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Connecting (supersymmetry) CERN LHC measurements with high scale theories

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If supersymmetry is discovered at the LHC, the measured spectrum of superpartner masses and couplings will allow us to probe the origins of supersymmetry breaking. However, to connect the collider-scale Lagrangian soft parameters to the more fundamental theory from which they arise, it is usually necessary to evolve them to higher scales. The apparent unification of gauge couplings restricts the possible forms of new physics above the electroweak scale, and suggests that such an extrapolation is possible. Even so, this task is complicated if the low-scale spectrum is not measured completely or precisely, or if there is new physics at heavy scales beyond the reach of collider experiments. In this work we study some of these obstacles to running up, and we investigate how to get around them. Our main conclusion is that even though such obstacles can make it very difficult to accurately determine the values of all the soft parameters at the high scale, there exist a number of special combinations of the soft parameters that can avoid these difficulties. We also present a systematic application of our techniques in an explicit example.

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I. INTRODUCTION

It is a remarkable feature of the minimal supersymmetric extension of the standard model (MSSM) that the $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ gauge couplings unify at a very high scale, of order 10^{16} GeV [1,2]. Furthermore, the matter fields of the MSSM, with the exception of the pair of Higgs doublets, have precisely the quantum numbers of three sets of $\bar{\mathbf{5}} \oplus \mathbf{10}$ representations of $SU(5)$. These properties may be purely accidental, but they do suggest a more symmetric unified structure at energies only slightly below the Planck scale. They also offer a tantalizing hint of the structure of nature at scales well above what we will be able to probe directly with colliders such as the LHC.

Taking these features as being more than accidental, we obtain significant constraints on the types of new physics that can arise between the electroweak and the grand unification (GUT) scales. Any new phenomenon that enters the effective theory in this energy range ought to maintain the unification of couplings, and should be consistent with a (possibly generalized) GUT interpretation. The simplest scenario is a *grand desert*, in which there is essentially no new physics at all below the unification scale $M_{\rm GUT}$. In this case, if supersymmetry is discovered at the Tevatron or the LHC, it will be possible to extrapolate the measured soft supersymmetry breaking parameters to much higher scales using the renormalization group (RG). Doing so may help to reveal the details of supersymmetry breaking, and possibly also the fundamental theory underlying it.

If supersymmetry is observed in a collider experiment, it will be challenging to extract all the supersymmetry breaking parameters from the collider signals. While some work has been put into solving this problem [3], there is still a great deal more that needs to be done. The parameters extracted in this way will be subject to experimental uncertainties, especially if the supersymmetric spectrum is relatively heavy. There will also be theoretical uncertainties from higher loop corrections in relating the physical masses to their running values [4]. These uncertainties in the supersymmetry breaking parameters, as well as those in the supersymmetric parameters, will complicate the extrapolation of the soft masses to high energies [5,6]. Much of the previous work along these lines has focused on running the parameters of particular models from the high scale down. This is useful only if the new physics found resembles one of the examples studied. Our goal is to study the running from low to high [7].

Evolving the soft parameters from collider energies up to much higher scales can also be complicated by new physics at intermediate energies below $M_{\rm GUT}$. The apparent unification of gauge couplings suggests that if this new physics is charged under the MSSM gauge group, it should come in the form of complete GUT multiplets or gauge singlets. Indeed, the observation of very small neutrino masses already suggests the existence of new physics in the form of very heavy gauge singlet neutrinos [8]. With new physics that is much heavier than the electroweak scale, it is often very difficult to study it experimentally, or to even deduce its existence. If we extrapolate the MSSM parameters without including the effects of heavy new physics, we will obtain misleading and incorrect values for the high-scale values of these parameters [9].

In the present work we study some of these potential obstacles to the RG evolution of the MSSM soft parame-

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ters. In Sec. II we investigate how uncertainties in the low-scale parameter values can drastically modify the extrapolated high-scale values. We focus on the so-called S term (*a.k.a.* the hypercharge D term) within the MSSM, which depends on all the soft masses in the theory, and can have a particularly large effect on the running if some of these soft masses go unmeasured at the LHC. In Secs. III and IV, we study two possible examples of heavy new physics. Section III investigates the effects of adding complete vectorlike GUT multiplets on the running of the soft parameters. Section IV describes how including heavy Majorana neutrinos to generate small neutrino masses can alter the running of the MSSM soft parameters. In Sec. V we combine our findings and illustrate how they may be put to use with an explicit example. Finally, Sec. VI is reserved for our conclusions. A summary of some useful combinations of scalar soft masses is given in the appendix.

Our main result is that the high-scale values of many Lagrangian parameters can be very sensitive to uncertainties in their low-scale values, or to the presence of heavy new physics. However, in the cases studied we also find that there are particular combinations of the Lagrangian parameters that are stable under the RG evolution, or that are unaffected by the new physics. These special parameter combinations are therefore especially useful for making a comparison with possible high-scale theories.

Throughout our analysis, we simplify the RG equations by setting all flavor nondiagonal soft terms to zero and keeping only the (diagonal) Yukawa couplings of the third generation. Under this approximation, we work to two-loop order for the running of the MSSM parameters, and interface with Suspect 2.3.4 [10] to compute one-loop threshold corrections at the low scale. For concreteness, we take this scale to be 500 GeV. The additional running due to new physics introduced at scales much larger than the weak scale is only performed at one loop. We also implicitly assume that the mass scale of the messengers that communicate supersymmetry breaking to the visible sector lies at or above the GUT scale, $M_{\mathrm{GUT}} \simeq 2.5 \times 10^{16}$ GeV. Even so, our methods and general analysis will also be applicable to scenarios that have lighter messenger particles, such as gauge mediation [11,12]. We also neglect the effects of hidden sector running, which can be significant if there are interacting states in the hidden sector significantly lighter than M_{GUT} [13–15]. While this manuscript was in preparation, methods similar to those considered in the present work were proposed in Ref. [15] to deal with these additional uncertainties in the high-scale values of the soft parameters.

In this work, we focus on low-energy supersymmetric models, and particular forms of intermediate scale new physics. Despite this restriction, we expect that our general techniques will be applicable to other solutions of the gauge hierarchy problem, or to more exotic forms of new intermediate scale physics.

II. UNCERTAINTIES DUE TO THE S TERM

The one-loop renormalization group (RG) equations of the MSSM soft scalar masses have the form [16]

$$(16\pi^2)\frac{dm_i^2}{dt} = \tilde{X}_i - \sum_{a=1,2,3} 8g_a^2 C_i^a |M_a|^2 + \frac{6}{5}g_1^2 Y_i S, \quad (1)$$

where $t = \ln(Q/M_Z)$, $\tilde{X}_i$ is a function of the soft squared masses and the trilinear couplings, M_a denotes the ath gaugino mass, and the S term is given by

$$\begin{aligned} S &= \mathrm{Tr}(Ym^2) \\ &= m_{H_u}^2 - m_{H_d}^2 + \mathrm{tr}(m_Q^2 - 2m_U^2 + m_E^2 + m_D^2 - m_L^2), \end{aligned} \quad (2)$$

where the first trace runs over all hypercharge representations, and the second runs only over flavors.

The S term is unusual in that it connects the running of any single soft mass to the soft masses of every other field with nonzero hypercharge. Taking linear combinations of the RG equations for the soft scalar masses, the one-loop running of S in the MSSM is given by

$$(16\pi^2)\frac{dS}{dt} = -2b_1 g_1^2 S, \quad (3)$$

where $b_1 = -33/5$ is the one-loop beta-function coefficient. Using Eq. (3) in Eq. (1) and neglecting the Yukawa-dependent terms $\tilde{X}_i$ (which are expected to be small for the first and second generations) the effect of $S \neq 0$ is to shift the high-scale value the soft mass would have had were $S(t_0) = 0$ by an amount

$$\Delta m_i^2(t) = \frac{Y_i}{\mathrm{Tr}(Y^2)}\left[\frac{g_1^2(t)}{g_1^2(t_0)} - 1\right]S(t_0). \quad (4)$$

The one-loop RG equation for S is homogeneous. Thus, if S vanishes at any one scale, it will vanish at all scales (at one loop). In both minimal supergravity (mSUGRA) and simple gauge-mediated models, S does indeed vanish at the (high) input scale, and for this reason the effects of this term are often ignored.

From the low-energy perspective, there is no reason for $S(t_0)$ to vanish, and in many cases its effects can be extremely important. Since g_1 grows with increasing energy, the mass shift due to the S term grows as well. For $t_0 \simeq t_{M_Z}$ and $t \to t_{\mathrm{GUT}}$, the prefactor in Eq. (4) is about $(0.13)Y_i$. The value of $S(t_0)$ depends on all the scalar soft masses, and the experimental uncertainty in its value will therefore be set by the least well-measured scalar mass. In particular, if one or more of the soft masses are not measured at all, $S(t_0)$ is unbounded other than by considerations of naturalness. Fortunately, this uncertainty only affects the soft scalar masses. The S term does not enter directly into the running of the other soft parameters until three-loop order [16,17], and therefore its effects on these parameters is expected to be mild.

There is also a theoretical uncertainty induced by the S term. Such a term is effectively equivalent to a Fayet-Iliopoulos (FI) D term for hypercharge [17]. To see how this comes about, consider the hypercharge D-term potential including a FI term,

$$\mathcal{L} = \ldots + \frac{1}{2}D_1^2 + \xi D_1 + \sqrt{\frac{3}{5}}g_1 D_1 \sum_i \bar{\phi}_i Y_i \phi^i - \sum_i \tilde{m}_i^2 |\phi^i|^2, \tag{5}$$

where we have also included the soft scalar masses. Eliminating the D_1 through its equation of motion, we find

$$\mathcal{L} = \ldots - \frac{1}{2}\xi^2 - \frac{g_1^2}{2}\Big(\sum_i \bar{\phi}_i Q_i \phi^i\Big)^2 - \sum_i \bar{\phi}_i\Big(\tilde{m}_i^2 + \sqrt{\frac{3}{5}}g_1 Y_i \xi\Big)\phi^i. \tag{6}$$

Thus, except for the constant addition to the vacuum energy, the effect of the FI term is to shift each of the soft squared masses by an amount

$$\tilde{m}_i^2 \to m_i^2 := \tilde{m}_i^2 + \sqrt{\frac{3}{5}}g_1 Y_i \xi. \tag{7}$$

The low-energy observable quantities are the $\{m_i^2\}$, not the $\{\tilde{m}_i^2\}$. Since we cannot extract the shift in the vacuum energy, the low-energy effects of the FI term are therefore invisible to us, other than the shift in the soft masses. This shift is exactly the same as the shift due to the S term. Let us also mention that the S term, as we have defined it in Eq. (2), runs inhomogeneously at two loops and above, so the exact correspondence between a hypercharge D term and the S term of Eq. (2) does not hold beyond one-loop order.

A simple way to avoid both the large RG uncertainties in the soft masses and the theoretical ambiguity due to the S term is apparent from Eq. (4). Instead of looking at individual soft masses, it is safer to consider the mass differences

$$Y_j m_i^2 - Y_i m_j^2, \tag{8}$$

for any pair of fields. These differences are not affected by the mass shifts of Eq. (4). They are also independent of the value of the FI term.

In the rest of this section, we show how the S term can complicate the running of the soft masses to high energies with a particular example. If one of the scalar soft masses is unmeasured, it is essential to use the linear mass combinations given in Eq. (8) instead of the individual masses themselves. We also discuss how the special RG properties of the S term provide a useful probe of high-scale physics if all the scalar soft masses are determined experimentally.

A. Example: SPS-5 with an unmeasured Higgs soft mass

To illustrate the potential high-scale uncertainties in the RG-evolved soft parameters due to the S term, we study the sample mSUGRA point SPS-5 [18] under the assumption that one of the scalar soft masses goes unmeasured at the LHC. If this is the case, the S parameter is undetermined, and the precise values of the high-scale soft terms are no longer precisely calculable. Even if the value of the S term is bounded by considerations of naturalness, the uncertainties in the high-scale values of the soft scalar masses can be significant.

The SPS-5 point is defined by the mSUGRA input values $m_0 = 150$ GeV, $m_{1/2} = 300$ GeV, $A_0 = -1000$, $\tan\beta = 5$, and $\mathrm{sgn}(\mu) > 0$, at $M_{\rm GUT}$. The mass spectrum for this point has relatively light sleptons around 200 GeV, and somewhat heavier squarks with masses near 400–600 GeV. The lightest superpartner particle (LSP) of the model is a mostly Bino neutralino, with mass close to 120 GeV. The perturbation we consider for this point is a shift in the down-Higgs soft mass, $m_{H_d}^2$.

Of the soft supersymmetry breaking parameters in the MSSM, the soft terms associated with the Higgs sector can be particularly difficult to deduce from LHC measurements. At tree level, the independent Lagrangian parameters relevant to this sector are [1]

$$v, \quad \tan\beta, \quad \mu, \quad M_A, \tag{9}$$

where $v \simeq 174$ GeV is the electroweak breaking scale, $\tan\beta = v_u/v_d$ is the ratio of the H_u and H_d VEVs, μ is the supersymmetric μ-term, and M_A is the pseudoscalar Higgs boson mass. Other Higgs-sector Lagrangian parameters, such as $m_{H_d}^2$ and $m_{H_u}^2$, can be expressed in terms of these using the conditions for electroweak symmetry breaking in the MSSM.

Among the Higgs-sector parameters listed in Eq. (9), only the value of v is known. The value of μ can potentially be studied independently of the Higgs scalar sector by measuring neutralino and chargino masses and couplings [3,18,19], although it is likely to be poorly determined if only hadron colliders are available. A number of observables outside the Higgs sector may also be sensitive to $\tan\beta$, especially if it is large, $\tan\beta \gtrsim 20$. For example, the dilepton invariant mass distributions in the inclusive $2\ell + \text{jets} + \not{E}_T$ channel can vary significantly depending on the value of $\tan\beta$, but this dependence is such that the value of $\tan\beta$ can at best only be confined to within a fairly broad range [20]. Determining M_A at the LHC typically requires the discovery of one of the heavier MSSM Higgs boson states. Finding these states can also help to determine $\tan\beta$ [21]. Unfortunately, the LHC reach for the heavier Higgs states is limited, especially for larger values of M_A and intermediate or smaller values of $\tan\beta \lesssim 20$ [22]. If none of the heavier Higgs bosons are found at the

LHC, it does not appear to be possible to determine both of the Higgs soft masses, $m_{H_u}^2$ and $m_{H_d}^2$.

For the low-scale parameters derived from SPS-5, the pseudoscalar Higgs mass M_A is about 700 GeV. With such a large value of M_A, and $\tan\beta = 5$, only the lightest SM-like Higgs boson is within the reach of the LHC [22]. Motivated by this observation, we examine the effect of changing the low-scale value of $m_{H_d}^2$ on the running of the other soft parameters. The actual low-scale value of $m_{H_d}^2$ is about $(235\ \text{GeV})^2$. The perturbation we consider is to set this value to $(1000\ \text{GeV})^2$, while keeping $\tan\beta$ fixed. Such a perturbation does not ruin electroweak symmetry breaking, and tends to push the heavier Higgs masses to even larger values. In the present case, the heavier Higgs masses increase from about 700 GeV to over 1200 GeV.[1]

The effects of this perturbation in $m_{H_d}^2$ on some of the soft scalar masses are shown in Figs. 1 and 2. In these plots, we show $m_i = m_i^2/\sqrt{|m_i^2|}$. The deviations in the soft masses are substantial, and the S parameter is the source of this uncertainty. In addition to the S term, varying $m_{H_d}^2$ can also modify the running of the Higgs mass parameters and the down-type squarks and sleptons through the Yukawa-dependent terms X_i in the RG equations, Eq. (1). In the present case these Yukawa-dependent effects are very mild since for $\tan\beta = 5$, the b and τ Yukawas are still quite small. This can be seen by noting the small difference between the perturbed running of $m_{U_{1,2}}^2$ and $m_{U_3}^2$ in Fig. 2. For larger values of $\tan\beta$, the down-type Yukawa couplings can be enhanced and this non-S effect from $m_{H_d}^2$ can be significant. However, we also note that as these Yukawa couplings grow larger, it becomes much easier for the LHC to detect one or more of the heavier Higgs states. To the extent that the Yukawa-dependent shifts can be neglected, the linear mass combinations of Eq. (8) remove most of the uncertainty due to an unknown $m_{H_d}^2$ in the running of the soft masses that are measured. The effect of not knowing $m_{H_d}^2$ has only a very small effect on the running of the gaugino masses and the trilinear terms.

In this example we have assumed that $m_{H_d}^2$ is the only unmeasured soft scalar mass. Several of the other soft scalar masses may be difficult to reconstruct from LHC data as well. For example, within many SUSY scenarios the third-generation squarks and some of the heavier sleptons have very small LHC production rates. If there are other unmeasured soft scalar masses besides $m_{H_d}^2$, the uncertainties due to the S term in the extrapolation of the measured scalar soft masses will be even greater than what we have presented here. The mass combinations of Eq. (8) will be necessary to study the high-scale supersymmetry breaking spectrum in this case.

[1]The values of μ and $B\mu$ also change, although the variation in μ is very mild: $\mu \simeq 640\ \text{GeV} \to 670\ \text{GeV}$.

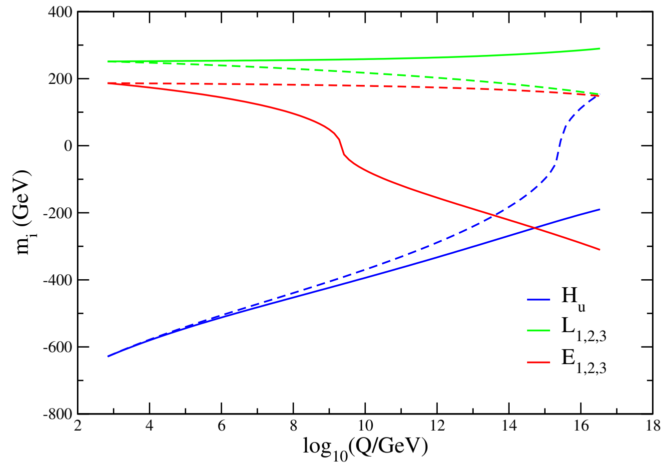

FIG. 1 (color online). Deviations in the running of some of the SPS-5 soft masses due to setting $m_{H_d}^2 = (1000\ \text{GeV})^2$ at the low scale. The solid lines show the running of $m_{H_u}^2$, m_E^2, and m_L^2 with this perturbation, while the dashed lines show the unperturbed running of these soft masses. The unperturbed low-scale value of the down-Higgs soft mass is $m_{H_d}^2 \simeq (235\ \text{GeV})^2$.

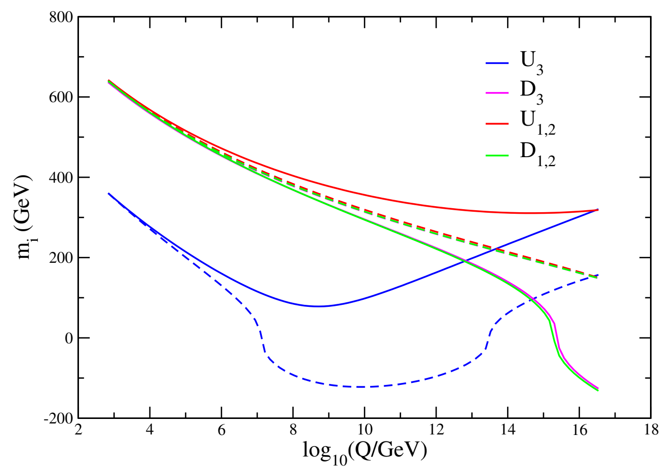

FIG. 2 (color online). Deviations in the running of some of the SPS-5 soft masses due to setting $m_{H_d}^2 = (1000\ \text{GeV})^2$ at the low scale. The solid lines show the running of $m_{U_{1,2}}^2$, $m_{D_{1,2}}^2$, $m_{U_3}^2$, and $m_{D_3}^2$ with this perturbation, while the dashed lines show the unperturbed running of these soft masses.

B. Origins and uses of the S term

While the S term can complicate the extrapolation of the soft scalar masses if one of them goes unmeasured, the simple scale dependence of this term also makes it a useful probe of the high-scale theory if all the masses are determined. The essential feature is the homogeneous RG evolution of the S term, given in Eq. (3), which is related to the nonrenormalization of FI terms in the absence of supersymmetry breaking.

A nonvanishing S term can arise from a genuine FI term present in the high-scale theory, or from nonuniversal scalar soft masses at the high input scale. The size of a fundamental hypercharge FI term, ξ, is naively on the order of the large input scale. Such a large value would either destabilize the gauge hierarchy or lead to $U(1)_Y$ breaking at high energies. However, the nonrenormalization of FI terms implies that it is technically natural for ξ to take on much smaller values. In this regard, a small value for ξ is analogous to the μ problem in the MSSM. Adding such a FI term to mSUGRA provides a simple one-parameter extension of this model, and can have interesting effects [23,24]. Note, however, that in a GUT where $U(1)_Y$ is embedded into a simple group, a fundamental hypercharge FI term in the full theory is forbidden by gauge invariance.

It is perhaps more natural to have the S term emerge from nonuniversal scalar soft masses [25,26]. This is true even in a GUT where $U(1)_Y$ is embedded into a simple group. Within such GUTs, the contribution to S from complete GUT multiplets vanishes. However, nonzero contributions to S can arise from multiplets that are split in the process of GUT breaking. For example, in $SU(5)$ with H_u and H_d states embedded in $\mathbf{5}$ and $\bar{\mathbf{5}}$ multiplets, a nonzero low-energy value of S can be generated when the heavy triplet states decouple provided the soft masses of the respective multiplets are unequal. Whether it is zero or not, the low-scale value of the S term provides a useful constraint on the details of a GUT interpretation of the theory.

So far we have only considered the S term corresponding to $U(1)_Y$. If there are other gauged $U(1)$ symmetries, there will be additional S termlike factors for these too. In fact, the homogeneity of the S term evolution also has a useful implication for any nonanomalous *global* $U(1)$ symmetry in the theory. The only candidate in the MSSM is $U(1)_{B-L}$, up to linear combinations with $U(1)_Y$ [27]. Let us define S_{B-L} by the combination

$$\begin{aligned} S_{B-L} &= \mathrm{Tr}(Q_{B-L} m^2) \\ &= \mathrm{tr}(2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2), \end{aligned} \quad (10)$$

where the second trace runs only over flavors.[2] We can think of S_{B-L} as the effective S term for a gauged $U(1)_{B-L}$ in the limit of vanishing coupling. At one-loop order, the RG running of S_{B-L} is given by

$$(16\pi^2)\frac{dS_{B-L}}{dt} = \frac{3}{5}\,\mathrm{Tr}(Q_{B-L}Y)g_1^2 S = n_g\frac{16}{5}g_1^2 S, \quad (11)$$

where $n_g = 3$ is the number of generations. If S is measured and vanishes, S_{B-L} provides a second useful combination of masses that is invariant under RG evolution,[3] and yields an additional constraint on possible GUT embeddings of the theory.

III. NEW PHYSICS: COMPLETE GUT MULTIPLETS

As a second line of investigation, we consider the effects of some possible types of new intermediate scale physics on the running of the MSSM soft terms. If this new physics is associated with supersymmetry breaking as in gauge mediation [11], then it is of particular interest in its own right. Indeed, in this case the low-energy spectrum of soft terms may point towards the identity of the new physics after RG evolution. On the other hand, there are many kinds of possible new intermediate scale physics that are not directly related to supersymmetry breaking. The existence of this type of new phenomena can make it much more difficult to deduce the details of supersymmetry breaking from the low-energy soft terms.

A useful constraint on new physics is gauge coupling unification. To preserve unification, the SM gauge charges of the new physics should typically be such that all three MSSM gauge beta functions are modified in the same way.[4] This is automatic if the new matter fills out complete multiplets of a simple GUT group into which the MSSM can be embedded. Such multiplets can emerge as remnants of GUT symmetry breaking.

As an example of this sort of new physics, we consider vectorlike pairs of complete $SU(5)$ multiplets. For such multiplets, it is possible to write down a supersymmetric mass term of the form

$$\mathcal{W} \supset \tilde{\mu}\,\bar{X}X, \quad (12)$$

where X and $\bar{X}$ denote the chiral superfields of the exotic multiplets. We also assume that the exotic multiplets have no significant superpotential (Yukawa) couplings with the MSSM fields. Under these assumptions, the exotic $SU(5)$ multiplets can develop large masses independently of the details of the MSSM. They will interact with the MSSM fields only through their gauge interactions.

If the supersymmetric mass $\tilde{\mu}$ is much larger than the electroweak scale, it will be very difficult to deduce the presence of the additional GUT multiplets from low-energy data alone. Moreover, an extrapolation of the measured soft masses using the RG equations appropriate for the MSSM will lead to incorrect values of the high-scale parameters. In this section, we characterize the sizes and patterns of the deviations in the high-scale soft spectrum induced by additional vectorlike GUT multiplets. We also

[2] Up to flavor mixing, we can also define an independent S_{B-L} for each generation.

[3] This nonevolution of mass combinations corresponding to nonanomalous global symmetries persists at strong coupling. In models of conformal sequestering, this can be problematic [13].

[4] For an interesting exception, see Ref. [28].

show that even though the new matter interferes with the running of the MSSM soft parameters, it is often still possible to obtain useful information about the input spectrum, such as the relative sizes of the gaugino masses and whether there is intergenerational splitting between the soft scalar masses.

A. Shifted gauge running

The main effect of the exotic GUT multiplets is to shift the running of the $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ gauge couplings. Recall that in the MSSM, the one-loop running of these couplings is determined by

$$\frac{dg_a^{-2}}{dt} = \frac{b_a}{8\pi^2}, \tag{13}$$

with $(b_1, b_2, b_3) = (-33/5, -1, 3)$. The presence of a massive GUT multiplet shifts each of the b_a up by an equal amount above the heavy threshold at $t = t_I = \ln(\tilde{\mu}/M_Z)$,

$$\Delta b = -N_{\mathbf{5}\oplus\bar{\mathbf{5}}} - 3N_{\mathbf{10}\oplus\overline{\mathbf{10}}} + \ldots, \tag{14}$$

where $N_{\mathbf{5}\oplus\bar{\mathbf{5}}}$ is the number of additional $\mathbf{5}\oplus\bar{\mathbf{5}}$ representations and $N_{\mathbf{10}\oplus\overline{\mathbf{10}}}$ is the number of $\mathbf{10}\oplus\overline{\mathbf{10}}$'s. The modified one-loop solution to the RG equations is therefore,

$$g_a^{-2}(t) = \begin{cases} g_a^{-2}(t_0) + \frac{b_a}{8\pi^2}(t - t_0) & t < t_I, \\ g_a^{-2}(t_0) + \frac{b_a}{8\pi^2}(t - t_0) + \frac{\Delta b}{8\pi^2}(t - t_I) & t > t_I. \end{cases} \tag{15}$$

It follows that the unification scale is not changed, but the value of the unified gauge coupling is increased. Note that the number of new multiplets is bounded from above for a given value of $\tilde{\mu}$ if gauge unification is to be perturbative.[5]

The change in the gauge running shifts the running of all the soft parameters, but the greatest effect is seen in the gaugino masses. At one loop, these evolve according to

$$\frac{dM_a}{dt} = -\frac{b_a}{8\pi^2} g_a^2 M_a. \tag{16}$$

It follows that M_a/g_a^2 is RG invariant above and below the heavy threshold. If the threshold is also supersymmetric, M_a will be continuous across it at tree level. Since g_a is also continuous across the threshold at tree level, the addition of heavy vectorlike matter does not modify the one-loop scale invariance of the ratio M_a/g_a^2. This holds true whether or not the new matter preserves gauge unification, but it is most useful when unification holds. When it does, the measurement of the low-energy gaugino masses immediately translates into a knowledge of their ratio at $M_{\rm GUT}$ [29].

From Eq. (15) and the one-loop scale invariance of M_a/g_a^2, the shift in the gaugino masses due to the additional matter is

$$M_a(t) = \bar{M}_a(t)\left[1 + \frac{\Delta b \bar{g}_a^2}{8\pi^2}(t - t_I)\right]^{-1}, \tag{17}$$

where $\bar{g}_a$ and $\bar{M}_a$ denote the values these parameters would have for $\Delta b = 0$ (i.e. the values obtained using the MSSM RG equations). For $t = t_{\rm GUT}$, the shift coefficient is identical for $a = 1, 2, 3$ provided gauge unification occurs. The shift in the running of the gauge couplings and the gaugino masses due to seven sets of $\mathbf{5}\oplus\bar{\mathbf{5}}$'s at 10^{11} GeV is illustrated in Fig. 3. An unperturbed universal gaugino mass of $M_{1/2} = 700$ GeV is assumed. Both the values of the unified gauge coupling and the universal gaugino mass at $M_{\rm GUT}$ are increased by the additional multiplets.

The running of the soft masses also depends on the running of the gauge couplings and gaugino masses, and is modified by the appearance of new matter. At one-loop order, in the limit that we can neglect the Yukawa couplings, it is not hard to find the shifts in the soft masses. We can divide these shifts into two contributions,

$$m_i^2(t) = \bar{m}_i^2 + \Delta m_{i_\lambda}^2 + \Delta m_{i_S}^2, \tag{18}$$

where $\bar{m}_i^2$ is the value of the soft mass obtained by running the measured value up in the absence of the new matter, $\Delta m_{i_\lambda}^2$ is the shift due to the modified gaugino masses, and $\Delta m_{i_S}^2$ is due to the change in the running of the S term.

The first shift, $\Delta m_{i_\lambda}^2$, can be obtained straightforwardly from Eqs. (1), (13), and (16), and is given by

$$\Delta m_{i_\lambda}^2 = \sum_a 2C_i^a \left|\frac{M_a}{g_a^2}\right|^2 \Delta I_a, \tag{19}$$

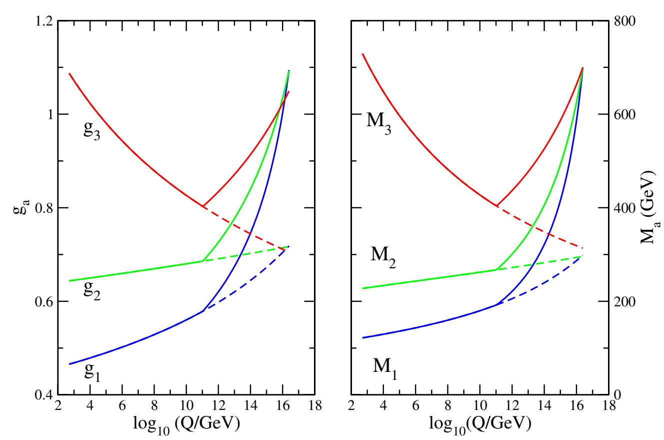

FIG. 3 (color online). The shift in the running of the gauge couplings (left) and the gaugino masses (right) due to 7 sets of $\mathbf{5}\oplus\bar{\mathbf{5}}$'s with mass $\tilde{\mu} = 10^{11}$ GeV. The universal gaugino mass is taken to be 700 GeV.

[5]Note that the parts of $\tilde{\mu}$ corresponding to the doublet and triplet components of the $\mathbf{5}$ will run differently between $M_{\rm GUT}$ and the intermediate scale. This will induce additional threshold corrections that we do not include in our one-loop analysis.

where C_i^a is the Casimir invariant of the ath gauge group and

$$\Delta I_a = \frac{1}{(b_a + \Delta b)}(g_a^4 - g_{a_I}^4) - \frac{1}{b_a}(\bar{g}_a^4 - g_{a_I}^4), \quad (20)$$

with g_a the actual gauge coupling at scale $t > t_I$ (including the extra matter), g_{a_I} the gauge coupling at the heavy threshold t_I, and $\bar{g}_a$ the gauge coupling at scale $t > t_I$ in the absence of new matter.

New chiral matter can also modify the running of the soft scalar masses through the S parameter. The new matter shifts the running of S by changing the running of g_1, but it can also contribute to the S term directly at the heavy threshold. Combining these effects, the value of S above the threshold is

$$\begin{aligned} S(t) &= \left(\frac{g_1}{g_{1_0}}\right)^2 S(t_0) + \left(\frac{g_1}{g_{1_I}}\right)^2 \Delta S \\ &= \bar{S}(t) + \left(\frac{g_1^2 - \bar{g}_1^2}{g_{1_0}^2}\right) S(t_0) + \left(\frac{g_1}{g_{1_I}}\right)^2 \Delta S, \end{aligned} \quad (21)$$

where ΔS is the shift in the value of S at the threshold, g_{1_0} is the low-scale value of the gauge coupling, and $\bar{S}(t)$ is the value the S term would have in the absence of the new matter. Inserting this result into Eq. (1), the effect of the new matter on the running of the soft scalar masses through the S term is

$$\begin{aligned} \frac{5}{3Y_i}\Delta m_{i_S}^2 &= \frac{1}{b_1}\left[\left(\frac{\bar{g}_1}{g_{1_0}}\right)^2 - \left(\frac{g_{1_I}}{g_{1_0}}\right)^2\right] S_0 - \frac{1}{(b_1 + \Delta b)} \\ &\quad \times \left[\left(\frac{g_1}{g_{1_0}}\right)^2 - \left(\frac{g_{1_I}}{g_{1_0}}\right)^2\right] S_0 - \frac{1}{(b_1 + \Delta b)} \\ &\quad \times \left[\left(\frac{g_1}{g_{1_I}}\right)^2 - 1\right] \Delta S. \end{aligned} \quad (22)$$

As before, the effects of the S term on the soft masses are universal up to the hypercharge prefactor. Thus, they still cancel out of the linear combinations given in Eq. (8).

B. Yukawa effects and useful combinations

We have so far neglected the effects of the MSSM Yukawa couplings on the modified running of the soft masses. As a result, the shifts in the running of the soft scalar masses written above are family universal. There are also nonuniversal shifts in the soft scalar masses. These arise from the Yukawa-dependent terms in the soft scalar mass beta functions, which themselves depend on the Higgs and third-generation soft scalar masses. As a result, the low-energy spectrum obtained from a theory with universal scalar masses at the high scale and additional GUT multiplets can appear to have nonuniversal soft masses at the high scale if the extra GUT multiplets are not included in the RG evolution. These nonuniversal shifts are usually a subleading effect, but as we illustrate below they can be significant when the supersymmetric mass of the new GUT multiplets is within a few orders of magnitude of the TeV scale.

Nonuniversal mass shifts obscure the relationship between the different MSSM generations and the source of supersymmetry breaking. This relationship is closely linked to the SUSY flavor problem [30], and possibly also to the origin of the Yukawa hierarchy. For example, third-generation soft masses that are significantly different from the first and second generation values is one of the predictions of the model of Ref. [31], in which strongly coupled conformal dynamics generates the Yukawa hierarchy and suppresses flavor-mixing soft terms. The relative sizes of the high-scale soft masses for different families is therefore of great theoretical interest.

Even when there are nonuniversal shifts from new physics, it is sometimes still possible to obtain useful information about the flavor structure of the soft scalar masses at the high scale. To a very good approximation, the flavor nonuniversal contributions to the RG evolution of the soft masses are proportional to the third-generation Yukawa couplings or the trilinear couplings. There is also good motivation (and it is technically allowed) to keep only the trilinear couplings for the third generation. In this approximation, the Yukawa couplings and the A terms only appear in the one-loop RG equations for the soft masses through three independent linear combinations. Of the seven soft masses whose running depends on these combinations, we can therefore extract four mass combinations whose evolution is independent of Yukawa effects at one-loop order [32].[6] They are:

$$\begin{aligned} m_{A_3}^2 &= 2m_{L_3}^2 - m_{E_3}^2, \qquad & m_{B_3}^2 &= 2m_{Q_3}^2 - m_{U_3}^2 - m_{D_3}^2, \\ m_{X_3}^2 &= 2m_{H_u}^2 - 3m_{U_3}^2, \qquad & m_{Y_3}^2 &= 3m_{D_3}^2 + 2m_{L_3}^2 - 2m_{H_d}^2. \end{aligned} \quad (23)$$

The cancellation in the first two terms occurs because the linear combinations of masses correspond to L and B global symmetries. They run only because these would-be symmetries are anomalous under $SU(2)_L$ and $U(1)_Y$. The other mass combinations can also be related to anomalous global symmetries of the MSSM.

These mass combinations have the same one-loop RG running as certain combinations of masses involving only the first and second generations. For example, the $m_{B_3}^2$ combination runs in exactly the same way at one loop as

$$m_{B_i}^2 = 2m_{Q_i}^2 - m_{U_i}^2 - m_{D_i}^2, \quad (24)$$

for $i = 1, 2$. If these linear combinations are unequal at the low scale, the corresponding soft masses will be nonun-

[6] We also assume implicitly that the soft masses are close to diagonal in the super Cabibbo-Kobayashi-Maskawa (CKM) basis, as they are quite constrained to be [30].

iversal at the high scale. This holds in the MSSM, as well as in the presence of any new physics that is flavor universal and respects baryon number. On the other hand, $m^2_{B_3} = m^2_{B_1}$ does not imply family-universal high-scale masses. For example, within a $SO(10)$ GUT, a splitting between the soft masses of the **16**'s containing the first, second, and third generations will not lead to a difference between the mass combinations in Eq. (23) at the low scale. A similar conclusion holds for the mass combinations $m^2_{A_i}$.

In the case of $m^2_{X_3}$ and $m^2_{Y_3}$, it is less obvious what to compare them to. The trick here is to notice that in the absence of Yukawa couplings, $m^2_{H_d}$ runs in the same way as $m^2_{L_1}$ since they share the same gauge quantum numbers. If the S term vanishes as well, $m^2_{H_u}$ also has the same RG evolution as $m^2_{L_1}$. This motivates us to define

$$m^2_{X_i} = 2m^2_{L_i} - 3m^2_{U_i}, \qquad m^2_{Y_i} = 3m^2_{D_i}, \tag{25}$$

for $i = 1, 2$. These mass combinations can be compared with $m^2_{X_3}$ and $m^2_{Y_3}$ in much the same way as for $m^2_{B_i}$ and $m^2_{A_i}$ (although comparing the $m^2_{X_i}$'s is only useful for $S = 0$). They also correspond to anomalous global symmetries in the limit that the first and second generation Yukawa couplings vanish.

The mass combinations listed above can be useful if there is heavy new physics that hides the relationships between the high-scale masses. For instance, suppose the high-scale soft spectrum obtained using the MSSM RG equations applied to the measured soft scalar masses shows a large splitting between $m^2_{Q_3}$ and $m^2_{Q_1}$. If the corresponding splitting between $m^2_{B_3}$ and $m^2_{B_1}$ (at any scale) is very much smaller, this feature suggests that there is new physics that should have been included in the RG running, or that there exists a special relationship between $m^2_{Q_i}$, $m^2_{U_i}$, and $m^2_{D_i}$ at the high scale. A similar conclusion can be made for the other mass combinations.

C. Some numerical results

In our numerical analysis, we follow a similar procedure to the one used in the previous section. The MSSM running is performed at two-loop order, and we interface with Suspect 2.3.4 [10] to compute the low-scale threshold corrections. New physics, in the form of vectorlike GUT multiplets at an intermediate scale is included only at the one-loop level. Unlike the previous section, we define our high-energy spectrum using a simple mSUGRA model in the $\Delta b \neq 0$ theory, and include the new physics effects in generating the low-energy spectrum. We then evolve this spectrum back up to the unification scale using the MSSM RG evolution, with $\Delta b = 0$. Our goal is to emulate evolving the MSSM soft parameters in the presence of unmeasured and unknown high-scale new physics.

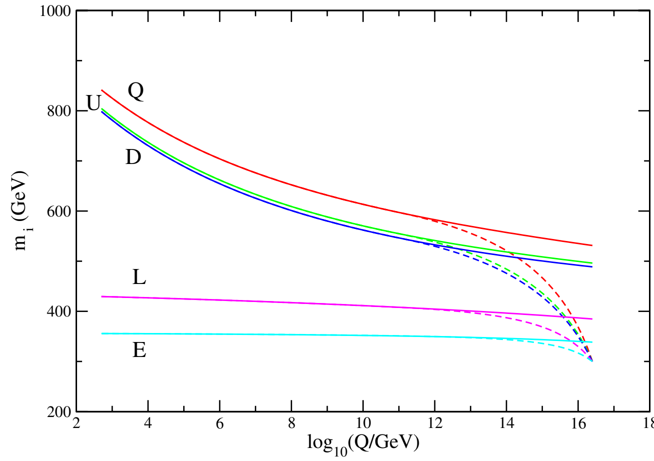

FIG. 4 (color online). Running of the first generation soft scalar masses with $N_{5\oplus\bar{5}} = 7$ and $\tilde{\mu} = 10^{11}$ GeV for the mSUGRA input parameters $m_0 = 300$ GeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$, and $A_0 = 0$. The dashed lines show the actual running of these parameters, while the solid lines show the running from low to high using the RG equations of the MSSM, ignoring the additional heavy multiplets.

The running of the soft masses with $N_{5\oplus\bar{5}} = 7$ sets of $\mathbf{5} \oplus \bar{\mathbf{5}}$ multiplets with an intermediate scale mass of $\tilde{\mu} = 10^{11}$ GeV is shown in Fig. 4 for the mSUGRA parameters $m_0 = 300$ GeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$, and $A_0 = 0$. These parameters are used to find the low-energy spectrum, which is then RG evolved back up to the high scale with $\Delta b = 0$. From this figure, we see that a naive MSSM extrapolation of the soft parameters (i.e. with $\Delta b = 0$) yields predictions for the high-scale soft scalar masses that are significantly larger than the actual values. In the same way, the MSSM predicted values of the high-scale gaugino masses are smaller than the correct values, as can be seen in Fig. 3. Note that for $\tilde{\mu} = 10^{11}$ GeV, $N_{5\oplus\bar{5}} = 7$ is about as large as possible while still keeping the gauge couplings perturbatively small all the way up to $M_{\rm GUT}$.

Besides confusing the relationship between the high-scale gaugino and scalar soft masses, heavy GUT multiplets can also obscure the comparison of the high-scale scalar masses from different generations. As discussed above, this arises from the backreaction in the Yukawa-dependent terms in the RG equations for the third-generation soft scalar masses. Numerically, we find that the splitting is quite small compared to the absolute scale of the masses for $\tilde{\mu} \gtrsim 10^{11}$ GeV. This is illustrated in Fig. 5. We also find that an approximate preservation of universality persists for other values of m_0, A_0, and $\tan\beta$ as well. The reason for this appears to be that for $\tilde{\mu} \gtrsim 10^{11}$ GeV, the Yukawa couplings are smaller than the gauge couplings by the time the new physics becomes relevant.

A much greater splitting between the high scale values of $m^2_{Q_1}$ and $m^2_{Q_3}$, and $m^2_{U_1}$ and $m^2_{U_3}$, is obtained for lower

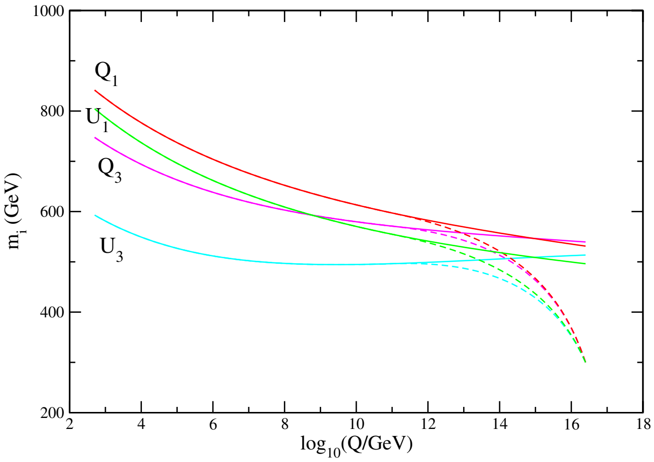

FIG. 5 (color online). Running of the soft scalar masses of $Q_{1,3}$ and $U_{1,3}$ with $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 7$ and $\tilde{\mu} = 10^{11}$ GeV for the mSUGRA input parameters $m_0 = 300$ GeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$, and $A_0 = 0$. The dashed lines show the actual running of these parameters, while the solid lines show the running from low to high using the RG equations of the MSSM, ignoring the additional heavy multiplets.

values of $\tilde{\mu}$. This effect is shown in Fig. 6 for $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 3$ sets of $\mathbf{5} \oplus \bar{\mathbf{5}}$ multiplets with an intermediate scale mass of $\tilde{\mu} = 10^4$ GeV, and the mSUGRA parameters $m_0 = 300$ GeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$, and $A_0 = 0$. The value of the gauge couplings at unification here is very similar to the $\tilde{\mu} = 10^{11}$ GeV and $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 7$ case. As might be expected, the Yukawa-dependent terms in the soft scalar mass RG running become important at lower scales where the top Yukawa approaches unity.

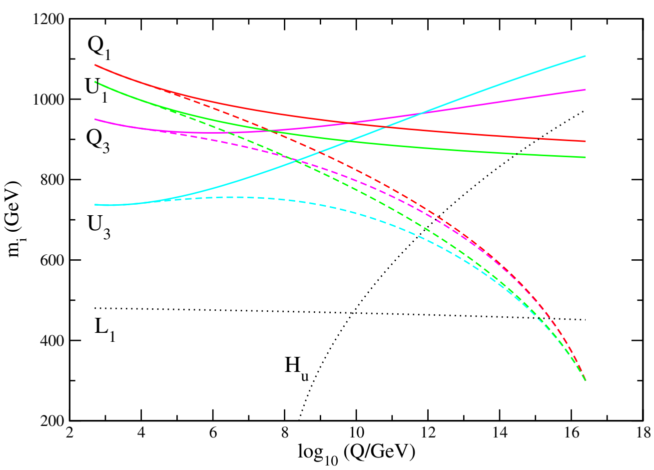

FIG. 6 (color online). Running of the soft scalar masses of $Q_{1,3}$ and $U_{1,3}$ with $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 3$ and $\tilde{\mu} = 10^4$ GeV for the mSUGRA input parameters $m_0 = 300$ GeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$, and $A_0 = 0$. The dashed lines show the actual running of these parameters, while the solid lines show the running from low to high using the RG equations of the MSSM, ignoring the additional heavy multiplets.

It is also interesting to note that in both Figs. 5 and 6, the soft masses appear to take on family-universal values, $m^2_{Q_1} = m^2_{Q_3}$ and $m^2_{U_1} = m^2_{U_3}$, at the same scale, near 10^{15} GeV in Fig. 5, and close to 10^{10} GeV in Fig. 6. It is not hard to show, using the mass combinations in Eqs. (23)–(25), that this feature holds exactly at one-loop order provided $S = 0$, the high-scale masses are family universal, and the only relevant Yukawa coupling is that of the top quark. In this approximation, all the family-dependent mass splittings are proportional to $(m^2_{H_u} - m^2_{L_1})$, and hence vanish when $m^2_{H_u} = m^2_{L_1}$. This relationship can be seen to hold approximately in Fig. 6, which also includes two-loop and bottom Yukawa effects.

In Figs. 7 and 8 we show the running of the mass combinations $m_{B_{1,3}}$ and $m_{X_{1,3}}$ (where $m_i = m_i^2/\sqrt{|m_i^2|}$) for $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 3$ and $\tilde{\mu} = 10^4$ GeV with the high-scale mSUGRA input values $m_0 = 300$ GeV, $M_{1/2} = 700$ GeV, $\tan\beta = 10$, and $A_0 = 0$. These figures also show the values of $m_{B_{1,3}}$ and $m_{X_{1,3}}$ that would be obtained by running up without including the effects of the heavy new physics. Comparing these figures to Figs. 5 and 6, it is apparent that the splittings between $m^2_{B_1}$ and $m^2_{B_3}$, and $m^2_{X_1}$ and $m^2_{X_3}$, are very much less than the high-scale splittings between the Q and U soft masses.

This relationship between the B and X soft mass combinations from different families is a footprint left by the full theory (including the heavy GUT multiplets) on the low-energy spectrum. Since the scalar masses in the full theory are universal at the high scale, the low-scale splittings between the B and X soft mass combinations are very

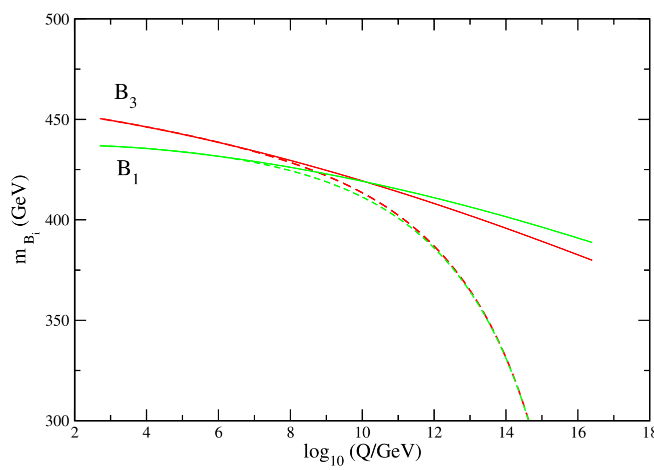

FIG. 7 (color online). Running of the soft scalar mass combinations m_{B_3} and m_{B_1} with $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 3$ and $\tilde{\mu} = 10^4$ GeV for the mSUGRA input parameters $m_0 = 300$ GeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$, and $A_0 = 0$. The dashed lines show the actual running of these parameters, while the solid lines show the running from low to high using the RG equations of the MSSM, ignoring the additional heavy multiplets. The small deviations in these figures arise from higher loop effects.

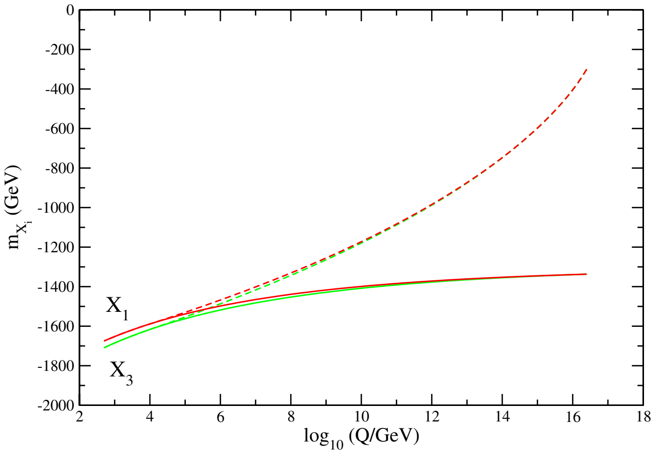

FIG. 8 (color online). Running of the soft scalar mass combinations m_{X_3} and m_{X_1} with $N_{5\oplus\bar{5}} = 3$ and $\tilde{\mu} = 10^4$ GeV for the mSUGRA input parameters $m_0 = 300$ GeV, $m_{1/2} = 700$ GeV, $\tan\beta = 10$, and $A_0 = 0$. The dashed lines show the actual running of these parameters, while the solid lines show the running from low to high using the RG equations of the MSSM, ignoring the additional heavy multiplets. The small deviations in these figures arise from higher loop effects.

small. On the other hand, running the low-scale Q and U scalar soft masses up within the MSSM does not suggest any form of family universality among these masses. Therefore, as we proposed above, the low-energy values of these particular combinations of soft scalar masses can provide evidence for heavy new physics.[7]

The analysis in this section shows that even though new physics in the form of additional heavy GUT multiplets can significantly disrupt the predictions for the high-scale soft spectrum obtained by running in the MSSM, certain key properties about the input spectrum can still be deduced using collider-scale measurements. Most significantly, the low-scale values of the gaugino masses and gauge couplings can be used to predict the approximate ratios of the high-scale values, provided gauge unification is preserved.

The effect of additional GUT multiplets on the scalar soft masses is more severe. Extrapolating the soft masses without including the contributions from the heavy GUT multiplets leads to a prediction for the input soft masses that are generally too low. The splittings between the soft masses from different generations can be shifted as well. Despite this, some of the flavor properties of the input soft mass spectrum can be deduced by comparing the evolution of the mass combinations in Eqs. (23)–(25). For example, $m^2_{B_3} = m^2_{B_1}$ and $m^2_{L_1} = m^2_{L_3}$ suggests some form of flavor universality (or an embedding in $SO(10)$), even if the scalar masses extrapolated within the MSSM do not converge at $M_{\rm GUT}$. We expect that these special mass combinations could prove useful for studying other types of heavy new physics as well.

[7]We have checked that the small splittings between m_{B_1} and m_{B_3}, as well as between m_{X_1} and m_{X_3}, arise from higher loop effects.

IV. THE (S)NEUTRINO CONNECTION

We have seen in the previous sections that taking the unification of the gauge couplings as a serious theoretical input still leaves considerable room for experimental uncertainties and new physics to modify the extrapolated values of the soft supersymmetry breaking parameters at very high energies. For example, a Fayet-Iliopoulos D term for hypercharge or additional complete GUT multiplets with intermediate scale masses will not disrupt gauge coupling unification, but will in general change the running of the parameters of the model. In this section we wish to study the effect of additional intermediate scale singlet matter with significantly large Yukawa couplings to the MSSM matter fields. A particularly well-motivated example of this, and the one we consider, are heavy singlet neutrino multiplets.

The observed neutrino phenomenology can be accommodated by extending the matter content of the MSSM to include at least two right-handed (RH) neutrino supermultiplets that are singlets under the SM gauge group [8]. Throughout the present work, we will assume there are three RH neutrino flavors. The superpotential in the lepton sector is then given by

$$\mathcal{W}_l = \mathbf{y_e} L H_d E + \mathbf{y}_\nu L H_u N_R - \tfrac{1}{2}\mathbf{M_R} N_R N_R, \tag{26}$$

where L, E, and N_R are, respectively, the $SU(2)_L$ doublet, $SU(2)_L$ singlet, and neutrino chiral supermultiplets, each coming in three families. The quantities $\mathbf{y_e}$, $\mathbf{y}_\nu$, and $\mathbf{M_R}$ are 3×3 matrices in lepton family space. The H_d and H_u fields represent the usual Higgs multiplets. The gauge-invariant interactions among leptons and Higgs superfields are controlled by the family-space Yukawa matrices $\mathbf{y_e}$ and $\mathbf{y}_\nu$. As is conventional, we shall implicitly work in a basis where $\mathbf{y_e}$ is diagonal. Since the N_R are singlets, we can also add to the superpotential a Majorana mass $\mathbf{M_R}$ for these fields.

Assuming the eigenvalues of $\mathbf{M_R}$ lie at a large intermediate mass scale, 10^9–10^{14} GeV, we can integrate out the RH neutrino superfields and obtain a term in the effective superpotential that leads to small neutrino masses through the seesaw mechanism,

$$\mathcal{W}_{m_\nu} = -\tfrac{1}{2}\mathbf{y}_\nu^{\mathbf{T}} \mathbf{M_R}^{-1} \mathbf{y}_\nu L H_u L H_u. \tag{27}$$

After electroweak symmetry breaking, the neutrino mass matrix becomes

$$(\mathbf{m}_\nu) = \mathbf{y}_\nu^{\mathbf{T}} \mathbf{M_R}^{-1} \mathbf{y}_\nu v_u^2, \tag{28}$$

where $v_u = \langle H_u \rangle$. The mass matrix can be conveniently diagonalized by the transformation

$$(\mathbf{m}_\nu^{\mathbf{diag}}) = \mathbf{U}^{\mathbf{T}} (\mathbf{m}_\nu) \mathbf{U}, \tag{29}$$

with $\mathbf{U}$ a unitary matrix. This matrix $\mathbf{U}$ is the usual Pontecorve-Maki-Nakagawa-Sakata (PMNS) matrix that describes lepton mixing relative to the flavor basis where the charged lepton Yukawa matrix $\mathbf{y_e}$ is diagonal.

One can also write the neutrino Yukawa matrix as [33]

$$\mathbf{y}_\nu = \frac{1}{v_u}\sqrt{\mathbf{M_R^{diag}}}\mathbf{R}\sqrt{\mathbf{m_\nu^{diag}}}\mathbf{U}^\dagger, \tag{30}$$

where $v_u = \langle H_u \rangle$ and $\mathbf{R}$ is a complex orthogonal matrix that parametrizes our ignorance of the neutrino Yukawas. As an estimate, Eq. (30) shows that the size of the neutrino Yukawa couplings will be on the order of

$$y_\nu \simeq \frac{0.57}{\sin\beta}\left(\frac{M_R}{10^{14}\ \mathrm{GeV}}\right)^{1/2}\left(\frac{m_\nu}{0.1\ \mathrm{eV}}\right)^{1/2}. \tag{31}$$

Thus, the neutrino Yukawa couplings can take large $\mathcal{O}(1)$ values comparable to the top Yukawa coupling for $M_R \sim 10^{14}$ GeV. If this is the case, then above the seesaw mass threshold the effects on the RG running of the MSSM soft parameters due to the neutrino Yukawas can be substantial.

The addition of RH neutrinos to the MSSM can lead to lepton flavor violation (LFV) through the RG running of the off-diagonal slepton mass terms [34,35]. In this work we will only consider simple scenarios of neutrino phenomenology in which the amount of lepton flavor violation induced by the heavy neutrino sector is small. However, the observation of LFV signals could potentially provide information about a heavy neutrino sector [34–37]. Precision measurements of the slepton mass matrices can also be used to constrain possible heavy neutrino sectors [38,39]. Heavy singlet neutrinos may also be related to the source of the baryon asymmetry through the mechanism of leptogenesis [40].

Running up

The strategy we use in this section is similar to the one followed in the previous sections. We assume a universal high-scale mass spectrum at M_{GUT}, and RG evolve the model parameters down to the low scale $M_{\mathrm{low}} = 500$ GeV including the additional effects of the neutrino sector parameters. The resulting low-scale spectrum is then run back up to M_{GUT} using the RG equations for the MSSM without including the neutrino sector contributions. As before, we use this procedure to illustrate the discrepancy between the extrapolated parameter values and their true values if the new physics effects are not included in the running.

To simplify the analysis, we make a few assumptions about the parameters in the neutrino sector. We choose the complex orthogonal matrix $\mathbf{R}$ to be purely real, implying that it is unitary, and we take the heavy neutrino mass matrix $\mathbf{M_R}$ to be proportional to the unit matrix, $\mathbf{M_R} = M_R\mathbb{I}$. We also take the physical neutrino masses to be degenerate. This allows us to write

$$\mathbf{y}_\nu = \frac{0.57}{\sin\beta}\left(\frac{M_R}{10^{14}\ \mathrm{GeV}}\right)^{1/2}\left(\frac{m_\nu}{0.1\ \mathrm{eV}}\right)^{1/2}\mathbf{R}\mathbf{U}^\dagger := y_\nu \mathbf{R}\mathbf{U}^\dagger, \tag{32}$$

so that the neutrino Yukawa couplings have the form of a universal constant multiplying a unitary matrix. To fix the value of y_ν, we will set $m_\nu = 0.1$ eV.[8] This choice is close to being as large as possible while remaining consistent with the cosmological bounds on the sum of the neutrino masses, $\sum_\nu < 0.68$ eV [41]. Note that larger values of the neutrino masses tend to maximize the size of the resulting neutrino Yukawa couplings.

We also need to impose boundary conditions on the RH sneutrino masses and trilinear couplings. We take these new soft parameters to have universal and diagonal boundary conditions at the unification scale,

$$m^2_{N_{ij}} = m_0^2\delta_{ij} \quad \text{and} \quad a_{\nu_{ij}} = A_0 y_{\nu_{ij}}, \tag{33}$$

where m_0 and A_0 are the same universal soft scalar mass and trilinear coupling that we will apply to the MSSM fields in the analysis to follow. With these assumptions about the neutrino and soft parameters, the effects of the neutrino sector on the (one-loop) RG running of the MSSM soft terms take an especially simple form. In particular, the amount of leptonic flavor mixing induced is expected to be very small, and the diagonal and universal form of the neutrino sector soft terms, Eq. (33), is approximately maintained at lower scales. The leading logarithmic contributions of the neutrino Yukawa couplings to the renormalization of the soft supersymmetry breaking terms are proportional to [42]

$$(\mathbf{y}_\nu^\dagger \mathbf{y}_\nu)_{ij}\ln\left(\frac{M_{\mathrm{GUT}}}{M_R}\right), \tag{34}$$

where i and j are leptonic flavor indices. Since $\mathbf{y}_\nu$ is a constant multiplying a unitary matrix under our assumptions, Eq. (32), we have that $(\mathbf{y}_\nu^\dagger \mathbf{y}_\nu)_{ij} \propto \delta_{ij}$, and therefore no lepton flavor mixing is induced at the leading order. However, let us emphasize that our assumptions about the structure of the neutrino sector were chosen for convenience. These assumptions need not hold in more realistic scenarios or, in particular, models [43], and the amount of the lepton flavor violation in the more general case can be significant when M_R is large.

To illustrate the effects of the neutrino sector, we set the high-scale spectrum to coincide with the SPS-5 benchmark point, and we extend the corresponding soft terms to the neutrino sector. The input values at M_{GUT} for this point are $m_0 = 150$ GeV, $m_{1/2} = 300$ GeV, $A_0 = -1000$ GeV,

[8]The diagonal neutrino mass matrix $\mathbf{m_\nu^{diag}}$ and the lepton mixing matrix $\mathbf{U}$ are measured at low scales, and one should really evaluate them at the intermediate scale M_R by running the Yukawa couplings up [37]. Since we are most interested in the effect of the neutrino Yukawas after reaching the intermediate scale, we will neglect this additional running below M_R.

and $\tan\beta = 5$ with μ positive. These input values tend to magnify the effects of the neutrino sector because the large value of A_0 feeds into the running of the soft masses. Large neutrino Yukawa couplings alter the running of the top Yukawa coupling as well.

Of the MSSM soft parameters, the greatest effects of the heavy neutrino sector are seen in the soft scalar masses and the trilinear A terms. The gaugino masses are only slightly modified. The evolution of the soft masses for the H_u, L_3, and U_3 fields from low to high are shown in Fig. 9, both with and without including the effects of the neutrino sector for $M_R = 3 \times 10^{14}$ GeV. For this value of M_R and with $\tan\beta = 5$, the Yukawa coupling is close to being as perturbatively large as possible. The extrapolated values of $m^2_{H_u}$ and $m^3_{L_3}$ deviate significantly from the actual input values if the effects of the neutrino sector are not taken into account in the RG evolution. These fields are particularly affected because they couple directly to the heavy neutrino states through the neutrino Yukawa coupling. The shift in the running of $m^2_{U_3}$ arises indirectly from the effect of the neutrino Yukawas on $m^2_{H_u}$ and the top Yukawa coupling y_t.

In Fig. 10 we show the size of the discrepancies in the extrapolated high-scale values of a few of the soft scalar masses if the neutrino sector effects are not included in the running. These discrepancies are plotted as a function of the heavy neutrino mass scale M_R. As above, the high-scale input spectrum consists of the SPS-5 values. Again, the soft masses $m^2_{H_u}$ and $m^2_{L_i}$ are altered the most, although the third-generation squark soft masses also get shifted somewhat as a backreaction to the changes in $m^2_{H_u}$ and the top Yukawa coupling. This plot also shows that the sizes of the discrepancies remain quite small for M_R less

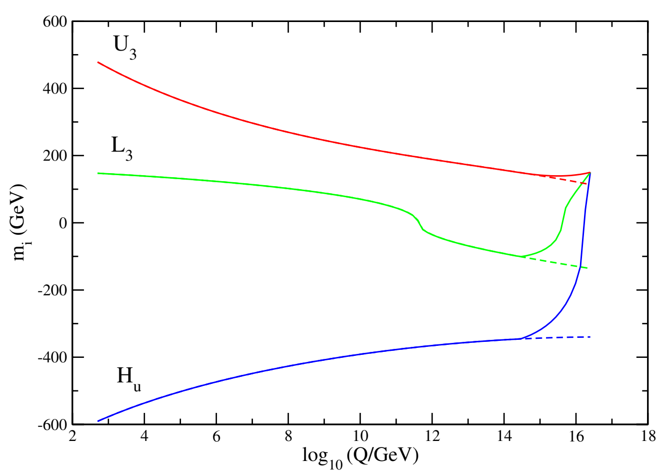

FIG. 9 (color online). Running of the soft scalar masses of H_u, L_3, and U_3 for SPS-5 input parameters at $M_{\rm GUT}$ with three additional heavy RH neutrinos of mass $M_R = 10^{14}$ GeV. The solid lines show the full running, including the neutrino sector effects, while the dashed lines show the low-to-high running of the soft masses in the MSSM, with the neutrino sector effects omitted.

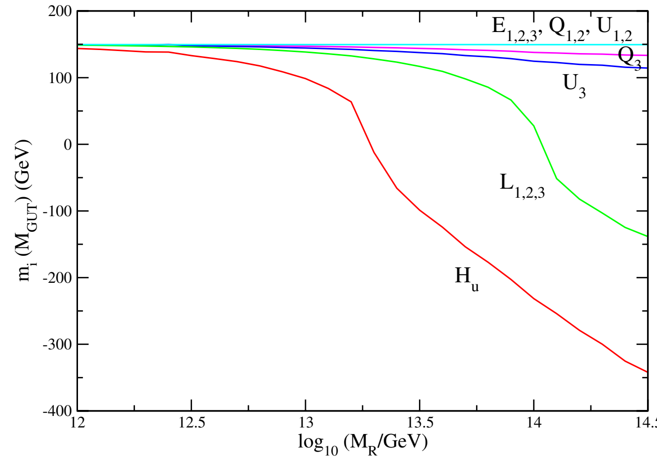

FIG. 10 (color online). The high-scale ($M_{\rm GUT}$) values of the soft scalar masses extrapolated using the MSSM RG equations, without including neutrino sector effects. The low-scale (500 GeV) values of the soft masses used in the extrapolation were obtained from SPS-5 input parameters at $M_{\rm GUT}$ with three additional heavy RH neutrinos of mass M_R. The deviations from $m_i(M_{\rm GUT}) = 150$ GeV represent the discrepancy between the MSSM extrapolated values and the correct value in the full theory with heavy RH neutrinos. These discrepancies are shown as a function of the heavy neutrino scale M_R.

than 10^{13} GeV. Values of M_R considerably less than this are favored if leptogenesis is to be the source of the baryon asymmetry [40]. Neutrino masses well below 0.1 eV would also lead to less pronounced deviations in the extrapolated soft masses.

The deviations induced by not including the new neutrino sector physics in the running take a similar form to those obtained by not taking account of heavy GUT multiplets. For both cases, the gaugino mass running is only modified in a very controlled way, while the soft scalar masses and trilinear couplings deviate more unpredictably. In particular, the high-scale flavor structure of the soft masses can be obscured. With large neutrino Yukawa couplings, the third-generation squark masses receive additional contributions to their running relative to the first and second generations due to the potentially large effect of the neutrino Yukawa couplings on $m^2_{H_u}$. The sizes of these additional family-dependent shifts tend to be fairly small, as can be seen in Fig. 10. Furthermore, these effects cancel out in the mass combinations $m^2_{B_i}$ defined in Eqs. (23) and (24), and we find $m^2_{B_1} \simeq m^2_{B_3}$ at all scales, regardless of whether or not the neutrino effects are included. On the other hand, the running of $m^2_{A_3}$ relative to $m^2_{A_1}$, $m^2_{X_3}$ relative to $m^2_{X_1}$, and $m^2_{Y_3}$ relative to $m^2_{Y_1}$ need no longer coincide if there is a heavy neutrino sector.

Finally, let us also mention that for more general neutrino sector parameters than those we have considered, there can arise significant lepton flavor-mixing couplings in the MSSM slepton soft terms from the RG running [34]. Measurements of this mixing in lepton flavor violating processes can therefore provide an experimental probe of the heavy neutrino multiplets [34–37]. A measured splitting among the three slepton masses $m^2_{L_i}$ would also constitute another indication of the existence of a neutrino sector with sizeable Yukawa couplings and nontrivial flavor structure. Both high and intermediate energy data may be complementary and very useful in extrapolating the MSSM soft terms to high energies.

V. PUTTING IT ALL TOGETHER: AN EXAMPLE

In this section we summarize some of our previous results with an explicit example. We begin with a low-energy spectrum of MSSM soft supersymmetry breaking parameters that we assume to have been measured at the LHC to an arbitrarily high precision. We then attempt to deduce the essential features of the underlying high-scale structure by running the low-energy parameters up and applying some of the techniques discussed in the previous sections.

In our example we will make the following assumptions:

(1) The possible types of new physics beyond the MSSM are:
   - (a) Complete $\mathbf{5} \oplus \bar{\mathbf{5}}$ GUT multiplets with a common (SUSY) mass scale $\tilde{\mu}$.
   - (b) Three families of heavy singlet (RH) neutrinos at the mass scale M_R.
   - (c) A fundamental hypercharge D term.

In the case of complete GUT multiplets, we will assume further that there are no superpotential interactions with the MSSM states as in Sec. III. For heavy RH neutrinos, we will make the same set of assumptions about the form of the mixing and mass matrices as in Sec. IV.

(2) The high-scale spectrum has the form of a minimal SUGRA model (up to the scalar mass shifts due to a hypercharge D term) at the high scale $M_{\rm GUT} \simeq 2.5 \times 10^{16}$ GeV.

(3) This mSUGRA spectrum also applies to the soft parameters corresponding to any new physics sectors. For example, a trilinear A term in the RH neutrino sector has the form $\mathbf{a}_\nu = A_0 \mathbf{y}_\nu$ at $M_{\rm GUT}$, where A_0 is the universal trilinear parameter.

These assumptions are not entirely realistic, but they make the analysis tractable. Moreover, even though this exercise is highly simplified compared to what will be necessary should the LHC discover supersymmetry, we feel that it illustrates a number of useful techniques that could be applied in more general situations. With this set of assumptions, the underlying free parameters of the theory are:

$$m_0, \quad m_{1/2}, \quad A_0, \quad \xi, \quad N_{\mathbf{5}\oplus\bar{\mathbf{5}}}, \quad \tilde{\mu}, \quad M_R, \qquad (35)$$

where m_0, $m_{1/2}$, and A_0 are common mSUGRA inputs at $M_{\rm GUT}$, ξ is the fundamental hypercharge D term, $N_{\mathbf{5}\oplus\bar{\mathbf{5}}}$ is the number of additional $\mathbf{5} \oplus \bar{\mathbf{5}}$ multiplets in the theory with a supersymmetric mass $\tilde{\mu}$, and M_R is the heavy neutrino scale.

A. Step 1: Running up in the MSSM

As a first step, we run the low-energy spectrum up to the high scale $M_{\rm GUT}$ using the RG equations for the MSSM, without including any potential new physics effects. The low-energy MSSM soft spectrum we consider, defined at the low scale $M_{\rm low} = 500$ GeV, is given in Table I. In addition to these soft terms, we also assume that $\tan\beta = 7$ has been determined, and that the first and second generation soft scalar masses are equal. With this set of soft terms, we have verified that the low-energy superpartner mass spectrum is phenomenologically acceptable using Suspect 2.3.4 [10]. The lightest Higgs boson mass is 114 GeV for a top quark mass of $m_t = 171.4$ GeV.

Even before extrapolating the soft parameters, it is possible to see a number of interesting features in the spectrum. The most obvious is that the low-scale gaugino masses have ratios close to $M_1{:}M_2{:}M_3 \simeq 1{:}2{:}6$. This suggests that the high-scale gaugino masses have a universal value $m_{1/2}$, and provide further evidence for gauge unification. The low-energy value of the S term, as defined in Eq. (2), is also nonzero and is in fact quite large, $S(M_{\rm low}) \simeq (620\ {\rm GeV})^2$. This indicates that there are sig-

TABLE I. The low-energy scale ($M_{\rm low} = 500$ GeV) soft supersymmetry breaking spectrum used in our analysis. The soft scalar masses listed in the table correspond to the signed square roots of the actual masses squared. In this table we also use the high-scale values of these soft parameters obtained by running them up to $M_{\rm GUT} \simeq 2.5 \times 10^{16}$ GeV using the RG evolution appropriate for the MSSM.

Soft parameter	Low-scale value (GeV)	High-scale value (GeV)
M_1	146	356
M_2	274	355
M_3	859	370
A_t	−956	−766
A_b	−1755	−818
A_τ	−737	−524
m_{H_u}	−700	419
m_{H_d}	350	236
m_{Q_3}	821	549
m_{U_3}	603	445
m_{D_3}	884	501
m_{L_3}	356	213
m_{E_3}	349	404
m_{Q_1}	934	532
m_{U_1}	872	402
m_{D_1}	888	501
m_{L_1}	357	213
m_{E_1}	352	404

nificant contributions to the effective hypercharge D term in the high-scale theory. Since the S term is nonzero, it is also not surprising that $S_{B-L} \simeq (446\ \mathrm{GeV})^2$, as defined in Eq. (10), is nonzero as well.

The values of the soft parameters extrapolated to M_{GUT} within the MSSM are listed in Table I. The MSSM running of the soft scalar masses is also shown in Fig. 11. As anticipated, the gaugino masses unify approximately to a value $M_1 \simeq M_2 \simeq M_3 \simeq m_{1/2} = 350$ GeV at M_{GUT}. The high-scale pattern of the soft scalar masses (and the trilinear A terms) shows less structure, and is clearly inconsistent with mSUGRA high-scale input values.

Since $S(M_{\mathrm{low}})$ is large and nonzero, we are motivated to look for a hypercharge D term contribution to the soft scalar masses. Such a contribution would cancel in the mass combinations

$$\Delta m_{ij}^2 = (Y_j m_i^2 - Y_i m_j^2)/(Y_j - Y_i). \tag{36}$$

If the high-scale soft masses have the form $m_i^2 = m_0^2 + \sqrt{\frac{3}{5}} g_1 Y_i \xi$, as in mSUGRA with a hypercharge D term, these combinations will all be equal to m_0^2 at this scale. The high-scale soft masses here, extrapolated within the MSSM, exhibit no such relationship. Even so, these mass combinations will prove useful in the analysis to follow.

It is also interesting to compare the pairs of mass combinations $m_{A_i}^2$, $m_{B_i}^2$, $m_{X_i}^2$, and $m_{Y_i}^2$ for $i = 1, 3$, as defined in Sec. III. Of these, the most useful pair is $m_{B_1}^2$ and $m_{B_3}^2$. At the low and high scales (extrapolating in the MSSM) this pair obtains the values

$$\begin{aligned} m_{B_1}^2(M_{\mathrm{low}}) &\simeq (441\ \mathrm{GeV})^2, \\ m_{B_3}^2(M_{\mathrm{low}}) &\simeq (452\ \mathrm{GeV})^2, \\ m_{B_1}^2(M_{\mathrm{GUT}}) &\simeq (392\ \mathrm{GeV})^2, \\ m_{B_3}^2(M_{\mathrm{GUT}}) &\simeq (392\ \mathrm{GeV})^2. \end{aligned} \tag{37}$$

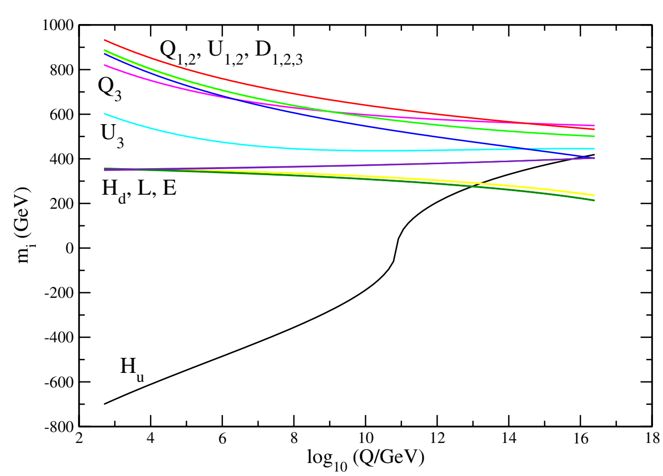

FIG. 11 (color online). Scale dependence of the soft scalar masses for the input soft parameters given in Table I. No new physics effects beyond the MSSM were included in the running.

The near equality of $m_{B_1}^2$ and $m_{B_3}^2$ at the high scale is particularly striking. By comparison, there is a significant interfamily splitting that occurs between the high-scale soft masses of U_1 and U_3,

$$\begin{aligned} m_{U_1}^2(M_{\mathrm{GUT}}) &= (402\ \mathrm{GeV})^2, \\ m_{U_3}^2(M_{\mathrm{GUT}}) &= (445\ \mathrm{GeV})^2. \end{aligned} \tag{38}$$

This apparent fine-tuning among the soft masses that make up the B-type combinations is suggestive of an underlying structure in the theory. However, based on the values of the individual soft masses extrapolated within the MSSM, such a structure is not obvious. Instead, we can interpret this as a hint for new intermediate scale physics.

Without our guiding assumptions, the high-scale spectrum listed in Table I obtained by running up in the MSSM does not exhibit any particularly remarkable features aside from the universality of the gaugino masses. Even so, the curious relationship between the $m_{B_1}^2$ and $m_{B_3}^2$ mass combinations provides a strong hint that we are missing something. It is not clear how strong this hint would have been had we also included reasonable uncertainties in the low-scale parameter values.

B. Step 2: Adding GUT multiplets

As a first attempt to fit the low-energy soft spectrum to the class of models outlined above, let us consider adding additional vectorlike GUT multiplets to the theory at the scale $\tilde{\mu}$. We try this first because, as we found in Secs. III and IV, the contributions from such multiplets are potentially much larger than those due to heavy singlet neutrinos.

In adding the new GUT multiplets, we will make use of our starting assumptions about the possible forms of new physics. Given the large value of $S(M_{\mathrm{low}})$, there appears to be significant hypercharge D term. Also from our assumptions, this D term will contribute to the soft scalar masses of the heavy GUT multiplets, which will in term feed into the running of the MSSM scalar masses through the S term above the scale $\tilde{\mu}$. For this reason, it is safer to work with the mass differences defined in Eq. (36) whose running (to one loop) does not depend on the S term.

Among the low-scale soft masses listed in Table I, we expect the slepton soft mass $m_{E_1}^2$ to be among the easiest to measure, and the least susceptible to new physics effects. Thus, we will use it as a reference mass in all but two of the differences we choose. The mass differences we consider are

$$\begin{matrix} \Delta m_{Q_1E_1}^2, & \Delta m_{U_1E_1}^2, & \Delta m_{D_1E_1}^2, & \Delta m_{L_1E_1}^2, \\ \Delta m_{H_dE_1}^2, & \Delta m_{H_uE_1}^2, & \Delta m_{Q_1D_1}^2, & \Delta m_{Q_3D_1}^2, \\ \Delta m_{Q_3E_1}^2, & \Delta m_{U_3E_1}^2, & \Delta m_{D_3E_1}^2, & \Delta m_{L_3E_1}^2. \end{matrix} \tag{39}$$

These depend on several independent mass measurements.

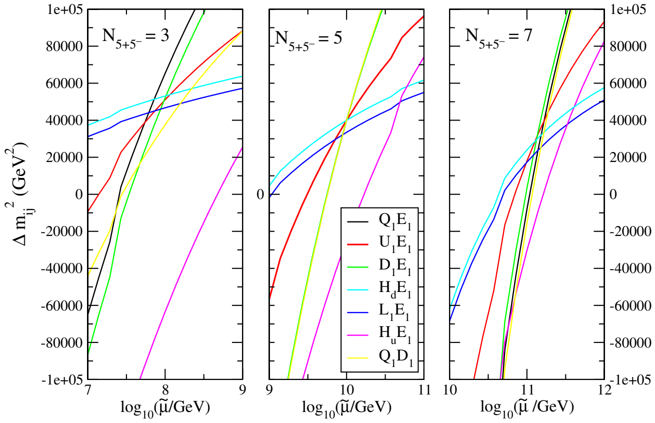

FIG. 12 (color online). High-scale values for the soft scalar mass differences Δm_{ij}^2 defined in Eq. (39) as a function of the mass scale $\tilde{\mu}$ of the $N_{\mathbf{5}\oplus\bar{\mathbf{5}}}$ heavy GUT multiplets for $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 3, 5, 7$.

In Fig. 12 we show the high-scale values of these mass differences obtained by running up the low-scale soft masses while including a given number of $N_{\mathbf{5}\oplus\bar{\mathbf{5}}}$ additional $\mathbf{5} \oplus \bar{\mathbf{5}}$ GUT multiplets at the scale $\tilde{\mu}$. For each of the plots, nearly all the mass differences unify approximately, as they would be expected to do if the underlying theory has a mSUGRA spectrum. The best agreement with a mSUGRA model is obtained for $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 5$ with $\tilde{\mu} = 10^{10}$ GeV. (More precisely, the agreement is obtained when a shift $\Delta b = -5$ is applied to the gauge beta-function coefficients b_i at the scale $\tilde{\mu} = 10^{10}$ GeV).

Taking $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 5$ and $\tilde{\mu} = 10^{10}$ GeV, the high-scale values of the mass differences are

$$\Delta m^2_{Q_1E_1} = (200\text{ GeV})^2 = \Delta m^2_{U_1E_1} = \Delta m^2_{D_1E_1} = \Delta m^2_{H_dE_1},$$
$$\Delta m^2_{Q_3E_1} = (197\text{ GeV})^2, \qquad \Delta m^2_{D_3E_1} = (200\text{ GeV})^2,$$
$$\Delta m^2_{H_uE_1} = -(157\text{ GeV})^2, \qquad \Delta m^2_{L_{1,3}E_1} = (183\text{ GeV})^2. \tag{40}$$

Most of these high-scale values coincide, suggesting a mSUGRA value for the universal soft scalar mass of about $m_0 = 200$ GeV. On the other hand, the soft mass differences involving the H_u and L fields show a significant deviation from this near-universal value. Based on the results of Sec. IV, these are precisely the scalar masses that are most sensitive to a heavy singlet neutrino sector.

Using this same choice of new physics parameters, we can also estimate the values of the other mSUGRA parameters. Heavy singlet neutrinos are not expected to significantly alter the running of the gaugino soft mass parameters. If we run these up to $M_{\rm GUT}$ including $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 5$ additional GUT multiplets at $\tilde{\mu} = 10^{10}$ GeV, we find $m_{1/2} \simeq 700$ GeV. Doing the same for the trilinear couplings, we do not find a unified high-scale value for them. Instead, we obtain $A_t(M_{\rm GUT}) = -401$ GeV, $A_\tau = -407$ GeV, and $A_b = -500$ GeV. This is not surprising since a heavy RH neutrino sector would be expected to primarily modify A_t and A_τ, while having very little effect on A_b. Thus, we also expect $A_0 \simeq -500$ GeV.

It is possible to estimate the value of the hypercharge D term as well. Using the hypothesis $m_i^2 = m_0^2 + \sqrt{\frac{3}{5}} g_1 Y_i \xi$ at the high scale, we find

$$\xi(M_{\rm GUT}) = \sqrt{\frac{5}{3}}\frac{1}{g_1}(m^2_{E_1} - m^2_{H_d})/(Y_E - Y_{H_d}) \simeq (494\text{ GeV})^2. \tag{41}$$

We obtain similar values from the corresponding combinations of other mass pairs with the exception of L and H_u. Based on our previous findings, we suspect that the L and H_u soft masses are modified by a heavy RH neutrino sector.

Note that had we included experimental and theoretical uncertainties it would have been considerably more difficult to distinguish different values of $N_{\mathbf{5}\oplus\bar{\mathbf{5}}}$ and $\tilde{\mu}$. Instead of finding a single value for $\mathbf{5} \oplus \bar{\mathbf{5}}$ and a precise value for $\tilde{\mu}$, it is likely that we would have only been able to confine $N_{\mathbf{5}\oplus\bar{\mathbf{5}}}$ and $\tilde{\mu}$ to within finite ranges.

C. Step 3: Adding a heavy neutrino sector

By adding $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 5$ complete $\mathbf{5} \oplus \bar{\mathbf{5}}$ multiplets at the scale $\tilde{\mu} = 10^{10}$ GeV and a hypercharge D term, we are nearly able to fit the low-scale spectrum given in Table I to a mSUGRA model with $m_0 = 200$ GeV, $m_{1/2} = 700$ GeV, and $A_0 = -500$ GeV. However, there are several small deviations from this picture, most notably in the soft masses for H_u and L as well as the trilinear couplings A_t and A_τ. We attempt to fix these remaining discrepancies by including heavy RH neutrinos at the scale M_R.

Given our initial assumptions about the form of a possible RH neutrino sector, the only independent parameter

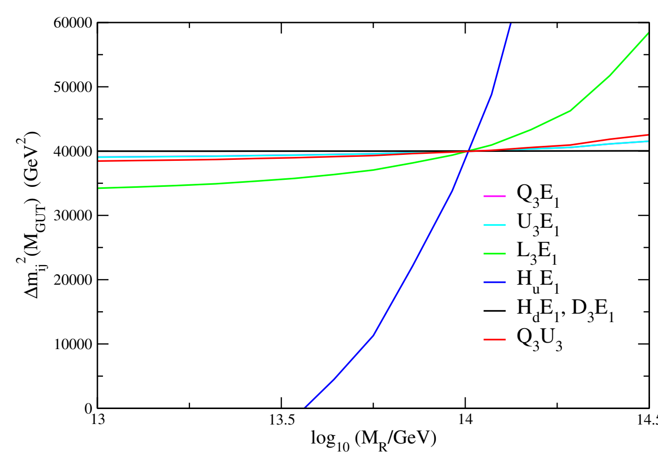

FIG. 13 (color online). High-scale values for third family and Higgs boson soft scalar mass differences Δm_{ij}^2 (as defined in Eq. (36)) as a function of the mass scale M_R of the heavy RH neutrinos. $N_{\mathbf{5}\oplus\bar{\mathbf{5}}} = 5$ GUT multiplets were included in the running above the scale $\tilde{\mu} = 10^{10}$ GeV.

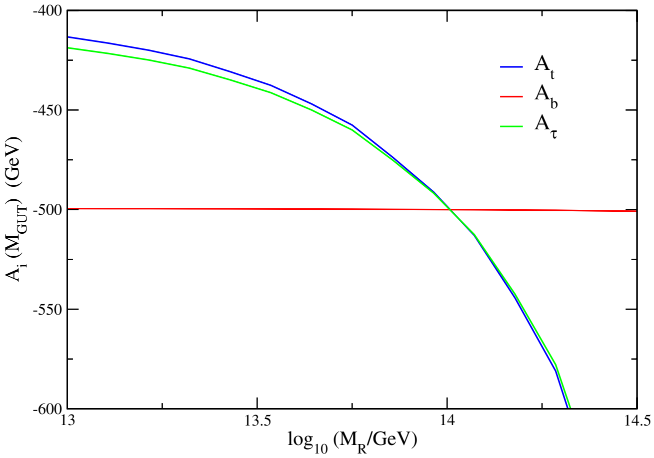

FIG. 14 (color online). High-scale values for the soft trilinear A terms as a function of the mass scale of the heavy RH neutrinos. $N_{5\oplus\bar{5}} = 5$ GUT multiplets were included in the running above the scale $\tilde{\mu} = 10^{10}$ GeV.

in this sector is the heavy mass scale M_R. To investigate the effects of RH neutrinos, we examine the high-scale values of the mass differences given in Eq. (36) for the third-generation scalars and H_u, using $m^2_{E_1}$ as a reference mass. We add a RH neutrino sector with heavy mass M_R and run the low-scale parameters listed in Table I subject to the additional neutrino effects, as well as those from $N_{5\oplus\bar{5}} = 5$ heavy GUT multiplets with $\tilde{\mu} = 10^10$ GeV. The result is shown in Fig. 13, in which we scan over M_R. This plot shows that if we include heavy RH neutrinos at the mass scale near $M_R = 10^{14}$ GeV, all the mass differences will flow to a universal value of about $m_0 = 200$ GeV at the high scale.

We can further confirm this result by examining the high-scale trilinear couplings obtained by this procedure. These also attain a universal value, $A_0 = -500$ GeV, at the high scale for $M_R = 10^{14}$ GeV as shown in Fig. 14. These universal values are consistent with those we hypothesized before the inclusion of heavy neutrino sector effects. Similarly, we can also study the value of ξ obtained using Eq. (41), but using the high scale L and H_u soft masses computed by including heavy RH neutrinos in their RG evolution. As before, we obtain a value $\xi \simeq (494\ \text{GeV})^2$.

D. Summary

We have succeeded in deducing a high-scale mSUGRA model augmented by heavy new physics effects that reproduces the soft spectrum in Table I. The relevant parameter values are:

$$m_0 = 200\ \text{GeV}, \qquad m_{1/2} = 700\ \text{GeV},$$
$$A_0 = -500\ \text{GeV}, \qquad N_{5\oplus\bar{5}} = 5, \qquad \tilde{\mu} = 10^{10}\ \text{GeV},$$
$$M_R = 10^{14}\ \text{GeV}, \qquad \xi = (494\ \text{GeV})^2. \tag{42}$$

Our example did not include potential uncertainties in the input soft parameter values. It is likely that such uncertainties would make the analysis more challenging.

Running the low-scale soft parameters up to M_{GUT} within the MSSM, and without including any new physics, we obtained a reasonable but mostly unremarkable high-scale soft spectrum. The most obvious feature of this spectrum is the unification of the gaugino masses. A more subtle aspect of the high-scale spectrum is the small splitting between $m^2_{B_1}$ and $m^2_{B_3}$ relative to that between the $m^2_{Q_1}$ and m_{Q_3}, and $m^2_{U_1}$ and $m^2_{U_3}$. This feature hinted at an underlying family-universal flavor spectrum obscured by new physics effects. It is not clear whether this hint would survive in a more complete treatment that included uncertainties in the input parameter values. By adding new physics, in the form of heavy GUT multiplets and RH neutrinos, a simple mSUGRA structure emerged.

VI. CONCLUSIONS

If supersymmetry is discovered at the LHC, the primary challenge in theoretical particle physics will be to deduce the source of supersymmetry breaking. By doing so, we may perhaps learn about the more fundamental theory underlying this source. In most models of supersymmetry breaking, the relevant dynamics take place at energies much larger than those that will be directly probed by the LHC. It is therefore likely that the soft supersymmetry breaking parameters measured by experiment will have to be extrapolated to higher scales using the renormalization group. Given the apparent unification of gauge couplings in the MSSM only slightly below M_{Pl}, we may hope that there is little to no new physics between the LHC scale and the supersymmetry breaking scale so that such an extrapolation can be performed in a straightforward way.

Gauge unification still allows for some types of new physics at intermediate scales such as complete GUT multiplets and gauge singlets. If this new physics is present, RG evolving the MSSM soft parameters without including the new physics effects can lead to an incorrect spectrum of soft parameters at the high scale. Even without new intermediate physics, if some of the MSSM soft parameters are only poorly determined at the LHC, or not measured at all, there can arise significant uncertainties in the RG running of the soft masses that have been discovered.

In the present work we have investigated both of these potential obstacles to running up in the MSSM. The soft scalar masses are particularly sensitive to these effects, but we find that the gaugino soft masses, and their ratios, in particular, are considerably more robust. If any one of the scalar soft masses goes unmeasured at the LHC, the running of the remaining of these soft terms can be significantly modified by the effects of the hypercharge S term. These effects are especially severe for the slepton soft masses, which otherwise do not tend to run very strongly at all. We find that the uncertainties due to the S term can be avoided if we consider the soft mass differences $(Y_j m_i^2 - Y_i m_j^2)$, where Y_i denotes the hypercharge of the corresponding field. If all the soft mass is measured, the soft scalar mass combinations S and S_{B-L}, defined in Eqs. (2) and (10), provide useful information about potential GUT embeddings of the theory.

We have also investigated the effects of two plausible types of new physics beyond the MSSM that preserve consistency with gauge unification; namely, complete vectorlike GUT multiplets and heavy singlet neutrinos. In the case of complete GUT multiplets, extrapolating the measured low-energy soft parameter values without including the additional charged matter in the RG running leads to high-scale predictions for the gaugino masses that are too small, and soft scalar masses that are too large. Even so, the ratios of the gaugino masses at the high scale are not modified at leading order, and can be predicted from the low-energy measured values provided gauge unification occurs. The extrapolated values of the scalar masses are shifted in more complicated ways, and relationships such as family universality at the high scale can be obscured. Despite this, certain hints about the underlying flavor structure of the soft masses can still be deduced from the properties of special linear combinations of the soft masses, such as $(2m_{Q_3}^2 - m_{U_3}^2 - m_{D_3}^2)$ relative to $(2m_{Q_1}^2 - m_{U_1}^2 - m_{D_1}^2)$.

The running of the MSSM soft masses can also be modified if there are heavy singlet neutrino chiral multiplets in the theory. These can induce small masses for the standard model neutrinos through the seesaw mechanism. If the singlet neutrino scale is very heavy, greater than about 10^{13} GeV, the corresponding neutrino Yukawa couplings can be large enough to have a significant effect on the running of the soft masses of H_u and L. We have studied the size of these effects, as well as the shifts in the other soft masses. The extrapolated values of the gaugino masses and the squark soft masses are only weakly modified by heavy neutrino sector effects.

In Sec. V we applied the methods described above to a specific example. In this example, the scalar masses have a common high-scale value, up to a hypercharge D term. However, because of the presence of heavy new physics, this simple structure does not emerge when the low-scale soft scalar masses are extrapolated up to $M_{\rm GUT}$ using the RG equations of the MSSM. Based on the low-energy spectrum alone, we were able to deduce the presence of the hypercharge D term. The presence of additional new physics was suggested by the fact that the splitting between $m_{U_3}^2$ and $m_{U_1}^2$ was considerably larger than the related splitting between $m_{B_3}^2$ and $m_{B_1}^2$. By studying the scalar mass combinations of Eq. (36) and including heavy GUT multiplets and a right-handed neutrino sector, we were able to reproduce the low-energy soft spectrum with an underlying mSUGRA model.

One can also invert this perspective of overcoming new physics obstacles, and instead view these obstacles as providing information and opportunities. As the example of Sec. V illustrates, analyses of the kind we consider here probe new physics in indirect ways that can lead to convincing arguments for its existence or absence. Such analyses may be the main way to learn about new physics that cannot be studied directly.

If the LHC discovers new physics beyond the standard model, it will be a challenge to extract the Lagrangian parameters from the data. It may also be difficult to correctly extrapolate these parameters to higher scales in order to deduce the underlying theory that gives rise to the low-energy Lagrangian. In the present work we have begun to study this second aspect of the so-called LHC inverse problem, and we have found a few techniques to address some of the potential obstacles. However, our study is only a beginning. We expect that a number of additional techniques for running up could be discovered with more work. A similar set of techniques could also be applied to understanding the high-scale origin of other types of new physics beyond the standard model. These techniques deserve further study.

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APPENDIX: USEFUL COMBINATIONS OF SCALAR MASSES

In this appendix we collect some combinations of soft scalar masses that are particularly useful for running up.

1. *S*-term effects

The mass differences

$$\Delta m_{ij}^2 = (Y_j m_i^2 - Y_i m_j^2)/(Y_j - Y_i) \tag{A1}$$

are useful when there is a nonvanishing hypercharge D term. A hypercharge D term can shift the low-energy values of the soft scalar masses, and can also modify their RG running through the S term, as discussed in Sec. II, and in Refs. [25,26,44]. The effects of a hypercharge D term cancel out these mass differences, as well as in the RG equations for them.

This feature is helpful for running up because the effect of the D term on the running is determined by the low-scale value of the S term, which depends on all the soft scalar masses (of hypercharged fields) in the theory. If one of these soft masses goes unmeasured, there will be a large uncertainty in the value of the S term, and this in turn will induce a significant uncertainty in the high-scale values of the soft scalar masses after RG evolution. By focusing on the mass differences of Eq. (A1), these ambiguities cancel each other out.

On the other hand, if all the MSSM soft scalar masses are measured, the low-scale values of the soft scalar mass combinations

$$S = \mathrm{Tr}(Ym^2), \qquad S_{B-L} = \mathrm{Tr}[(B-L)m^2], \tag{A2}$$

provide useful information about the high-scale theory, and can be used to test possible GUT embeddings of the MSSM [26].

2. Flavor splitting effects

New physics can obscure the underlying flavor structure of the soft scalar masses. Family-universal soft masses derived from a theory containing new physics can generate a low-energy spectrum that does not appear to be family universal after it is evolved back up to the high scale without including this new physics. This is true even if the new physics couples in a flavor universal way to the MSSM. We presented a particular example of this in Sec. III, where the new physics took the form of complete GUT multiplets having no superpotential couplings with the MSSM sector.

There are four pairs of soft mass combinations that are helpful in this regard [32]. By comparing these pairs (at any given scale), it is sometimes possible to obtain clues about the underlying flavor structure of the MSSM soft masses. These combinations are:

$$\begin{aligned} m_{A_3}^2 &= 2m_{L_3}^2 - m_{E_3}^2 \leftrightarrow m_{A_1}^2 = 2m_{L_1}^2 - m_{E_1}^2, \\ m_{B_3}^2 &= 2m_{Q_3}^2 - m_{U_3}^2 - m_{D_3}^2 \leftrightarrow m_{B_1}^2 \\ &\quad = 2m_{Q_1}^2 - m_{U_1}^2 - m_{D_1}^2, \\ m_{X_3}^2 &= 2m_{H_u}^2 - 3m_{U_3}^2 \leftrightarrow m_{X_1}^2 = 2m_{L_1}^2 - 3m_{U_1}^2, \\ m_{Y_3}^2 &= 3m_{D_3}^2 + 2m_{L_3}^2 - 2m_{H_d}^2 \leftrightarrow m_{Y_1}^2 = 3m_{D_1}^2. \end{aligned} \tag{A3}$$

If the high-scale soft scalar masses are family universal, we expect each of these pairs, with the possible exception of the $m_{X_i}^2$, to be roughly equal at the low scale in the MSSM. The $m_{X_i}^2$ combinations are expected to match only if $S = 0$ as well.

To apply the soft mass combinations in Eq. (A3), one should compare them to the splitting between individual soft masses after running all soft masses up to the high scale within the MSSM (without new physics). For instance, an inequality of the form

$$|m_{B_3}^2 - m_{B_1}^2| \ll \max\{|m_{Q_3}^2 - m_{Q_1}^2|, |m_{U_3}^2 - m_{U_1}^2|, |m_{D_3}^2 - m_{D_1}^2|\}, \tag{A4}$$

is suggestive of high-scale family universality or a particular relationship between the Q, U, and D soft masses that has been obscured by new physics. This can arise from GUT multiplets as in Sec. III, or from a heavy RH neutrino sector as in Sec. IV. Note that heavy neutrinos can disrupt the relationships between the A, X, and Y pairs.

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