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Minimal model of fermionic dark matter

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We explore a minimal extension of the standard model with fermionic cold dark matter by introducing a gauge singlet Dirac fermion. The interactions between the dark matter and the standard model matters are described by the nonrenormalizable dimension-5 term. We show that the measured relic abundance of the cold dark matter can be explained in our model and predict the direct detection cross section. The direct search of the dark matter provides severe constraints on the mass and coupling of the minimal fermionic dark matter with respect to the Higgs boson mass.

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I. INTRODUCTION

Recently there has been a growing interest in the dark matter (DM) of the Universe as an interface between the cosmology and the particle physics, and between the astrophysical observation and the collider experiment. The precise measurement of the relic abundance of the cold dark matter (CDM) in the Universe has been obtained from the Wilkinson microwave anisotropy probe (WMAP) data on the cosmic microwave background radiation as [1]

$$0.085 < \Omega_{\rm CDM} h^2 < 0.119 \ (2\sigma \ \text{level}), \qquad (1)$$

where Ω is the energy density of the Universe normalized by the critical density and $h \simeq 0.7$ is the scaled Hubble constant in units of 100 km/sec/Mpc.

Lots of CDM candidates have been suggested in various new physics models beyond the standard model (SM). In the supersymmetric model with conserved R parity, the lightest supersymmetric particle (LSP) is stable and a good candidate of the CDM. [2,3]. The KK parity of the extra dimensional model suggests a Kaluza-Klein dark matter [4]. When the T parity is conserved in the little Higgs model, the lightest T-odd particle, likely the heavy photon, can be the dark matter candidate [5].

Since there is no CDM candidate in the SM contents, the SM Lagrangian should be extended by adding one or more new particles to explain the observed $\Omega_{\rm CDM} h^2$. The minimal extension of the SM with the CDM is achieved by introducing a singlet particle under the SM gauge group for the new particle to be protected from the gauge interaction. The model with a singlet scalar field with Z_2 parity has been considered as the simplest candidate of the nonbaryonic CDM [6–8]. Such a model contains three new parameters and a neutral scalar degree of freedom which interacts with the SM matter only via the coupling to the Higgs boson. It is shown that the model with a singlet scalar can explain $\Omega_{\rm CDM} h^2$, and satisfies the present experimental bounds for direct detection and collider signature. A general classification of the extra gauge multiplets as a minimal dark matter candidate has been performed in Ref. [9]. On the other hand, incorporating the gauge coupling unification with the dark matter issue, a minimal fermionic extension which introduces fermions with the quantum numbers of SUSY higgsinos, and a singlet [10] and a minimal scalar extension with multiple Higgs doublets [11] are suggested.

In this work, we propose a model with a Dirac fermion which is a singlet under the SM gauge group as a minimal model of fermionic dark matter. We require that new fermion number be conserved in order to avoid the mixing between the new fermion and the SM fermions. Then the dark matter can couple to only the Higgs boson among the SM matter. The leading interaction between the singlet fermion and the SM matters is given by the dimension five term $\mathcal{L}_{\rm int} \sim -(1/\Lambda)|H|^2 \bar{\psi}\psi$ by introducing the new scale Λ, where H is the SM Higgs doublet and ψ the dark matter fermion.

This paper is organized as follows: In Sec. II, we review the model by adding a singlet fermion to the SM. The relic density of the CDM is evaluated in Sec. III and the direct detection of the CDM is investigated in Sec. IV. Finally we conclude in Sec. V.

II. THE MODEL

We introduce a singlet fermion ψ under the SM gauge group. The Lagrangian consists of

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm DM} + \mathcal{L}_{\rm int}, \qquad (2)$$

where the dark matter Lagrangian

$$\mathcal{L}_{\rm DM} = \bar{\psi} i \gamma^\mu \partial_\mu \psi - m_0 \bar{\psi}\psi. \qquad (3)$$

The leading interaction term between the dark matter and the SM fields is given by the dimension-5 term

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$$\mathcal{L}_{\text{int}} = -\frac{1}{\Lambda} H^\dagger H \bar{\psi}\psi, \tag{4}$$

where the new scale Λ absorbs the $\mathcal{O}(1)$ coupling constant. The dimension-6 term described by the four-fermion interaction $\sim 1/\Lambda^2(\bar{Q}_L\gamma^\mu Q_L + \bar{q}_R\gamma^\mu q_R)(\bar{\psi}\gamma_\mu\psi)$ is possible and contributes to the annihilation cross section through $\bar{\psi} + \psi \to f + \bar{f}$. However it is suppressed by the factor of M^2/Λ^2 where M is the physical mass of ψ in most of the region of the allowed parameter space and we ignore the dimension-6 term in this work. With the vacuum expectation value (VEV) of the electroweak symmetry breaking

$$H \to H + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{5}$$

we rewrite the Lagrangian

$$\mathcal{L}_{\text{DM}} + \mathcal{L}_{\text{int}} = \bar{\psi} i\gamma^\mu \partial_\mu \psi - M\bar{\psi}\psi - g_\psi h\bar{\psi}\psi - \frac{1}{2\Lambda} h^2\bar{\psi}\psi, \tag{6}$$

where h is the SM Higgs boson and the Yukawa type coupling g_ψ and shifted mass M are given by

$$M = m_0 + \frac{v^2}{2\Lambda}, \qquad g_\psi = \frac{v}{\Lambda}, \tag{7}$$

with the electroweak scale $v = 246$ GeV. The coupling g_ψ is the only bridge between the dark matter sector and the SM sector. In this model, we introduce two new parameters m_0 and Λ, or equivalently the physical mass of the singlet fermion M and the coupling g_ψ. Although the dimension-5 term $-(1/2\Lambda)h^2\bar{\psi}\psi$ in Eq. (6) contributes to the $\bar{\psi} + \psi \to h + h$ annihilation process, this channel is not so important for the parameter space considered in this work, i.e., $M < m_h$.

III. COSMOLOGICAL IMPLICATION

In the early Universe, the fermion ψ as a CDM is assumed to be in thermal equilibrium. When the temperature T of the Universe is larger than the DM mass M, the DM number density is given by $n \propto T^3$. Once the temperature drops below the DM mass, the number density is suppressed exponentially, $n \propto e^{-M/T}$, so that the DM annihilation rate becomes smaller than the Hubble expansion rate at a certain point. Then the DM particles fall out of equilibrium and the DM number density in a comoving volume remains constant. Therefore, the cosmological abundance depends upon the annihilation cross section of ψ into the SM matters. Since ψ couples only to the Higgs boson through the Yukawa type $g_\psi h\bar{\psi}\psi$, the annihilation processes are the Higgs-mediated s-channel processes and the dominant final states of the annihilation are $b\bar{b}$, W^+W^-, ZZ, and $t\bar{t}$ depending upon the center-of-momentum energy s. We have the cross sections

$$\begin{aligned}
v_{\text{rel}} \cdot \sigma(\bar{\psi}\psi \to b\bar{b}) &= \frac{G_F N_c g_\psi^2 m_b^2}{4\sqrt{2}\pi}\beta_b^{3/2}\frac{(s-4M^2)}{(s-m_h^2)^2 + m_h^2\Gamma_h^2},\\
v_{\text{rel}} \cdot \sigma(\bar{\psi}\psi \to W^+W^-) &= \frac{G_F g_\psi^2}{8\sqrt{2}\pi} s\beta_W^{1/2}\left(1 - x_W + \frac{3}{4}x_W^2\right)\frac{(s-4M^2)}{(s-m_h^2)^2 + m_h^2\Gamma_h^2},\\
v_{\text{rel}} \cdot \sigma(\bar{\psi}\psi \to ZZ) &= \frac{G_F g_\psi^2}{16\sqrt{2}\pi} s\beta_Z^{1/2}\left(1 - x_Z + \frac{3}{4}x_Z^2\right)\frac{(s-4M^2)}{(s-m_h^2)^2 + m_h^2\Gamma_h^2},
\end{aligned} \tag{8}$$

where $\beta_X = \sqrt{1 - 4m_X^2/s}$. The pair annihilation cross sections of ψ are thermally averaged over s. The thermal average is given by

$$\langle \sigma_{\text{ann.}} v_{\text{rel}} \rangle = \frac{1}{8M^4 T K_2^2(M/T)} \int_{4M^2}^\infty ds\, \sigma_{\text{ann.}}(s)(s - 4M^2) \times \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right), \tag{9}$$

where $K_{1,2}$ are the modified Bessel functions.

The evolution of the relic density is described by the Boltzmann equation in terms of the $\langle \sigma_{\text{ann.}} v_{\text{rel}} \rangle$ and the equilibrium number density

$$\frac{dn_\psi}{dt} + 3Hn_\psi = -\langle \sigma_{\text{ann.}} v_{\text{rel}} \rangle [n_\psi^2 - (n_\psi^{\text{EQ}})^2], \tag{10}$$

where H is the Hubble parameter and n_ψ^{EQ} the equilibrium number density of ψ. After the freeze-out of the annihilation processes, the actual number of ψ per comoving volume, $n_\psi/S = n_{\bar{\psi}}/S$ becomes constant and the present relic density $\rho_\psi = Mn_\psi$ is determined. Approximately we have

$$\Omega_\psi h^2 \approx \frac{(1.07 \times 10^9)x_F}{\sqrt{g_*} m_{\text{Pl}}\ \ (\text{GeV})\langle \sigma_{\text{ann.}} v_{\text{rel}} \rangle}, \tag{11}$$

where g_* counts the effective degrees of freedom in equilibrium. The inverse freeze-out temperature $x_F \equiv M/T_f$ is determined by the iterative equation

$$x_F = \ln\left(\frac{M}{2\pi^3}\sqrt{\frac{45 m_{\text{Pl}}^2}{2g_* x_F}}\langle \sigma_{\text{ann.}} v_{\text{rel}} \rangle\right). \tag{12}$$

Figure 1 shows the allowed parameter set (M, g_ψ) or equivalently (M, Λ) which satisfy the WMAP measure-

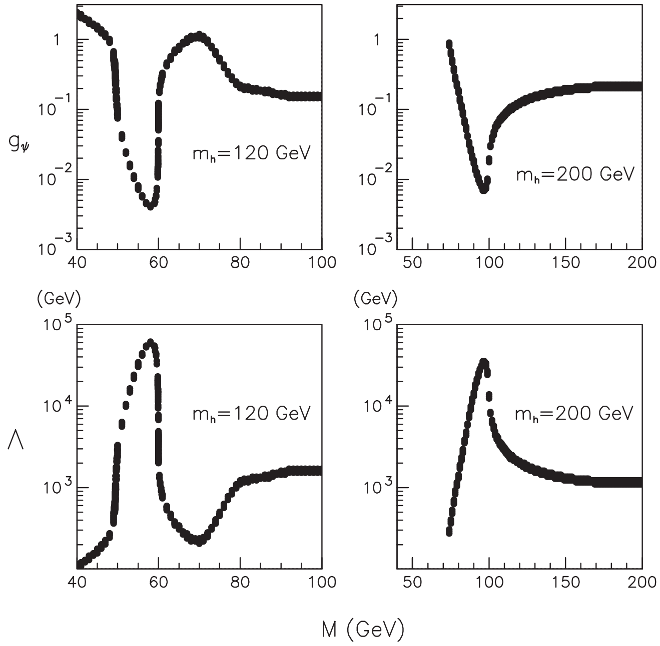

FIG. 1. Allowed parameter set (M, g_ψ) and (M, Λ) for Higgs masses $m_h = 120$ and 200 GeV.

ment of $\Omega_{\rm CDM}h^2$. The valley in (M, g_ψ) space or the peak in (M, Λ) space indicates the resonant region of the Higgs boson exchange, where $2M \simeq m_h$. In that region, the coupling constant g_ψ should be small in order to compensate the enhancement of the cross section by the Higgs resonance effect. In the case of $m_h = 120$ GeV, one more step appears when $M \sim 80$ GeV. It denotes that the annihilation channel $\bar\psi\psi \to W^-W^+/ZZ$ opens as M increases. In the case of $m_h = 200$ GeV, the step is hidden by the Higgs resonance effect. We find that the dark matter with the mass $M > \mathcal{O}(10\text{ GeV})$ can explain the measured relic density and implies the new physics with $O(\Lambda) \sim 1$ TeV.

IV. DIRECT DETECTION

We can detect the weakly interacting massive particle (WIMP) type CDM directly through its elastic scattering on target nuclei at underground experiments [12]. The elastic cross section for the scattering off a nucleon is described by the effective Lagrangian at the hadronic level,

$$\mathcal{L}_{\rm eff} = f_p(\bar\psi\psi)(\bar p p) + f_n(\bar\psi\psi)(\bar n n), \tag{13}$$

where the coupling constant f_p is given by [13,14]

$$\frac{f_{p,n}}{m_{p,n}} = \sum_{q=u,d,s} f_{Tq}^{(p,n)}\frac{\alpha_q}{m_q} + \frac{2}{27}f_{Tg}^{(p,n)}\sum_{q=c,b,t}\frac{\alpha_q}{m_q}, \tag{14}$$

with the matrix elements $m_{(p,n)}f_{Tq}^{(p,n)} \equiv \langle p, n|m_q\bar q q|p, n\rangle$ for $q = u, d, s$ and $f_{Tg}^{(p,n)} = 1 - \sum_{q=u,d,s}f_{Tq}^{(p,n)}$. The numerical values of the hadronic matrix elements $f_{Tq}^{(p,n)}$ are determined [14]

$$f_{Tu}^{(p)} = 0.020 \pm 0.004, \qquad f_{Td}^{(p)} = 0.026 \pm 0.005, \qquad f_{Ts}^{(p)} = 0.118 \pm 0.062, \tag{15}$$

and

$$f_{Tu}^{(n)} = 0.014 \pm 0.003, \qquad f_{Td}^{(n)} = 0.036 \pm 0.008, \qquad f_{Ts}^{(n)} = 0.118 \pm 0.062. \tag{16}$$

Actually the dominant contribution comes from $f_{Ts}^{(p,n)}$ and therefore we may set $f_p \approx f_n$. The effective coupling constant α_q is defined by the spin-independent four-fermion interaction of quarks and the dark matter fermion in our model. The relevant Lagrangian is given by

$$\mathcal{L}_{\rm int} = \sum_q \alpha_q(\bar\psi\psi)(\bar q q), \tag{17}$$

where α_q is obtained from the Higgs exchange t-channel diagram, such as

$$\alpha_q = \frac{1}{2}\frac{g g_\psi}{M^2 - m_h^2}\frac{m_q}{m_W}. \tag{18}$$

We obtain the elastic scattering cross section from the effective Lagrangian equation (13), which is given by

$$\sigma = \frac{4M_r^2}{\pi}[Zf_p + (A - Z)f_n]^2 \approx \frac{4M_r^2A^2}{\pi}f_p^2, \tag{19}$$

where M_r is the reduced mass of ψ and the target defined

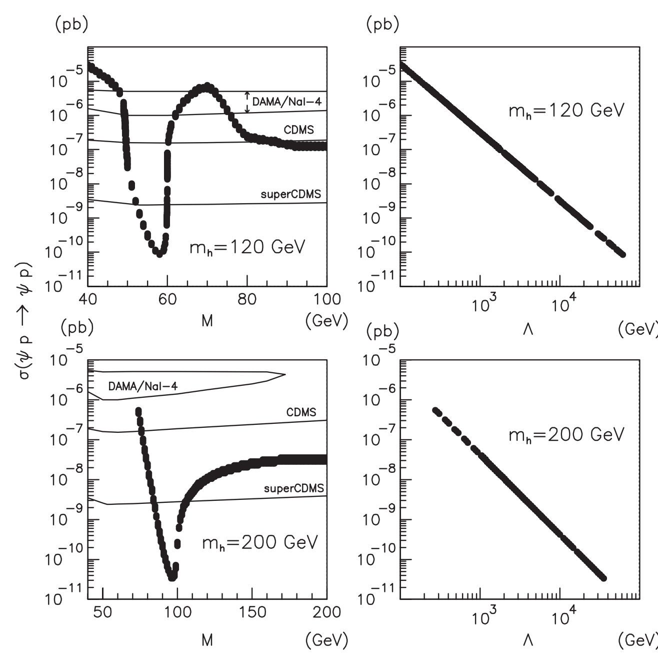

FIG. 2. Predictions of cross section $\sigma(\psi p \to \psi p)$ with respect to M and Λ for allowed parameter set (M, g_ψ) or (M, Λ). Higgs masses are assumed to be $m_h = 120$ and 200 GeV. The thin lines denote the allowed region of DAMA/NaI-4, present bounds of CDMS and superCDMS.

by $1/M_r = 1/m_\psi + 1/m_{\rm nuclei}$. It is convenient to consider the cross section with the single nucleon for comparing with the experiment. Then the target mass is the nucleon mass and the cross section is given by

$$\sigma(\psi p \to \psi p) \approx \frac{4m_r^2}{\pi} f_p^2, \quad (20)$$

where

$$\frac{1}{m_r} = \frac{1}{m_\psi} + \frac{1}{m_p}. \quad (21)$$

The predicted cross sections $\sigma(\psi p \to \psi p)$ with the allowed parameter set given in the previous section are shown in Fig. 2. The allowed region of DAMA/NaI-4 [15], the experimental bound obtained by the CDMS collaboration [16], and the expected reach of the superCDMS collaboration [17] are also presented. At the present the CDMS bound tells us that the lower bound of the new scale is $\mathcal{O}$(TeV), while the DAMA/NaI-4 result prefers the WIMP mass, $M < 50$ GeV and 60 GeV $< M <$ 75 GeV. We find that the future superCDMS experiment can probe the most parameter set of the model except for the Higgs resonance region.

V. CONCLUDING REMARKS

We propose the minimal model of fermionic cold dark matter. The gauge singlet fermion is introduced and coupled to the SM sector through the higher dimensional operator. By assigning the additional fermion number, the singlet fermion couples to the SM matters through the higher dimensional $\mathcal{L}_{\rm int} \sim -(1/\Lambda)|H|^2\bar{\psi}\psi$ term in the leading order. The physical interaction is described by the Yukawa type interaction after the electroweak symmetry breaking. Only two parameters are involved in our model, one denotes the new physics scale and the other the mass of the dark matter. We show that our ''minimal'' model can explain the current experimental data on the relic abundance and satisfy the bound of direct detection. In the future, we expect that the direct search of the superCDMS can probe a significant portion of the model parameter space.

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