

**Hurdles for recent measures in eternal inflation**Anthony Aguirre,<sup>1,\*</sup> Steven Gratton,<sup>2,†</sup> and Matthew C Johnson<sup>1,‡</sup><sup>1</sup>*SCIPP, University of California, Santa Cruz, California 95064, USA*<sup>2</sup>*Institute of Astronomy, Madingley Road, Cambridge, CB3 0HA, United Kingdom*

(Received 23 March 2007; published 4 June 2007)

In recent literature on eternal inflation, a number of measures have been introduced which attempt to assign probabilities to different pocket universes by counting the number of each type of pocket according to a specific procedure. We give an overview of the existing measures, pointing out some interesting connections and generic predictions. For example, *pairs* of vacua that undergo fast transitions between themselves will be strongly favored. The resultant implications for making predictions in a generic potential landscape are discussed. We also raise a number of issues concerning the types of transitions that observers in eternal inflation are able to experience.

DOI: [10.1103/PhysRevD.75.123501](https://doi.org/10.1103/PhysRevD.75.123501)

PACS numbers: 98.80.Cq, 98.80.Bp, 98.80.Qc

**I. INTRODUCTION**

In eternal inflation, different post-inflationary regions may have different properties. How—even in principle—to statistically describe these properties so as to make probabilistic cosmological predictions is a major outstanding problem in current cosmology. Recently, a number of proposals have been advanced for “gauge-independent” measures that do not depend on the choice of a time coordinate [1–4]. In this paper we compare, contrast, and assess the existing proposals, and point out some predictions that they seem to share. We focus here on eternal inflation as driven by a potential with multiple minima; transitions between these correspond to the nucleation of bubbles or “pocket universes” containing a new phase of different vacuum energy [5,6]. If transitions are sufficiently slow, the growing bubbles never percolate, and inflation is eternal.

A form of predictions in a multiverse is a set of statements such as “the probability that a randomly chosen  $X$  is in a region with properties  $\alpha$  is  $\mathcal{P}_X(\alpha)$ ,” where  $X$  is some “conditionalization object” such as a point in space, a baryon, a galaxy, or an “observer” that arguably makes  $\mathcal{P}_X$  relevant to what we will actually observe in some future experiment (see, e.g., [7,8]). This probability is generally split into two components:

$$\mathcal{P}_X(\alpha) \propto P_p(\alpha) n_{X,p}(\alpha). \quad (1)$$

Here,  $P_p$  is a “prior” probability distribution defined in terms of some type of object  $p$  regardless of the conditionalization object  $X$ , and  $\alpha$  is a vector of properties we might hope to compare to locally observed properties of our universe. For example, if  $p$  = “pocket universe” then  $P_p(\alpha)$  describes the probability that a randomly chosen bubble has low-energy observable properties  $\alpha$ . The factor  $n_{X,p}$  conditions these probabilities by the requirement that

some  $X$  object exists; for example, with  $X$  = “galaxy,”  $n_{X,p}(\alpha)$  might count the ( $\alpha$ -dependent) number of galaxies in a pocket with properties  $\alpha$ .

The measures discussed here are proposals for calculating  $P_p$  (though we will also discuss some relevant issues concerning  $n_{X,p}$ ). We shall see that many of the measures share some properties—for example, they all accord very high probability to regions in a potential landscape which allow for very rapid transitions between nearby minima. Unfortunately, these regions of the landscape look nothing like our universe: the resulting spacetimes would almost certainly be dominated by a very high vacuum energy and be devoid of structure. This, of course, is nothing new—the whole idea of the “anthropic” approach to explaining our observed universe is that  $n_{X,p}$ , where  $X$  = “observer,” will “unweight” such states. But we shall see that employing the measures under consideration makes the problem very acute.

More generally, while the measures we discuss are all stated and formulated in rather different ways, many of them are, in fact, either fully or partially equivalent (as acknowledged by the authors in some cases); we will attempt to sort out these relations comprehensively. Differences do exist, however, and we will also see that certain “desirable” properties hold in some measures and not others.

In Sec. II, we present a “scorecard” of features that might be desirable in a measure, and we summarize a number of recent measures and the connections between them. We then compute the prior distribution for a number of sample landscapes in Sec. III and use the results to highlight important predictions and connections. The implications of these predictions are discussed in Sec. IV. Section V describes some problems associated with the assumptions usually made about the global picture of an eternally inflating spacetime, and we conclude in Sec. VI. In the Appendix, a quick matrix method for calculating bubble abundances is introduced and a number of the more technical results of the paper are derived.

\*Electronic address: [aguirre@scipp.ucsc.edu](mailto:aguirre@scipp.ucsc.edu)†Electronic address: [stg20@cam.ac.uk](mailto:stg20@cam.ac.uk)‡Electronic address: [mjohnson@physics.ucsc.edu](mailto:mjohnson@physics.ucsc.edu)

## II. MEASURE DESIDERATA AND PROPOSALS

### A. Desirable measure properties: A scorecard

To test a theory of eternal inflation yielding diverse post-inflationary predictions, we would like to know “what physical properties are most likely,” and compare them to our local observations. This question, however, is simply ambiguous—any answerable version of this question will entail a tacit choice of a conditionalization  $X$  and calculation of  $\mathcal{P}_X$  as described above. The measures we will discuss correspond to different attempts to (at least implicitly) propose a plausible candidate for  $X$ , and to calculate the prior distribution  $P_p$  that might be used in calculating  $\mathcal{P}_X$  for that  $X$ .

A fundamental property that a well-defined measure should have is that its answer should be gauge-invariant, by which we simply mean that its answer can be calculated in any coordinate system we choose. This is distinct from “gauge independence” as we shall discuss shortly.

Beyond this, it is important to consider what properties we might want a sensible measure to have. Some such desiderata, either stressed previously in the literature or first mentioned here, are given below. We note, however, that it is quite possible that the “correct” measure (if it exists) does not satisfy every item.

- (1) **Physicality**—The  $p$  to which the measure applies, and the choice of  $P_p$ , should be such that (a) the probabilities do not appear to have been “picked out of a hat,” and (b)  $n_{X,p}$  is plausibly calculable. For example, we might choose  $p = \text{“vacuum”}$  and set  $P_p$  proportional to the tenth power of the hyperbolic tangent of the energy of the vacuum in Planck units. However, (a) this measure is obviously rather arbitrary, and (b) since there is no physical process behind the creation of regions described by the different vacua, the measure seems useless in calculating  $n_{X,p}$  for, say,  $X = \text{“baryon.”}$  Note, however, that different physically reasonable conditionalization objects may require different  $P_p$ —for example, were  $X = \text{“vacuum,”}$  then the measure would still violate condition (a), but would satisfy condition (b) by definition.
- (2) **Gauge independence**—The relative probabilities should not depend on an arbitrary decomposition of spacetime into space and time. For instance, it has been shown [9–12] that measures that weight based on the physical volume in a given state at late times give a result that depends sensitively on the assumed foliation of spacetime into equal-time hypersurfaces. In the absence of a strong physical reason for choosing a particular decomposition, such measures thus seem ambiguous.
- (3) **Ability to cope with varieties of transitions and vacua**—The measure should be general enough to treat all of the types of vacua (e.g. positive, negative,

or zero energy) and the various types of transitions between them.

- (4) **Independence of initial conditions**—It is often argued that eternal inflation approaches a steady state, and that essentially all observers exist “at late times,” so a physically reasonable measure should become independent of initial conditions. This criterion is not obviously necessary; although it may be appropriate for a particular conditionalization object (e.g.  $X = \text{“a randomly chosen observer”}$ ), it may not be appropriate for others. For example, if one were interested in knowing what a *given* observer (or worldline) will experience in the future, then a dependence on initial conditions seems quite reasonable.
- (5) **Ability to cope with various and/or varying topological structures**—The measure should potentially be applicable to spacetimes with nontrivial topological structures as may arise in eternal inflation (as discussed at length in Sec. V).
- (6) **Accurate and robust treatment of “states” and “transitions”**—This entails several subcriteria:
  - (a) **General principles**—The basic ideas behind the measure should allow it to be used (in theory) for the complicated “spacetimes” of landscapes that cannot simply be encapsulated by transition rates between vacua.
  - (b) **Physical description of transitions**—Transition rates must be clearly linked to the physical process that describes the transition (e.g. Coleman-De Luccia bubble nucleation).
  - (c) **Reasonable treatment of “split” states**—The measure should properly deal with very similar states and/or very large transition rates. (For example, a vacuum split by the insertion of a small potential barrier should, in the limit of an infinitesimal barrier, act just as a single vacuum.)
  - (d) **Continuity in transition rates**—When transition rates are used, the measure should be continuous in these rates. For example, there should be no discontinuity in the probabilities between a stable vacuum and a metastable vacuum with a lifetime  $\tau$ , in the limit  $\tau \rightarrow \infty$ .

We would argue that all of these potentially pleasing features are absent in at least one measure proposal in the literature, and that no extant proposal clearly fulfills them all. But the good news is that the bubble-counting procedures discussed here satisfy many of them, so let us summarize these measures and provide a listing of connections between them.

### B. The measures and their properties

We now examine the various measures under consideration. All of these have subtleties, so we refer the reader to

the original papers, and also to the review by Vilenkin [13] and to the lectures of Shenker [14]. Here, we will mainly provide brief summaries, but will also add extended comments on some measures.

Restricting the discussion to eternal inflation as driven by a potential with multiple minima, it is useful to classify vacua as “terminal” or “recycling”: terminal vacua can be reached, but never exited; recycling vacua can exit to the state from which they originated, and may also transition to other states. Following [1], we can also label entire landscapes as terminal or recycling; the former contain at least one terminal vacuum whereas the latter do not.

As a first step in this analysis, we can divide the measures into three categories: first, those that calculate volumes in different vacua on some equal-time surface; second, those that count individual bubbles; third, those that focus on the vacua experienced by an observer following a single worldline.

There are two basic volume-counting methods, counting either physical volume (i.e.  $p = \text{“unit of physical volume”}$ ) or comoving volume ( $p = \text{“unit of comoving volume”}$ ). See, e.g., [9–12,15] for the former; here we focus on the following:

- (i) *The comoving volume (CV) method*: Put forward by Garriga and Vilenkin [16], this method might be considered the counterpart for bubble nucleations (in comoving volume) to the work of Linde, Linde, and Mezhlumian [9] in stochastic inflation. One starts with some region on an initial spacelike surface and considers a congruence of hypersurface-orthogonal geodesics (the “comoving observers”) emanating from that region. As a function of some global time coordinate  $t$ , the number of worldlines (to which the comoving volume fraction is defined to be proportional) in different vacua is calculated. The probability,  $P^{\text{cv}}$ , to be in a given vacuum is then defined to be proportional to the fraction of comoving volume (or number of worldlines),  $f_i(t)$ , in that pocket, in the  $t \rightarrow \infty$  limit. Note that if there are terminal vacua, then as  $t \rightarrow \infty$  all of the comoving volume will be distributed among the terminal vacua, except for a set of measure zero (albeit one that corresponds to infinite physical volume). Metastable vacua are thus accorded zero weight. This measure depends heavily on initial conditions, because the fraction of comoving volume in a given terminal vacuum can only increase with time [17].

The next two methods, rather than counting total relative volume in different bubble types, count relative total *numbers* of bubbles, i.e.  $p = \text{“bubble.”}$

- (ii) *The comoving horizon cutoff (CHC) method*: In the proposal of Garriga *et al.* [3], the measure is defined by directly counting bubbles of a given phase. One has in mind performing the count at late times, or “future infinity.” We follow the most recent descrip-

tion of this procedure as given by Vilenkin [13]. First, just as in the CV method, a spacelike hypersurface in the spacetime is chosen, and a congruence of geodesics is extended from this hypersurface. The geodesics are followed arbitrarily far into the future, passing into any bubbles they may encounter. These lines are used to project bubbles in the spacetime back onto the initial hypersurface as “colored shadows.” The relative frequency of bubbles of different colors is defined to be the ratios of the numbers of their shadows on the initial hypersurface. The shadows are very clumped, gathering around the rare regions where inflation continues longest, with an arbitrarily large number of arbitrarily small overlaid shadows surrounding the set (of measure zero) of points on the surface where inflation continues forever. Thus, all counts are infinite numbers and require regularization to be well defined. The authors propose only counting shadows larger than a size  $\epsilon$  and then taking the limit  $\epsilon \rightarrow 0$ . This measure is argued to be independent of initial conditions on the surface and applies to terminal and recycling vacua. It also has the important feature of giving metastable states nonzero weight. While the idea of “counting bubbles at future infinity” is intuitively clear, it is somewhat unclear that the “shadow counting” used to actually implement the cutoff is particularly physical.

Moreover, converting this idea into an actual calculation is a subtle matter. To date, such calculations have been performed in a rate-equation framework in which one follows the fractions of comoving volume in the various vacua and then effectively “divides through” by the bubble volume in order to obtain the bubble count. The shadow-size cutoff is then implemented by imposing a *set* of late-time cutoffs, one for each bubble type out of which the counted bubbles are nucleated (on the assumption that this determines the “comoving size” of the nucleated bubbles, and thus the size of the shadow, to which the cutoff applies). This cutoff,  $t_{ij}^{(\epsilon)}$ , for transitions out of vacuum  $j$  into vacuum  $i$ , is given by [3]

$$t_{ij}^{(\epsilon)} = -\ln(\epsilon H_j), \quad (2)$$

and is designed so that, when bubbles intersecting the cutoff surface are projected back onto the initial surface, only bubbles of size exceeding  $\epsilon$  will be obtained.

There are, however, some features of this calculation that warrant a closer look. For example, the formalism allows situations in which a bubble formed soon after its parent can be assigned a larger asymptotic comoving size than the parent (we thank Alex Vilenkin for discussions of this point) and may there-

fore be included in the counting while its parent is not. It is, however, unclear if or how the nucleation of bubbles larger than their parent actually occurs, or what asymptotic size should really be assigned to them. One might hope that such events lead to a small error, but this is not clear because the ratio of four-volume between the cutoff surfaces to the full four-volume before the cutoffs may be large. Thus, rather than the time period between the cutoffs being unimportant, nucleations during this period may actually dominate the bubble statistics. Details of this sort should serve to encourage the development of calculational techniques in which spacetime dependence is more explicitly taken into account.

- (iii) *The worldline (W) method*: Easter *et al.* [2], whose measure we denote the W method, assume that, at some initial time (defined by a spacelike hypersurface), the universe is in some places in a nonterminal vacuum. They then suggest considering a finite number of randomly chosen points on this initial data surface and following forward worldlines with randomly chosen velocities [18] from these initial data points. Only bubbles that are encountered by at least one of these worldlines are counted in determining the relative bubble abundance (no bubble is counted more than once, even if multiple worldlines enter it). One then takes the total number of worldlines to infinity. Like CHC, this measure is claimed to be essentially independent of initial conditions as long as inflation is eternal. It was argued in [3] that the CHC and W methods of bubble counting yield identical answers for terminal landscapes (the W method is ill defined for fully recycling landscapes as discussed in [4]).

The remaining two measures focus on the transitions between vacua experienced by a single eternal worldline, and accord a probability to a vacuum that is proportional to the relative frequency with which it is entered ( $p =$  “segment of a worldline between vacuum transitions”).

- (iv) *The recycling transition (RT) method*: The proposal of Vanchurin and Vilenkin [4], which we will refer to as the RT method, is to follow the evolution of a given geodesic observer and set the probability to be in a given vacuum proportional to the frequency with which this vacuum is entered, in the limit where the proper time elapsed goes to infinity. As presented, the method only applies to landscapes with no terminal vacua, and was argued to be equivalent to the CHC method in that case [4].
- (v) *The recycling and terminal transition (RTT) method*: The Bousso proposal [1], which we denote the RTT method, covers the cases of terminal and recycling vacua. Here, one chooses an initial condition for the worldline (the predictions of this measure are dependent on initial conditions), and considers the relative

probabilities of the worldline entering various other vacua, averaging over possible realizations. This is equivalent to the RT measure in the case where there are no terminal vacua.

The focus in RTT on the worldline of an observer is presented as being motivated by holography and the desire to only consider regions of spacetime that an observer can signal to and receive signals from (the “causal diamond”). However, this viewpoint makes essentially no difference to the mathematics and—as mentioned below—the time average over histories for Bousso’s observer could equally well be thought of as spatial averages over widely separated worldlines in any of the above approaches. A similar observation is made in [15]. Of course, a holographic point of view might lead one to strongly disfavor further possible weighting factors to apply such as volume weighting.

Although we will not treat them further, let us also mention some other approaches to asking about predictions in eternal inflation. In [12], Tegmark advances a simple and direct possible answer to the question of the relative numbers of different vacuum regions: because eternal inflation should produce a countably infinite number of each type of vacuum region, and because all countable infinities are equal in the sense of being relatable by a one-to-one mapping, each vacuum should be assigned equal weight. In [19], the authors put a measure on the space of classical Friedmann Robertson Walker (FRW) solutions to the Einstein plus scalar field equations. If this could be extended to allow for quantum jumps analogous to bubble nucleations, it might help address the distribution of vacua within and amongst solutions. In [20], the authors focus on histories that might be/might have been observed, in the context of single-field inflation with a monotonic potential.

### C. Relations between the measures

Although the methods, both in their motivation and in their presentation here, have been categorized into “volume counting,” “bubble counting,” and “worldline following,” there are relations between them that cross these divisions, so that in fact there are actually very few essentially different measures under consideration.

Some of the relations between measures (as presented by their authors) have been mentioned above (e.g. the equality of CHC and W for terminal landscapes, and the equality of CHC and RT for “fully recycling” landscapes with no terminal vacua). More, however, exist.

In particular, the RTT method accords the same relative probabilities to terminal vacua as does the CV method (though the methods differ for nonterminal vacua, which have zero probability in CV and nonzero probability in RTT). To see this, consider a congruence of comoving worldlines starting in some vacuum. Now, as  $t \rightarrow \infty$ , every worldline that will eventually end up in a terminal vacuum



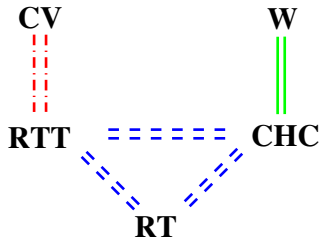


FIG. 1 (color online). A summary of the connections between the various measures. Solid green lines indicate equivalence between the measures for a terminal landscape. Dashed blue lines indicate equivalence in the case of a fully recycling landscape. Dashed-dotted red lines indicate that the measures assign the same relative weights to terminal vacua.

will do so (by definition); moreover, each terminal vacuum will only be entered once (also by definition). Since RTT accords relative probability to two terminal vacua, A and B, equal to the relative probability of a worldline entering them, this will be equal to the relative numbers of worldlines terminating in A versus B, which is in turn equal to the relative  $t \rightarrow \infty$  comoving volume fractions as defined in the CV method. In the Appendix, we show this correspondence by directly comparing the results of the RTT and CV methods in the context of a specific model. More generally, the results of the RTT method, for terminal as well as recycling landscapes, can be obtained by integrating the incoming probability current into the various vacua [15,21].

These relations between the measures (as formulated in the original papers) are summarized in Fig. 1. It also appears possible to use what is understood about these connections to devise some hybrid or generalized versions of the methods.

For example, take the CV procedure, where only a single late-time hypersurface is considered, and attempt to count the number of bubbles intersecting this surface from the volume distribution and some appropriately defined cutoff. This is *not* quite the CHC method since, as described above, the CHC calculation requires a different time cutoff for bubbles formed in different parent vacua. But this CV-CHC “hybrid” prescription does not seem any less reasonable to us. One could also generalize the CHC prescription to obtain an infinite number of related measures by altering the limiting procedure: rather than only counting shadows larger than a size independent of the bubble type, one could instead only count shadows larger than a given size relative to, say, some function of their Hubble radius. That is, for bubbles of type  $P$ , rather than only counting those that have shadows larger than  $\epsilon$  on the initial surface, count those that have shadows larger than, say,  $\epsilon' H_P$  or  $\epsilon''/H_P$ . This would correspond to replacing the time cutoff of  $-\ln(\epsilon H_M)$  for bubbles of type  $P$  forming out of bubbles of type  $M$  with  $-\ln(\epsilon' H_M H_P)$  or  $-\ln(\epsilon'' H_M/H_P)$ . It would be interesting to investigate how (in)sensitive the

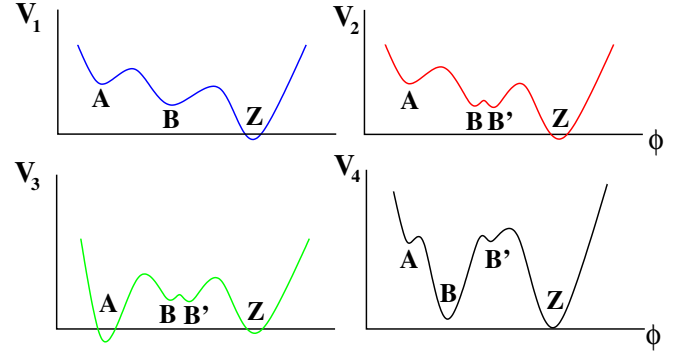


FIG. 2 (color online). Some sample landscapes. Potential  $V_1$  depicts the  $ABZ$  example discussed by Bousso [1].  $V_2$  splits the  $B$  vacuum by introducing a small barrier. The potential  $V_3$  lowers the  $A$  vacuum to zero or negative energy, so that it becomes terminal. The potential  $V_4$  has a low-energy minimum with high-energy neighbors that have short lifetimes (relative to other vacua in the landscape).

probabilities are to the choice of a particular cutoff procedure.

Having described the various bubble-counting measures and their connections, we now use a set of sample landscapes to illustrate some of their predictions.

### III. SOME SAMPLE LANDSCAPES

Consider the related one-dimensional landscapes pictured in Fig. 2. They all contain both terminal and recycling vacua (where we assume here that a vacuum is terminal if and only if its energy is zero or negative), and we now discuss the predictions made by the RTT method for each. In light of the close connections between the measures, many of the conclusions drawn from these calculations will hold more generally.

Following Bousso, we define the relative probability  $\mu_{NM}$  to transition from vacuum  $M$  to vacuum  $N$  as

$$\mu_{NM} \equiv \frac{\kappa_{NM}}{\sum_P \kappa_{PM}} \quad (3)$$

where  $P$  is summed over all decay channels out of  $M$ , and  $\kappa_{NM}$  is the probability per unit time of tunneling from vacuum  $M$  to vacuum  $N$ . Note that all summations in this paper are expressly indicated.  $\kappa_{NM}$  typically takes the form of a three-volume times a nucleation rate per unit four-volume, the latter being calculated using semiclassical instanton techniques. Note that  $\sum_P \mu_{PM} = 1$  if  $M$  is metastable and  $\mu_{PM} = 0$  if  $M$  is terminal, and also that  $\mu_{MN} \neq \mu_{NM}$ , in general. Bousso introduces the concepts of trees and pruned trees in order to calculate the prior distribution in the RTT method. He also presents a matrix formulation, which we develop further in the Appendix.

It will be important for what follows to obtain an indication of the magnitudes of tunneling rates in a typical landscape. We model this landscape by a single scalar field

$\phi$  with a potential  $V(\phi)$  expressed as  $V(\phi) = \mu^4 v(\phi/m)$ . We further assume that  $v$  is a smooth function that varies over a range of order unity as its argument changes by order unity, and  $\mu$  sets the energy scale. For the semiclassical approximation that we are working in to make sense, we must have  $\mu^4 \ll M_{\text{Pl}}^4$ , where  $M_{\text{Pl}}$  is the Planck mass. For Coleman-De Luccia instantons to exist,  $m$  must be less than some  $O(1)$  multiple of  $M_{\text{Pl}}$ . See [22] for more on the motivation for this form of the potential.

As mentioned above, we will estimate tunneling rates between the potential minima using semiclassical instanton techniques, notwithstanding thorny issues of interpretation, particularly for upward transitions. Then  $\kappa_{NM} \propto e^{-(S_{(NM)} - S_M)}$ , the bracketed exponential factor being the difference between the action  $S_{(NM)}$  of the Coleman-De Luccia or Hawking-Moss instanton linking the two vacua and the action  $S_M$  of the Euclidean four-sphere corresponding to the tunneled-from spacetime. Note that the same instanton applies to uphill and downhill transitions (hence the use of symmetrizing brackets in its label). Using the Euclidean equations of motion,  $S_{(NM)}$  can be written as

$$S_{(NM)} = - \int \sqrt{g} V(\phi) d^4x \quad (4)$$

where the integral is performed over the Euclidean manifold of the instanton. The background subtraction term (which is negative and larger in magnitude than the instanton action) is given by the same expression and evaluates to

$$S_M = - \frac{3M_{\text{Pl}}^4}{8V(\phi_M)}, \quad (5)$$

where  $V(\phi_M)$  is the value of the potential of the pretunneling vacuum  $M$  at  $\phi = \phi_M$ .

From these formulas we can immediately deduce two important facts. First, we can compare uphill and downhill rates between two vacua. In the ratio of the rates the instanton part cancels out, and only the background parts are left. If  $V(\phi_M) = V(\phi_N) + \Delta V$ , then

$$\frac{\kappa_{MN}}{\kappa_{NM}} \sim \exp \frac{-3M_{\text{Pl}}^4}{8} \frac{\Delta V}{V^2(\phi_M)} = \exp \frac{-3}{8} \frac{\Delta v}{v_M^2} \left( \frac{M_{\text{Pl}}}{\mu} \right)^4. \quad (6)$$

So, unless  $\Delta v$  is tuned to be much smaller than  $v$ , the uphill rate is exponentially smaller than the downhill rate.

Second, we can compare the rates to two vacua  $N$  and  $P$  from the same parent vacuum  $M$ . This time the background parts cancel and we are left with the exponential of the difference of the instanton actions:

$$\frac{\kappa_{PM}}{\kappa_{NM}} \sim \exp -(S_{(PM)} - S_{(NM)}). \quad (7)$$

Both instanton actions will be of order  $(M_{\text{Pl}}/\mu)^4$ , so we typically expect the tunneling rates to differ exponentially. In particular, if  $V_N$  and  $V_M$  are somewhat atypically similar and there is only a small barrier between the two, then, as

long as  $V_P$  is not atypically close to  $V_M$  also, tunneling from  $M$  to  $P$  will be exponentially disfavored relative to tunneling to  $N$ . This holds even if the tunneling from  $M$  to  $N$  is uphill and that from  $M$  to  $P$  is downhill. This difference in tunneling rates can be extreme: for a typical inflationary energy scale of  $\mu \sim 10^{16}$  GeV,  $\kappa_{PM}/\kappa_{NM} \sim e^{-10^{12}}$ .

### A. Coupled pairs dominate in terminal landscapes

We begin by considering the potential  $V_2$  depicted in Fig. 2. We assume that the barrier separating  $B$  and  $B'$  is very small, so that rapid transitions occur between the two wells. Thus we take  $\kappa_{B'B} \gg \kappa_{AB}$  and  $\kappa_{BB'} \gg \kappa_{ZB'}$ . Using the results of the Appendix, in the limit we obtain

$$\begin{pmatrix} P_A^{A,B,B'} \\ P_B^{A,B,B'} \\ P_{B'}^{A,B,B'} \\ P_Z^{A,B,B'} \end{pmatrix} \propto \begin{pmatrix} \kappa_{BB'} \kappa_{AB} \\ \kappa_{BB'} \kappa_{B'B} \\ \kappa_{BB'} \kappa_{B'B} \\ \kappa_{B'B} \kappa_{ZB'} \end{pmatrix} \quad (8)$$

where  $P_N^M$  is the prior probability of Eq. (1) (with subscript  $p$  dropped) to be in the vacuum  $N$ , given an initial state in vacuum  $M$ . A multiple superscript indicates that the same distribution applies to the listed initial states for the transition rates under consideration.

There are a number of interesting points to note here. First

$$\frac{P_B^{A,B,B'}}{P_A^{A,B,B'}} = \frac{\kappa_{B'B}}{\kappa_{AB}} \gg 1, \quad (9)$$

$$\frac{P_B^{A,B,B'}}{P_Z^{A,B,B'}} = \frac{\kappa_{BB'}}{\kappa_{ZB'}} \gg 1. \quad (10)$$

These ratios hold independent of initial conditions. Vacuum  $B'$  is similarly weighted relative to  $A$  and  $Z$ . We therefore see that (as might be expected in a measure that counts transitions) metastable vacua participating in fast transitions with their neighbors are weighted very heavily. Such regions certainly exist in a landscape with sufficient complexity, and it is these regions that the prior distribution in the RTT method will favor. From our above estimates of typical transition rates in regimes with energies somewhat below the Planck scale, factors of order  $e^{10^{12}}$  should be commonplace.

Of course, *arbitrarily* fast transitions between  $B$  and  $B'$  (which give arbitrarily high weighting to both vacua) are unrealistic. In reality, bubble collisions will become important, and at high enough nucleation rates there will be percolation. In this limit, there should then be a transition to a treatment in terms of field-rolling and diffusion. In this regard, it would be desirable to treat field diffusion as described by the stochastic formalism and bubble nucleation (with collisions taken into account) in a unified way (see [21] for work in this direction).

Although the CHC measure is inequivalent to the RTT measure in landscapes with terminal vacua, it (and hence the W method) nevertheless gives similar qualitative predictions. We can see this by analyzing the ‘‘FABI’’ model of [3], which, in the limit where  $\kappa_{B'B} \gg \kappa_{AB}$  and  $\kappa_{BB'} \gg \kappa_{ZB'}$ , gives the same ratios as Eqs. (9) and (10). Thus the CHC and W proposals weight fast-transitioning states exponentially more than others in exactly the same way the RTT method does. The weighting can easily be large enough to dominate any volume factors, which appear in the full probability defined using the CHC method [3], unless the number of e-folds during the slow-roll period after a transition is extreme.

We have seen that pairs of vacua undergoing fast transitions in both directions are weighted very heavily, but what about transitions that are fast in one direction only? For example, consider  $V_4$  in Fig. 2, where there are quick transitions into  $B$ , but transitions out of  $B$  are strongly suppressed. Requiring only  $\kappa_{BB'} \gg \kappa_{ZB'}$  in the probability tables from the Appendix yields

$$\begin{pmatrix} P_A^{A,B,B'} \\ P_B^{A,B,B'} \\ P_{B'}^{A,B,B'} \\ P_Z^{A,B,B'} \end{pmatrix} \propto \begin{pmatrix} \kappa_{BB'} \kappa_{AB} \\ \kappa_{BB'} (\kappa_{AB} + \kappa_{B'B}) \\ \kappa_{BB'} \kappa_{B'B} \\ \kappa_{B'B} \kappa_{ZB'} \end{pmatrix}. \quad (11)$$

It is apparent that vacuum  $B$  will be the most probable vacuum in this sample landscape. The relative weight of  $A$  to  $B'$  is very sensitive to the details of the potential since, as shown above, there is an exponential dependence on the difference in instanton actions (which itself tends to be quite large). In the absence of extremely fine-tuned cancellation in this difference (which would be required to make  $\kappa_{AB} \sim \kappa_{B'B}$ ), one of the two will be vastly more probable than the other. We have already considered the case where vacuum  $B'$  is much more likely than vacuum  $A$  with landscape  $V_2$  above. So the other generic alternative is for vacua  $A$  and  $B$  to have probabilities very close to one-half, vacuum  $B'$  to be exponentially suppressed, and vacuum  $Z$  to be even more suppressed.

These two examples together make it clear that, in order to obtain the large weighting observed for potentials  $V_2$  and  $V_3$ , there must be pairs of vacua which undergo fast transitions in *both* directions. This allows for closed loops that produce large numbers of bubbles of each of the vacua in the pair; in such cases the probabilities of both vacua scale with the product of the transition rates between them.

### B. Coupled pairs dominate in cyclic landscapes

As one might expect, the extreme weighting of coupled pairs persists if we raise the height of the  $Z$  well of  $V_2$  in Fig. 2 so that it is no longer terminal. From the calculations in the Appendix, we find

$$\frac{P_B^{A,B,B',Z}}{P_A^{A,B,B',Z}} \simeq \frac{\kappa_{B'B}}{\kappa_{AB}}, \quad (12)$$

$$\frac{P_B^{A,B,B',Z}}{P_Z^{A,B,B',Z}} \simeq \frac{\kappa_{BB'}}{\kappa_{ZB'}} \quad (13)$$

with the same results for the ratios of  $P_{B'}$  in place of  $P_B$  to  $P_A$  and  $P_Z$ . This is of special interest because for cyclic landscapes the predictions of the RTT method agree with those of the CHC and RT methods (see Fig. 1). Thus all of these measures will weight rapidly transitioning vacua heavily.

### C. Splitting vacua

A closely related ‘‘test’’ to which we can put the RTT method to is to consider the situation where potential  $V_2$  is obtained from potential  $V_1$  (the ‘‘ABZ’’ example of [1]) by inserting a small potential barrier in the middle ( $B$ ) well. The ratio of weights in the  $A$  and  $Z$  wells in potential  $V_1$  is given by

$$\frac{P_A^{A,B}}{P_Z^{A,B}} = \frac{\kappa_{AB}}{\kappa_{ZB}}, \quad (14)$$

which can be found from the result of [1] by substituting  $\epsilon = \kappa_{AB}/(\kappa_{AB} + \kappa_{ZB})$  and  $1 - \epsilon = \kappa_{ZB}/(\kappa_{AB} + \kappa_{ZB})$ . Now let us insert the barrier in such a way that the transition rates into and out of the  $A$  and  $Z$  wells remain unaffected. After the insertion, the relative weights of vacuum  $A$  and  $Z$  (in potential  $V_2$ ) are then found from Eq. (8) to be

$$\frac{P_A^{A,B,B'}}{P_Z^{A,B,B'}} = \frac{\kappa_{BB'} \kappa_{AB}}{\kappa_{B'B} \kappa_{ZB'}}. \quad (15)$$

Now we can consider two cases. First, if there is no symmetry as  $B$  is interchanged with  $B'$ , then we see that inserting the barrier has changed *both* the absolute probabilities (which are now strongly weighted toward  $B$  and  $B'$ ) and also the *relative* weights of the other vacua. Second, if the problem is symmetric under interchange of  $B$  and  $B'$  (so that  $\kappa_{BB'} = \kappa_{B'B}$  and  $\kappa_{ZB'} = \kappa_{AB}$ ), then the relative weights of  $A$  and  $Z$  are unaffected; however, the absolute weights of both are still altered drastically by this decomposition of  $B$  into two identical vacua with fast transitions between them. This is somewhat disturbing, and again points to the need for a smooth connection between ‘‘vacuum transitions’’ and ‘‘field evolution.’’

### D. Continuity of predictions

The next landscape we wish to consider is one of the simplest imaginable—just a double well potential. In this example, the predicted ratio of weights in vacuum  $A$  to that in  $Z$  (in the case of full recycling) is identical for the CHC, RT, and RTT methods, with  $P_A/P_Z = 1$ , independent of

the relative lifetimes of the states. The ratio of weights predicted by the CV method is  $P_A/P_Z = (H_A/H_Z)^4 e^{S_A - S_Z}$  [4], where  $H_{A,Z}$  is the Hubble constant and  $S_{A,Z}$  the entropy of vacuum  $A$  and  $Z$ , respectively. The difference is due to the fact that the CHC, RT, and RTT methods count the frequency of transitions while the CV method weights according to the time spent in a given vacuum [4].

Now consider shifting the entire potential down, such that the lower well becomes a terminal vacuum. The predictions of the CHC, RT, and RTT methods will remain identical until the lower well is exactly terminal, at which point the CHC and RTT methods (the RT method breaks down when the lower well becomes terminal) predict  $P_A = 0$ ,  $P_Z = 1$  [23]. Were this a correct description of relevant probabilities, it would be very important in making predictions to know if the energy of a minimum were zero or different from zero by one part in  $10^{10^{100}}$ . The CV method will predict this distribution as well, but will approach it in a *continuous* manner ( $S_Z \rightarrow \infty$ , sending the ratio  $P_A/P_Z$  to zero). The predictions of the CV method are for this reason much more robust under small changes of the potential.

One possible way to avoid this discontinuity might be to reverse the order of limits  $t \rightarrow \infty$  and  $\kappa_{AZ}^{-1} \rightarrow \infty$ . All of the measures discussed in this paper take the  $t \rightarrow \infty$  limit first, but one could perhaps define a measure where the duration in time is held finite while  $\kappa_{AZ}^{-1} \rightarrow \infty$ . Applying this to the two-well example, as the lifetime of the lower well goes to infinity, the expectation value of the number of transitions observed would smoothly go to zero. Alternatively, it may be the case that there are no truly terminal vacua (with strictly zero probability of being tunneled from) [24]. Finally, it may be that there is simply something conceptually flawed in the way bubble-counting measures treat the borderline between a vacuum being terminal and nonterminal.

#### IV. CONSEQUENCES FOR PREDICTIONS IN A LANDSCAPE

The previous section pointed out some interesting features of bubble-counting measures (all the measures here save CV) as somewhat abstract procedures applied to small “toy” landscapes. What might these features imply for predictions (in the form of  $P_p$  or  $\mathcal{P}_X$ ) in a more realistic landscape with many, many vacua and transitions connecting them?

Without a well-specified model of such a landscape this is a difficult question to answer; however, the strong preference for pairs of fast-transitioning vacua does suggest some general—and possibly troubling—predictions. Within a landscape, imagine the set of all pairs of neighboring vacua ( $M, N$ ) with similar pairs of energies ( $V_M, V_N$ ), and suppose that for each pair, the barrier between  $M$  and  $N$  is independent of the barriers separating  $M$  and  $N$  from other nearby vacua. Then we might expect that members of different pairs will be accorded exponentially

differing probabilities depending on the details of the barrier. In Sec. III we found in our sample landscapes that the probabilities for the vacua in a fast-transitioning pair ( $N, M$ ) are approximately proportional to the product  $\kappa_{NM}\kappa_{MN}$  of the transition rates between them. What determines this product? We fix  $V_M$  and  $V_N$ , and imagine the possible potentials  $v$  in between (i.e. we consider many pairs in the landscape). We have

$$\kappa_{MN}\kappa_{NM} \sim e^{-2S_{(MN)}(v)} e^{S_M + S_N}, \quad (16)$$

where  $S_{(MN)}(v)$  is the instanton action of Eq. (4) and  $S_{M,N}$  are the background subtractions for vacua  $M$  and  $N$ , given by Eq. (5). With  $S_M$  and  $S_N$  fixed, the product then depends just on  $S_{(MN)}$ . As argued above, this action will be of order  $(M_{\text{Pl}}/\mu)^4$ , and vary by order unity as the parameters governing the potential  $v$  are varied. Thus the weightings of the members of each pair do appear to be exponentially sensitive to the shape of the potential in between.

Now imagine that our vacuum is one tunnel away from one of the vacua with energy  $V_N$ . All other things being equal, we should be likely to come from any given one according to its weight. The evolution towards our vacuum depends on the shape of the potential, and because  $v$  is smooth this will not be independent of the shape of the potential between the endpoints of the instanton. If an observable  $\alpha$  depends on the shape of the potential as our vacuum is approached, then this raises the possibility of it having an exponentially varying prior over an observationally relevant range. A good example might be the number of post-tunneling e-folds, which might possess a prior exponentially favoring a particular number.

One might hope to compensate the prior probabilities  $P_p$  favoring cosmologies unlike ours using a conditionalization factor  $n_{X,p}$  that disfavors them (e.g. conditionalizing on the existence of a galaxy). In some cases, this seems plausible. For example, if we consider the cosmological constant  $\Lambda$  and (unrealistically) assume that all other cosmological parameters stay fixed to our observed values, then  $n_{X,p}(\Lambda)$  decreases as an exponential in  $\Lambda/\xi^4 Q^3$ , where  $Q \sim 10^{-5}$  is the fluctuation amplitude and  $\xi \sim 10^{-28}$  is the matter mass per photon in Planck masses (e.g., [25]). Because this scale is so much smaller than the scale over which the parameters of the potential vary (i.e.  $\xi^4 Q^3 \ll M$ ), the exponential variations of  $P_p(\Lambda)$  are likely to be nearly constant over a range of order  $\xi^4 Q^3$ , so  $n_{X,p}(\Lambda)$  would be effective in forcing  $\mathcal{P}_X$  to give most weight to a region of parameter space near to what we observe [26,27]. But in other cases this is far from clear; for example, the number of inflationary e-folds is determined by the *high-energy* structure of the potential at and near tunneling, and the number of e-folds is linked to the field value to which tunneling occurs, which is in turn linked to the instanton solution and hence the tunneling rate. Thus  $n_{X,p}$  and  $P_p$  might easily vary over the same scale in the parameters governing the landscape potential, and the



conditionalization may be ineffective at forcing  $\mathcal{P}_X$  to peak in the observed range.

## V. OBSERVERS IN ETERNAL INFLATION

Measures relying on properties experienced by a local observer (generally equated with a causal worldline) require that observers can actually transition between the different vacua. It is not, however, clear that this is always the case. In [28], two of the authors found that in semiclassical Hamiltonian descriptions of thin-wall tunneling, there are always two qualitatively different types of transitions described by the same formalism.

One, called the ‘‘R’’ tunneling geometry, is a generalization of Coleman-De Luccia [5]/Lee-Weinberg [6] (CDL/LW) true and false vacuum bubbles. It corresponds to the fluctuation of a bubble of the new phase which is always in causal contact with the background region, in the sense that worldlines in the old phase can both ‘‘tunnel with’’ the bubble and also enter the bubble of the new phase soon after it forms.

In the other, which was called the ‘‘L’’ tunneling geometry (a generalization of the Farhi-Guth-Guven mechanism [29]), the bubble of the new phase lies behind a wormhole separating it from the original background spacetime. In this case, no causal curve from the original phase can enter the new phase after the tunneling event (in marked contrast to the usual picture of an expanding bubble of new phase, or to the R mechanism). Some rare worldlines might tunnel with the bubble, but the physical connection between pre- and post-tunneling phases represented by such worldlines is obscure at best; moreover, such worldlines do not exist in the (highest probability) limit in which the bubble has zero mass.

If both L and R processes occur, then the L mechanism is the most probable path by which regions of *higher* vacuum energy emerge, while the R geometry dominates decay to a lower vacuum [28]; both processes are dominated by the lowest-mass bubbles.

At the semiclassical level of these calculations, the authors of [28] found no convincing reason that one but not the other of these two tunneling processes would occur. Holographic considerations would seem to conflict with the L geometries (at least for transitions to higher vacuum energy), and [30] argued using AdS/CFT that such tunneling events from AdS to dS would correspond to nonunitary processes; however, the question has not been settled with any clarity. (See [31] for another treatment of L-tunneling geometries using AdS/CFT.) In this section we will therefore consider how the L-tunneling process would impact eternal inflation, and the measures as applied to it.

Let us consider an initial parcel of comoving volume in a metastable state residing in an arbitrary potential landscape. This is shown at the bottom of Fig. 3. As time goes on, bubbles of either higher or lower vacuum energy will nucleate by either the L or R tunneling geometries.

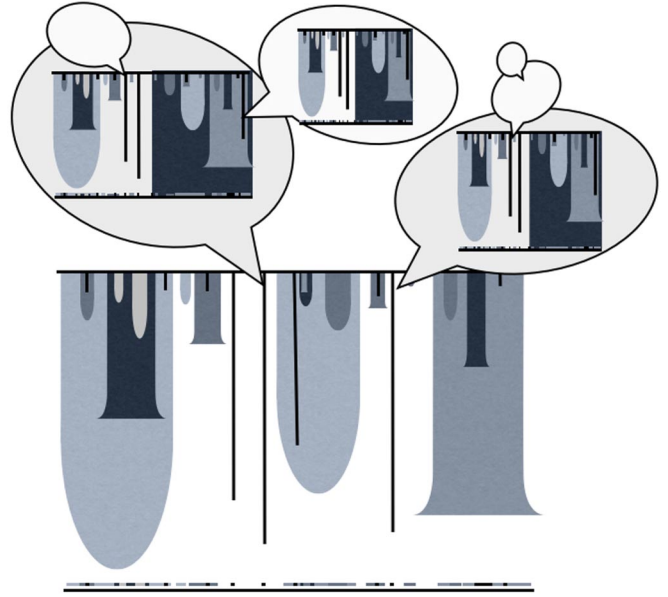


FIG. 3 (color online). A picture of an eternally inflating universe which takes into account both L and R tunneling geometries. At the bottom, there is an ‘‘original’’ parcel of comoving volume (defined by the horizontal spacelike slice at the bottom of the figure), which evolves in time (vertically). True and false vacuum bubble nucleation events occur via the R geometry in this volume, denoted by the shaded regions which in the case of true vacuum bubbles grow to a comoving Hubble volume and in the case of false vacuum bubbles shrink to a comoving Hubble volume. The vertical black lines denote the black holes formed during L-geometry tunneling events. On the other side of a wormhole (inside the captions), the initial distribution, which is fixed by the tunneling geometry, undergoes L and R tunneling events as well, spawning more disconnected parcels of volume in which this process repeats. The original parcel of comoving volume will spawn an infinite amount of new comoving volume via L-geometry tunneling events. Shown on the bottom of each parcel is the set of bubble shadows that might be used in the CHC method to calculate probabilities  $P^{\mathcal{V}_i}$  for each region  $\mathcal{V}_i$ .

Since low-mass bubbles are most probable, most downward transitions will be CDL bubbles (the R geometry in the zero-mass limit), and most upward transitions will be L-geometry tunneling events corresponding to a very small mass black hole forming in the background spacetime. Such small black holes affect the background spacetime in a completely negligible way as long as the nucleation rate is rather small [32]. In particular, these upward nucleations remove zero comoving volume from the old phase.

The pre- and post-tunneling spacetimes in an L-tunneling event are described comprehensively in, e.g., [28]; the portion of the post-tunneling spacetime existing behind the wormhole consists of regions with both new and old vacuum energy separated by a thin wall, and in the zero-mass limit is just the Lorentzian CDL bounce geometry. Both vacuum regions are larger than their corresponding Hubble radii and so will unavoidably continue to

inflate, independent of the precise details of the initial nucleated space (i.e. how the instanton is “sliced” to be continued into Lorentzian space; see [21] for the corresponding issue concerning the CDL instanton).

The result is that an entirely new “branch” of eternal inflation is created, with some initial physical volume, having essentially no effect on the original spacetime. If a comoving volume is assigned to this physical volume using the “scale factor time” of the background geometry near the nucleation event, then the effect will be to create *new comoving volume* [33]. The new branch will in turn spawn more branches—and more comoving volume—via L events, so that the comoving volume appears to actually grow exponentially (though in what “time” this occurs is unclear since there is no foliation of the entire spacetime). This process is shown in Fig. 3.

How do the measures we have been discussing connect with this new picture? Consider first the measures RTT, RT, and W that explicitly follow causal worldlines. As formulated, these measures would essentially “ignore” L transitions. This seems quite artificial, however, as regions with high vacuum energy (reached by upward transitions) would almost all arise from this process; put another way, choosing a random point in the entire spacetime (including the tree of new universes formed by the L-tunneling geometry) and projecting any geodesic back, it would almost certainly hit an L-geometry nucleation surface in the past rather than the assumed initial slice.

Now consider the CV and CHC prescriptions. As stated, the idea is to count the relative comoving volume or number of bubbles of different types “on future null infinity.” But as described in Sec. II B and in [3,13,16], these measures are actually *calculated* with very strong reliance on a congruence of geodesics emanating from an initial surface; thus as calculated in this formulation they would be as unaffected by L-geometry events as RTT, RT, and W. It is interesting, however, to speculate about taking these prescriptions seriously as counting bubbles on future infinity, as this would actually include the bubbles in the other branches created by L events.

Consider, then, a volume  $\mathcal{V}_i$  nucleated by an L event (with the subscript  $i$  labeling the particular region under consideration), and imagine a congruence of geodesics emanating from it, denoting by  $\mathcal{J}^+(\mathcal{V}_i)$  the part of the spacetime’s future null infinity reachable by these geodesics. Then we might “count bubbles of comoving size exceeding  $\epsilon$ ” (for CHC) or “count comoving volume” (for CV) on  $\mathcal{J}^+(\mathcal{V}_i)$ , to define a set of relative probabilities  $P^{\mathcal{V}_i}$ .

Now, it is very unclear how precisely to combine the  $P^{\mathcal{V}_i}$  in all of the branches  $i$  formed from L-tunnelings out of both the original spacetime and out of the future of  $\mathcal{V}_i$ , and from the descendants of these branches, etc. Nonetheless, some general statements might be made even in the absence of such precision.

Consider first CHC. Since its probabilities are essentially independent of  $\mathcal{V}_i$ , it seems that  $P^{\mathcal{V}_i}$  will be the same in all branches, so it is hard to see how anything else could result from combining them.

Now consider CV, which *is* dependent on the initial conditions for  $\mathcal{V}_i$ . Here, the “initial” conditions for a branch are not provided by the original spacetime, but rather by the dynamics of the L-tunneling process, with a different set corresponding to each pair of vacua between which the nucleations can occur. Whatever way we calculate all of the  $P^{\mathcal{V}_i}$ , it seems likely that the original spacetime’s initial conditions will be completely overwhelmed by those of all of the branches in the infinite self-similar tree depicted in Fig. 3. One might then imagine that the total prior distribution  $P$  is given by a weighted sum of these separate distributions and is independent of the initial conditions of the original spacetime.

We also point out that these questions may apply to “stochastic” eternal inflation as well. It is generally implicitly assumed in these models that the global spacetime is causally connected, but this is far from proven. Indeed, large fluctuations generically cause a large backreaction, and it is not obvious that the large stochastic fluctuations driving eternal inflation do not cause the production of universes behind a wormhole (this is suggested by singularity theorems [34–36]). This discussion is also relevant for hypothetical transitions out of negative energy minima. While no instanton has been constructed for such a transition (see [37] for a proposal concerning the probability of such a process), if one exists then (considering thin-wall constructions [30]) it would have to be an L-geometry. Based on the considerations above, it is unclear how or if including such transitions would change the predictions of extant measures.

## VI. DISCUSSION AND CONCLUSIONS

We have analyzed a number of existing measures for eternal inflation, exploring connections that exist between them, and highlighting some generic predictions that they make. With this perspective, let us return to the list of desiderata presented in Sec. II A. Shown in Table I is a scorecard detailing which of the measures, in at least a majority of the authors’ humble and irresolute opinions, satisfy the properties listed in Sec. II A.

First, which measures are “physical,” in the sense of providing a nonarbitrary prior probability  $P_p$ , for some “counting object”  $p$ , useful for calculating  $\mathcal{P}_X$ ? Physical volume weighting (discussed little here) would seem quite physical but appears to lead to gauge dependence [9,10], and incorrect predictions in at least some gauges (see [11,12]). The related CV ( $p =$  “unit of comoving volume”) method may avoid some of this difficulty, but at some cost to physicality: comoving volumes are generally meaningful only insofar as they are reconverted to physical ones, or if there are conserved

TABLE I. Properties of bubble-counting measures—Y = yes, N = no, P = partial.

Property	CV	CHC	W	RT	RTT
Physicality	P	P	P	P	P
Gauge independence	P	Y	Y	Y	Y
Independence of initial conditions	N	Y	Y	Y	N
Copes with varieties of transitions and vacua	P	Y	N	N	Y
Copes with nontrivial topologies	P	P	N	N	N
Treatment of states and transitions:					
General principles	P	P	P	P	P
Physical description of transitions	P	P	P	N	N
Reasonable treatment of split states	Y	N	N	N	N
Continuity in transition rates	Y	N	N	N	N

objects (baryons, galaxies, etc.) with fixed density per unit comoving volume. The latter may be true after reheating, but it is unclear to us that comoving volume is as meaningful *during* a complex, inhomogeneous inflationary period. Another option is to weight according to the integrated incoming probability current [15,38] across reheating surfaces, which can be found directly from volume distributions. This proposal, which is tied more closely to the conditionalization, avoids the gauge dependence and spurious predictions of standard volume weighting (as discussed above, this prescription can reproduce the results of the RTT method [15,21]).

The CHC and W methods have  $p =$  “bubbles,” which might be tied to conditionalization objects associated with the various reheating surfaces (though this involves considerable uncertainty since those reheating surfaces are generically infinite). However, the objects (worldlines and shadows) actually used to arrive at a bubble count seem rather less physical, particularly as they demand a cutoff prescription that—while natural—also seems as if it could easily be different. The RT and RTT methods use  $p =$  “segment of a worldline between vacuum transitions,” and have been suggested as an appropriate measure if we identify  $X =$  “unit of entropy production” [1,39]. This connection is not entirely compelling, however, as the results of these “holographic” measures can be found by considering an ensemble of observers (as noted in Sec. II B and by [15]). These connections suggest that CV, RT, and RTT are very closely related, but with a consistent and appropriate physical interpretation somewhat lacking.

Consider now gauge independence. Physical volume weighting is gauge dependent, but the other measures appear gauge independent, albeit with some caveats. For RT, RTT, W, and CHC, gauge independence stems from their counting of objects (bubbles) or events (transitions); in CV it occurs via use of a congruence of geodesics, which are also then “counted” to obtain comoving volume. The caveats stem from subtleties—connected with a time variable choice—in defining cutoffs, transition rates, and initial conditions, and we hope to elucidate some of these further in future work. (We single out CV as partially gauge

dependent because the results will depend on the time slicing used to characterize the initial value surface.)

Drawing on the description of the various measures presented in Sec. II B, we can see that not all of the measures under discussion have the ability to cope with all types of transitions and vacua. For instance, the CV method accords zero weight to metastable minima (particularly disturbing as we may live in one), and the RT method in its current formulation is not able to describe a landscape with terminal vacua. We also note that the CV and RTT methods are dependent on initial conditions.

In Sec. V, we argued that it is possible—if certain types of L bubble nucleation events occur—for different regions of the eternally inflating multiverse to be separated by wormholes, and therefore causally disconnected. None of the evaluated measures are, as formulated, equipped to deal with such spacetimes in a reasonable way. The “philosophy” behind CV and CHC—of counting bubbles or volume on future infinity—might reasonably apply to such spacetimes, and if this could be implemented technically we argued that in this case CV would probably become independent of initial conditions. The philosophy behind RTT and RT would suggest simply ignoring these events (as indeed those measures effectively do) but it is rather unclear to us that this is appropriate. Accounting for such tunneling events in measure prescriptions is very difficult—but this merely highlights the possible importance of such transitions, and of determining whether or not they occur.

Even thornier problems might arise from considering transitions in greater generality. All of the measures considered rely on a congruence of worldlines and a fairly straightforward spacetime structure. Were we to include transitions between different string/M theory flux vacua, including even different numbers of large spacetime dimensions, it is unclear whether the principles of extant measures would apply. Without having a well-defined description of such transitions this is difficult to assess; hence we do not consider this in our table.

But even confining our attention to (relatively) well-understood spacetime evolution in a general scalar poten-

tial landscape, the measures differ somewhat in how generally and robustly they treat “vacua” and transitions. All of the measures under discussion have been applied to the brand of eternal inflation driven by metastable minima. However, it would be desirable to include the effects of all the dynamics of an eternally inflating universe, and the effective scalar fields that are imagined to drive it. This includes a description of the diffusion and classical rolling of the field that will occur. There has been work extending CV and CHC methods to these cases, but little so far in making such an extension to RT or RTT.

In terms of connecting transition rates to physical transitions, all of the measures ignore the small-scale details of vacuum transitions (i.e. within a few Hubble volumes). This may be relatively benign, but bears investigation. For example, in RTT, transitions are thought of as something that occurs to a worldline within its causal diamond—but these transitions could occur via the encounter of a bubble formed in a nucleation process *outside* the causal diamond.

More trouble occurs when we consider nearby vacua separated by a small barrier. The main observations of this paper centered around a study of the sample landscapes shown in Fig. 2 using the RTT method. In Sec. III A it was found that pairs of vacua that undergo fast transitions will be very strongly weighted. Using order of magnitude estimates of the transition rates, we argued that the probability ratio of such pairs to other vacua in the sample landscape can be exponentially large. This effect occurs in both terminal and recycling landscapes. Using the equivalences between the various measures noted in Sec. II C (for a summary, see Fig. 1), and an explicit example for the CHC method, we have shown that the weighting of fast-transitioning pairs occurs in the CHC, W, and RT methods as well. As discussed in Sec. III C, because of this effect, by inserting a small barrier in an intermediate state, the absolute weight assigned to each vacuum is affected drastically. Therefore, the RTT, RT, W, and CHC methods are only partially robust in their definition of transitions; the undivided-well distribution is not recovered as the barrier disappears. This situation might be remedied if, as bubble collisions become more and more important, the diffusion analysis replaces bubble nucleation (giving further impetus to generalizing the measures to treat this). In contrast, the CV method *does* approach the undivided-well weight as the small barrier disappears.

Lastly, we considered continuity in transition rates, which was studied using a two-well landscape in Sec. III D. It was noted that the predictions of the CHC, RT, and RTT methods change discontinuously as a recycling vacuum is deformed into a terminal vacuum. This discontinuity makes the *exact* properties of vacua in a landscape important. Such a discontinuity could potentially be avoided if the order of limits in the cutoff procedure were modified.

Most of the discussion—and all of the scorecard—has focused on issues of principle concerning the measures as abstract procedures. Some of the features discussed have implications for what such assumed measures would mean *observationally*. In particular, we saw in Sec. IV that the exponential dependence of the prior distribution  $P_p$  on the details of the potential implies that making predictions using bubble counting measures may be very hard. This problem is particularly acute when, for some parameter  $\alpha$ , the factors  $P_p(\alpha)$  and  $n_{X,p}(\alpha)$  [these are the prior and conditionalization factors needed to produce a prediction in the form of Eq. (1)] vary appreciably over the same range in  $\alpha$ . This may be the case, for example, when  $\alpha$  is related to the number of e-folds during inflation. If the observation that fast-transitioning pairs are exponentially weighted generalizes to more complicated landscapes, then bubble-counting measures may in some cases lead to strongly exponential prior probabilities that would overwhelm any conditionalization factor  $n_{X,p}(\alpha)$ . This would lead to very strong predictions, which might be successful, or disastrous. More generally, this exponential dependence suggests that current measures seem to potentially call for a complete knowledge of the fine details of the entire landscape, a Herculean requirement.

Perhaps not surprisingly, we come to the conclusion that, while progress has been made towards predicting our place in the multiverse, we are far from finished. It would be desirable to find and explore other measures and see if they fall victim to any of the same problems that we have outlined.

## ACKNOWLEDGMENTS

We thank Raphael Bousso, Ben Freivogel, Antony Lewis, Andrei Linde, Alex Vilenkin, and Sergei Winitzki for helpful discussions. A.A. and M.J. were partially supported by a “Foundational Questions in Physics and Cosmology” grant from the John Templeton Foundation during the preparation of this work. S.G. is supported by PPARC.

## APPENDIX: MATRIX CALCULATIONS AND SNOWMAN DIAGRAMS

In this appendix we present a quick way of calculating normalized probabilities for terminal and cyclic landscapes in a unified manner, which also sheds light on the nature of the regularizing limit taken in the cyclic case.

First, assemble the relative transition probabilities  $\mu_{NM}$  into a matrix  $\boldsymbol{\mu}$  (equivalent to Bousso’s  $\eta$  matrix). Starting in an initial state represented by a vector  $\boldsymbol{q}$  with components  $q_N$  ( $\sum_N q_N = 1$ ), after one transition the mean number of entries (or “raw probability”) for each vacuum will be given by  $\boldsymbol{\mu}\boldsymbol{q}$ . At the second transition an additional  $\boldsymbol{\mu}^2\boldsymbol{q}$  entry will occur and so on. After  $n$  transitions the raw probability will be given by  $(\boldsymbol{\mu} + \boldsymbol{\mu}^2 + \dots + \boldsymbol{\mu}^n)\boldsymbol{q}$ . If we



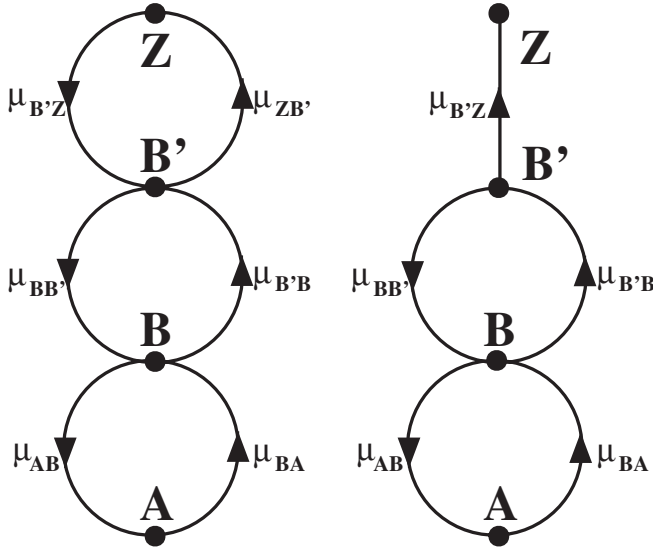


FIG. 4. Examples of “snowman diagrams” summarizing relative transition probabilities  $\mu_{NM}$ . The one on the left is for a recycling landscape and the one on the right is for a terminal landscape.

set  $S_n \equiv \mu + \mu^2 + \dots + \mu^n$ , then  $(1 - \mu)S_n = \mu(1 - \mu^n)$ . In the terminal case we can invert  $(1 - \mu)$  and take the  $n \rightarrow \infty$  limit to obtain  $S_\infty$  directly ( $\mu^n \rightarrow 0$  since asymptotically all the probability goes into the terminal vacua and so fewer and fewer vacuum entries occur). In the cyclic case  $\det(1 - \mu) = 0$  and  $\mu^n$  does not tend to zero, and things are not so simple. It is convenient to proceed by replacing  $\mu$  by  $(1 - \varepsilon)\mu$  (where  $\varepsilon$  is an auxiliary parameter to be taken to zero after the calculation), which can be inverted. Neglecting the troublesome determinant factor (since we shall later be normalizing to obtain probabilities from numbers of vacuum entries anyway), we take the limits  $n \rightarrow \infty$  and  $\varepsilon \rightarrow 0$  in that order, and for both terminal and recycling landscapes obtain the simple expression

$$S_\infty \propto T \equiv (\text{adj}(1 - \mu))\mu \quad (\text{A1})$$

where  $\text{adj}$  denotes the adjoint matrix operation (i.e. the transpose of the matrix of cofactors of the matrix in question). Multiplying  $T$  into  $q$  and normalizing yields the probabilities for the vacua given the initial state in question.

This procedure yields exactly the same results as the pruned tree method. We thus see that the latter procedure is equivalent to considering sequences of transitions up to some length  $n$  and then taking the limit  $n \rightarrow \infty$ .

The  $\mu_{NM}$ 's in question can conveniently be depicted in snowmanlike diagrams such as those shown in Fig. 4, which apply to the calculations in Sec. III. These diagrams emphasize that the path between any two vacua can involve an arbitrary number of circulations in closed loops between recycling vacua. In fact, we treat both cases at once by leaving  $\mu_{ZB'}$  arbitrary and only set it to 1 or 0 as appro-

priate after having calculated  $T$ . We also allow for the possibility of vacuum  $A$  being terminal in the same manner.

Suppressing the normalizing factor for clarity, we obtain

$$\begin{pmatrix} P_A^{A,B,B',Z} \\ P_B^{A,B,B',Z} \\ P_{B'}^{A,B,B',Z} \\ P_Z^{A,B,B',Z} \end{pmatrix} \propto \begin{pmatrix} \mu_{AB}(1 - \mu_{B'Z}\mu_{ZB'}) \\ 1 - \mu_{B'Z}\mu_{ZB'} \\ \mu_{B'B} \\ \mu_{B'B}\mu_{ZB'} \end{pmatrix} \quad (\text{A2})$$

in the recycling case with the full set of superscripts indicating that the results are independent of initial conditions.

In the terminal case we can only start in states  $A$ ,  $B$ , or  $B'$ , and we obtain

$$\begin{pmatrix} P_A^A \\ P_B^A \\ P_{B'}^A \\ P_Z^A \end{pmatrix} \propto \begin{pmatrix} \mu_{AB} \\ 1 \\ \mu_{B'B} \\ \mu_{ZB'}\mu_{B'B} \end{pmatrix}, \quad (\text{A3})$$

$$\begin{pmatrix} P_A^B \\ P_B^B \\ P_{B'}^B \\ P_Z^B \end{pmatrix} \propto \begin{pmatrix} \mu_{AB} \\ \mu_{AB}\mu_{BA} + \mu_{BB'}\mu_{B'B} \\ \mu_{B'B} \\ \mu_{B'B}\mu_{ZB'} \end{pmatrix}, \quad (\text{A4})$$

and

$$\begin{pmatrix} P_A^{B'} \\ P_B^{B'} \\ P_{B'}^{B'} \\ P_Z^{B'} \end{pmatrix} \propto \begin{pmatrix} \mu_{AB}\mu_{BB'} \\ \mu_{BB'} \\ \mu_{BB'}\mu_{B'B} \\ \mu_{ZB'}(1 - \mu_{AB}\mu_{BA}) \end{pmatrix}. \quad (\text{A5})$$

The relative transition probabilities are related to the transition rates by

$$\mu_{BA} = 0 \quad \text{or} \quad 1, \quad (\text{A6})$$

$$\mu_{AB} = \frac{\kappa_{AB}}{\kappa_{AB} + \kappa_{B'B}}, \quad (\text{A7})$$

$$\mu_{B'B} = \frac{\kappa_{B'B}}{\kappa_{AB} + \kappa_{B'B}}, \quad (\text{A8})$$

$$\mu_{BB'} = \frac{\kappa_{BB'}}{\kappa_{ZB'} + \kappa_{BB'}}, \quad (\text{A9})$$

$$\mu_{ZB'} = \frac{\kappa_{ZB'}}{\kappa_{ZB'} + \kappa_{BB'}} \quad (\text{A10})$$

where  $\mu_{BA} = 0$  if  $A$  is terminal and  $\mu_{BA} = 1$  if it is not. Substituting these expressions into Eqs. (A3)–(A5), we can then take the limits discussed in Sec. III to produce the appropriate probability tables.

In the case where vacuum  $A$  is terminal ( $\mu_{AB} = 0$ ), there are a number of ratios of interest. The probabilities assigned by the CV method to this sample landscape were

calculated in [3] (the FABI model), and using these results, we can directly compare the results of the CV and RTT methods. For initial conditions in  $B$  or  $B'$ , we find

$$\frac{P_A^B}{P_Z^B} = \frac{\kappa_{AB}(\kappa_{BB'} + \kappa_{ZB'})}{\kappa_{B'B}\kappa_{ZB'}}, \quad (\text{A11})$$

$$\frac{P_A^{B'}}{P_Z^{B'}} = \frac{\kappa_{AB}\kappa_{BB'}}{\kappa_{ZB'}(\kappa_{AB} + \kappa_{B'B})}. \quad (\text{A12})$$

As expected, given the argument of Sec. II C, these results agree with the predictions of the CV method.

- 
- [1] R. Bousso, Phys. Rev. Lett. **97**, 191302 (2006).
- [2] R. Easther, E. A. Lim, and M. R. Martin, J. Cosmol. Astropart. Phys. 03 (2006) 016.
- [3] J. Garriga, D. Schwartz-Perlov, A. Vilenkin, and S. Winitzki, J. Cosmol. Astropart. Phys. 01 (2006) 017.
- [4] V. Vanchurin and A. Vilenkin, Phys. Rev. D **74**, 043520 (2006).
- [5] S. R. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).
- [6] K.-M. Lee and E. J. Weinberg, Phys. Rev. D **36**, 1088 (1987).
- [7] A. Aguirre and M. Tegmark, J. Cosmol. Astropart. Phys. 01 (2005) 003.
- [8] A. Aguirre, *Universe or Multiverse* (Cambridge University Press, Cambridge, UK, 2007).
- [9] A. D. Linde, D. A. Linde, and A. Mezhlumian, Phys. Rev. D **49**, 1783 (1994).
- [10] S. Winitzki, Phys. Rev. D **71**, 123507 (2005).
- [11] A. H. Guth, Phys. Rep. **333**, 555 (2000).
- [12] M. Tegmark, J. Cosmol. Astropart. Phys. 04 (2005) 001.
- [13] A. Vilenkin, arXiv:hep-th/0609193.
- [14] S. Shenker, <http://online.itp.ucsb.edu/online/strings06/shenker/>.
- [15] A. Linde, J. Cosmol. Astropart. Phys. 01 (2007) 022.
- [16] J. Garriga and A. Vilenkin, Phys. Rev. D **64**, 023507 (2001).
- [17] Note that this is worse than it may sound, because the *same spacetime* might be sliced with different initial surfaces so as to lead to completely different probability distributions.
- [18] It is unclear the extent to which the velocities of individual points can be chosen at random, as discussed by Vilenkin in [13].
- [19] G. W. Gibbons and N. Turok, arXiv:hep-th/0609095.
- [20] S. Gratton and N. Turok, Phys. Rev. D **72**, 043507 (2005).
- [21] J. Garriga and A. Vilenkin, Phys. Rev. D **57**, 2230 (1998).
- [22] A. Aguirre, T. Banks, and M. Johnson, J. High Energy Phys. 08 (2006) 065.
- [23] It is worth noting that this is completely independent of the ratio of the lifetimes of the states, which might be arbitrarily large [22].
- [24] For example, if the L process describe below in Sec. V occurs, it might mediate transitions away from negative- or zero-energy vacua. A heuristic argument in favor of tunneling from negative “big crunch” vacua was given in [37]. Finally, we note that after tunneling to a negative vacuum, the spacetime is an open FRW model with energy density. Thus there may conceivably be tunneling before the “crunch” even if such tunneling is impossible from pure AdS or Minkowski space.
- [25] M. Tegmark, A. Aguirre, M. Rees, and F. Wilczek, Phys. Rev. D **73**, 023505 (2006).
- [26] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).
- [27] D. Schwartz-Perlov and A. Vilenkin, J. Cosmol. Astropart. Phys. 06 (2006) 010.
- [28] A. Aguirre and M. C. Johnson, Phys. Rev. D **73**, 123529 (2006).
- [29] E. Farhi, A. H. Guth, and J. Guven, Nucl. Phys. **B339**, 417 (1990).
- [30] B. Freivogel *et al.*, J. High Energy Phys. 03 (2006) 007.
- [31] G. L. Alberghi, D. A. Lowe, and M. Trodden, J. High Energy Phys. 07 (1999) 020.
- [32] In fact, even more probable is the zero-mass limit in which there is no black hole at all, which also clearly does not affect the background spacetime.
- [33] How to actually define “comoving volume” in the new phase is very unclear; comoving volume is related to a particular coordinatization of a spacetime, and its definition is tied to a congruence of geodesics; here, no such congruence continues through the nucleation to fill the initial slice.
- [34] E. Farhi and A. H. Guth, Phys. Lett. B **183**, 149 (1987).
- [35] T. Vachaspati and M. Trodden, Phys. Rev. D **61**, 023502 (1999).
- [36] A. Aguirre and M. C. Johnson, Phys. Rev. D **72**, 103525 (2005).
- [37] T. Banks and M. Johnson, arXiv:hep-th/0512141.
- [38] J. Garcia-Bellido, A. D. Linde, and D. A. Linde, Phys. Rev. D **50**, 730 (1994).
- [39] R. Bousso and B. Freivogel, arXiv:hep-th/0610132.