

Cuscuton cosmology: Dark energy meets modified gravityNiayesh Afshordi,^{1,*} Daniel J. H. Chung,^{2,†} Michael Doran,^{3,‡} and Ghazal Geshnizjani^{2,§}¹*Institute for Theory and Computation, Harvard-Smithsonian Center for Astrophysics, MS-51, 60 Garden Street, Cambridge, Massachusetts 02138, USA*²*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA*³*Institut für Theoretische Physik, Philosophenweg 16, 69120 Heidelberg, Germany*

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In a companion paper, we have introduced a model of scalar field dark energy, *Cuscuton*, which can be realized as the incompressible (or infinite speed of sound) limit of a *k*-essence fluid. In this paper, we study how *Cuscuton* modifies the constraint sector of Einstein gravity. In particular, we study *Cuscuton* cosmology and show that even though *Cuscuton* can have an arbitrary equation of state, or time dependence, and is thus inhomogeneous, its perturbations do not introduce any additional dynamical degree of freedom and only satisfy a constraint equation, amounting to an effective modification of gravity on large scales. Therefore, *Cuscuton* can be considered to be a minimal theory of evolving dark energy, or a minimal modification of a cosmological constant, as it has no internal dynamics. Moreover, this is the only modification of Einstein gravity to our knowledge, that does not introduce any additional degrees of freedom (and is not conformally equivalent to the Einstein gravity). We then study two simple *Cuscuton* models, with quadratic and exponential potentials. The quadratic model has the exact same expansion history as Λ CDM, and yet contains an early dark energy component with constant energy fraction, which is constrained to $\Omega_Q \lesssim 2\%$, mainly from WMAP Cosmic Microwave Background and Sloan Digital Sky Survey Lyman- α forest observations. The exponential model has the same expansion history as the Dvali-Gabadadze-Poratti self-accelerating brane-world model, but generates a much smaller Integrated Sachs-Wolfe effect, and is thus consistent with the Cosmic Microwave Background observations. Finally, we show that the evolution is local on superhorizon scales, implying that there is no gross violation of causality, despite *Cuscuton*'s infinite speed of sound.

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I. INTRODUCTION

The nature of the current acceleration of cosmic expansion is among the most outstanding puzzles in theoretical physics. Various cosmological observations such as the dimming of distant supernovae Ia [1,2], anisotropies in the Cosmic Microwave Background (CMB) [3], and the large scale structure of the Universe (e.g. [4]) can be most easily explained by having an exotic *dark energy* component with negative, nearly constant and uniform, pressure which dominates the energy density of the Universe.

While the simplest model of vanilla dark energy, i.e. a cosmological constant, remains consistent with all the present observations [4], many models of nonminimal dark energy have been developed in anticipation of any future failure of cosmological constant in explaining the data. However, all the models that predict an observable dark energy density evolution suffer from an extreme fine-tuning problem, requiring incredibly light mass scales ($\sim 10^{-33}$ eV) that are hard to protect from quantum corrections.

In a companion paper [5], we developed a new model of field theoretical dark energy, *Cuscuton*, that while gener-

ally nonuniform, lacks any dynamical degree of freedom. This protects the theory from quantum corrections at low energies, and thus makes it a perfect candidate for an evolving dark energy in the current era. The name *Cuscuton* (pronounced käs-kü-tän), is derived from the Latin name for the parasitic plant of dodder, *Cuscuta*. Classically, it is a new kind of constraint system, allowing a novel class of constrained dynamics.

The *Cuscuton* action for the scalar field φ can be written as

$$S = \int d^4x \sqrt{-g} [\mu^2 \sqrt{|g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi|} - V(\varphi)], \quad (1)$$

where μ is an (arbitrarily defined) energy scale. One can show that *Cuscuton* is an incompressible *k*-essence fluid [6,7], i.e. it has an infinite speed of sound, raising questions about the causality of the theory. However, in [5], we show that there is no breakdown of causality, as the perturbations lack (symplectic) dynamics, and thus do not carry information.

Given that *Cuscuton* acts as a constraint system, it is interesting to ask how a minimal coupling of *Cuscuton* to gravity will modify interaction of gravity with matter. In particular, as is well known, the scalar perturbations about the Friedmann-Robertson-Walker (FRW) metric represent the general relativistic space-time constraints for matter coupled to gravity. A nontrivial choice of *Cuscuton* potential will modify the constraint equations, leading to ob-

*Electronic address: nafshordi@cfa.harvard.edu

†Electronic address: Danielchung@wisc.edu

‡Electronic address: M.Doran@thphys.uni-heidelberg.de

§Electronic address: ghazal@physics.wisc.edu

servable consequences in cosmology. We aim to characterize these observables associated with the scalar perturbations.

We derive general scalar perturbation equations in the presence of Cuscuton and show analytically and numerically how the constraints are modified. We show that Cuscuton models can have an expansion history identical to that of Λ CDM but with different CMB and matter power spectra due to gravitational potential evolution mimicking that of tracker models of quintessence. We also find that Cuscuton can exactly replicate the expansion history of the Dvali-Gabadadze-Poratti (DGP) self-accelerating cosmology, while predicting similar small angle CMB and matter power spectra, at a few percent level. The main difference between the DGP self-accelerating cosmology and Cuscuton will show up due to the anisotropic-stress-induced boost in the Integrated Sachs-Wolfe (ISW) effect for the DGP model. Finally, we briefly consider the issue of causality in the scalar sector and find that a Yukawa-like exponential falloff of Cuscuton effects beyond the \sqrt{H} scale protects the gravitational phenomenology from gross anomalies in the long wavelength limit. We show that the usual Bardeen parameter remains approximately constant in the long wavelength limit.

The paper is organized as follows: In Sec. II, we compute the background evolution, followed by linear perturbations in Sec. III. We then study how the matter power spectrum and the ISW effect are affected by Cuscuton in Sec. IV, and discuss current observational constraints. Before presenting our conclusions in Sec. VI, we also discuss causality of linear Cuscuton perturbations in Sec. V. Throughout this paper, we use the reduced Planck's constant $M_p = (8\pi G_N)^{-1/2} \approx 2 \times 10^{18}$ GeV.

II. BACKGROUND EVOLUTION

Starting with the Cuscuton action [Eq. (1)] in a flat Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a(t)^2 dx^i dx^i, \quad (2)$$

and a homogenous field configuration, i.e. $\varphi = \varphi(t)$, the action takes the form

$$S = \int a^3 dt [\mu^2 |\dot{\varphi}| - V(\varphi)]. \quad (3)$$

Varying the action with respect to φ yields the background field equation

$$(3\mu^2 H) \text{sgn}(\dot{\varphi}) + V'(\varphi) = 0. \quad (4)$$

One implication of Eq. (4) is that $V(\varphi)$ always decreases (increases) with time in an expanding (collapsing) universe. Equation (4), in combination with the Friedmann (or G_0^0 Einstein) equation in a flat universe

$$H^2 = \frac{\rho_{\text{tot}}}{3M_p^2}, \quad (5)$$

yields

$$\left(\frac{M_p^2}{3\mu^4}\right)V'^2(\varphi) - V(\varphi) = \rho_m, \quad (6)$$

where ρ_m is the background density of ordinary matter in the Universe. Notice that the Cuscuton kinetic term does not contribute to its energy density.

In Fig. 1, we show different possibilities for the Cuscuton potential in the $V - V'$ phase space. Equation (6) implies that for a positive matter density ($\rho_m > 0$; weak energy condition), assuming a flat universe, we should have

$$V(\varphi) < \left(\frac{M_p^2}{3\mu^4}\right)V'^2(\varphi). \quad (7)$$

The excluded region is shown by the shaded area in Fig. 1, while the light line shows a constant $\rho_m > 0$ contour.

The continuity equation for matter density $\dot{\rho}_m = -3H(\rho_m + p_m)$, in combination with Eq. (4) and the time derivative of Eq. (6), assuming the null energy condition $\rho_m + p_m > 0$, gives:

$$3\mu^2 H |\dot{\varphi}| \left(\frac{2M_p^2}{3\mu^4} V''(\varphi) - 1\right) = -\dot{\rho}_m > 0, \quad (8)$$

which puts an additional constraint on the potential:

$$V''(\varphi) = \frac{1}{2} \frac{dV'^2(\varphi)}{dV(\varphi)} > \frac{3\mu^4}{2M_p^2}. \quad (9)$$

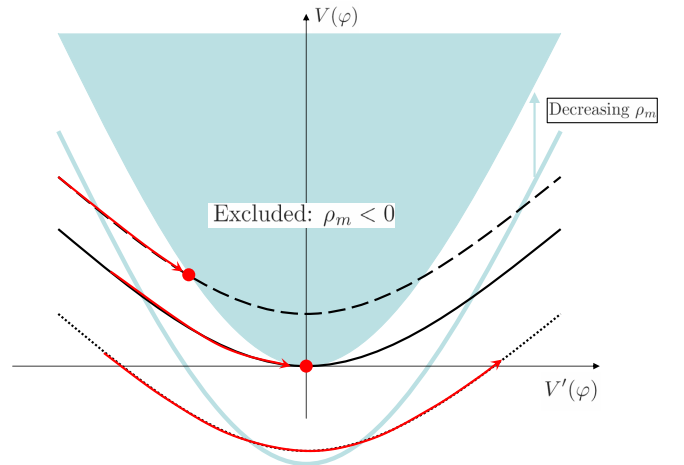


FIG. 1 (color online). Different possible Cuscuton potentials in the $V - V'$ phase space. The shaded region is excluded in a flat FRW cosmology, as it requires negative matter density, ρ_m [Eq. (6)]. The dark solid, dashed and dotted lines are quadratic potentials with zero, positive and negative bare cosmological constants. The dark arrows show the time evolution of the field in a flat universe, and the points show the asymptotic state of the field, if present. The light solid line shows a constant $\rho_m > 0$ contour. As we show in the text, in addition to avoiding the excluded region, the potentials should be shallower than the constant ρ_m contour in the $V - V'$ plane.

Therefore, all the allowed potentials need to be shallower than the constant density contours (light line) in Fig. 1.

Dark lines in Fig. 1 show different quadratic potentials of the form

$$V(\varphi) = V_0 + \frac{1}{2}m^2\varphi^2. \quad (10)$$

Interestingly—as we will show in Sec. IV—a quadratic potential leads to a tracking behavior. In this case, the quadratic term of the potential always maintains a constant fraction of the total density of the Universe. The dark solid line shows a quadratic potential with no bare cosmological constant ($V_0 = 0$). For a positive bare cosmological constant ($V_0 > 0$; dashed line), it is interesting to notice that the potential never reaches its minimum, and is stalled at the boundary of the $\rho_m < 0$ region (shaded area). Therefore, the effective value of the cosmological constant is larger than its bare value. Finally, a potential with a negative bare cosmological constant ($V_0 < 0$; dotted line) results in a bounce, as V' , and thus the expansion rate goes to zero, and changes sign subsequently.

Another interesting example is an exponential potential of the form

$$V(\varphi) = V_0 \exp\left[-\left(\frac{\mu^2 r_c}{M_p^2}\right)\varphi\right]. \quad (11)$$

Direct substitution in Eqs. (4) and (5) yields

$$H = \frac{1}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{\rho_m}{3M_p^2}}, \quad (12)$$

which is exactly the same as the background dynamics in a (flat) DGP [8] 5D self-accelerating brane-world model [9]. However, this is only a coincidence, as the detailed dynamics of metric perturbations cannot be identical. For example, the anisotropic stress always vanishes in Cuscuton models, which follows from the use of 3 + 1D Einstein equations for linear perturbations of a scalar field. On the contrary, the anisotropic stress is generically non-vanishing (see e.g. [10] and references therein) in DGP models. Nevertheless, this coincidence can be used to examine the observable differences between dark energy and modified gravity models with exact same background dynamics.

Before concluding this section, it is important to emphasize that Cuscuton is classically a theory of modified gravity, rather than Einstein gravity with additional scalar field degree of freedom. This can even be seen at the level of background equations, as combining Eqs. (4)–(6), (for $\dot{\varphi} < 0$) we can write the modified Friedmann equation as

$$H^2 = \frac{1}{3M_p^2} \{\rho_m + V[V'^{-1}(3\mu^2 H)]\}, \quad (13)$$

where V'^{-1} is the inverse function of $V'(\varphi)$. We see that, in general, H^2 is no longer linearly dependent on the energy density ρ_m and the exact nonlinearity is controlled by the

choice of function $V(\varphi)$. Moreover, unlike in ordinary Einstein-Hilbert action coupled to a homogeneous scalar degree of freedom, the modified Friedmann equation is fixed once ρ_m is fixed, i.e. one does not need to specify initial/boundary conditions for φ .

To obtain intuition for how the modification works, let us again consider the quadratic potential. If $V(\varphi) = \frac{1}{2}m^2\varphi^2$, it is simple to check that Eq. (13) is identical to the ordinary Einstein-Hilbert Friedmann equation with a renormalized Planck's constant

$$M_p^2 \rightarrow M_p^2 - \frac{3\mu^4}{2m^2}. \quad (14)$$

This is a manifestation of Cuscuton modification of gravity.

This renormalization of the Planck mass is reminiscent of a similar effect for Lorentz-Violating vector fields [11], as well as the exponential quintessence model (see Sec. IV B), although, in contrast to Cuscuton, both models do introduce an additional dynamical degree of freedom.

III. LINEAR PERTURBATIONS IN CUSCUTON COSMOLOGY

A. Linearized field equation

Varying the Cuscuton action with respect to φ in a general curved space-time yields

$$\left(g_{\mu\nu} - \frac{\partial_\mu\varphi\partial_\nu\varphi}{X}\right)\nabla^\mu\nabla^\nu\varphi + \mu^{-2}\sqrt{X}V'(\varphi) = 0, \quad (15)$$

where $X = \partial^\lambda\varphi\partial_\lambda\varphi$, and ∇^μ denotes covariant derivative. Assuming a linearly perturbed FRW metric in the longitudinal gauge

$$ds^2 = (1 + 2\Phi)dt^2 - a(t)^2(1 - 2\Phi)dx^i dx^i, \quad (16)$$

we can evaluate the field equation at the linear order in field/metric perturbations:

$$3\dot{\varphi}(\dot{\Phi} + H\Phi) + a^{-2}\nabla^2\delta\varphi - \mu^{-2}|\dot{\varphi}|V''(\varphi)\delta\varphi = 0. \quad (17)$$

Here, ∇^2 is the spatial Laplacian with respect to comoving coordinates. It is interesting to note that the linear perturbation equations do not include any second order time derivative.

Taking the time derivative of Eq. (4), we find

$$\frac{|\dot{\varphi}|V''(\varphi)}{\mu^2} = -3\dot{H}. \quad (18)$$

This yields the following form for Eq. (17) in the Fourier space:

$$\delta\varphi = \frac{3\dot{\varphi}(\dot{\Phi} + H\Phi)}{\frac{k^2}{a^2} - 3\dot{H}}, \quad (19)$$

explicitly showing that, as pointed out in [5], Cuscuton

perturbations simply follow metric perturbations in a nonlocal way, and do not introduce any additional dynamical degree of freedom. In other words, similar to the homogenous Eq. (4), *the field equation only amounts to a constraint condition that relates metric and field perturbations.*

B. Linear Einstein equations: Modified gravity

The Einstein equations for scalar metric perturbations in the presence of Cuscuton as well as ordinary (dust) matter inhomogeneities are simply written as:

$$a^{-2}\nabla^2\Phi = 3H(\dot{\Phi} + H\Phi) + (2M_p^2)^{-1}(\delta\rho_m + V'(\varphi)\delta\varphi), \quad (20)$$

$$\dot{\Phi} + H\Phi = (2M_p^2)^{-1}(\mu^2\delta\varphi + \rho_m\lambda), \quad (21)$$

$$\begin{aligned} \ddot{\Phi} + 4H\dot{\Phi} + (2\dot{H} + 3H^2)\Phi &= (2M_p^2)^{-1}(\mu^2 \operatorname{sgn}(\dot{\varphi})\delta\dot{\varphi} \\ &\quad - \mu^2|\dot{\varphi}|\Phi - V'(\varphi)\delta\varphi), \end{aligned} \quad (22)$$

where λ is the potential of the matter peculiar velocity $u^i = a^{-1}\partial\lambda/\partial x^i$.

Transforming to the Fourier space, it is interesting to note that $\delta\varphi$ can be eliminated by combining the field equations [Eqs. (4) and (19)] and the G_{00} equation [Eq. (20)] to yield a modified law of gravity

$$\begin{aligned} \left(\frac{k^2}{a^2}\right)\Phi + \left[3H + \frac{9H(2\dot{H} + 3H^2\Omega_m)}{2(\frac{k^2}{a^2} - 3\dot{H})}\right](\dot{\Phi} + H\Phi) \\ + (2M_p^2)^{-1}\delta\rho_m = 0, \end{aligned} \quad (23)$$

where $\Omega_m = \rho_m/(3M_p^2H^2)$ is the matter density in units of the critical density of the Universe.

Thus we notice that $\delta\varphi$ completely drops out of the linear gravity (or Poisson) equation, although it modifies the equation in a nonlocal way. This is another manifestation of Cuscuton being *a theory of modified gravity*, even though it is a particular limit of k -essence.

It is important to note that this modification (or screening of gravity) does not affect Newtonian gravity on small subhorizon scales, i.e. as long as $\frac{k^2}{a^2} \gg H^2, \dot{H}$.

We should point out that the only other modification of Einstein gravity that does not introduce a new degree of freedom (to the best of our knowledge), known as Modified-Source Gravity [12], is conformally equivalent to the Einstein gravity. Moreover, Modified-Source Gravity can be constructed by a nonlinear local modification of the matter Lagrangian, where, in terms of the modified Lagrangian, (and in contrast to Cuscuton) the gravity reduces to Einstein gravity.

C. Linear Einstein equations: Modified dynamics

Let us consider the modified dynamics of the gravitational potential, Φ , in the presence of Cuscuton and pressureless dark matter. Plugging Eq. (19) into Eq. (22), after straightforward manipulations, we arrive at

$$\begin{aligned} (1 + C_2)\ddot{\Phi} + (4H + C_1 + C_2H + C_3)\dot{\Phi} \\ + \left(3H^2 + \dot{H} - \frac{3}{2}\Omega_m H^2 + C_1H + C_2\dot{H} + C_3H\right)\Phi = 0, \end{aligned} \quad (24)$$

$$\begin{aligned} C_1 &\equiv \frac{3(\ddot{H} + 3H\dot{H})}{\frac{k^2}{a^2} - 3\dot{H}}, & C_2 &\equiv \frac{3(2\dot{H} + 3H^2\Omega_m)}{2(\frac{k^2}{a^2} - 3\dot{H})}, \\ C_3 &\equiv \frac{3[2H(\frac{k}{a})^2 + 3\dot{H}](2\dot{H} + 3H^2\Omega_m)}{2(\frac{k^2}{a^2} - 3\dot{H})^2}, \end{aligned} \quad (25)$$

which is the desired differential equation for Φ . Note that in the limit of pure matter dominated case without Cuscuton modification of gravity, we have

$$\Omega_m \rightarrow 1, \quad \text{and} \quad 2\dot{H} + 3H^2, \quad C_i \rightarrow 0, \quad (26)$$

implying that the solution to Eq. (24) asymptotically approaches $\Phi \rightarrow \text{constant}$. Also, note that any nontrivial scale dependence introduced by Cuscuton is characterized by the scale \dot{H} , and not H .

Consider the long wavelength limit $(k/a)^2 \ll \dot{H}$. First, let us ask whether a constant Φ is a solution to linearized Einstein equations with just pressureless dust field degrees of freedom. The Cuscuton modification that could prevent Φ from being a constant is the coefficient of Φ in Eq. (24):

$$\begin{aligned} 3H^2 + \dot{H} - \frac{3}{2}\Omega_m H^2 + C_1H + C_2\dot{H} + C_3H \\ = 3\Omega_m \left(\frac{H\ddot{H}}{2\dot{H}^2} - 1\right)H^2, \end{aligned} \quad (27)$$

which generically does not vanish unless the scale factor is of the form

$$a = a_i \left[1 + \frac{|\dot{H}_i|}{H_i}(t - t_i)\right]^{H_i^2/|\dot{H}_i|}, \quad (28)$$

corresponding to power-law expansion, or a constant effective equation of state. Therefore, having a constant effective equation of state will allow a constant Φ solution on long wavelengths.

Next, whether or not Φ is damped [due to the friction term in Eq. (24)] depends on the sign of

$$\frac{4H + C_1 + C_2H + C_3}{1 + C_2} = H + \frac{d}{dt} \ln \dot{H}^{-1}. \quad (29)$$

Hence, we see that whether the potential decays or not depends on how fast \dot{H} horizon grows/decays with time. In the case of a constant effective equation of state [or Eq. (28)], the potential will always decay until it asymp-

totically reaches a constant. As we will argue in Sec. V, the general behavior of Φ on superhorizon scales can be easily understood from the conservation of the Bardeen parameter.

In the opposite limit of $(k/a)^2 \gg \dot{H}$, the coefficient in Eq. (24) that prevents Φ from being constant becomes

$$3H^2\left(1 - \frac{\Omega_m}{2}\right) + \dot{H}, \quad (30)$$

which again does not necessarily vanish with the gravity modified by a nontrivial $V(\varphi)$, even though the form of the terms looks identical to that of Einstein gravity sourced by pressureless dust. That is because the background Einstein equations are modified by the presence of Cuscuton. Note that, as far as the damping coefficient is concerned, since $C_i \rightarrow 0$ in the short wavelength limit, the Φ term always acts as a damping term.

IV. OBSERVATIONAL SIGNATURES OF CUSCUTON COSMOLOGIES

In this section, we study the observational signatures of Cuscuton cosmology. First we consider the observables analytically, in a perturbative setting. Afterwards, we focus on different observational signatures and constraints for quadratic and exponential Cuscuton potentials.

A. Analytic treatment of general Cuscuton potentials

In addition to the matter power spectrum, the decaying gravitational potential caused by the Cuscuton modification will manifest itself in the anisotropy of the Cosmic Microwave Background. The induced CMB anisotropy due to the Fourier mode $\Phi_{\mathbf{k}}$ is [13]:

$$\begin{aligned} \Theta_{l,\mathbf{k}} = & \int_0^{\eta_0} d\eta g(\eta)(\Theta_0 + \Phi_{\mathbf{k}})j_l[k(\eta_0 - \eta)] \\ & + \frac{1}{ik} \int_0^{\eta_0} d\eta v_b(\mathbf{k})g(\eta)\frac{\partial}{\partial\eta}j_l[k(\eta_0 - \eta)] \\ & + 2 \int_0^{\eta_0} d\eta \frac{\partial\Phi_{\mathbf{k}}}{\partial\eta} e^{-\tau(\eta)}j_l[k(\eta_0 - \eta)], \end{aligned} \quad (31)$$

where η is the conformal time, Θ_0 is the monopole temperature fluctuation, $g(\eta)$ is the visibility function, v_b is the baryon velocity field, τ is the optical depth, and η_0 is the present day conformal time. The first term is known as the Sachs-Wolfe effect, while the second term is the dipole contribution, and the last term is the Integrated Sachs-Wolfe contribution, which depends on the time variation of the gravitational potential. All the integrals are taken over the light cone.

We can compute the ISW contribution to CMB anisotropies perturbatively. To first order in the Cuscuton potential, V , one can write $\partial\Phi/\partial\eta$, which appears in the ISW term of Eq. (31), as

$$\frac{\partial\Phi_{(1),\mathbf{k}}}{\partial\eta} = a_i\left(\frac{t(\eta)}{t_i}\right)^{-2} \int_{t_i}^{t(\eta)} dt' \left(\frac{t'}{t_i}\right)^{8/3} S_{\mathbf{k}}(t'), \quad (32)$$

where $t(\eta) \sim a_i^3 \eta^3 / 27 t_i^2$, $S(t)$ is given by

$$\begin{aligned} S_{\mathbf{k}}(t) \equiv & -\left\{3V + \frac{3t\dot{V}}{4} + \frac{3}{\frac{k^2}{a^2} + \frac{2}{r^2}} \left[\frac{\dot{V}}{t} + \frac{\ddot{V}}{2} + \left(\frac{\dot{V}}{2t}\right) \frac{4k^2}{3a^2} + \frac{4}{r^2} \right] \right\} \\ & \times \frac{\Phi_{(0),\mathbf{k}}}{3M_p^2}, \end{aligned} \quad (33)$$

where $\Phi_{(0),\mathbf{k}}$ is the zeroth order solution in V , to Eq. (24), which is a constant in a flat matter dominated universe. The key feature manifested in Eq. (33) is the mildness of the scale dependence. This is simply a result of the fact that $C_i \rightarrow 0$ in the limit of $k \rightarrow \infty$, while C_i does not vanish in the limit of $k \rightarrow 0$. Although more explicit expressions for Θ_l and C_l may be derived, general results deviating strongly away from cosmological-constant-induced ISW effect are complicated and unilluminating. Hence, in the subsequent subsections, we will numerically examine two interesting explicit potentials, and compare them against current cosmological observations.

B. Quadratic (tracking) potential

Using potentials of the form $V(\varphi) = V_0 + \frac{1}{2}m^2\varphi^2$, one finds solutions similar to that of a tracking dark energy [14–16] component plus a cosmological constant. While V_0 simply contributes towards the cosmological constant, the quadratic term $\frac{1}{2}m^2\varphi^2$ maintains a constant fraction of the total energy density of the Universe, much like models of early dark energy [17–19]. To see this, take the square of Eq. (4) together with the Friedmann Eq. (5) to get

$$\Omega_Q = -\frac{\Delta M_p^2}{M_p^2} = \frac{\frac{1}{2}m^2\varphi^2}{\rho_{\text{tot}}} = \frac{3\mu^4}{2M_p^2 m^2} = \text{const}, \quad (34)$$

where ΔM_p^2 denotes the equivalent change in the large scale Planck mass [Eq. (14)]. Therefore, assuming $V_0 \neq 0$, the expansion history $H(z)$ is exactly equivalent to that of a Λ CDM cosmology, as the extra quadratic component simply follows the rest of the energy content of the Universe. In fact, the expansion history $H(z)$ becomes independent of Ω_Q , provided one scales the other components of the Universe appropriately, i.e. simultaneously transforming $\Omega_i \rightarrow (1 - \Omega_Q)\Omega_i$, where i labels Λ , photons, baryons and dark matter while changing the number of relativistic neutrino species $N_\nu = 3.04(1 - 2.47\Omega_Q)$ leaves $H(z)$ invariant. This leads to two corollaries: First, there will be a degeneracy between N_ν and Ω_Q (at least, as long as we only consider the expansion history). Second, quadratic Cuscuton can only be constrained by studying subhorizon perturbations, provided one does not *a priori* know the number of neutrino species.

Current constraints on light element abundances, which are predicted from Big Bang Nucleosynthesis, already put

significant constraints on the number of relativistic neutrinos during the radiation era, $N_\nu = 3.1 \pm 0.7$ [20], which yields $\Omega_Q \lesssim 10\%$. However, as we see below, observational constraints on cosmological inhomogeneities can put much tighter constraints on Ω_Q .

Since Cuscuton perturbations have an infinite speed of sound, they do not cluster on subhorizon scales, and thus perturbations start to decay as they enter the horizon. This behavior is reminiscent of scalar field dark energy (or quintessence) for which, once inside horizon, the perturbations also free-stream with the speed of light.

One can work out this subhorizon decay analytically during matter domination where $a \propto t^{2/3}$. Then, ignoring $\delta\varphi$ in Eq. (22) (or using Eq. (24) with $C_i \rightarrow 0$), in the $k \gg aH$ regime, we end up with:

$$\ddot{\Phi} + 4H\dot{\Phi} + \frac{3}{2}H^2\Omega_Q\Phi = 0, \quad (35)$$

which can be easily solved if $H = \frac{2}{3t}$:

$$\Phi \propto a^\alpha; \quad \alpha = \frac{5}{4} \left(-1 \pm \sqrt{1 - \frac{24}{25}\Omega_Q} \right). \quad (36)$$

This result is identical to the growth suppression for a quintessence field with an exponential potential, which has a similar tracking behavior [21] (e.g. see Eq. 12 in [22]). For example, for $\Omega_Q \ll 1$, the dominant metric mode decays as $a^{-3\Omega_Q/5}$ during matter domination, which can also be obtained from Eqs. (32) and (33). This introduces a significant suppression of large scale structure power during the matter era:

$$\frac{\delta(\Omega_Q, z=0)}{\delta(0, z=0)} \approx_{z_{\text{eq}}}^{-3\Omega_Q/5} \approx 0.61\Omega_Q^{0.1}, \quad (37)$$

if we fix the amplitude at matter-radiation equality, assuming that it happened at $z_{\text{eq}} \approx 3500$. Of course, only modes inside the horizon at matter-radiation equality will feel this amount of suppression. Modes entering after equality will suffer less suppression the later they enter. This leads to a red tilt in the cold dark matter spectrum up to the scale of matter-radiation equality k_{eq} mimicking a running spectral index [23]. However, all scales $k > k_{\text{eq}}$ are suppressed by the same factor (37) which distinguishes quadratic Cuscuton from a simple running spectral index Λ CDM model. As the metric potential decays inside the horizon through the matter era, the CMB anisotropies receive an extra contribution from the Integrated Sachs-Wolfe. This leads to additional power, typically shifting the first Doppler peak of the CMB power spectrum towards slightly lower multipoles.

Fig. 2 shows the resulting matter and CMB power spectra for $\Omega_Q = 0, 0.05, \text{ and } 0.1$, where we have fixed the initial amplitude of scale-invariant adiabatic perturbations, as well as the background expansion history, to that of WMAP3 concordance model [3]. As expected, the

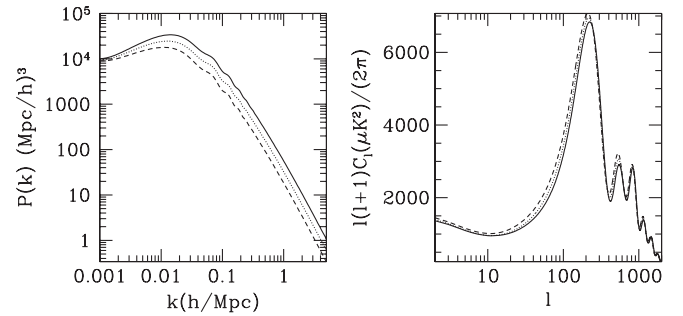


FIG. 2. Left panel: Dark matter power spectra for quadratic Cuscuton densities $\Omega_Q = 0, 0.05, 0.1$, shown as solid, dotted, and dashed lines, respectively. The initial amplitude of the scale-invariant adiabatic perturbations is kept constant. Right panel: The CMB anisotropy power spectrum for the same cosmologies.

changes in the amplitude of the matter power spectrum (left panel in Fig. 2) is most significant. However, direct measurement of the amplitude of the power spectrum (e.g. from galaxy surveys) may be problematic due to the ambiguity in the value of linear bias. Methods to measure the linear bias, while present, are not as reliable as the shape of the power spectrum, as they involve nonlinear physics, and thus are not widely used to obtain cosmological constraints. In addition, analytic calculations of nonlinear structure formation in a similar model of early dark energy yield considerable and quite unexpected deviations from the Λ CDM scenario precluding the use of standard approximations to infer the nonlinear spectrum given the linear evolution [24].

Thus, instead of galaxy catalogs, we use Lyman- α forest observations of quasar spectra from the Sloan Digital Sky Survey (SDSS) [4], which mainly constrains the linear overdensity Δ_L^2 and spectral index n_{eff} at a pivot scale of $k \approx 0.009 \text{ s/km}$ ($\approx 1 \text{ Mpc}^{-1}$), and a pivot redshift of $z \approx 3$ [4]. In this mildly nonlinear regime, we anticipate nonlinear effects of Ω_Q to play little role. This point is further strengthened from the low values of Ω_Q allowed by all other data without using Lyman- α (see below). At such low values of Ω_Q , the remaining nonlinear effects of Ω_Q will be negligible. Hence, Lyman- α forest observations seem well suited to constrain the effects of Ω_Q on growth of linear perturbations.

The changes in the CMB power spectrum are more subtle. As mentioned above, the main impacts of the Cuscuton quadratic potential can be seen in an additional ISW contribution and a slight suppression of small scale power in the CMB spectrum. We ran a Monte Carlo Markov Chain (MCMC) simulation to constrain this model using a modified version of cmbeasy [25]. For this, we used the three-year CMB data of WMAP [3], supernovae Ia measurements [26,27], constraints from baryonic acoustic oscillations [28], as well as the latest release of the SDSS galaxy survey [29] and SDSS Lyman- α forest observations [4].

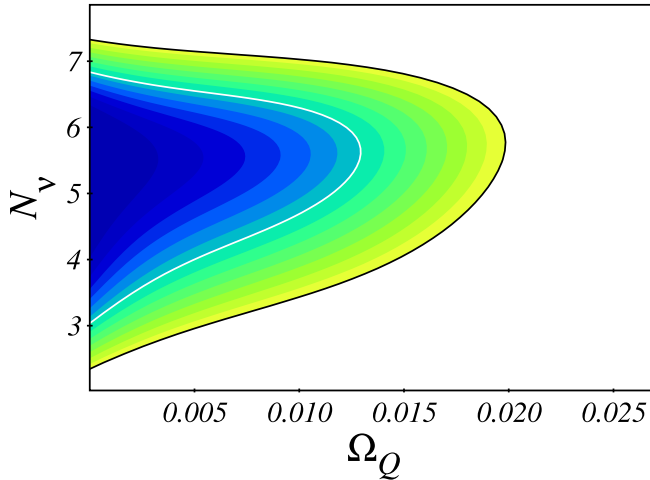


FIG. 3 (color online). Confidence contours for the abundance of the quadratic Cuscuton, Ω_Q and the number of relativistic neutrino species N_ν derived from using supernovae Ia, CMB, large scale structure and Lyman- α forest observations. The white and black boundaries indicate 68% and 95% likelihood regions. While in principle a degeneracy of the background evolution between Ω_Q and N_ν exists, it is too weak to be seen here, because the allowed abundance of Ω_Q is too small to have any significant effect on N_ν constraints (see the text).

The WMAP three-year data alone constrains $\Omega_Q < 3.6\%$ at 95% confidence level. Adding SNe Ia and large scale structure data improves this—already substantial—bound further to $\Omega_Q < 2.7\%$. Finally, adding Lyman- α forest data yields a tight limit of $\Omega_Q < 1.6\%$.

Hence, the background degeneracy of N_ν and Ω_Q while present has little effect on the contours in Fig. 3. The allowed abundance Ω_Q is very small such that the effect on N_ν is at most $\Delta N_\nu = -3.04 \times 2.47 \times \Omega_Q \approx -0.2$. This change in N_ν is rather small compared to the 68% confidence interval for N_ν , which is $N_\nu = 5.5_{-1.2}^{+0.9}$.

C. Exponential (DGP-like) potential

The DGP model of modified gravity [8], similar to other brane-world models, is inspired by nonperturbative objects in string theory. It posits that the observable universe lives on a $3 + 1$ D brane in $4 + 1$ D space-time. While matter fields are constrained to the brane, gravity can propagate into the bulk and thus can be sensitive to the full geometry of the Universe. The characteristic of the DGP model is that the brane action includes a term proportional to the volume integral of the induced Ricci scalar on our 3-brane. This is added to the ordinary Einstein-Hilbert action in the bulk, which integrates the full Ricci scalar. It turns out that at high energies, the induced action dominates the dynamics of the induced metric, leading to ordinary four dimensional gravity, while at low energies, the bulk action takes over and gravity becomes five dimensional. This may lead

to a self-accelerating phase of cosmic evolution, even in the absence of a cosmological constant.

Although it is argued that the self-accelerating phase of the DGP model contains ghosts [30–32], and thus may not be realized in a physically stable way (but see [33]), it still remains the most widely studied concrete example of a modified gravity model which competes with dark energy models as an explanation for the observed acceleration of the Universe.

As we mentioned in Sec. II, for a spatially flat 3-brane, the background evolution within a self-accelerating DGP model coincides with that of a Cuscuton model with an exponential potential [Eq. (12)]. Therefore, it is not possible to distinguish the two models based on the traditional geometrical tests of background cosmology, such as supernovae Ia [1,2], distance to the last scattering surface, or the scale of baryonic acoustic oscillations [28].

For subhorizon perturbations, References [34,35] compare the linear growth of structure in DGP model with a quintessence model with an identical expansion history. Since dark energy does not cluster on subhorizon scales for $c_s \geq 1$, such a quintessence model should have a very similar behavior to the exponential Cuscuton model. Reference [34] finds that the DGP model shows an extra 5–10% decay compared to the quintessence model (see their Fig. 2). Therefore, it is not easy to distinguish DGP and a DGP-like Cuscuton (or quintessence) model based on the growth of the large scale structure. Reference [36] estimates that this distinction can only be done at less 3σ level, with the next generation of supernovae, weak lensing, and (small angle) CMB observations.

More significant is the ISW effect induced by the anisotropic stress in the DGP model, which is a characteristic feature of modified gravity models. Reference [34] finds that, as a result of the ISW effect, the CMB anisotropy power spectrum is a factor of ~ 4 larger than Λ CDM cosmology on small ℓ 's [37]. Therefore, they advocate use of correlations between CMB and high redshift large scale structure surveys (see e.g. [38]) in order to detect signatures of the DGP model. As we show in Fig. 4, the CMB power spectrum for the exponential Cuscuton model is nearly identical to that of the DGP-like quintessence model, and is still much smaller than the prediction for the actual DGP model at low ℓ 's [39].

We thus conclude this section by pointing out that observational distinction between dark energy and modified gravity models may be significantly more difficult than advocated in the literature (e.g. [40] where the authors also point to a number of issues that need to be explored in such endeavor). A relatively simple and well-behaved dark energy model such as exponential Cuscuton can exactly replicate the expansion history of the DGP self-accelerating cosmology, while predicting similar small angle CMB and matter power spectra, at a few percent level. The smoking gun for the modified gravity models,

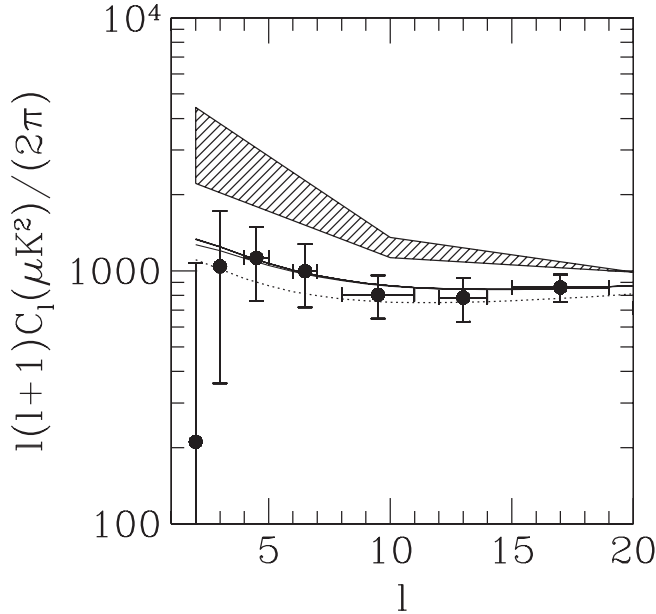


FIG. 4. CMB anisotropy power spectrum at low l 's: The solid curves show the CMB power spectrum for the DGP-like dark energy models with $c_s^2 = 1, 10, 100$, where larger sound speeds have slightly larger power. Exponential Cuscuton is realized in the $c_s^2 \rightarrow \infty$ limit, although the power spectra for $c_s^2 > 10$ are virtually indistinguishable. The dotted curve is the power spectrum for the Λ CDM cosmology, while the dashed region shows the prediction for the DGP self-accelerating model [34]. All the power spectra assume $\Omega_m = 0.26$ and $h = 0.66$. The points show the WMAP three yr observations, while the error bars indicate an estimate of the cosmic variance error based on the data [47].

thus, is their anisotropic stress, that can be potentially probed by the ISW effect in the low ℓ regime of the CMB power spectrum.

V. ON THE CAUSALITY OF CUSCUTON LINEAR PERTURBATIONS

In this section, we briefly comment on causal properties of the Cuscuton field theory with a minimal gravitational coupling.

Equation (19), which simply has a Yukawa screening form, can be written in the real space as:

$$\delta\varphi(\mathbf{x}, t) = 3\dot{\varphi}a^2 \int d^3\mathbf{x}' \frac{e^{-a|3\dot{H}|^{1/2}|\mathbf{x}-\mathbf{x}'|}}{4\pi|\mathbf{x}-\mathbf{x}'|} \times [\dot{\Phi}(\mathbf{x}', t) + H\Phi(\mathbf{x}', t)]. \quad (38)$$

Therefore, we see that the nonlocal dependence of Cuscuton on the metric perturbations is exponentially suppressed beyond a horizon that is defined by $|\dot{H}|$. It is interesting to notice that an \dot{H} horizon also naturally occurs in cosmological gravitomagnetism, beyond which, the coupling of a gyroscope precession to the rotation of its surrounding environment saturates [41].

Since metric scalar perturbations do not propagate, and tensor perturbations that do propagate do not couple to Cuscuton perturbations at the linear order, it is not possible to rigorously address the question of causality of Cuscuton coupled to gravity, in the context of what we have done thus far. However, it is clear that the naive picture of instantaneous interaction, due to the infinite sound speed of Cuscuton, does not hold in a general relativistic context, as Cuscuton is exponentially insensitive to metric fluctuations at separations larger than an \dot{H} horizon. By the same token, and direct substitution into Eq. (24), one can see that superhorizon perturbations in the Bardeen parameter [42],

$$\zeta = \Phi - \frac{H}{\dot{H}}(\dot{\Phi} + H\Phi) = \Phi + \frac{2(H^{-1}\dot{\Phi} + \Phi)}{3(1+w)}, \quad (39)$$

remain constant until the modes enter the \dot{H} horizon, where w is the effective equation of state. Therefore, *there is no gross violation of causality* in Cuscuton cosmology.

VI. CONCLUSIONS

In this paper, we have studied the physical features, as well as possible observational signatures of a cosmology with Cuscuton dark energy, which was first introduced in a companion paper [5], and could be realized as an incompressible k -essence fluid. We showed that Cuscuton perturbations have no independent dynamical degree of freedom and, in lieu of other couplings, simply follow the space-time metric. Therefore, Cuscuton can be considered to be a minimal theory of evolving dark energy, or a minimal modification of a cosmological constant. Because of lack of internal dynamics, Cuscuton only modifies (or dresses) the gravity of massive objects, and thus resembles a modified gravity theory. Indeed, to the best of our knowledge, this is the only modification of Einstein gravity that does not introduce any additional degree of freedom (and is not conformally equivalent to Einstein gravity). We then studied two specific Cuscuton cosmologies, with quadratic and exponential potentials.

We saw that the expansion history of Cuscuton cosmology with a quadratic potential is identical to that of Λ CDM, and thus geometrical tests such as supernovae Ia, or the angular scale of baryonic acoustic oscillations, are blind to a quadratic term in the Cuscuton potential. Nevertheless, the quadratic term acts as an early dark energy component with a constant energy fraction, and thus can be detected via its influence on the matter power spectrum, or through the ISW effect in the CMB. We find that joint constraints from supernovae Ia, CMB anisotropies, power spectra of galaxy surveys, and Lyman- α forest fluctuations limit this component to $\Omega_Q < 1.6\%$, which is the most stringent upper limit that has ever been put on an early dark energy component.

For the exponential Cuscuton model, we found that, surprisingly, the expansion history is identical to that of a flat DGP self-accelerating modified gravity model. The

only observational distinction between the two cosmologies is in the ISW effect at $\ell \lesssim 20$ in the CMB power spectrum, and is due to the anisotropic stress, present in the DGP modified gravity model.

Indeed, cosmologists have yet to develop efficient techniques to detect a smoking gun for modified gravity. Exponential Cuscuton cosmology is a clear example of where such a smoking gun may become necessary, and detection of a nonvanishing anisotropic stress (see e.g. [43]), through ISW effect, appears to be the only way to rule out any such (scalar) dark energy model. An independent motivation for why new physics might be at work in the generation of large angle ISW effect may also come from observations of large angle anomalies in the CMB sky (e.g. [44], and references therein).

Finally, we showed that, despite its infinite speed of sound, Cuscuton perturbations are exponentially insensitive to the metric perturbations beyond the Hubble radius, justifying why there is no gross violation of causality for superhorizon perturbations.

It is now becoming clear that the phenomenology of dark energy will be the central theme in theoretical and observational cosmology, over the next ten years [45]. Nevertheless, all the current theoretical models for any

observable deviation from a cosmological constant are at best ill-motivated, or at worst, already ruled out. Although falling short of solving the celebrated “cosmological constant problem” [46], Cuscuton dark energy provides an alternative which could be considered more natural, as it is protected against quantum corrections [5] (even in the presence of other couplings), and yet allows for a potentially observable evolving dark energy. The enthusiastic cosmologist may thus find some use for Cuscuton in trying to convince the skeptical theorist about the benefits of cosmological dark energy probes!

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