Thermodynamical properties of dark energy

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We have investigated the thermodynamical properties of dark energy. Assuming that the dark energy temperature $T \sim a^{-n}$ and considering that the volume of the Universe enveloped by the apparent horizon relates to the temperature, we have derived the dark energy entropy. For dark energy with constant equation of state $w > -1$ and the generalized Chaplygin gas, the derived entropy can be positive and satisfy the entropy bound. The total entropy, including those of dark energy, the thermal radiation, and the apparent horizon, satisfies the generalized second law of thermodynamics. However, for the phantom with constant equation of state, the positivity of entropy, the entropy bound, and the generalized second law cannot be satisfied simultaneously.

Results from numerous and complementary observations show an emerging paradigm ''concordance cosmology'' indicating that our Universe is spatially flat and composed of about 70% dark energy (DE) and about 25% dark matter. The weird DE is a major puzzle of physics now. Its nature and origin have been an intriguing subject of discussions in the past years. The DE has been sought within a wide range of physical phenomena, including a cosmological constant, quintessence, or an exotic field called phantom [1]. Except for the known fact that DE has a negative pressure causing the acceleration of the Universe, its nature still remains a complete mystery. In the conceptual set up of DE, one of the important questions concerns its thermodynamical properties. It is expected that the thermodynamical consideration might shed some light on the properties of DE and help us understand its nature.

The topic of the DE entropy, temperature, and their evolution by using the first law of thermodynamics was widely discussed in the literature [2−10]. It was found that the entropy of the phantom might be negative [6−8]. The existence of negative entropy of the phantom could be easily seen from the relation $Ts = \rho + p$ between temperature T, the entropy density s, the energy density ρ, and the pressure p. Negative entropy is problematic if we accept that the entropy is in association with the measure of the number of microstates in statistical mechanics. The intuition of statistical mechanics requires that the entropy of all physical components to be positive.

Besides, if we consider the Universe as a thermodynamical system, the total entropy of the Universe including DE and dark matter should satisfy the second law of thermodynamics. The generalized second law (GSL) for phantom and nonphantom DE has been explored in [8]. It was found that the GSL can be protected in the Universe with DE. The GSL of the Universe with DE has been investigated in [9,10] as well. In order to rescue the GSL of thermodynamics, Bekenstein conjectured that there exists an upper bound on the entropy for a weakly self-gravitating physical system [11]. Bekenstein's entropy bound has received independent support [12]. A holographic entropy bound [13] was subsequently built, and it was argued to be a real conceptual change in our thinking about gravity [14]. The idea of the holographic entropy bound was found to be a useful tool in studying cosmology [15].

In the discussion of thermodynamical properties of the Universe, it is usually assumed that the physical volume and temperature of the Universe are independent, and by using the integrability condition $\partial^2 S/\partial V \partial T = \partial^2 S/\partial T \partial V$ and the first law of thermodynamics, one obtains the constant comoving entropy density. However, if we apply this treatment in the Universe with DE, we find some problems of the DE thermodynamics [10]. Naively, we may think the DE temperature is equal or proportional to the horizon temperature T_H. It was found that the equation of state of DE is uniquely determined and the phantom entropy is negative [10]. Therefore, a general DE model is not in thermal equilibrium with the Hawking radiation of the horizon. Besides, although the GSL can be valid for $w > -1$, for the phantom with $w < -1$, it was found that the GSL breaks down due to the negative temperature deduced in the formalism where the volume and the tem-

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perature are assumed to be independent [10]. In summary, for the phantom, we either run into the negative entropy problem or the GSL is violated. It is more realistic to consider that the physical volume and the temperature of the Universe are related, since in the general situation they both depend on the scale factor $a(t)$. In the cosmological context, the apparent horizon is important, since on the apparent horizon there is the well-known correspondence between the first law of thermodynamics and the Einstein equation [16]. On the other hand, it was found that the apparent horizon is a good boundary for keeping thermo-dynamical laws [9]. Considering the apparent horizon as the physical boundary of the Universe, it was found that both the temperature and entropy can be positive for DE, including phantom. Furthermore, by considering the real-istic case that the physical volume and the temperature are related, the GSL is proved to be always satisfied within the volume of the apparent horizon [10]. Thus, in studying the DE thermodynamics, it is more appropriate to consider the Universe in which the volume and the DE temperature are related.

In this work we will investigate the thermodynamical properties of DE by assuming that the physical volume and the temperature are not independent. Now again it is natural to think that the DE is in thermal equilibrium with the Hawking radiation of the apparent horizon. In this case, we found that the DE entropy is the dominant entropy component and it becomes negative even for DE with $w > -1$ [10]. Recall that the radiation temperature in the Universe scales as $T \sim a^{-1}$, so we assume here that the DE temperature has a similar behavior $T \sim a^{-n}$ to avoid the negative entropy problem, where n is an arbitrary constant. It is not necessary to take $n = 1$ to ensure that the DE is in equilibrium with the thermal radiation, since their dispersion relations could be completely different [6,10]. From the above discussions, it is reasonable to expect that a physically acceptable entropy of DE should be positive and satisfy the entropy bound. It should also satisfy the property required by the GSL. Since the usual thermal radiation temperature in the Universe decreases as the Universe expands, we expect that the DE temperature also preserves this property.

By using the first law of thermodynamics $TdS = dE + pdV$ for the DE, and considering the volume of the Universe within the apparent horizon $V = 4\pi \tilde{r}_A^3/3$ and the total DE $E = \rho V$, we can express the DE entropy as [10]

$$TdS = -\frac{2\pi}{3}\left(\frac{8\pi G}{3}\right)^{-3/2} \rho_t^{-5/2}(\rho_t + 3p_t)d\rho, \quad (1)$$

where the Friedmann equation and the energy conservation law have been used in the derivation, and ρ_t and p_t denote the total energy density and pressure, respectively. Taking the derivative with respect to time on both sides of the above equation, we have

$$\dot{S} = 2\pi\left(\frac{8\pi G}{3}\right)^{-3/2} \rho_t^{-5/2}(\rho_t + 3p_t)H(\rho + p)/T. \quad (2)$$

It can be seen that $\dot{S} \geq 0$ (< 0) if $(\rho + p)/T \geq 0$ (< 0) during the radiation dominated era (RD) and the matter dominated era (MD), and $\dot{S} \leq 0$ (> 0) if $(\rho + p)/T \geq 0$ (< 0) during DE domination. In the phantom domination, the apparent horizon entropy decreases as the Universe expands [10]. This requires $\dot{S} > 0$ and $|1 + 3w|T_H/2 > T > 0$ to protect the GSL. Thus in the phantom domination era, the temperature of the phantom has to be positive to rescue the GSL.

The radiation entropy can be obtained as usual, $S_r = sV$, where $s = \sigma/a^3$ is the physical entropy density and σ is the constant comoving entropy. For DE with constant equation of state w, using the Friedmann equation, the entropies of the radiation and the apparent horizon are

$$S_r = S_{r0}x^{-3}\Omega_t(x)^{-3/2}, \quad (3)$$

$$S_A = S_{A0}\Omega_t(x)^{-1}, \quad (4)$$

where $\Omega_t(x) = \Omega_{m0}x^{-3} + \Omega_{r0}x^{-4} + \Omega_{w0}x^{-3(1+w)}$ and $x = a/a_0$. To get the DE entropy we need to solve Eq. (1) by assuming $T = T_0(a/a_0)^{-n}$. In the evolution of the Universe, the solution to Eq. (1) is given in the form

$$\frac{S_w}{S_{r0}}\frac{\Omega_{r0}T_{w0}}{\Omega_{w0}T_{r0}} = \begin{cases} \frac{9(1+w)}{4(n+3(1-w))}\Omega_{r0}^{-3/2}x^{n+3(1-w)}, \\ S_{w1} + \frac{9(1+w)}{8(n+3/2-3w)}\Omega_{m0}^{-3/2}x^{n+3/2-3w}, \\ S_{w2} + \frac{9(1+w)(1+3w)}{8(n+3(1+w)/2)}\Omega_{w0}^{-3/2}x^{n+3(1+w)/2}, \end{cases}$$

$$(5)$$

for the RD, MD, and DE domination, respectively, where S_{w1} and S_{w2} are integration constants.

As we mentioned previously, the intuition of the statistical mechanics requires positive entropy. We expect that this should also hold for the entropy of DE if it is supposed to keep the same microscopic meaning. From Eq. (5) we learn that for DE with constant equation of state $w > -1$, non-negative S_w can be obtained if $n > 3w - 3$ during RD. During DE domination, if $n > -3(1 + w)/2$, then $S_w \to -\infty$ when $a \to \infty$. Thus, to get positive entropy for the DE, the parameter n should be chosen within the range $-6 < 3w - 3 < n < -3(1 + w)/2 < 0$.

This parameter range of n can be further constrained if we express the solution of Eq. (1) as

$$\frac{S_w}{S_{r0}} = \frac{3}{4}(1 + w)\frac{\Omega_{w0}}{\Omega_{r0}}\frac{T_{r0}}{T_{w0}}\left[x^{-3+n-3w}\Omega_t(x)^{-3/2}\right.$$
$$\left. - (n - 3w)\int_0^{a/a_0} x^{-4+n-3w}\Omega_t(x)^{-3/2}dx\right]. \quad (6)$$

If $n > 3w$, the second term in the above equation is negative, which might lead S_w to be negative. Therefore, we need to restrict $3w - 3 < n < 3w$ to ensure the positivity of S_w. Note that for radiation, $n = 3w = 1$, Eq. (6) reduces to Eq. (3). Since $n < 0$, the dark energy temperature will

increase with the scale factor a, and at the present moment $T_{w0} \gg T_{r0}$. During RD and MD, it can be seen that both DE entropy and the radiation entropy increase. However, if one notes that $\dot{S}_w/\dot{S}_r = 3(1 + w)\rho_w T_r/(4\rho_r T_w) < 1$, the DE entropy increases slower than the radiation entropy. During the DE domination, both the DE entropy and the radiation entropy decrease, $S_w \to S_{w2} > 0$, and $S_r \to 0$ when $a \to \infty$, so $S_w > S_r$ in the future. Since the apparent horizon entropy increases during the DE domination, $\dot{S}_A = 3(1 + w)S_A H$, $\dot{S}_w = (n + 3(1 + w)/2)S_w H$, and $\dot{S}_r = 3(1 + 3w)S_r H/2$, so the GSL is always respected for DE with constant equation of state $w > -1$. To see these points more clearly, we solve Eq. (2) numerically by choosing $w = -0.9$, $\Omega_{m0} = 0.3$, $\Omega_{w0} = 0.7$, and $\Omega_{r0} = 8.35 \times 10^{-5}$. The results for $n = -5.0$ and $n = -3.5$ are shown in Fig. 1. The numerical results confirm that by constraining $3w - 3 < n < 3w$, S_w is positive. It is easy to see that the DE entropy and the radiation entropy are much smaller compared to the apparent horizon entropy, thus the entropy bound is always held. Although the radiation entropy and the DE entropy may decrease in the DE domination, due to their very small scale, their decreasing behaviors can be overcome by the increase of the entropy on the apparent horizon. Thus, including the total entropy in the Universe and the entropy of the apparent horizon, we find that the GSL is protected.

Now we come to consider the phantom with constant equation of state $w < -1$. In the RD and MD eras, if $n < 3w - 3$, S_w is positive but it decreases starting from ∞ as the Universe expands. The entropy bound is violated at very early times. If $n > -3(1 + w)/2$, S_w is negative during RD and MD and $S_w \to \infty$ when $a \to \infty$, so in the future although the GSL can be protected, the entropy bound will be violated. If $3w - 3 < n < -3(1 + w)/2$, S_w is negative. Thus, for the phantom with constant equation of state, it seems impossible to get a viable thermodynamics. The requirements of the positivity of DE entropy, the entropy bound, and the GSL cannot be met simultaneously. In [8], the authors used the future event horizon to study the phantom thermodynamics and found that the GSL could be respected if the phantom entropy is negative. The problem with the future event horizon is that for the Universe with DE with equation of state $w \neq -1$, the thermodynamical description breaks down on the event horizon [9].

FIG. 1 (color online). The evolution of S_w for $w = -0.9$ and S_r. The dotted line is for $(S_w/S_{r0}) \times (T_{w0} \times 10^{-16}/T_{r0})$ with $n = -5.0$, the dash-dotted line is for $(S_w/S_{r0}) \times (T_{w0} \times 10^{-8}/T_{r0})$ with $n = -3.5$, the dashed line is for S_r/S_{r0}, and the solid line is for the apparent horizon entropy S_A/S_{r0}.

Furthermore, the definitions of the event horizon temperature and entropy could be less certain than a guess. Even if we use the similar temperature and entropy definitions of the apparent horizon for the future event horizon, the first law of the thermodynamics is not satisfied [9].

In the above discussion we have concentrated on DE with a constant equation of state. To study the thermodynamics of a dynamic DE, we will use the generalized Chaplygin gas (GCG) [17] as an example. When the Universe is dominated by the GCG, the entropies of the apparent horizon and the radiation read [10]

$$S_A = S_{A0}\Omega_c^{-1}, \qquad (7)$$

$$S_r = S_{r0}\left(\frac{a}{a_0}\right)^{-3}\Omega_c^{-3/2}, \qquad (8)$$

where $\Omega_c = [-w_{c0} + (1 + w_{c0})(a/a_0)^{-3(1+\alpha)}]^{1/(1+\alpha)}$. The entropy for the GCG can be obtained by solving Eq. (1), which can be expressed as

$$\frac{S_c}{S_{r0}}\frac{T_{c0}}{T_{r0}} = \begin{cases} \frac{9}{4(n+3)}(1 + w_{c0})^{1/(1+\alpha)}\Omega_{r0}^{-5/2}x^{n+3}, & \text{RD}, \\ S_{c1} + \frac{9}{8(n+3/2)}(1 + w_{c0})^{-1/2(1+\alpha)}\Omega_{r0}^{-1}x^{n+3/2}, & a \ll a_0, \\ S_{c2} - \frac{9}{4(n-3-3\alpha)}(1 + w_{c0})(-w_{c0})^{-1-1/2(1+\alpha)}\Omega_{r0}^{-1}x^{n-3(1+\alpha)}, & a \gg a_0, \end{cases} \qquad (9)$$

where S_{c1} and S_{c2} are integration constants. To have $S_c \geq 0$, the parameter n must satisfy the condition $-3 < n < 3(1 + \alpha)$. Numerical results show that this condition is not enough. For example, if we choose $w_{c0} = -0.88$ and $\alpha = 1.57$, which are the best fitting values from observations
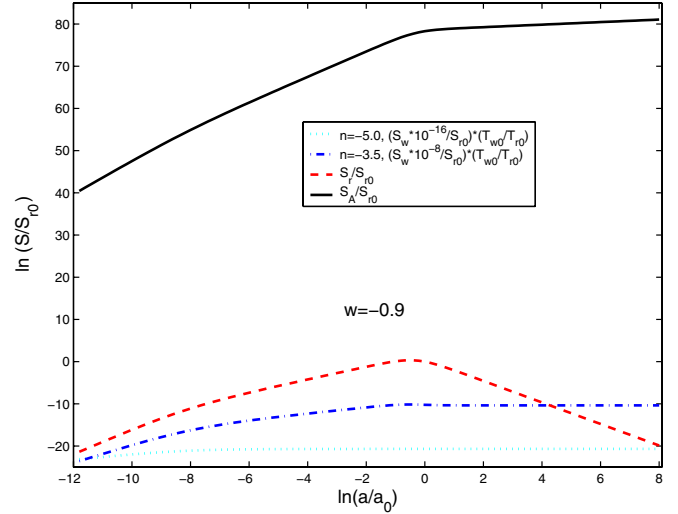
[17], we find that S_w is negative after MD when $n = 2$. At late times, $a \to \infty$ and $S_c \to S_{c2}$. For positive entropy, S_c will be greater than S_r at late times since $S_r \to 0$. The range of n to keep S_c positive can be more confined by numerical calculation. Choosing appropriate n to ensure S_c
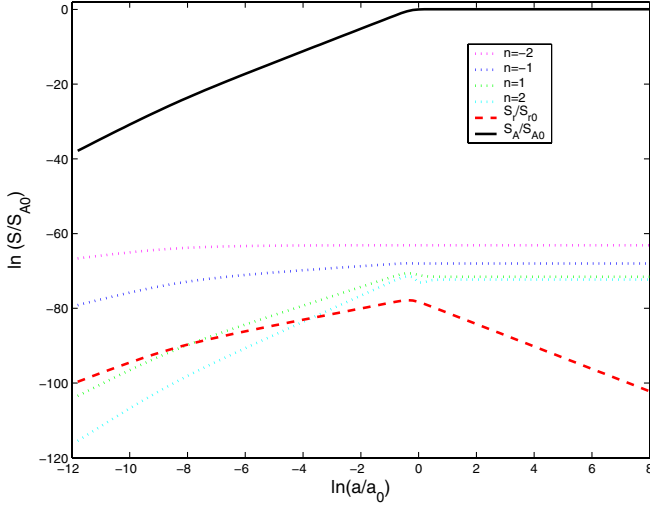
FIG. 2 (color online). The evolutions of S_c, S_r, and S_A with $w_{c0} = -0.88$ and $\alpha = 1.57$. The dotted lines, for $(S_c/S_{A0}) \times (T_{c0}/T_{r0})$, from top to down are $n = -2$, $n = -1$, $n = 1$, and $n = 2$. (Note that S_c is negative after MD when $n = 2$.) The dashed line is for S_r/S_{A0}, and the solid line is for S_A/S_{A0}.

to be positive, we have shown the numerical results in Fig. 2 on the evolution of entropies of GCG, radiation, and the apparent horizon. When $n < 0$, T_c increases with the expansion of the Universe and the numerical results show that S_c can be less than S_r during the RD and MD eras if T_{c0}/T_{r0} is large enough. If $n > 0$, then T_c decreases as the Universe expands and S_c increases faster than S_r during the RD and MD eras. When $n = 1$, the GCG and the radiation temperatures evolve in the same way and S_c can be larger than S_r during MD as shown in Fig. 2. It is clear from Fig. 2 that compared to the apparent horizon entropy, S_r and S_c are negligible, thus the entropy bound can be protected for the GCG case. In addition, the GSL can also be saved in the GCG case, since the total entropy evolves basically in the same way as the entropy of the apparent horizon. Though in the GCG dominated period S_r decreases as the Universe expands, owing to its negligible value compared to the apparent horizon entropy, its decrease can be overcome by the increase of the apparent horizon.

In summary, in this work we have investigated the thermodynamical properties of DE. In calculating the DE entropy we have considered the volume of the Universe enveloped by the apparent horizon and assumed that the physical volume and the temperature are related. The

apparent horizon is a good boundary for studying cosmology, since on the apparent horizon there is the well-known correspondence between the first law of thermodynamics and the Einstein equation [16]. Furthermore, it has been found that the apparent horizon is good in keeping thermodynamical laws [9]. Assuming that the temperature of the DE has the form $T \sim a^{-n}$, we have derived the evolution of the DE entropy. For DE with constant equation of state $w > -1$, we have found the appropriate range of n for keeping the DE entropy positive, which is the requirement of the statistical understanding of the concept of entropy. In this range of n, the entropy bound and the GSL can also be protected. The negative point is that the allowed range of n for giving a physically acceptable DE entropy leads the DE temperature to increase as the Universe expands, which is different from the behavior of the thermal temperature that decreases with the expansion of the Universe. This conflict could be overlooked since the DE temperature and the thermal temperature may have different dispersion relations [6,10], and it is not necessary that these two different temperatures behave accordingly. In the era of phantom domination, the GSL requires that the phantom entropy increases as the Universe expands, and the phantom temperature T satisfies the condition $|1 + 3w|T_H/2 > T > 0$. Since the horizon entropy decreases to zero as the Universe expands, the holographic entropy bound will be violated if the phantom entropy is positive. For the phantom with constant equation of state $w < -1$, we found that there is no common range of n so that the positivity of the entropy, the entropy bound, and the GSL can all be satisfied. The physical requirement on the DE entropy does not favor the phantom with constant equation of state. We have also extended our investigation to the dynamical DE by using the GCG as an example. We have found that by appropriately choosing parameters, we can have positive DE entropy, and meanwhile we can protect the holographic entropy bound and the GSL. Within the allowed parameter range for physically acceptable DE entropy, the DE temperature can decrease and it can even scale in the same way as the radiation temperature does as the Universe expands.

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