How far is it to a sudden future singularity of pressure?

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(Received 17 April 2007; published 29 June 2007)

We discuss the constraints coming from current observations of type Ia supernovae on cosmological models which allow sudden future singularities of pressure (with the scale factor and the energy density regular). We show that such a sudden singularity may happen in the very near future (e.g. within 10×10^6 years) and its prediction at the present moment of cosmic evolution cannot be distinguished, with current observational data, from the prediction given by the standard quintessence scenario of future evolution. Fortunately, sudden future singularities are characterized by a momentary peak of infinite tidal forces only; there is no geodesic incompleteness, which means that the evolution of the universe may eventually be continued throughout until another "more serious" singularity such as a big crunch or big rip.

DOI: 10.1103/PhysRevD.75.123524 PACS numbers: 98.80.−k, 95.36.+x

Over the past decade observations of high-redshift type Ia supernovae (SNIa) have provided strong evidence that the expansion of the universe is accelerating, driven in the standard paradigm by some form of dark energy [1,2]. Current data [2] continue to leave open the possibility that dark energy exists in the form of phantom energy, which may violate all energy conditions [3]: the null ($\varrho c^2 + p \geq 0$), weak ($\varrho c^2 \geq 0$ and $\varrho c^2 + p \geq 0$), strong ($\varrho c^2 + p \geq 0$ and $\varrho c^2 + 3p \geq 0$), and dominant energy ($\varrho c^2 \geq 0$, $-\varrho c^2 \leq p \leq \varrho c^2$) conditions (where c is the speed of light, ϱ is the mass density in kg m^{-3}, and p is the pressure). Phantom matter may dominate the universe in the future and drive it towards a big-rip (BR) singularity in which all matter will be dissociated by gravity [4]. This is dramatically different from the standard picture of future cosmic evolution which suggests an asymptotically empty de Sitter state driven by the cosmological constant or quintessence [5] and leading to the violation of the strong energy condition only.

Phantom-driven scenarios have encouraged the study of other exotic possibilities for the future evolution of the universe. One of these possibilities appears in those models which do not assume any explicit form for the equation of state $p = p(\varrho)$, leaving the evolution of the energy density and pressure unconstrained. This freedom may result in a so-called *sudden future singularity* (SFS) of pressure [6] which violates only the dominant energy condition. The nature of a sudden future singularity is different from that of a standard big-bang (BB) singularity, and also from a big-rip singularity, in that it does not exhibit geodesic incompleteness and the cosmic evolution may eventually be extended beyond it [7,8]. The only physical characteristic of these singularities is a momentarily infinite peak of the tidal forces in the universe. In more general models this peak may also appear in the derivatives of the tidal forces. It is interesting to note that these types of singularities are, in a way, similar to yet another type, which was termed finite density singularities [9]. However, the crucial difference is that finite density singularities occur as singularities in space rather than in time, which means that even at the present moment of cosmic evolution they could exist somewhere in the universe [10]. We will not discuss in detail finite density singularities in this paper since they basically appear in cosmological models without homogeneity. On the other hand, it is worth mentioning that the sudden future singularities are quite generic since they may arise in both homogeneous [11] *and* inhomogeneous [12] models of the universe.

In order to obtain a sudden future singularity, consider the simple framework of an Einstein-Friedmann cosmology governed by the standard field equations

$$\varrho = \frac{3}{8\pi G}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right), \tag{1}$$

$$p = -\frac{c^2}{8\pi G}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right), \tag{2}$$

where the energy-momentum conservation law

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$$\dot{\varrho} = -3\frac{\dot{a}}{a}\left(\varrho + \frac{p}{c^2}\right) \tag{3}$$

is trivially fulfilled due to the Bianchi identity. Here $a(t)$ is the scale factor, G is the gravitational constant, and the curvature index $k = 0, \pm 1$. What is crucial in order to obtain a sudden future singularity is that no link between the energy density and pressure (the equation of state) is specified. This allows us to integrate (3) only by quadratures as

$$\varrho a^3 = \exp\left[-\left(\frac{3p(t')}{c^2\varrho(t')}\ln a(t')\right)\Big|_{t_0}^{t} + \frac{3}{c^2}\int_{t_0}^{t}\left(\frac{p(t')}{\varrho(t')}\right)^{\cdot} \times \ln a(t')dt' \right]. \tag{4}$$

Of course (4) reduces to the standard expression for energy conservation, $\varrho a^{3(w+1)} = \mathrm{const}$, provided a barotropic equation of state, $p = w\varrho c^2$ for constant w, is assumed. (The condition for phantom models, for example, is $w < -1$.)

From Eqs. (1) and (2) one can easily see that a pressure singularity $p \to \mp\infty$ occurs when the acceleration $\ddot{a} \to \pm\infty$, notwithstanding that the values of the energy density ϱ and the scale factor $a(t)$ are regular. Since in that case $|p| > \varrho$, it is clear that the dominant energy condition is violated. This condition can be achieved if the scale factor takes the form [6]

$$a(t) = a_s[1 + (1-\delta)y^m - \delta(1-y)^n], \qquad y \equiv \frac{t}{t_s} \tag{5}$$

with the appropriate choice of the constants δ, t_s, a_s, m, n. Moreover, we can see that the rth derivative of the scale factor (5) is given by

$$a^{(r)} = a_s\left[\frac{m(m-1)\ldots(m-r+1)}{t_s^r}(1-\delta)y^{m-r} + (-1)^{r-1}\delta\frac{n(n-1)\ldots(n-r+1)}{t_s^r}(1-y)^{n-r}\right], \tag{6}$$

and is related to the appropriate pressure derivative $p^{(r-2)}$. Thus, in general, it is possible that one has a pressure derivative $p^{(r-2)}$ singularity which accompanies the blowup of the rth derivative of the scale factor $a^{(r)}$. Observationally this could be manifested in, for example, the blowup of the characteristics known as *statefinders*, such as jerk, snap, etc. [13]. The pressure derivative singularity $p^{(r-2)}$ appears when

$$r - 1 < n < r, \qquad r = \text{integer}, \tag{7}$$

and for any $r \geq 3$ it fulfills all energy conditions. These singularities are called generalized sudden future singularities (GSFS) and are possible, for example, in theories with higher-order curvature quantum corrections [14].

Let us now return to the case of $r = 2$, for which $1 < n < 2$ and we obtain sudden future singularity models of pressure (and obviously all of its higher derivatives) which lead to violation of the dominant energy condition. In such models, expressed in terms of the scale factor (5), the evolution begins with the standard BB singularity at $t = 0$ for $a = 0$, and finishes at SFS for $t = t_s$ where $a = a_s \equiv a(t_s)$ is a constant. [Note that we have changed the original parametrization of Ref. [6] for the scale factor (5) using $A = \delta a_s$.]

The standard Friedmann limit (i.e. models without an SFS) of (5) is achieved when $\delta \to 0$; hence δ becomes the "nonstandardicity" parameter of SFS models. Additionally, notwithstanding Ref. [6] and in agreement with the field equations (1) and (2), we assume that δ can be both positive and negative leading to a deceleration or an acceleration [cf. (6)] of the universe, respectively.

It is important to our discussion that the asymptotic behavior of the scale factor (5) close to the BB singularity at $t = 0$ is given by a simple power law $a_{\rm BB} = y^m$, simulating the behavior of flat $k = 0$ barotropic fluid models with $m = 2/[3(w+1)]$. This allows us to preserve all the standard observed characteristics of early universe cosmology—such as the cosmic microwave background, density perturbations, nucleosynthesis, etc.—provided we choose an appropriate value of m. On the other hand, close to an SFS the asymptotic behavior of the scale factor is nonstandard, $a_{\rm SFS} = a_s[1 - \delta(1-y)^n]$, showing that $a_{\rm SFS} = a_s$ for $t = t_s$ (i.e. $y = 1$) at the SFS. Notice that one does not violate the energy conditions if the parameter m lies in the range

$$0 < m \leq 1 \qquad (w \geq -1/3). \tag{8}$$

This range of values is, in fact, equivalent to a standard (neither quintessencelike nor phantomlike) evolution of the universe. However, with no adverse impact on the field equations (1) and (2), one could also extend the values of m to lie in the complementary ranges [7] $m > 1$ (i.e. $-1 < w < -1/3$) for quintessence, and $m < 0$ (i.e. $w < -1$) for phantom, although these ranges may lead to violation of the strong and weak energy conditions, respectively.

We will next calculate the luminosity distance as a function of redshift, and hence the redshift-magnitude relation, for SFS models. This will allow us to establish whether these models are a realistic possibility for the future evolution of the universe, and more specifically whether current cosmological observations of high-redshift supernovae are consistent with values of the constant n in the range $1 < n < 2$, as required in order that the scale factor will display an SFS (or, more generally, a GSFS for $r - 1 < n < r$). We will then explore the range of values for the other SFS model parameters which are consistent with current observational constraints on standard cosmology, and thus determine limits on how far into the future an SFS might occur. In fact, as we will see below, we need to consider only two further parameters: δ and $y_0 = t_0/t_S$, where t_0 is the current age of the universe in the SFS model. Notice that, in view of (8), it is reasonable to take $m = 2/3$ as is the case for the standard dust-dominated evolution. This implies that, at early times, our SFS model reduces to the Einstein–de Sitter universe.

We proceed within the framework of Friedmann cosmology, and consider an observer located at $r = 0$ at coordinate time $t = t_0$. The observer receives a light ray emitted at $r = r_1$ at coordinate time $t = t_1$. We then have a standard null geodesic equation

$$\int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}} = \int_{t_1}^{t_0} \frac{cdt}{a(t)}, \tag{9}$$

with the scale factor $a(t)$ given by (5). Using (5) again, the redshift is given by

$$1 + z = \frac{a(t_0)}{a(t_1)} = \frac{\delta + (1-\delta)y_0^m - \delta(1-y_0)^n}{\delta + (1-\delta)y_1^m - \delta(1-y_1)^n}, \tag{10}$$

where $y_0 = y(t_0)$ and $y_1 = y(t_1)$. The luminosity distance is defined as

$$D_L = r_1 a(t_0)(1 + z). \tag{11}$$

Neglecting extinction and k corrections, the observed and absolute magnitudes of a source at redshift z and luminosity distance D_L are related by

$$m(z) = M - 5\log_{10} H_0 + 25 + 5\log_{10} D_L(z), \tag{12}$$

which, with the help of Eqs. (9)–(11), allows a redshift-magnitude relation for SFS cosmological models to be constructed. It is obvious that Eq. (9) has to be integrated numerically in order to establish the relation between t_0 and t_1, which can then be inserted into (10) and (11) to constrain the SFS model parameters. As a first step we determine the dependence on the SFS model parameters of the Hubble law, which replaces Eq. (12) when $z \approx 0$, i.e. $cz \approx H_0 D_L$, where

$$H_0(\mathrm{km\,s^{-1}\,Mpc^{-1}}) = \left(\frac{\dot a}{a}\right)_0 = \frac{3.09 \times 10^{19}}{t_0(\mathrm{sec}) y_0} \times \left[\frac{m(1-\delta)y_0^{m-1} + n\delta(1-y_0)^{n-1}}{\delta + (1-\delta)y_0^m - \delta(1-y_0)^n}\right] \tag{13}$$

is the present value of the Hubble parameter, which we can take as 72 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [1].

Similarly, we could derive an expression, in terms of the SFS model parameters, for the deceleration parameter $q_0 = -(\ddot a a/\dot a^2)_0$. However, in order to search the parameter space for models which are admissible by current observations, we write the product of H_0 and q_0 as

$$q_0 H_0 = -\left(\frac{\ddot a}{\dot a}\right)_0 = -\frac{t_0}{y_0} \times \frac{m(m-1)(1-\delta)y_0^{m-2} - \delta n(n-1)(1-y_0)^{n-2}}{m(1-\delta)y_0^{m-1} + n\delta(1-y_0)^{n-1}}. \tag{14}$$

In order to obtain an accelerated universe at the present moment of the evolution, this product should be negative. Figure 1 shows an example plot of the product $H_0 q_0$ as a function of δ and y_0, with the other parameters fixed at $m = 2/3$, $n = 1.9993$, $t_0 = 13.2457$ Gyr. From the plot we see that there are large regions of the parameter space which admit cosmic acceleration. We have explored the parameter space further with various configurations of m, n, δ, y_0, t_0, q_0, and H_0, and obtained the general conclusion that there is a large class of SFS models which are compatible with current acceleration.

Out of these admissible models we then searched for those which are compatible with the redshift-magnitude relation (12) observed for recent SNIa data [2], and hence with the derived parameters of the standard "concordance cosmology" (CC). We were able to identify SFS models that are in remarkably tight agreement with current SNIa data. As an illustrative example Fig. 2 shows luminosity distance as a function of redshift for the CC model with $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_{m0} = 0.26$, and $\Omega_{\Lambda 0} = 0.74$, and an SFS model with parameters $m = 2/3$, $y_0 = 0.999\,36$, $\delta = -0.471$, $n = 1.9999$. We see that the SFS model mimics the CC model very closely over a wide range of redshifts. In particular, it is clear that recent SNIa data from the Tonry *et al.* "Gold" sample [1] and Supernova Legacy Survey (SNLS) sample [2] cannot yet discriminate between the CC and SFS models.

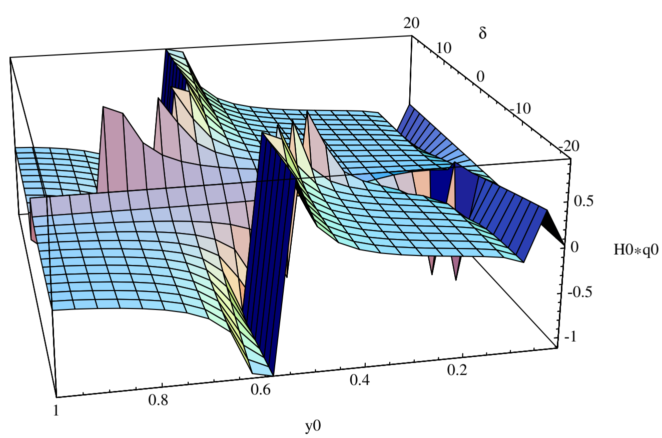

FIG. 1 (color online). Parameter space $(H_0 q_0, \delta, y_0)$ for fixed values of $m = 2/3$, $n = 1.9993$, $t_0 = 13.3547$ Gyr of the sudden future singularity models. There are large regions of the parameter space which admit cosmic acceleration.

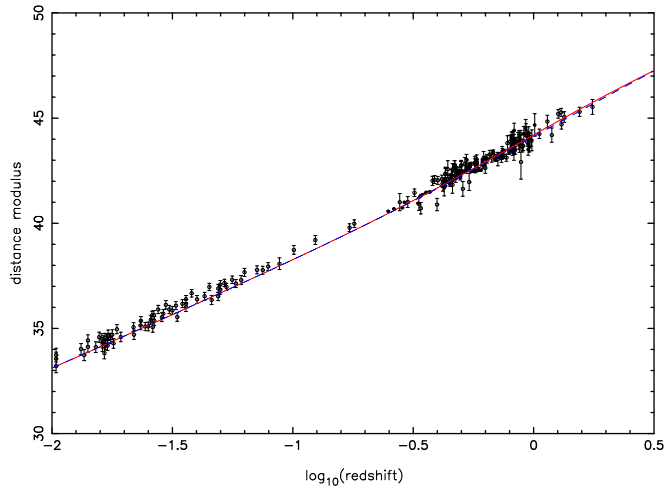

FIG. 2 (color online). The distance modulus $\mu_L = m - M$ for the CC model with $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_{m0} = 0.26$, $\Omega_{\Lambda 0} = 0.74$ (dashed curve) and the SFS model for $m = 2/3$, $n = 1.9999$, $\delta = -0.471$, $y_0 = 0.999\,36$ (solid curve). Also shown are the "Gold" (open circles) and SNLS (filled circles) SNIa data. Taking the age of the SFS model to be equal to that of the CC model, i.e. $t_0 = 13.6$ Gyr, one finds that an SFS is possible in only 8.7×10^6 years.

Taking the current age of the universe in the SFS model to be equal to the age of the CC model, i.e. $t_0 = 13.6$ Gyr, we find that the time to the sudden singularity is $t_s - t_0 \approx 8.7$ Myr, which is *amazingly* close to the present epoch. In that context it is no wonder that these singularities are called "sudden." We have also checked that the larger the value of r in (7) the later into the future a GSFS appears. This means that the strongest of these singularities which violates the dominant energy condition (i.e. an SFS) is more likely to become reality.

Our remark about the effect of the sudden pressure singularity seems in agreement with the result of Ref. [15] which showed that the dominant energy condition is now violated and that it became violated quite recently (at redshift $z \sim 0.2$). Of course this violation may also be due to phantom energy [3].

In conclusion, we have shown that a sudden future singularity may happen in the comparatively near future (e.g. within 10×10^6 years) and its prediction at the present moment of cosmic evolution cannot be distinguished, with current observational data, from the prediction given by the standard quintessence scenario of future evolution in the concordance model. Fortunately, sudden future singularities are characterized by a momentary peak of infinite tidal forces only; there is no geodesic incompleteness which means that the evolution of the universe may eventually be continued beyond the SFS until another "more serious" singularity such as a big crunch or a big rip. One could then consider, more generally, a scale factor of the form [7,16]

$$a(t) = A + [(a_s - A) - D(t_r - t_s)^p - Et_s^o]y^m - (A + Dt_r^p)(1-y)^n + D(t_r - t_s y)^p + Et_s^o y^o, \tag{15}$$

where the constants m, o, p, A, D, E are chosen so that the universe begins with a big bang at $t = 0$ where $a = 0$, next faces a sudden future singularity at $t = t_s$ where $a(t_s) = a_s$, and then eventually continues to a big rip at $t = t_r$ where $a(t_r) \to \infty$. All of the matter sources may be involved since the constants in (15) can be taken as $0 < m \leq 1$ (quintessence), $p < 0$ (phantom), and $o > 1$ (standard positive matter pressure).

Whether the universe will end in a big rip or a big crunch is an open question. Moreover, unlike a sudden future singularity, both a big-rip and a big-crunch singularity would represent the real end of the universe. Fortunately, as was shown in Refs. [4,17], a big-rip singularity is not possible in the very near future: in order to reach it one must wait about the same time as the current age of the universe. Apart from that, it is still possible to avoid it due to a negative tension brane contribution in a turnaround cyclic cosmology [18].

M. P. D. and T. D. acknowledge the support of the Polish Ministry of Education and Science Grant No. 1 P03B 043 29 (years 2005–2007).

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