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Positivity constraints on spin observables in hadronic inclusive reactions

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We study the implications on experimental data and theoretical models of the positivity constraints on spin observables for the reaction $A(\text{spin}1/2) + B(\text{unpolarized}) \to C(\text{spin}1/2) +$ anything.

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The positivity constraints have been widely used in hadron physics to restrict, in a model-independent way, the domain of allowed values for spin observables [1,2]. Consider the following parity conserving reaction:

$$A(\text{spin } 1/2) + B(\text{unpolarized}) \to C(\text{spin } 1/2) + X. \quad (1)$$

For this process one can define *eight* independent observables, which depend on three kinematic variables, $\sqrt{s}$ the center-of-mass energy and x_F, and p_T the longitudinal and transverse momenta of the final particle C. In order to define these observables, we recall the standard notation already used in Ref. [1] (Appendix 3), $(AB|CX)$, where the spin directions of A, B, C, and X are specified in one of the three possible directions: L, N, S. Since only one initial and one final spin are observed, we have in fact $(A0|C0)$, and $\mathbf{L}$, $\mathbf{N}$, $\mathbf{S}$ are unit vectors, in the center-of-mass system, along the incident momentum, along the normal to the scattering plane which contains A, B, and C, and along $\mathbf{N} \times \mathbf{L}$, respectively. In addition to the unpolarized cross section, $\sigma_0 = (00|00)$, there are *seven* spin-dependent observables, *two* single transverse spin asymmetries,

$$A_N = (N0|00) \quad \text{and} \quad P_C = (00|N0), \quad (2)$$

and *five* depolarization parameters,

$$\begin{aligned} D_{LL} &= (L0|L0), \\ D_{SS} &= (S0|S0), \\ D_{LS} &= (L0|S0), \\ D_{NN} &= (N0|N0), \quad \text{and} \\ D_{SL} &= (S0|L0). \end{aligned} \quad (3)$$

Several years ago, Doncel and Méndez [3] derived very general inequalities constraining these parameters which read:

$$(1 \pm D_{NN})^2 \geq (P_C \pm A_N)^2 + (D_{LL} \pm D_{SS})^2 + (D_{LS} \mp D_{SL})^2. \quad (4)$$

Let us first concentrate on a particular reaction $p^{\uparrow}p \to \Lambda^{\uparrow}X$, where the incoming proton beam is polarized and the polarization of the outgoing Λ is measured. We consider the case where the particle spins are normal to the scattering plane, then the inequalities (4) give

$$1 \pm D_{NN} \geq |P_{\Lambda} \pm A_N|. \quad (5)$$

These constraints must be satisfied for any kinematic values of the variables x_F, p_T, $\sqrt{s}$.

These inequalities involve three spin parameters; once we fix the value of one parameter, the other two are restricted to lie in a certain domain. For instance, Fig. 1 shows that for $D_{NN} = 0$, P_{Λ} and A_N are correlated within the shaded area of a square with boundaries $(-1, +1)$. This domain is more restricted when $D_{NN} = 1/3$ (see Fig. 1), and in the limit $D_{NN} = 1$ we immediately deduce from Eq. (5) that $P_{\Lambda} = A_N$. In the case where D_{NN} is negative, one obtains the same regions, but P_{Λ} and A_N are interchanged with respect to their axes.

Various applications can be envisaged with the above inequalities: in particular, testing, on the one hand, the consistency of experimental data and, on the other hand, the validity of the spin observables predicted by theoretical models.

At Fermilab, the experiment E-704 [4] has performed the measurement of P_{Λ}, A_N, and D_{NN} with a transversely polarized proton beam at 200 GeV/c, in the kinematic range $0.2 \leq x_F \leq 1.0$ and $0.1 \leq p_T \leq 1.5$ GeV/c. Their data indicate a negative A_N and a value of D_{NN} up to 30%. If we take, for instance, $p_T \sim 1$ GeV/c, $x_F \sim 0.8$, then $D_{NN} \sim 30\%$, for these kinematical values, they have $A_N \sim -10\%$ and $P_{\Lambda} \sim -30\%$. In this case the inequalities (5) are well satisfied.

The DISTO Collaboration [5] has measured the A_N and D_{NN} parameters, for three beam momenta 3.67, 3.31, and 2.94 GeV/c, as a function of x_F, p_T, for the exclusive reaction $p^{\uparrow}p \to pK^{+}\Lambda^{\uparrow}$. In the range $-0.6 \leq x_F \leq 0.8$,

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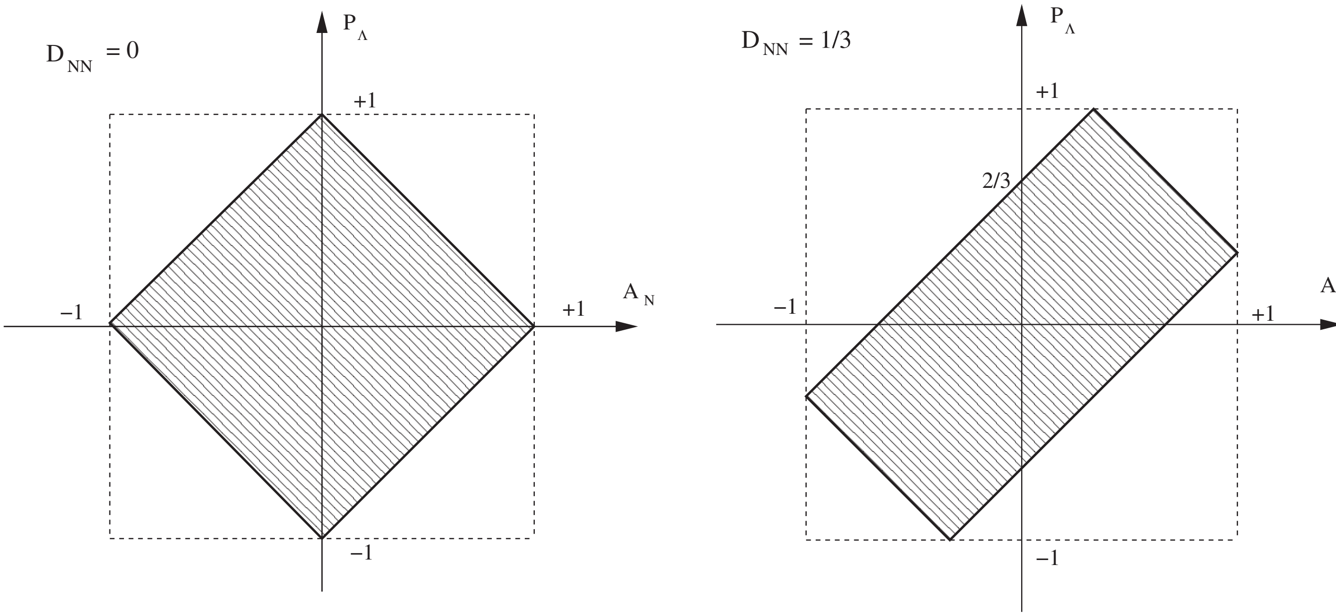

FIG. 1. The polarization P_Λ versus the analyzing power A_N in the cases $D_{NN} = 0, 1/3$.

they find $D_{NN} \approx -50\%$ and $-15\% \leq A_N \leq 0$, so by using the above inequalities we obtain the constraint $-35\% \leq P_\Lambda \leq 50\%$. They also have [6] at 3.67 GeV/c and $x_F = -0.45$, $D_{NN} = -0.57 \pm 0.07$ and $A_N = -0.27 \pm 0.02$, which gives a stronger constraint on P_Λ.

Finally, let us mention the measurement of D_{NN} in the inclusive process $p^\uparrow p \to p^\uparrow X$ at 6 GeV/c incident momentum [7]. For the momentum transfer $-t = 0.20\ \mathrm{GeV}^2$, D_{NN} was found rather large and close to $+1$, so positivity implies $P_p \sim A_N$, as indicated above. Unfortunately the data is unavailable, so we were not able to check this expectation.

We see that these constraints are useful to check experimental data, which must lie inside the domain allowed by positivity.

On the theoretical side, different phenomenological models have been proposed to understand the important polarization effects in hyperon production [8]. In Ref. [9], a semiclassical picture is proposed where the direction of the orbital motion of the polarized valence quarks in the initial proton induces the left-right asymmetry for inclusive Λ. These calculations lead to a reasonable agreement for P_Λ, A_N, and D_{NN} in the inclusive Λ production, which satisfies the positivity constraints. This model was also applied successfully to the diffractive process $pp \to \Lambda K^+ p$ with $D_{NN} = +1$, which implies $P_\Lambda = A_N$ [10].

When the polarization is not normal to the scattering plane, one has new depolarization parameters. In the case where both p and Λ are longitudinally polarized, one can measure D_{LL}, and when each particle has a transverse polarization in the scattering plane, one can measure D_{SS}. In this case, one gets the nontrivial constraint

$$1 \pm D_{NN} \geq |D_{LL} \pm D_{SS}|. \tag{6}$$

Note that the two depolarization parameters in the right-hand side of Eq. (6) have not been measured. The RHIC pp collider at Brookhaven National Laboratory is certainly the most appropriate machine for these experiments at high energy, due to the existence of both longitudinally and transversely polarized proton beams. Positivity was used to determine the allowed domains for D_{LL} [11] and D_{NN} [12].

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[1] C. Bourrely, E. Leader, and J. Soffer, Phys. Rep. **59**, 95 (1980).
[2] For a review, see X. Artru, M. Elchikh, J.-M. Richard, J. Soffer, and O. Teryaev (unpublished).

[3] M. R. Doncel and A. Méndez, Phys. Lett. **41B**, 83 (1972).
[4] A. Bravar *et al.* (Fermilab E-704 Collaboration), Phys. Rev. Lett. **75**, 3073 (1995); **78**, 4003 (1997). See also the review by L. G. Pondrom, Phys. Rep. **122**, 57 (1985).
[5] R. Bertini (DISTO Collaboration), Cesk. Cas. Fyz. **51**, 77 (2001); R. Bertini, Nucl. Phys. **A680**, 342 (2001); M. Maggiora, Nucl. Phys. **A691**, 329 (2001).
[6] R. Bertini and M. Maggiora (private communication).
[7] H. Courant *et al.*, Phys. Rev. Lett. **44**, 1373 (1980).
[8] J. Félix, Mod. Phys. Lett. A **14**, 827 (1999).
[9] C. Boros and Liang Zuo-tang, Phys. Rev. D **53**, R2279 (1996); Phys. Rev. Lett. **79**, 3608 (1997).
[10] Liang Zuo-tang and C. Boros, Phys. Rev. D **61**, 117503 (2000).
[11] D. de Florian, M. Stratman, and W. Vogelsang, Phys. Rev. Lett. **81**, 530 (1998).
[12] D. de Florian, J. Soffer, M. Stratman, and W. Vogelsang, Phys. Lett. B **439**, 176 (1998).