integrals are well defined. In fact $W(\theta)$ suppresses the cross section for center-of-mass energy beyond the value of noncommutative scale; therefore, working with truncated power series expansion of functions in $\theta_{\mu\nu}$ is permitted. Now the properties of $W(\theta)$ and the definition of the operator trace allows one to obtain the Lagrangian for the Lorentz-conserving noncommutative field theory

$$\mathcal{L}(x) = \int d^6\theta W(\theta) \mathcal{L}(\phi, \partial\phi)_*, \tag{9}$$

in which $\mathcal{L}(\phi, \partial\phi)_*$ depends on both x and $\theta_{\mu\nu}$, and its subscript indicates the $*$ product defined in Eq. (4).

III. $\theta^{\mu\nu}$-EXPANDED FIELDS UP TO THE SECOND ORDER

A non-Abelian gauge theory, based on a Lie group, for example SU(n), in the noncommutative space cannot be constructed in the same way as the commutative one. In fact the main difference is that for every two gauge parameters, Λ and Λ', one has

$$[\Lambda \stackrel{*}{,} \Lambda'] = \tfrac{1}{2}[T^a, T^b]\{\Lambda_{1,a}(x) \stackrel{*}{,} \Lambda_{2,b}(x)\} + \tfrac{1}{2}\{T^a, T^b\} \times [\Lambda_{1,a}(x) \stackrel{*}{,} \Lambda_{2,b}(x)], \tag{10}$$

where $\Lambda = \Lambda_a(x)T^a$. Obviously, the anticommutators of T^a's do not close the Lie algebra of a non-Abelian gauge theory except for U(N), and they reproduce all the higher powers of the generators. Meanwhile, the enveloping algebra consists of all ordered tensor powers of the generators T^a and seems to be a proper choice for such a gauge theory [5,6]. However, the enveloping algebra is infinite dimensional and as a consequence the enveloping algebra-valued noncommutative gauge parameter and fields would have an infinite number of degrees of freedom which can be considered as follows:

$$\hat{\Lambda}_\alpha = \alpha + \hbar\Lambda^1_\alpha + \hbar^2\Lambda^2_\alpha + \ldots, \tag{11}$$

$$\hat{\Psi} = \Psi^0 + \hbar\Psi^1 + \hbar^2\Psi^2 + \ldots, \tag{12}$$

$$\hat{A}_\mu = A^0_\mu + \hbar A^1_\mu + \hbar^2 A^2_\mu + \ldots, \tag{13}$$

$$\hat{\Phi} = \Phi^0 + \hbar\Phi^1 + \hbar^2\Phi^2 + \ldots, \tag{14}$$

where α is the ordinary gauge parameter and Ψ^0, A^0_μ, and Φ^0 are, respectively, the commutative fermion fields, gauge fields and Higgs fields, and the superscript i stands for their order in the expansion. The infinite number of degrees of freedom can be restricted demanding that the enveloping algebra-valued quantities (such as gauge and matter fields and so on) depend on the algebra-valued ones and their space-time derivatives only. This requirement, based on existence of the Seiberg-Witten map to all orders, reduces the number of degrees of freedom of the NC gauge theory to the same one of the gauge theory of the commutative space. In fact, Seiberg and Witten have shown that there is an equivalence between ordinary and noncommutative gauge fields to any finite order in $\theta_{\mu\nu}$ which is realized by a map in a way that preserves the gauge equivalence relation. In other words if $\hat{A}_\mu$ and $\hat{A}'_\mu$ are equivalent gauge fields in noncommutative space-time, the corresponding ordinary gauge fields A_μ and A'_μ should be equivalent too. This means

$$\delta\hat{A}_\mu = \hat{\delta}\hat{A}_\mu, \qquad \delta\hat{\Psi} = \hat{\delta}\,\hat{\Psi}. \tag{15}$$

In the ordinary space the commutator of two infinitesimal gauge transformations are closed. Therefore, it is necessary to consider the following consistency condition for the noncommutative gauge parameter:

$$i\delta_\alpha\hat{\Lambda}_\beta - i\delta_\beta\hat{\Lambda}_\alpha + [\hat{\Lambda}_\alpha \stackrel{\star}{,} \hat{\Lambda}_\beta] = i\hat{\Lambda}_{-i[\alpha,\beta]}. \tag{16}$$

By substituting the expansion of $\hat{\Lambda}_\alpha$ from (11) in (16), one up to the first order of $\theta_{\mu\nu}$ finds

$$i(\delta_\alpha\Lambda^1_\beta - \delta_\beta{\Lambda^1}_\alpha) + [\alpha, {\Lambda^1}_\beta] + [{\Lambda^1}_\alpha, \beta] - i\Lambda^1_{-i[\alpha,\beta]} = \frac{i}{2}\theta^{\mu\nu}\{\partial_\mu\alpha, \partial_\nu\beta\}, \tag{17}$$

and after a little algebra up to the second order of $\theta_{\mu\nu}$ one has

$$\begin{aligned} i(\delta_\alpha\Lambda^2_\beta - \delta_\beta{\Lambda^2}_\alpha) + [\alpha, \Lambda^2_\beta] + [{\Lambda^2}_\alpha, \beta] - i\Lambda^2_{-i[\alpha,\beta]} &= \frac{1}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}[\partial_\mu\partial_\kappa\alpha, \partial_\nu\partial_\lambda\beta] - [{\Lambda^1}_\lambda, {\Lambda^1}_\beta] \\ &\quad - \frac{i}{2}\theta^{\mu\nu}(\{\partial_\mu{\Lambda^1}_\alpha, \partial_\nu\beta\} - \{\partial_\mu{\Lambda^1}_\beta, \partial_\nu\alpha\}), \end{aligned} \tag{18}$$

which in terms of ${\Lambda^1}_\alpha$ and ${\Lambda^2}_\alpha$ are inhomogeneous linear equations with the following solutions, respectively [5,6]:

$$\Lambda^1_\alpha = -\tfrac{1}{4}\theta^{\mu\nu}\{A^0_\mu, \partial_\nu\alpha\}, \tag{19}$$

and

$$\begin{aligned} \Lambda^2_\alpha = \frac{1}{32}\theta^{\mu\nu}\theta^{\kappa\lambda}(&\{A^0_\mu, \{\partial_\nu A^0_\kappa, \partial_\lambda\alpha\}\} + \{A^0_\mu, \{A^0_\kappa, \partial_\nu\partial_\lambda\alpha\}\} \\ &+ \{\{A^0_\mu, \partial_\nu A^0_\kappa\}, \partial_\lambda\alpha\} - \{\{F^0_{\mu\kappa}, A^0_\nu\}, \partial_\lambda\alpha\} \\ &- 2i[\partial_\mu A^0_\kappa, \partial_\nu\partial_\lambda\alpha]). \end{aligned} \tag{20}$$

In the second step the noncommutative field $\hat{\Psi}$ can be determined by replacing the expansion of $\hat{\Lambda}_\alpha$ and $\hat{\Psi}$ which is given, respectively, in (11) and (12) in the gauge transformation $\delta\hat{\Psi} = i\hat{\Lambda}_\alpha * \hat{\Psi}$ [5,6]. It can be easily shown that up to the first order of $\theta_{\mu\nu}$ one has

$$\delta_\alpha\Psi^1 - i\alpha\Psi^1 = i\Lambda^1_\alpha\Psi^0 - \tfrac{1}{2}\theta^{\mu\nu}\partial_\mu\alpha\partial_\nu\Psi^0, \tag{21}$$

and up to the second order of $\theta_{\mu\nu}$, it results in

$$\begin{aligned} \delta_\alpha\Psi^2 - i\alpha\Psi^2 = i\Lambda^2_\alpha\Psi^0 + i\Lambda^1_\alpha\Psi^1 - \frac{1}{2}\theta^{\mu\nu}\partial_\mu\Lambda^1_\alpha\partial_\nu\Psi^0 \\ - \frac{1}{2}\theta^{\mu\nu}\partial_\mu\alpha\partial_\nu\Psi^1 \\ - \frac{i}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}\partial_\mu\partial_\kappa\alpha\partial_\nu\partial_\lambda\Psi^0. \end{aligned} \tag{22}$$

The first equation can be solved by

$$\Psi^1 = -\frac{1}{2}\theta^{\mu\nu}A^0_\mu\partial_\nu\Psi^0 + \frac{i}{4}\theta^{\mu\nu}A^0_\mu A^0_\nu\Psi^0, \tag{23}$$

and the solution of the second equation can be obtained as follows:

$$\begin{aligned} \Psi^2 = -\frac{i}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}(&\partial_\kappa A^0_\mu\partial_\nu\partial_\lambda\Psi^0 + iA^0_\kappa A^0_\mu\partial_\nu\partial_\lambda\Psi^0 - i\partial_\kappa A^0_\mu A^0_\nu\partial_\lambda\Psi^0 + iF^0_{\kappa\mu}A^0_\nu\partial_\lambda\Psi^0 - iA^0_\nu\partial_\kappa A^0_\mu\partial_\lambda\Psi^0 + 2iA^0_\nu F^0_{\kappa\mu}\partial_\lambda\Psi^0 \\ &+ 2A^0_\mu A^0_\kappa A^0_\nu\partial_\lambda\Psi^0 - A^0_\mu A^0_\nu A^0_\kappa\partial_\lambda\Psi^0 - \frac{i}{4}(2\partial_\kappa A^0_\mu\partial_\lambda A^0_\nu\Psi^0 - 2i\partial_\kappa A^0_\mu A^0_\lambda A^0_\nu\Psi^0 + 2iA^0_\nu A^0_\lambda\partial_\kappa A^0_\mu\Psi^0 \\ &+ i[[\partial_\kappa A^0_\mu, A^0_\nu], A^0_\lambda]\Psi^0 + 4iA^0_\nu F^0_{\kappa\mu}A^0_\lambda\Psi^0 - A^0_\kappa A^0_\lambda A^0_\mu A^0_\nu\Psi^0 + 2A^0_\kappa A^0_\mu A^0_\nu A^0_\lambda\Psi^0)). \end{aligned} \tag{24}$$

The enveloping algebra gauge potential can be found by inserting $\hat{A}$ and $\hat{\Lambda}_\alpha$ from (11) and (13), respectively, in the transformation $\delta\hat{A}_\mu = \partial_\mu\hat{\Lambda}_\alpha - i[\hat{A}_\mu \stackrel{\star}{,} \hat{\Lambda}_\alpha]$, and retaining the coefficients up to the first order of $\theta_{\mu\nu}$ leads to

$$\delta_\alpha A^1_\sigma - i[\alpha, A^1_\sigma] = \partial_\sigma\Lambda^1_\alpha - i[A^0_\sigma, \Lambda^1_\alpha] + \tfrac{1}{2}\theta^{\mu\nu}\{\partial_\mu A^0_\sigma, \partial_\nu\alpha\}, \tag{25}$$

which has the solution

$$A^1_\sigma = -\tfrac{1}{4}\theta^{\mu\nu}(\{A^0_\mu, \partial_\nu A^0_\sigma\} - \{F^0_{\mu\sigma}, A^0_\nu\}). \tag{26}$$

For the next order of $\theta_{\mu\nu}$, after some manipulation one has

$$\begin{aligned} \delta_\alpha A^2_\sigma - i[\alpha, A^2_\sigma] = \partial_\sigma\Lambda^2_\alpha - i[A^0_\sigma, \Lambda^2_\alpha] - i[A^1_\sigma, \Lambda^1_\alpha] \\ + \frac{1}{2}\theta^{\mu\nu}\{\partial_\mu A^1_\sigma, \partial_\nu\alpha\} \\ + \frac{1}{2}\theta^{\mu\nu}\{\partial_\mu A^0_\sigma, \partial_\nu\Lambda^1_\alpha\} \\ + \frac{i}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}[\partial_\kappa\partial_\mu A^0_\sigma, \partial_\lambda\partial_\nu\alpha], \end{aligned} \tag{27}$$

with the solution given in the following equation. There are differences in signs of some terms of given equation in comparison with the corresponding one given in Ref. [6]. The misprinting in the signs can be verified easily by considering the reduction of the equation to the Abelian case

$$A^2_\sigma = \tfrac{1}{32}\theta^{\mu\nu}\theta^{\kappa\lambda}(\{\{A^0_\kappa, \partial_\lambda A^0_\mu\}, \partial_\nu A^0_\sigma\} - \{\{F^0_{\kappa\mu}, A^0_\lambda\}, \partial_\nu A^0_\sigma\} - 2i[\partial_\kappa A^0_\mu, \partial_\lambda\partial_\nu A^0_\sigma] - \{A^0_\mu, \{\partial_\nu F^0_{\kappa\sigma}, A^0_\lambda\}\} - \{A^0_\mu, \{F^0_{\kappa\sigma}, \partial_\nu A^0_\lambda\}\} + \{A^0_\mu, \{\partial_\nu A^0_\kappa, \partial_\lambda A^0_\sigma\}\} + \{A^0_\mu, \{A^0_\kappa, \partial_\nu\partial_\lambda A^0_\sigma\}\} - \{\{A^0_\kappa, \partial_\lambda F^0_{\mu\sigma}\}, A^0_\nu\} + \{\{\mathcal{D}^0_\kappa F^0_{\mu\sigma}, A^0_\lambda\}, A^0_\nu\} + 2\{\{F^0_{\mu\kappa}, F^0_{\sigma\lambda}\}, A^0_\nu\} + 2i[\partial_\kappa F^0_{\mu\sigma}, \partial_\lambda A^0_\nu] - \{F^0_{\mu\sigma}, \{A^0_\kappa, \partial_\lambda A^0_\nu\}\} + \{F^0_{\mu\sigma}, \{F^0_{\kappa\nu}, A^0_\lambda\}\}), \quad (28)$$

where $F^0_{\mu\nu} = \partial_\mu A^0_\nu - \partial_\nu A^0_\mu - i[A^0_\mu, A^0_\nu]$ is the Lie algebra field strength. Finally, to construct LCNCSM we need to determine the hybrid Seiberg-Witten map for Higgs fields. The gauge transformation for Higgs field which transforms on the left and on the right under two arbitrary gauge groups with corresponding gauge potentials is

$$\delta\hat{\Phi} = i\hat{\Lambda}_\alpha * \hat{\Phi} - i\hat{\Phi} * \hat{\Lambda}'_\gamma, \quad (29)$$

in which $\hat{\Phi}$, $\hat{\Lambda}_\alpha$, and $\hat{\Lambda}'_\gamma$ are defined by the Eqs. (11) and (14). Therefore, the noncommutative Higgs transformation at the first order of $\theta_{\mu\nu}$ reduces to

$$\delta\Phi^1 - i\alpha\Phi^1 + i\Phi^1\gamma = -\tfrac{1}{2}\theta^{\mu\nu}\partial_\mu\Phi^0\partial_\nu\Phi^0 + \tfrac{1}{2}\theta^{\mu\nu}\partial_\mu\Phi^0\partial_\nu\gamma + i\Lambda^1_\alpha\Phi^0 - i\Phi^0\Lambda^{1'}_\gamma, \quad (30)$$

with the solution

$$\Phi^1 = \frac{1}{2}\theta^{\mu\nu}A^0_\nu\Big(\partial_\mu\Phi^0 - \frac{i}{2}(A^0_\mu\Phi^0 - \Phi^0A^{0'}_\mu)\Big) + \frac{1}{2}\theta^{\mu\nu}\Big(\partial_\mu\Phi^0 - \frac{i}{2}(A^0_\mu\Phi^0 - \Phi^0A^{0'}_\mu)\Big)A^{0'}_\nu, \quad (31)$$

and at the next order leads to

$$\delta\Phi^2 - i\alpha\Phi^2 + i\Phi^2\gamma = \frac{i}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}\partial_\mu\partial_\kappa\Phi^0\partial_\nu\partial_\lambda\gamma - \frac{i}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}\partial_\mu\partial_\kappa\alpha\partial_\nu\partial_\lambda\Phi^0 - \frac{1}{2}\theta^{\mu\nu}\partial_\mu\Lambda^1_\alpha\partial_\nu\Phi^0 - \frac{1}{2}\theta^{\mu\nu}\partial_\mu\alpha\partial_\nu\Phi^1 + \frac{1}{2}\theta^{\mu\nu}\partial_\mu\Phi^0\partial_\nu\Lambda^{1'}_\gamma + \frac{1}{2}\theta^{\mu\nu}\partial_\mu\Phi^1\partial_\nu\gamma + i\Lambda^1_\alpha\Phi^1 + i\Lambda^2_\alpha\Phi^0 - i\Phi^0\Lambda^{2'}_\gamma - \Phi^1\Lambda^{1'}_\gamma, \quad (32)$$

which can be solved up to this order as

$$\Phi^2 = \Phi^2_1 + \Phi^2_2 + \Phi^2_3, \quad (33)$$

where $\Phi^2_1 = \Psi^2$ and is given in Eq. (24),

$$\Phi^2_2 = \theta^{\mu\nu}\theta^{\kappa\lambda}\Big(-\frac{3}{32}\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\kappa A^{0\prime}{}_\nu A^{0\prime}{}_\mu + \frac{1}{8}\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\nu A^{0\prime}{}_\kappa A^{0\prime}{}_\mu - \frac{1}{16}\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\nu A^{0\prime}{}_\mu A^{0\prime}{}_\kappa - \frac{5i}{32}\Phi^0A^{0\prime}{}_\lambda\partial_\mu A^{0\prime}{}_\kappa A^{0\prime}{}_\nu - \frac{i}{32}\Phi^0\partial_\mu A^{0\prime}{}_\lambda A^{0\prime}{}_\kappa A^{0\prime}{}_\nu - \frac{i}{8}\partial_\mu\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\kappa A^{0\prime}{}_\nu - \frac{i}{16}\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\nu\partial_\mu A^{0\prime}{}_\kappa - \frac{i}{16}\Phi^0\partial_\mu A^{0\prime}{}_\lambda A^{0\prime}{}_\nu A^{0\prime}{}_\kappa + \frac{i}{8}\partial_\mu\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\nu A^{0\prime}{}_\kappa + \frac{i}{32}\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\nu\partial_\kappa A^{0\prime}{}_\mu - \frac{3i}{32}\Phi^0A^{0\prime}{}_\lambda\partial_\kappa A^{0\prime}{}_\nu A^{0\prime}{}_\mu - \frac{i}{8}\partial_\kappa\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\nu A^{0\prime}{}_\mu - \frac{1}{16}\Phi^0\partial_\nu A^{0\prime}{}_\lambda\partial_\mu A^{0\prime}{}_\kappa - \frac{1}{8}\partial_\nu\Phi^0\partial_\mu A^{0\prime}{}_\lambda A^{0\prime}{}_\kappa + \frac{1}{8}\partial_\mu\partial_\kappa\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\nu + \frac{1}{4}\partial_\kappa\Phi^0\partial_\mu A^{0\prime}{}_\lambda A^{0\prime}{}_\nu + \frac{1}{8}\partial_\mu\Phi^0A^{0\prime}{}_\lambda\partial'_\kappa A^{0\prime}{}_\nu - \frac{i}{8}\partial_\nu\partial_\kappa\Phi^0\partial_\mu A^{0\prime}{}_\lambda\Big), \quad (34)$$

and Φ^2_3 is

$$\Phi^2_3 = \theta^{\mu\nu}\theta^{\kappa\lambda}\Big(-\frac{i}{4}A^0{}_\nu\partial_\mu A^0{}_\kappa\Phi^0A^{0\prime}_\lambda + \frac{i}{8}A^0{}_\mu A^0{}_\nu\partial_\kappa\Phi^0A^{0\prime}{}_\lambda - \frac{i}{8}A^0{}_\kappa A^0{}_\nu\Phi^0\partial_\mu A^{0\prime}{}_\lambda - \frac{i}{4}A^0{}_\kappa\Phi^0\partial_\mu A^{0\prime}{}_\lambda A^{0\prime}{}_\nu + \frac{i}{8}A^0{}_\nu\partial_\mu\Phi^0A^{0\prime}{}_\kappa A^{0\prime}{}_\lambda - \frac{i}{8}\partial_\mu A^0{}_\kappa\Phi^0A^{0\prime}{}_\nu A^{0\prime}{}_\lambda + \frac{1}{8}A^0{}_\mu A^0{}_\nu A^0{}_\kappa\Phi^0A^{0\prime}{}_\lambda - \frac{1}{8}A^0{}_\nu\Phi^0A^{0\prime}{}_\mu A^{0\prime}{}_\kappa A^{0\prime}{}_\lambda + \frac{1}{4}\partial_\mu A^0{}_\kappa\Phi^0\partial_\nu A^{0\prime}{}_\lambda + \frac{1}{4}A^0{}_\nu\partial_\mu\partial_\alpha\Phi^0A^{0\prime}{}_\beta + \frac{1}{8}\partial_\kappa A^0{}_\nu\Phi^0\partial_\mu A^{0\prime}{}_\lambda - \frac{1}{8}A^0{}_\mu A^0{}_\kappa\Phi^0A^{0\prime}{}_\nu A^{0\prime}{}_\lambda - \frac{1}{16}A^0{}_\mu A^0{}_\nu\Phi^0A^{0\prime}{}_\kappa A^{0\prime}{}_\lambda + \frac{i}{8}F^0{}_{\kappa\mu}A^0{}_\nu\Phi^0A^{0\prime}{}_\lambda + \frac{i}{8}A^0{}_\kappa\Phi^0A^{0\prime}{}_\nu F^{0\prime}{}_{\lambda\mu} - \frac{i}{4}A^0{}_\kappa\partial_\mu\Phi^0A^{0\prime}{}_\lambda A^{0\prime}{}_\nu + \frac{1}{8}A^0{}_\kappa\partial_\mu\Phi^0F^{0\prime}{}_{\nu\lambda} + \frac{i}{4}A^0{}_\mu A^0{}_\kappa\partial_\nu\Phi^0A^{0\prime}{}_\lambda + \frac{1}{8}F^0{}_{\mu\kappa}\partial_\nu\Phi^0A^{0\prime}{}_\lambda\Big). \quad (35)$$

It should be noted that the Seiberg-Witten maps cannot be obtained uniquely. Indeed the maps for each noncommutative fields are arbitrary up to a solution of the corresponding homogeneous equations with undetermined coefficient. However, the physical results do not depend on this freedom, therefore, those terms which are solutions of the homogeneous equation are physically irrelevant [6].

IV. CONSTRUCTING LCNCSM ACTION

Lorentz-conserving noncommutative standard model can be constructed in three steps:

(i) Replacing the ordinary products with the star products.
(ii) Substituting the noncommutative fields for each corresponding commutative one.
(iii) Performing the trace with respect to the noncommutative tensor with even weight function to make the theory Lorentz invariant.

As was mentioned in the previous section, the noncommutative fields cannot be uniquely determined by the Seiberg-Witten map i.e. there is freedom in construction of the noncommutative gauge parameter and fields by the Seiberg-Witten map. Therefore, the fields can be appropriately redefined to neglect physically irrelevant terms in the action. In this paper we extend the electroweak sector of the standard model to the noncommutative space. One can easily follow the three steps prescription to derive the action of the LCNCSM for this sector. To this end we separate the action into four parts as

$$S_{\mathrm{LCNCSM}} = S_{\mathrm{fermion}} + S_{\mathrm{gauge}} + S_{\mathrm{Higgs}} + S_{\mathrm{yukawa}}, \tag{36}$$

in which each term will be explained in the following.

(i) S_{fermion}: This part describes the fermion interaction in the electroweak sector of the LCNCSM and can be written easily as

$$S_{\mathrm{fermion}} = \int d^6\theta \int d^4x W(\theta)(\bar{\hat{L}} * i\hat{\not{\mathcal{D}}}\hat{L} + \bar{\hat{R}} * i\hat{\not{\mathcal{D}}}\hat{R}), \tag{37}$$

with $\hat{L} = \hat{L}_l$ or $\hat{L}_Q$ where

$$\hat{L}_l = \begin{pmatrix} \hat{\Psi}_{L_{\nu_l}} \\ \hat{\Psi}_{L_l} \end{pmatrix}, \qquad \hat{L}_Q = \begin{pmatrix} \hat{\Psi}_{L_u} \\ \hat{\Psi}_{L_d} \end{pmatrix}, \tag{38}$$

and

$$\hat{R} = \hat{\Psi}_{R_l}, \hat{\Psi}_{R_u}, \hat{\Psi}_{R_d}, \tag{39}$$

in which subscripts u and d, respectively, refer to up-type and down-type quarks for all generations and the subscripts l and Q stand for the leptons and quarks, respectively. The covariant derivative $\hat{\mathcal{D}}_\mu$ in terms of the gauge fields W_μ, B_μ, and G_μ is defined as

$$\hat{\mathcal{D}}_\mu \hat{L} = \Big(\partial_\mu - igT^a\hat{W}^a_\mu - ig'\frac{Y}{2}\hat{B}_\mu - ig_sT^a_s\hat{G}^a_\mu\Big)\hat{L}, \tag{40}$$

and

$$\hat{\mathcal{D}}_\mu \hat{R} = \Big(\partial_\mu - ig'\frac{Y}{2}\hat{B}_\mu - ig_sT^a_s\hat{G}^a_\mu\Big)\hat{R}, \tag{41}$$

where T^a_s, T^a, and $\frac{Y}{2}$ are the generators of the gauge groups $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$, respectively. The gauge eigenstate weak bosons are related to the mass eigenstates (i.e. the electroweak gauge bosons $(W^\pm, Z)$ and the photon (A)) by

$$W^\pm_\mu = \frac{W^1_\mu \mp W^2_\mu}{\sqrt{2}}, \tag{42}$$

$$Z_\mu = \frac{-g'B_\mu + gW^3_\mu}{\sqrt{g^2+g'^2}} = -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu, \tag{43}$$

$$A_\mu = \frac{gB_\mu + g'W^3_\mu}{\sqrt{g^2+g'^2}} = cos\theta_W B_\mu + \sin\theta_W W^3_\mu, \tag{44}$$

where the electric charge is $e = g\sin\theta_w = g'\cos\theta_w$. It should be noted that the covariant derivative acts on the fermion or Higgs fields according to their representations given in Table I. Furthermore, the fields with a hat are the noncommutative fields and should be replaced by the appropriate expressions in terms of the ordinary fields which are obtained up to the second order of $\theta_{\mu\nu}$ by using the Seiberg-Witten maps in Eqs. (23) and (24), Eqs. (26), (28), and (31)–(33) for the fermion field, gauge boson, and Higgs fields, respectively. One also can see that the commutative gauge potential A^0_μ appears in the expansion of all quantities of noncommutative gauge theory; therefore, for the matter fields we have to use the appropriate vector fields corresponding to

TABLE I. Matter and Higgs fields content of the standard model and their representations.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
e_R	1	1	-2
$L_l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	1	2	-1
u_R	3	1	$\frac{4}{3}$
d_R	3	1	$\frac{-2}{3}$
$L_q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	$\frac{1}{3}$
$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	1

their representations. Namely,

$$\hat{L}_l[L_l^0, A_\mu^0] = \hat{L}_l\bigg[L_l^0, gT^a W_\mu^a + g'\frac{Y}{2}B_\mu\bigg], \quad (45)$$

$$\hat{L}_Q[L_Q^0, A_\mu^0] = \hat{L}_Q\bigg[L_Q^0, g_s T_s^a G_\mu^a + gT^a W_\mu^a + g'\frac{Y}{2}B_\mu\bigg], \quad (46)$$

$$\hat{\Psi}_{R_l}[\Psi_{R_l}, A_\mu^0] = \hat{\Psi}_{R_l}\bigg[\Psi_{R_l}, g'\frac{Y}{2}B_\mu\bigg], \quad (47)$$

$$\hat{\Psi}_{R_Q}[\Psi_{R_Q}, A_\mu^0] = \hat{\Psi}_{R_Q}\bigg[\Psi_{R_Q}, g_s T_s^a G_\mu^a + g'\frac{Y}{2}B_\mu\bigg], \quad (48)$$

where A_μ^0 for the left handed sector is

$$A_\mu^0 = \tfrac{1}{2}g'YB_\mu + gT^aW_\mu^a + g_sT_s^aG^a, \quad (49)$$

and for the right handed sector is

$$A_\mu^0 = \tfrac{1}{2}g'YB_\mu + g_sT_s^aG^a. \quad (50)$$

Now inserting the appropriate expansion of the noncommutative fields in terms of the ordinary fields in (37) leads to the following equation for S_{fermion} at the leading order of $\theta_{\mu\nu}$ as

$$\begin{aligned} S_{\text{fermion}} = {}& \int d^4x(\bar{L}i\not{D}L + \bar{R}i\not{D}R) + \int d^6\theta \int d^4x\theta^{\mu\nu}\theta^{\kappa\lambda}W(\theta)\Big(-\frac{i}{8}\bar{L}\gamma^\rho F^0_{\mu\kappa}F^0_{\lambda\rho}\mathcal{D}^0_\nu L - \frac{i}{4}\bar{L}\gamma^\rho F^0_{\mu\rho}F^0_{\nu\kappa}\mathcal{D}^0_\lambda L \\ & - \frac{1}{8}\bar{L}\gamma^\rho(\mathcal{D}^0_\mu F^0_{\kappa\rho})\mathcal{D}^0_\nu\mathcal{D}^0_\lambda L - \frac{i}{8}\bar{L}\gamma^\rho F^0_{\mu\nu}F^0_{\kappa\rho}\mathcal{D}^0_\lambda L\Big) + \int d^6\theta\int d^4x\theta^{\mu\nu}\theta^{\kappa\lambda}W(\theta) \\ & \times\Big(-\frac{i}{8}\bar{R}\gamma^\rho F^0_{\mu\kappa}F^0_{\lambda\rho}\mathcal{D}^0_\nu R - \frac{i}{4}\bar{R}\gamma^\rho F^0_{\mu\rho}F^0_{\nu\kappa}\mathcal{D}^0_\lambda R - \frac{1}{8}\bar{R}\gamma^\rho(\mathcal{D}^0_\mu F^0_{\kappa\rho})\mathcal{D}^0_\nu\mathcal{D}^0_\lambda R - \frac{i}{8}\bar{R}\gamma^\rho F^0_{\mu\nu}F^0_{\kappa\rho}\mathcal{D}^0_\lambda R\Big). \end{aligned} \quad (51)$$

In obtaining (51) the irrelevant terms are ignored by redefinition of the fields via the freedom in determining of the Seiberg-Witten maps. Equation (51) shows that besides the usual standard model and the NCSM interactions, there are new couplings between the fermions and the electroweak gauge bosons such as $ff\gamma\gamma\gamma$, $ff\gamma\gamma Z$, $ff\gamma ZZ$, $ffZZZ$, and so on. The vertex $ff\gamma\gamma\gamma$ is one of the vertices of LCNCQED [7,8]. In the noncommutative space, a neutral particle can interact with photon in the adjoint representation [3,15]. These interactions are proportional to the odd power of $\theta_{\mu\nu}$; therefore, they are absent in the Lorentz-invariant noncommutative field theory, see the action (51). Nevertheless, in contrast to the minimal NCSM there is no photon-neutrino coupling in the LCNCSM [3].

(ii) S_{gauge}: This term contains the kinetic terms for the gauge bosons of the standard model. The general form of the gauge invariant action for the gauge sector of the LCNCSM can be written as follows:

$$S_{\text{gauge}} = -\frac{1}{2}\int d^6\theta \int d^4xW(\theta)\,\text{Tr}\bigg\{\frac{1}{G^2}\hat{F}^{\mu\nu} * \hat{F}_{\mu\nu}\bigg\}, \quad (52)$$

where Tr is trace over all representations. G is an operator which determines the coupling constants of the theory and commutes with all generators of SU(2) and SU(3) and is defined as [16]

$$\frac{1}{g_I^2} = \text{Tr}\bigg\{\frac{1}{G^2}T_I^aT_I^a\bigg\}, \quad (53)$$

where g_I and T_I^a are the ordinary coupling constants and generators of the gauge group, respectively. Since the gauge group is extended to incorporate the noncommutative corrections, according to the Seiberg-Witten map and using the enveloping algebra [5], one encounters all ordered tensor powers of the generators T^a in the trace in (52). Therefore, the trace in the kinetic terms for gauge bosons in contrast to the ordinary case is not unique and depends on a choice of the representation of the gauge group [16–18]. The minimal choice can be the simplest choice in which Tr is a sum of three traces over the U(1), SU(2), and SU(3) sectors with the fundamental representations for SU(2) and SU(3) generators in the corresponding traces and [16]

$$Y = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (54)$$

However, the freedom in the choice of the traces can be used to construct new versions of the LCNCSM. Since the fermion-gauge boson interactions remain the same regardless of the choice of traces in the gauge sector, the matter sector of the action is the same for all versions of the LCNCSM. Nevertheless, in the nonminimal versions of the theory, new parameters appear which cannot be uniquely obtained in the theory [16,18]. The general form of S_{gauge} in terms of SM fields can be obtained by inserting the expansion of the field strength $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i[\hat{A}_\mu, \hat{A}_\nu]$ in terms of the commutative one in (52) as follows:

$$
\begin{aligned}
S_{\mathrm{gauge}} = &-\frac{1}{2}\int d^6\theta \int d^4x W(\theta)\,\mathrm{Tr}\bigg\{\frac{1}{G^2}\bigg\{-\frac{1}{2}F^0_{\mu\nu}F^{0\mu\nu} + \theta^{\mu\nu}\theta^{\kappa\lambda}\bigg(\frac{1}{8}F^0_{\mu\nu}F^0_{\kappa\lambda}F^0_{\rho\sigma}F^{0\rho\sigma} - \frac{i}{4}F^0_{\mu\nu}(\mathcal{D}^0_\kappa F^0_{\rho\sigma})(\mathcal{D}^0_\lambda F^{0\rho\sigma}) \\
&- \frac{1}{8}(\mathcal{D}^0_\mu \mathcal{D}^0_\kappa F^0_{\rho\sigma})(\mathcal{D}^0_\nu \mathcal{D}^0_\lambda F^{0\rho\sigma}) + \frac{i}{2}(\mathcal{D}^0_\mu F^0_{\rho\kappa})(\mathcal{D}^0_\nu F^0_{\sigma\lambda})F^{0\rho\sigma} + \frac{1}{2}F^0_{\mu\rho}F^0_{\nu\sigma}F^{0\rho}_{\kappa}F^{0\sigma}_{\lambda} + \frac{1}{2}F^0_{\mu\rho}F^0_{\nu\sigma}F^{0\sigma}_{\kappa}F^{0\rho}_{\lambda} \\
&- \frac{1}{2}F^0_{\mu\nu}F^0_{\kappa\rho}F^0_{\lambda\sigma}F^{0\rho\sigma} - \frac{1}{2}F^0_{\kappa\rho}F^0_{\lambda\sigma}F^0_{\mu\nu}F^{0\rho\sigma} + \frac{1}{2}(F^0_{\mu\kappa}F^0_{\nu\rho}F^0_{\lambda\sigma} + 2F^0_{\nu\rho}F^0_{\mu\kappa}F^0_{\lambda\sigma} + F^0_{\lambda\sigma}F^0_{\nu\rho}F^0_{\mu\kappa})F^{0\rho\sigma}\bigg)\bigg\}\bigg\},
\end{aligned}
\tag{55}
$$

where $F^0_{\mu\nu} = \partial_\mu A^0_\nu - \partial_\nu A^0_\mu - i[A^0_\mu, A^0_\nu]$ is the field strength in the usual space and

$$
A_\mu = \tfrac{1}{2}g' Y B_\mu + g T^a W^a_\mu + g_s T^a_s G^a_{s\mu}. \tag{56}
$$

In Eq. (55) there are new vertices in all versions of LCNCSM in comparison with the commutative standard model. Meanwhile, in contrast to the minimal noncommutative standard model, new interactions appear in the electroweak part of the LCNCSM at the leading order of $\theta_{\mu\nu}$. However, inserting the electroweak part of (56) in (55) and performing the trace operation in the minimal case leads to the electroweak sector of the minimal version of S_{gauge}:

$$
\begin{aligned}
S^{m\mathrm{LCNCSM}}_{\mathrm{gauge}} = &-\frac{1}{2}\int d^6\theta \int d^4x W(\theta)\bigg(\frac{1}{2}B_{\mu\nu}B^{\mu\nu} + W^a_{\mu\nu}W^{a\mu\nu} + \theta^{\mu\nu}\theta^{\kappa\lambda}\bigg(g'^2\bigg(\frac{1}{64}B_{\mu\nu}B_{\kappa\lambda}B_{\rho\sigma} - \frac{1}{8}B_{\mu\nu}B_{\kappa\rho}B_{\lambda\sigma} \\
&+ \frac{1}{4}B_{\mu\kappa}B_{\nu\rho}B_{\lambda\sigma}\bigg)B^{\rho\sigma} + \frac{g^2}{4}\bigg(\frac{1}{8}(W^a_{\mu\nu}W^a_{\kappa\lambda})(W^b_{\rho\sigma}W^{b\rho\sigma}) + (W^a_{\mu\rho}W^{a\rho}_{\lambda})(W^b_{\nu\sigma}W^{b\sigma}_{\kappa}) \\
&+ (W^a_{\mu\rho}W^{a\sigma}_{\lambda})(W^b_{\nu\sigma}W^{b\rho}_{\kappa}) - (W^a_{\mu\nu}W^{a\rho\sigma})(W^b_{\kappa\rho}W^b_{\lambda\sigma}) + 2(W^a_{\mu\kappa}W^a_{\nu\rho})(W^b_{\lambda\sigma}W^{b\rho\sigma})\bigg) \\
&+ g'g\bigg(\frac{1}{32}(B_{\mu\nu}B_{\kappa\lambda})(W^a_{\rho\sigma}W^{a\rho\sigma}) + \frac{1}{32}(B_{\rho\sigma}B^{\rho\sigma})(W^a_{\mu\nu}W^a_{\kappa\lambda}) + \frac{1}{8}(B_{\mu\nu}B_{\rho\sigma})(W^a_{\kappa\lambda}W^{a\rho\sigma}) \\
&- \frac{1}{2}(B_{\mu\nu}B_{\kappa\rho})(W^a_{\lambda\sigma}W^{a\rho\sigma}) - \frac{1}{2}(B_{\lambda\sigma}B^{\rho\sigma})(W^a_{\mu\nu}W^a_{\kappa\rho}) - \frac{1}{4}(B_{\mu\nu}B^{\rho\sigma})(W^a_{\kappa\rho}W^a_{\lambda\sigma}) - \frac{1}{4}(B_{\kappa\rho}B_{\lambda\sigma})(W^a_{\mu\nu}W^{a\rho\sigma}) \\
&+ \frac{1}{2}(B_{\mu\kappa}B^{\rho\sigma})(W^a_{\nu\rho}W^a_{\lambda\sigma}) + \frac{1}{2}(B_{\nu\rho}B_{\lambda\sigma})(W^a_{\mu\kappa}W^{a\rho\sigma}) + (B_{\mu\kappa}B_{\nu\rho})(W^a_{\lambda\sigma}W^{a\rho\sigma}) + (B_{\lambda\sigma}B^{\rho\sigma})(W^a_{\mu\kappa}W^a_{\nu\rho})\bigg) \\
&- \frac{g^3}{8}\epsilon^{abc}W^a_\nu \partial_\lambda W^{b\rho\sigma}\partial_\mu\partial_\kappa W^c_{\rho\sigma} - \frac{g^4}{16}\epsilon^{abc}\epsilon^{dce}W^a_\kappa W^{b\rho\sigma}W^d_\nu \partial_\mu\partial_\kappa W^e_{\rho\sigma} - \frac{g^4}{16}\epsilon^{abc}\epsilon^{dec}\partial_\mu(W^a_\kappa W^{b\rho\sigma})W^d_\nu\partial_\lambda W^e_{\rho\sigma} \\
&- \frac{g^4}{16}\epsilon^{abc}\epsilon^{dec}W^a_\mu \partial_\kappa W^{b\rho\sigma}\partial_\nu W^d_\kappa W^e_{\rho\sigma} - \frac{g^4}{16}\epsilon^{abc}\epsilon^{dec}W^a_\mu\partial_\kappa W^{b\rho\sigma}W^d_\nu\partial_\lambda W^e_{\rho\sigma} \\
&- \frac{g^4}{16}\epsilon^{abc}\epsilon^{dec}W^a_\kappa W^{b\rho\sigma}W^d_\mu \partial_\nu u \partial_\lambda W^e_{\rho\sigma} - \frac{g^5}{16}\epsilon^{abc}\epsilon^{dce}\epsilon^{fge}W^a_\lambda W^{b\rho\sigma}W^d_\nu W^f_\mu \partial_\kappa W^g_{\rho\sigma} \\
&- \frac{g^5}{16}\epsilon^{abc}\epsilon^{dce}\epsilon^{fge}W^a_\lambda W^{b\rho\sigma}W^d_\nu \partial_\mu(W^f_\kappa W^g_{\rho\sigma}) - \frac{g^5}{16}\epsilon^{abc}\epsilon^{dce}\epsilon^{fge}W^a_\kappa W^{b\rho\sigma}W^d_\mu \partial_\nu(W^f_\lambda W^g_{\rho\sigma}) \\
&- \frac{g^5}{16}\epsilon^{abc}\epsilon^{dce}\epsilon^{fge}W^a_\kappa W^{b\rho\sigma}W^d_\mu W^f_\nu \partial_\lambda W^g_{\rho\sigma} - \frac{g^6}{16}\epsilon^{abc}\epsilon^{dce}\epsilon^{fgh}\epsilon^{ihe}W^a_\kappa W^{b\rho\sigma}W^d_\mu W^f_\kappa W^g_{\rho\sigma}W^i_\nu\bigg)\bigg),
\end{aligned}
\tag{57}
$$

where for the hypercharge, $B_{\mu\nu}$ is defined as

$$
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{58}
$$

and for SU(2) gauge fields we define

$$
W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu. \tag{59}
$$

As (57) shows in the minimal version of LCNCSM (mLCNCSM) at the lowest order there are vertices such as $\gamma\gamma\gamma\gamma$, $\gamma\gamma\gamma Z$, $\gamma\gamma ZZ$, γZZZ, $ZZZZ$, and so on.

(iii) S_{Higgs}: This part of the action is responsible for breaking of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge symmetry of the standard model via the Higgs mechanism which in turn generates masses for the gauge bosons. The Higgs action in the Lorentz-invariant noncommutative space can be written as

$$S_{\text{Higgs}} = \int d^6\theta \int d^4x W(\theta)((\hat{\mathcal{D}}_\mu \hat{\Phi})^\dagger * (\hat{\mathcal{D}}^\mu \hat{\Phi}) - \mu^2 \hat{\Phi}^\dagger * \hat{\Phi} - \lambda(\hat{\Phi}^\dagger * \hat{\Phi})^2), \tag{60}$$

where the noncommutative Higgs field is given by the hybrid Seiberg-Witten map which is obtained already up to the second order of $\theta_{\mu\nu}$ in the previous section. In the leading order of the expansion in $\theta_{\mu\nu}$, one explicitly obtains

$$\begin{aligned} S_{\text{Higgs}} = \int d^6\theta \int d^4x W(\theta)&((\mathcal{D}^0_\alpha \Phi^0)^\dagger (\mathcal{D}^{0\alpha}\Phi^0) - \mu^2 \Phi^{0\dagger}\Phi^0 - \lambda(\Phi^{0\dagger}\Phi^0)^2 \\ &+ \theta^{\mu\nu}\theta^{\kappa\lambda}((\mathcal{D}^0_\alpha \Phi^0)^\dagger (\mathcal{D}^0_\alpha \Phi^2_{\mu\nu\kappa\lambda}) + (\mathcal{D}^0_\alpha \Phi^2_{\mu\nu\kappa\lambda})^\dagger (\mathcal{D}^0_\alpha \Phi^0) + (\mathcal{D}^0_\alpha \Phi^1_{\mu\nu})^\dagger (\mathcal{D}^0_\alpha \Phi^1_{\kappa\lambda}) \\ &- i(\mathcal{D}^0_\alpha \Phi^0)^\dagger A^{2\alpha}{}_{\mu\nu\kappa\lambda}\Phi^0 + i(\Phi^0)^\dagger A^{2\alpha}{}_{\mu\nu\kappa\lambda}(\mathcal{D}^0_\alpha \Phi^0) - i(\mathcal{D}^0_\alpha \Phi^0)^\dagger A^{1\alpha}{}_{\mu\nu}\Phi^1_{\kappa\lambda} \\ &+ i(\Phi^1_{\mu\nu})^\dagger A^{1\alpha}{}_{\kappa\lambda}(\mathcal{D}^0_\alpha \Phi^0) + i\Phi^{0\dagger}A^{1\alpha}{}_{\mu\nu}(\mathcal{D}^0_\alpha \Phi^1_{\kappa\lambda}) - i(\mathcal{D}^0_\alpha \Phi^1_{\mu\nu})^\dagger A^{1\alpha}{}_{\kappa\lambda}\Phi^0 + \Phi^{0\dagger}A^{1\alpha}{}_{\mu\nu}A^1_{\alpha\kappa\lambda}\Phi^0)). \end{aligned} \tag{61}$$

Here $\mathcal{D}_\mu$ can be appropriately defined very similarly to (40) and (41) according to the representations given in Table I. The functions $\Phi^1_{\mu\nu}$, $\Phi^2_{\mu\nu\kappa\lambda}$, $A^1_{\alpha\mu\nu}$, and $A^2_{\alpha\mu\nu\kappa\lambda}$ can be obtained easily by comparing (26), (28), (31), and (33), respectively, with

$$\begin{aligned} &\Phi^1(\Phi^0, A^0_\mu, A^{0\prime}_\mu) = \theta^{\mu\nu}\Phi^1_{\mu\nu}\left(\Phi^0, \frac{g'}{2}YB_\mu + gT^aW^a_\mu, 0\right), \quad \Phi^2(\Phi^0, A^0_\mu, A^{0\prime}_\mu) = \theta^{\mu\nu}\theta^{\kappa\lambda}\Phi^2_{\mu\nu\kappa\lambda}\left(\Phi^0, \frac{g'}{2}YB_\mu + gT^aW^a_\mu, 0\right), \\ &A^1_\alpha(A^0_\mu) = \theta^{\mu\nu}A^1_{\alpha\mu\nu}\left(\frac{g'}{2}YB_\mu + gT^aW^a_\mu\right), \quad A^2_\alpha(A^0_\mu) = \theta^{\mu\nu}\theta^{\kappa\lambda}A^2_{\alpha\mu\nu\kappa\lambda}\left(\frac{g'}{2}YB_\mu + gT^aW^a_\mu\right). \end{aligned} \tag{62}$$

The $\theta_{\mu\nu}$-independent part of (61) is the same as the usual action for the Higgs part of the standard model. Meanwhile, the remaining part of the action contains the derivative of the Higgs field (which does not have contribution to the minimum value of the potential) and terms containing the gauge and Higgs fields both. The latter terms are also vanished by fixing the vacuum expectation value. Therefore, the spontaneous symmetry breaking occurs according to the commutative standard model but in contrast to the standard model, numerous new couplings between the Higgs and the electroweak gauge bosons appear in this theory. One can see easily from (61) that among these new interactions there are couplings solely between the gauge fields with the coupling strength proportional to $(\langle\Phi\rangle_0)^2$.

(iv) S_{yukawa}: This part describes the Yukawa interactions between fermions and Higgs field which lead to the mass generation for fermions after the symmetry breaking. The Yukawa action of LCNCSM can be written as

$$S_{\text{yukawa}} = -\int d^6\theta \int d^4x W(\theta)(G_{ij}\bar{\hat{L}}^i * \hat{\Phi} * \hat{R}^j + G_{ij}\bar{\hat{R}}^i * \hat{\Phi}^\dagger * \hat{L}^j), \tag{63}$$

where i and j refer to the different generations. Once again the noncommutative fields have to be expanded in terms of the corresponding ordinary fields up to the second order of $\theta_{\mu\nu}$, but one should note that $\hat{\Phi}$ has to be written appropriately with respect to the representation of its left and right fields according to Table I. After some algebra, the general form for the Yukawa action, up to the leading order, is

$$\begin{aligned} S_{\text{yukawa}} = -\int d^6\theta \int d^4x W(\theta)G_{ij}\Big(&\bar{L}^i\Phi R^j + \theta^{\mu\nu}\theta^{\kappa\lambda}\Big(\bar{L}^i\Phi^1_{\mu\nu}R^{1j}_{\kappa\lambda} + \bar{L}^{1i}_{\mu\nu}\Phi^1_{\kappa\lambda}R^j + \bar{L}^{1i}_{\mu\nu}\Phi R^{1j}_{\kappa\lambda} \\ &+ \bar{L}^{2i}_{\mu\nu\kappa\lambda}\Phi R^j + \bar{L}^i\Phi R^{2j}_{\mu\nu\kappa\lambda} + \bar{L}^i\Phi^2_{\mu\nu\kappa\lambda}R^j + \frac{i}{2}\bar{L}^{1i}_{\mu\nu}\partial_\kappa\Phi\partial_\lambda R^j + \frac{i}{2}\bar{L}^i\partial_\mu\Phi^1_{\kappa\lambda}\partial_\nu R^j \\ &+ \frac{i}{2}\bar{L}^i\partial_\mu\Phi\partial_\nu R^{1j}_{\kappa\lambda} - \frac{1}{8}\bar{L}^i\partial_\mu\partial_\kappa\Phi\partial_\nu\partial_\lambda R^j\Big)\Big) + \text{C.C.}, \end{aligned} \tag{64}$$

where L, R, and Φ are ordinary fields and for leptons we define

$$R^1(R^0, A^0_\mu) = \theta^{\mu\nu} R^1_{\mu\nu}\Big(R^0, \frac{g'}{2} Y B_\mu\Big), \qquad R^2(R^0, A^0_\mu) = \theta^{\mu\nu}\theta^{\kappa\lambda} R^2_{\mu\nu\kappa\lambda}\Big(R^0, \frac{g'}{2} Y B_\mu\Big),$$
$$L^1(L^o, A^0_\mu) = \theta^{\mu\nu} L^1_{\mu\nu}\Big(L^0, \frac{g'}{2} Y B_\mu + g T^a W^a_\mu\Big), \qquad L^2(L^o, A^0_\mu) = \theta^{\mu\nu}\theta_{\kappa\lambda} L^2_{\mu\nu\kappa\lambda}\Big(L^0, \frac{g'}{2} Y B_\mu + g T^a W^a_\mu\Big),$$
$$\Phi^1(\Phi^0, A^0_\mu, A'^0_\mu) = \theta^{\mu\nu}\Phi^1_{\mu\nu}\Big(\Phi^0, \frac{1}{2} g' Y B_\mu + g T^a W^a_\mu, \frac{1}{2} g' Y B_\mu\Big),$$
$$\Phi^2(\Phi^0, A^0_\mu, A'^0_\mu) = \theta^{\mu\nu}\theta^{\kappa\lambda}\Phi^2_{\mu\nu\kappa\lambda}\Big(\Phi^0, \frac{1}{2} g' Y B_\mu + g T^a W^a_\mu, \frac{1}{2} g' Y B_\mu\Big), \tag{65}$$

while for quarks $g_s T^a_s G^a_s$ should be added to gauge fields to define

$$R^1(R^0, A^0_\mu) = \theta^{\mu\nu} R^1_{\mu\nu}\Big(R^0, \frac{g'}{2} Y B_\mu + g_s T^a_s G^a_\mu\Big), \qquad R^2(R^0, A^0_\mu) = \theta^{\mu\nu}\theta^{\kappa\lambda} R^2_{\mu\nu\kappa\lambda}\Big(R^0, \frac{g'}{2} Y B_\mu + g_s T^a_s G^a_\mu\Big),$$
$$L^1(L^o, A^0_\mu) = \theta^{\mu\nu} L^1_{\mu\nu}\Big(L^0, \frac{g'}{2} Y B_\mu + g T^a W^a_\mu + g_s T^a_s G^a_\mu\Big),$$
$$L^2(L^o, A^0_\mu) = \theta^{\mu\nu}\theta\kappa\lambda L^2_{\mu\nu\kappa\lambda}\Big(L^0, \frac{g'}{2} Y B_\mu + g T^a W^a_\mu + g_s T^a_s G^a_\mu\Big), \tag{66}$$
$$\Phi^1(\Phi^0, A^0_\mu, A'^0_\mu) = \theta^{\mu\nu}\Phi^1_{\mu\nu}\Big(\Phi^0, \frac{1}{2} g' Y B_\mu + g T^a W^a_\mu + g_s T^a_s G^a, \frac{1}{2} g' Y B_\mu + g_s T^a_s G^a\Big),$$
$$\Phi^2(\Phi^0, A^0_\mu, A'^0_\mu) = \theta^{\mu\nu}\theta^{\kappa\lambda}\Phi^2_{\mu\nu\kappa\lambda}\Big(\Phi^0, \frac{1}{2} g' Y B_\mu + g T^a W^a_\mu + g_s T^a_s G^a, \frac{1}{2} g' Y B_\mu + g_s T^a_s G^a\Big).$$

L and R in the left-hand sides of (65) and (66) are defined in (38) and (39) and can be used to obtain $L^1_{\mu\nu}$, $R^1_{\mu\nu}$ and so on by expanding the left-hand side of the equations up to the desired order of $\theta_{\mu\nu}$ and comparing the result with the right- hand sides of (65) and (66). One can see that (64) reproduces its counterpart in the standard model and many new couplings between the standard model fields. Finally, we note that in the LCNCSM, as in the standard model, a Cabibbo-Kobayashi-Maskawa matrix in the charged currents can be obtained by diagonalizing the Yukawa coupling matrices using biunitary transformations.

V. THE PHENOMENOLOGICAL TEST OF LCNCSM

In constructing LCNCSM one finds besides the usual standard model many new couplings between the ordinary fields of the standard model. For instance, here one has four gauge boson couplings such as 4-γ, 4-Z, ...; fermion-gauge boson couplings such as $ff\gamma\gamma$, $ff\gamma\gamma\gamma$, ... and so on. Furthermore, each usual vertex in the standard model receives corrections from the LCNCSM. Therefore, there are many measurable quantities in the LCNCSM which can show, if they exist, the effects of noncommutative space in future experiments. To this end we consider the neutral current interaction with Z_0 for leptons which can read from (51) as

$$\int d^6\theta \int d^4x W(\theta) \frac{g}{2\cos\theta}\Big(\bar\nu_L \not{Z}_0 \nu_L - \frac{1}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}\bar\nu_L \times \gamma^\rho \partial_\mu Z_{0\kappa\rho}\partial_\nu\partial_\lambda \nu_L + \Big(2\sin^2\theta - \frac{1}{2}\Big)\bar e \not{Z}_0 e - \frac{1}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}\Big(2\sin^2\theta - \frac{1}{2}\Big)\bar e \gamma^\rho \partial_\mu Z_{0\kappa\rho}\partial_\nu\partial_\lambda e + \Big(2\sin^2\theta - \frac{1}{2}\Big)\bar e \not{Z}_0 \gamma_5 e - \frac{1}{8}\theta^{\mu\nu}\theta^{\kappa\lambda}\Big(2\sin^2\theta - \frac{1}{2}\Big) \times \bar e \gamma^\rho\gamma_5 \partial_\mu Z_{0\kappa\rho}\partial_\nu\partial_\lambda e\Big), \tag{67}$$

where $Z_{0\mu\nu} = \partial_\mu Z_{0\nu} - \partial_\nu Z_{0\mu}$. This action leads to the following Feynman rule for eeZ-vertex, Fig. 1:

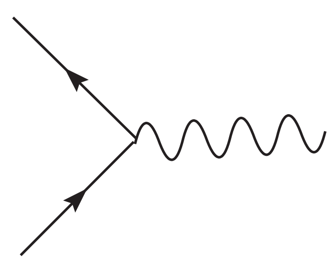

FIG. 1. Zee vertex. The bold and wavy lines show the electron and Z gauge boson, respectively. In the LCNCSM, the vertex is corrected with respect to the standard model as $\Gamma^{NC}_\mu = \Gamma_\mu(1 + \frac{\langle\theta^2\rangle}{96}(\frac{k^4}{4} - m_f^2 k^2))$, see (68).

$$\frac{ig}{\sin(2\theta_W)}\gamma^\mu(-2Q_f\sin^2\theta_W + T_3(1-\gamma_5)) \times\left(1+\frac{\langle\theta^2\rangle}{96}\left(\frac{k^4}{4}-m_f^2k^2\right)\right). \qquad (68)$$

Thus the vector coupling constant C_V and the scalar one C_A are corrected by the new term proportional to Q^4. On the other hand, according to the implemented experiments, such dependency was not confirmed. The noncommutative correction of the coupling constants cannot absorb through their redefinitions; therefore, this correction must be smaller than the resolution of the experiments. Comparing with [19], we find the noncommutative scale $\Lambda_C = \sqrt{\frac{12}{\langle\theta^2\rangle}}$ must be larger than 112 GeV. To find a better bound on the noncommutative scale, we should use linear colliders with a center-of-mass energy around a few TeV's. The QED sector of the LCNCSM has been investigated by the authors of [8] which leads to $\Lambda_C \sim 300$ GeV and by the authors of [9] which leads to $\Lambda_C \sim 160$ GeV.

VI. CONCLUSIONS

In this paper we have constructed the Lorentz-conserving version of the noncommutative standard model. For this purpose the $\theta_{\mu\nu}$ expansion of the standard model fields, up to the second order of $\theta_{\mu\nu}$ is obtained as given in (24), (28), and (33). The Seiberg-Witten map for the Higgs field, up to the second order of $\theta_{\mu\nu}$, is calculated for the first time, see (33). While (24) and (28) have been calculated already but there are differences in signs of (28) in comparison with the corresponding equation given in Ref. [6]. The misprinting can be understood easily by considering the reduction of the equation to the Abelian case, though to find its correct form we have recalculated the equation. Consequently, the action of the LCNCSM is introduced in terms of four terms, see (36). It is shown that in all versions of the LCNCSM new vertices appear in comparison with the ordinary standard model. In the minimal version of LCNCSM there are also new point interactions in contrast to the minimal NCSM. For instance, in the minimal LCNCSM besides the usual standard model and the NCSM interactions, there are new couplings between the fermions and the electroweak gauge bosons such as $ff\gamma\gamma\gamma$, $ff\gamma\gamma Z$, $ff\gamma ZZ$, and $ffZZZ$ and so on. The vertex $ff\gamma\gamma\gamma$ is one of the vertices of LCNCQED [7,8]. Nevertheless in contrast to the minimal NCSM there is not any photon-neutrino coupling in the LCNCSM [3]. Indeed such interactions are proportional to the odd power of $\theta_{\mu\nu}$ which are absent in the Lorentz-invariant noncommutative field theory, see the action (51). Furthermore, it is shown that in the mLCNCSM at the lowest order there are vertices such as $\gamma\gamma\gamma\gamma$, $\gamma\gamma\gamma Z$, $\gamma\gamma ZZ$, γZZZ, $ZZZZ$, and so on, see (57).

In constructing the LCNCSM besides many new couplings between the ordinary fields of the standard model, each usual vertex in the standard model receives corrections from the LCNCSM. Therefore, there are many measurable quantities in the LCNCSM which can show the effects of noncommutative space in future experiments. However, among the measurable quantities, the coupling constant C_V and C_A can be corrected by considering the eeZ vertex which leads to the value $\Lambda_C \sim 112$ GeV for the noncommutative scale.

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