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k_T factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

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We show that hard-scattering factorization is violated in the production of high-p_T hadrons in hadron-hadron collisions, in the case that the hadrons are back-to-back, so that k_T factorization is to be used. The explicit counterexample that we construct is for the single-spin asymmetry with one beam transversely polarized. The Sivers function needed here has particular sensitivity to the Wilson lines in the parton densities. We use a greatly simplified model theory to make the breakdown of factorization easy to check explicitly. But the counterexample implies that standard arguments for factorization fail not just for the single-spin asymmetry but for the unpolarized cross section for back-to-back hadron production in QCD in hadron-hadron collisions. This is unlike corresponding cases in e^+e^- annihilation, Drell-Yan, and deeply inelastic scattering. Moreover, the result endangers factorization for more general hadroproduction processes.

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I. INTRODUCTION

The great importance of hard-scattering factorization in high-energy phenomenology hardly needs emphasis. Essential to its application and predictiveness is the universality of parton densities (and fragmentation functions, etc.) between different reactions. However, as can be seen from [1–4], process-dependent Wilson lines appear to be needed in the inclusive production of two high-transverse-momentum particles in hadron-hadron collisions, i.e., in the process

$$H_1 + H_2 \to H_3 + H_4 + X. \tag{1.1}$$

In this paper we will show that this situation definitively leads to a breakdown of factorization.

The standard expectation is that the cross section is a convolution of a hard-scattering coefficient $d\hat{\sigma}$, parton densities, fragmentation functions, and a possible soft function:

$$E_3E_4\frac{d\sigma}{d^3\boldsymbol{p}_3d^3\boldsymbol{p}_4} = \sum\int d\hat{\sigma}_{i+j\to k+l+X}f_{i/1}f_{j/2}d_{3/k}d_{4/l} + \text{power-suppressed correction.} \tag{1.2}$$

Here the sum and integral are over the flavors and momenta of the partons of the hard scattering, $f_{i/H}$ denotes a parton density, and $d_{H/i}$ a fragmentation function.

It is noteworthy that the classic published proofs for factorization in hadron-hadron scattering [5,6] only concerned the Drell-Yan process. There are a number of difficult issues in the proof that are highly nontrivial to extend to other reactions in hadron-hadron collisions, even though Eq. (1.2) is a standard expectation.

We will examine the case that the produced hadrons are almost back-to-back. Then the appropriate factorization property is k_T factorization, which entails [7] the use of transverse-momentum dependent (TMD) parton densities and fragmentation functions. However, the issues raised by our counterexample to factorization are sufficiently general that they create a need to examine very carefully the arguments for factorization in hadroproduction of hadrons even in situations where ordinary collinear factorization with integrated densities is appropriate. In the case of k_T factorization with TMD densities, the factorization formula needs the insertion of a soft factor S, not shown in Eq. (1.2).

The problems concern gluon exchanges between different kinds of collinear line, as in Fig. 7 below. To obtain factorization, the gluon attachments must be converted to Wilson lines in gauge-invariant definitions of the parton densities and fragmentation functions. This relies [6] on the use of Ward identities applied to approximations to the amplitudes. But the approximations are only valid after certain contour deformations on the loop momenta.

Bacchetta, Bomhof, Mulders, and Pijlman [1–4] argued that because of the complicated combination of initial- and final-state interactions, the Wilson lines must be modified. What is not so clear is the interpretation of their result. So in the present paper we present an argument to make fully explicit the failure of factorization.

Since the issue is one of factorization in general, and not just specifically in QCD, we clarify the issue by examining

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a particular process in a model field theory. The process is a transverse single-spin asymmetry of the kind controlled by a Sivers function. This is a case where problems in the contour deformation directly affect the value of the cross section at the lowest possible order of perturbation theory. Our model field theory is simple enough that the calculations and their interpretation as implying factorization violation are unambiguous. But, as we will explain in the final section, we expect the failure of factorization to be more general: our particular process and model simply make it very easy to see the failure.

II. CONSTRUCTION OF MODEL

Since proofs of factorization apply to quantum field theories in general (if they are renormalizable), the construction of a counterexample, to demonstrate and to *understand* a failure of the normal methods of proof, is conveniently done in a simple model theory.

Our model resembles the one used by Brodsky, Hwang, and Schmidt [8] in their discussion of single-spin asymmetries. It is defined as follows:

(i) The gauge group is Abelian. This simplifies the graphs, and allows the next feature.

(ii) The gluon is massive. This avoids the discussion being confused by actual infrared divergences in the S matrix.

(iii) The initial-state particles correspond to Dirac fields that are neutral under the gauge group. We will call them hadrons. The fields need to be Dirac fields in order to have the single transverse spin asymmetries that we will examine. We will use two types of hadrons.

(iv) Each "hadron field" H_i will have a coupling to a Dirac field ψ_i and a scalar field ϕ_i, which in [8] would be called a diquark field:

$$\lambda_i(\bar{H}_i\psi_i\phi_i^\dagger + \bar{\psi}_i H_i\phi_i). \tag{2.1}$$

The quark field ψ_i has a coupling g_i to the gauge field, and the scalar field ϕ_i has the opposite coupling.

(v) All the masses in the theory are comparable, to avoid confusing the calculation with logarithmic dependence on large ratios of masses.

In our analog of hadroproduction, Fig. 7, the two initial-state hadrons are of the two different types, and we use this to simplify our argument for nonfactorization. The lines in the lower part of the graphs are chosen to be those of gauge coupling g_1, while the lines in the upper part of the graph are those with coupling g_2. But if the attachments of the gluon to the upper lines were in some way to correspond to a Wilson line in the parton density in the hadron in the lower part of the graph, the charge would have to be g_1. Since g_1 and g_2 are arbitrary, there is no way to make a correspondence between the graph and the Wilson line formalism, provided that the contribution is nonzero, as we will demonstrate. This will also insulate us against sign errors and the like.

We will also choose the detected outgoing particles H_3 and H_4 to correspond to the scalar fields. The sole purpose here is to simplify the Dirac algebra slightly, thereby making the calculations more transparent and elementary.

One feature of our counterexample appears to be very special to an Abelian gauge theory. This is that the two couplings g_1 and g_2 need have no relation to each other: there is a continuous infinity of representations of the gauge group. In contrast, there is a single value of the coupling g for all the fields in a non-Abelian theory. The role of the ratio g_2/g_1 is now taken over by the representation matrices for the different fields in QCD (triplet, antitriplet, octet), with the different couplings related by factors of rational numbers. So in any particular example there is a potential for a numerical coincidence between the sizes of the numerical values of the graphs, which could then appear to give consistency with factorization. A counterexample to factorization would then be more complicated, with a comparison of cases with different kinds of partons (quarks, antiquarks, or gluons), cf. [1–4].

III. REVIEW OF SIDIS AND DY

We now review how [8] a transverse single-spin asymmetry (SSA) arises in semi-inclusive deep-inelastic scattering (SIDIS), at the level of one-gluon exchange, and how it determines [9] the Wilson line that defines parton densities. Then we review the differences that give factorization with an exact sign reversal in the Sivers function for the Drell-Yan process [9,10]. This will give us methods of calculation that will give us a very elementary way to obtain the SSA for the process (1.1).

A. SIDIS

With the electromagnetic part of the scattering factored out, SIDIS is the process

$$\gamma^*(q) + H(p) \longrightarrow H'(r) + X. \tag{3.1}$$

We use light-front coordinates in which the incoming momenta are

$$p = \left(p^+, \frac{m_H^2}{2p^+}, 0_T\right), \qquad q = \left(-xp^+, \frac{Q^2}{2xp^+}, 0_T\right). \tag{3.2}$$

The detected outgoing particle is defined by a longitudinal momentum fraction z and a transverse momentum r_T:

$$r = \left(xp^+\frac{r_T^2 + m_\phi^2}{zQ^2}, \frac{zQ^2}{2xp^+}, r_T\right). \tag{3.3}$$

We will assume that Q is large and that the detected transverse momentum r_T is of order a hadronic mass scale m.

The lowest-order graph in Fig. 1 gives the following contribution to the differential structure tensor:

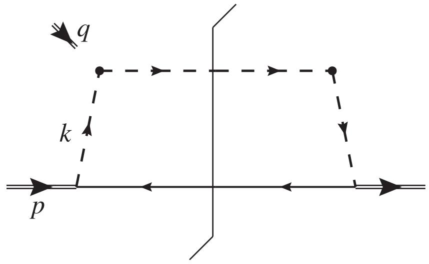

FIG. 1. Lowest-order graph for SIDIS in our model. The initial-state particle is a color-singlet Dirac particle. The spectator line is for a Dirac "quark" field, and the active parton is for a scalar "diquark" field, as can be enforced by a condition on the detected outgoing particle of momentum $q + k$. The arrows on the internal lines indicate the flow of color charge, and the vertical line cutting the graph denotes the final state.

$$\frac{dW^{\mu\nu}}{dzd^2r_T} = \frac{\lambda^2}{4\pi}\int\frac{dk^+dk^-}{(2\pi)^4}\delta\Big(z - \frac{k^- + q^-}{q^-}\Big) \times \frac{(2k^\mu + q^\mu)(2k^\nu + q^\nu)}{(k^2 - m_\phi^2)^2}(2\pi)^2\delta((q+k)^2 - m_\phi^2) \times \delta((p-k)^2 - m_\psi^2) \times \frac{1}{2}\mathrm{Tr}[(\not p + m_H)(1 + \gamma_5 \not s) \times (\not p - \not k + m_\psi)]. \tag{3.4}$$

The internal partons are all collinear to the target, i.e., $k^+ \sim p^+$, $k^- \sim m^2/p^+$, $k_T \sim m$, and to leading power in Q, parton model kinematics apply, so that $k^+ \simeq xp^+$ and $z \simeq 1$. We will assume throughout that the spin vector s corresponds to a transverse spin (in the (q, p) frame), and that it is normalized so that its extremal value obeys $s^2 = -1$. Since there is only one initial-state hadron, we do not bother with labels to indicate the kind of hadron (e.g., λ_1 or λ_2).

The above formula is simply related to a parton density:

$$\frac{dW^{\mu\nu}}{dzd^2r_T} = \frac{(p^\mu - q^\mu p\cdot q/q^2)(p^\nu - q^\nu p\cdot q/q^2)}{p\cdot q} \times \delta(z-1)P_{\phi/H}(x, r_T) + \text{power-suppressed correction}, \tag{3.5}$$

where the parton density at lowest order is

$$P_0(x, k_T) = \frac{\lambda^2 x(1-x)}{16\pi^3} \times \frac{\frac{1}{2}\mathrm{Tr}[(\not p + m_H)(1+\gamma_5\not s)(\not p - \not k + m_\psi)]}{[k_T^2 + m_\phi^2(1-x) + m_\psi^2 x - m_H^2 x(1-x)]^2}, \tag{3.6}$$

as follows from the conventional operator definition.

There is in fact no polarization dependence in this order, i.e., the Sivers function vanishes. At a simple calculational level, this occurs for two reasons. One is that γ_5 gives a nonzero contribution only when multiplied by at least four regular Dirac matrices,

$$\mathrm{tr}\,\gamma_5 \not a \not b \not c \not d = 4i\epsilon_{\kappa\lambda\mu\nu}a^\kappa b^\lambda c^\mu d^\nu, \tag{3.7}$$

while in (3.6) there are at most three. This reason for a vanishing SSA would no longer apply in a more complicated model or with higher order graphs. The second reason for the vanishing is that the trace (3.7) is imaginary, while the rest of the graph is real, so that the contribution to a cross section must be zero.

The lowest-order graphs for a nonzero SSA are the one-gluon-exchange graphs in Fig. 2, which get an imaginary part from an intermediate state that can go on shell; this state is made by the lines with momenta $q + k - l$ and $p - k + l$. Standard power counting shows that the exchanged gluon can only be collinear to the target or soft. The minus momentum of the gluon is trapped in the region $l^- \sim m^2/p^+$ by the other target-collinear lines. The on-shell intermediate state corresponds to small angle elastic scattering, and so to very small l^+, of order p^+m^2/Q^2. There the only significant dependence on l^+ is in the upper parton propagator. Multiplied by the neighboring gluon vertex this gives

$$\frac{-g(2q^\mu + 2k^\mu - l^\mu)}{(q+k-l)^2 - m_\phi^2 + i\epsilon} \simeq \frac{-g\delta^\mu_-}{-l^+ + \text{other terms} + i\epsilon}, \tag{3.8}$$

where the "other terms" are small or independent of l^+, and we have taken a leading-power approximation for the momenta in the numerator. The contour of integration of l^+

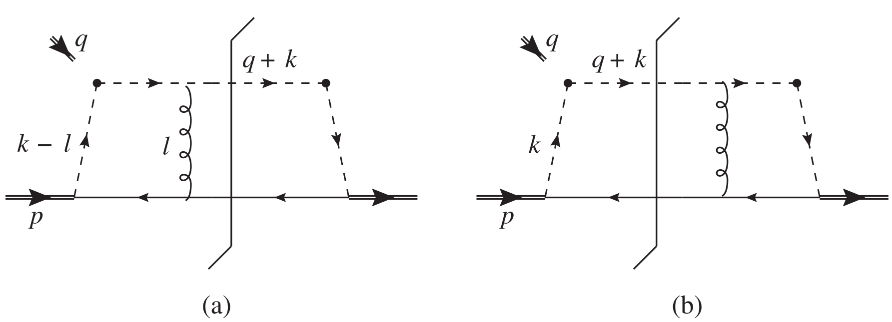

FIG. 2. Virtual one-gluon-exchange corrections to Fig. 1 that give a SSA.

can therefore be deformed into the lower half-plane until l is target collinear. In that case the "other terms" in (3.8) are negligible, and the denominator can be replaced by its eikonal approximation $1/(-l^+ + i\epsilon)$. This, together with a leading-power approximation in the numerator, shows [9] that the gluon exchange correction is equivalent to a contribution to the parton density with a suitable Wilson line, Fig. 3.

Let us perform the k^+ and k^- integrals by the on-shell conditions on the final state, and let us perform the l^- integration by contour integration. Then the necessary imaginary part comes simply from the imaginary part of (3.8) and thus from the replacement

$$\frac{-g(2q^\mu + 2k^\mu - l^\mu)}{(q+k-l)^2 - m_\phi^2 + i\epsilon} \mapsto ig\pi\delta^\mu_- \delta(l^+). \tag{3.9}$$

This gives rise to an SSA with the aid of the trace

$$\frac{1}{2}\,\mathrm{Tr}[(\not p + m_H)\gamma_5 \not s(\not p - \not k + \not l + m_\psi)\gamma^+(\not p - \not k + m_\psi)]$$
$$\simeq 2i\epsilon_{jk}s^j l^k p^+[m_H(1-x) + m_\psi] \tag{3.10}$$

where the approximation, good to leading power, arises from the neglect of the small components of l with respect to the transverse components. The two-dimensional ϵ tensor obeys $\epsilon_{12} = 1$. Since the denominator in the integrand is not azimuthally symmetric in l_T, the integral over l gives a nonzero result for the SSA from the whole graph.

The two graphs in Fig. 2 are related by Hermitian conjugation and so they give equal contributions to the SSA.

B. Drell-Yan

The Drell-Yan (DY) process,

$$H_A(s) + H_B \to \gamma^*(q) + X, \tag{3.11}$$

is treated quite similarly. We examine the cross section differential in q_T, and investigate a possible SSA, with H_A having a transverse spin vector s.

In our model the lowest-order graph is Fig. 4. It is readily shown to be the (convolution) product of two transverse-

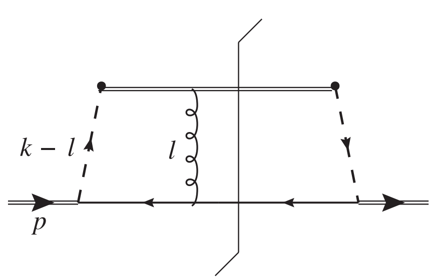

FIG. 3. Virtual one-gluon-exchange correction to parton density. The upper double line is the Wilson line, and the graph shown, together with its Hermitian conjugate gives the first contribution to the Sivers function.

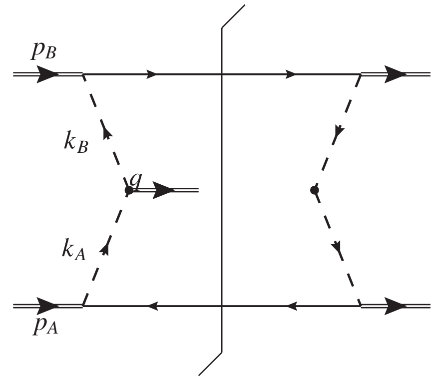

FIG. 4. Lowest-order graph for the Drell-Yan process.

momentum-dependent parton densities, and, just like the SIDIS process, it has no SSA at this order. Note that to have the process occur at the order shown within our model, the initial-state hadrons H_A and H_B must be antiparticles of each other.

The imaginary part in the amplitude needed to get an SSA with respect to the transverse spin s of the lower hadron H_A arises from graphs such as those in Fig. 5. Graphs (a) and (b) work just like those in Fig. 2 for SIDIS, except that the gluon couples to the incoming scalar antiparton instead an outgoing scalar parton, with the necessary reversal in sign of the coupling. Thus Eq. (3.8) is replaced by

$$\frac{g(2k_B^\mu + l^\mu)}{(k_B + l)^2 - m_\phi^2 + i\epsilon} \simeq \frac{g\delta^\mu_-}{l^+ - \text{other terms} + i\epsilon}, \tag{3.12}$$

which gives an imaginary part exactly opposite to that for SIDIS. The relative sign of the l^+ term and the $i\epsilon$ is now that for an initial-state interaction, so that the Wilson line in the operator definition of the parton density is now past-pointing instead of future-pointing [9]. As shown in [9], an exact reversal of sign of the Sivers function between SIDIS and DY follows from the time-reversal symmetry of QCD.

No contribution to the SSA is given by graphs, like Fig. 5(c), in which both ends gluons couple the active partons, or where the gluon couples the *upper* spectator quark to the lower active parton. In our model this is trivial: there are too few Dirac matrices on the lower line to give spin dependence. In a more general case, the annihilating partons could be Dirac fields, and then the lack of spin dependence arises because of the eikonalization of those parton lines on the lower side of the graph that would contribute to the imaginary part.

But spectator interaction graphs, (d) and (e), do individually contribute to the SSA. The imaginary part arises from putting all four spectator lines on shell. Provided the gluon momentum is routed the same way in both graphs before integration, e.g., to the left as shown, the two contributions are equal, but exactly opposite in sign. The cancellation is exactly one of those needed to prove facto-

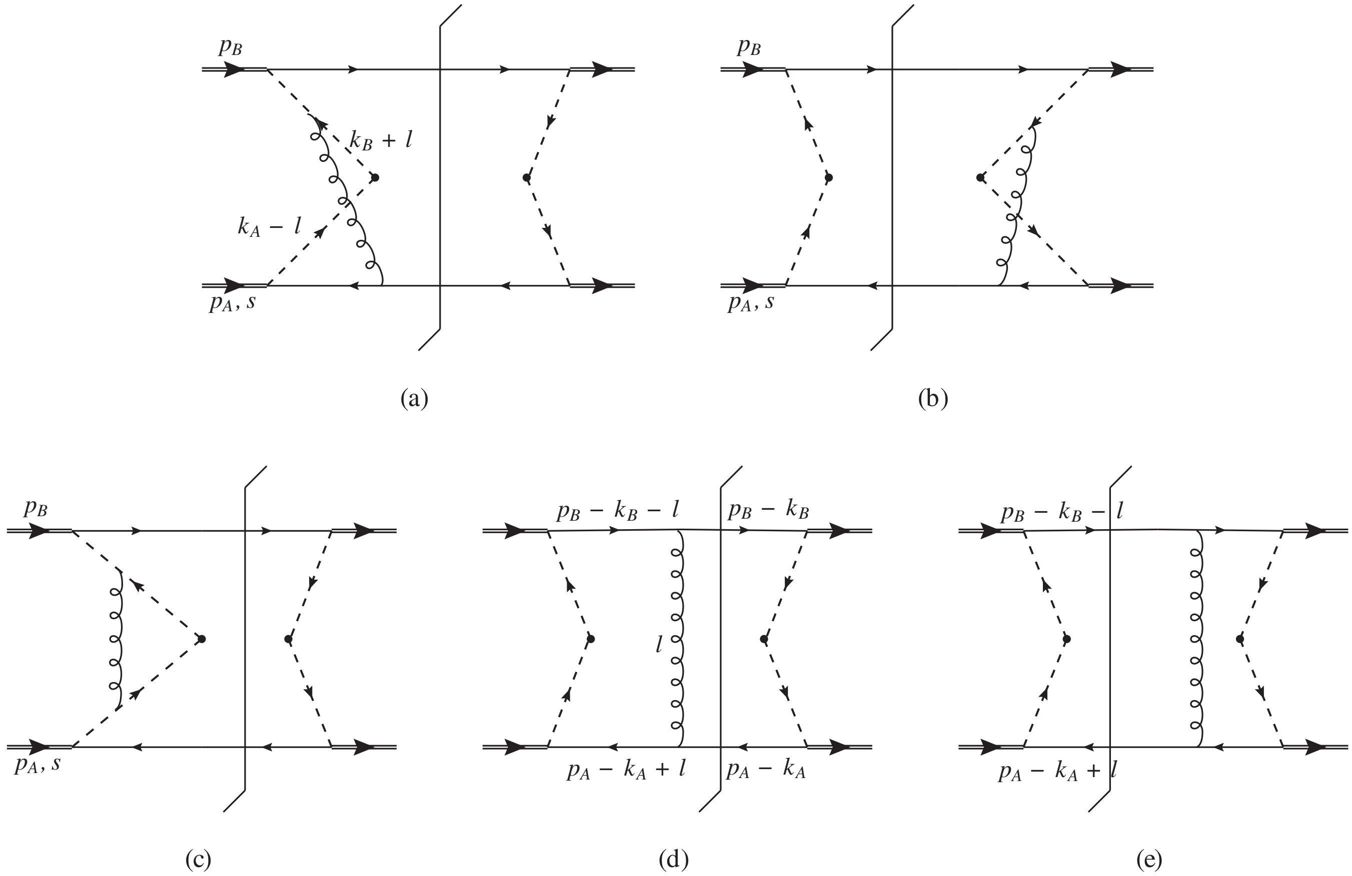

FIG. 5. Virtual one-gluon-exchange corrections to Fig. 4 relevant for a SSA when the lower hadron has transverse spin s. Graph (a) and its Hermitian conjugate (b) have imaginary parts (at the amplitude level) that give a nonzero SSA. Gluon exchange between the active partons, graph (c) and its not-shown conjugate, gives an imaginary part in the vertex correction that does not give a SSA. Spectator-spectator gluon exchange graphs, (d) and (e), do contribute individually to the SSA, but the two contributions cancel (at leading power).

rization [6], and involves a sum over all allowed cuts of a particular graph.

However, the two graphs have final states with particles of different momenta. This accounts for the difference compared with the noncancellation of the graphs in Fig. 2 for SIDIS, where there is also a sum over cuts. For the SIDIS graphs there is a requirement on the transverse momentum of the struck parton. In contrast, the cancellation between the two spectator-spectator graphs in the DY process occurs because our cross section is the fully inclusive DY cross section; no requirement was placed on the final state in the target fragmentation regions. (As is well known from the case of diffractive hard scattering, both theoretically [11,12] and experimentally [13,14], factorization fails when target-relative restrictions are imposed.)

IV. HADROPRODUCTION OF HADRONS

We now have the tools to make an extremely streamlined construction of a counterexample to factorization for the process of hadroproduction of high-transverse-momentum hadrons, $H_1 + H_2 \to H_3 + H_4 + X$. We again use an SSA because nonfactorization occurs with one gluon exchanged beyond the lowest order in which the reaction occurs at all. We choose H_1 to be the polarized hadron, and we choose the hadrons H_1 and H_2 to be of the two different flavors in our model. We also choose the detected final-state particles H_3 and H_4 to correspond to the two flavors of scalar parton. The high-transverse-momentum particles H_3 and H_4 are chosen to be almost back-to-back azimuthally (relative to the collision axis), so that transverse-momentum-dependent parton densities and fragmentation functions are needed for describing a factorized cross section.

The single lowest-order graph for the process is shown in Fig. 6. Its hard scattering is just the gluon-exchange subgraph. The cross section is the convolution of the hard scattering with a transverse-momentum-dependent parton density in each hadron. The fragmentation functions in this order are trivial delta functions. Although the longitudinal momenta of the incoming partons for the hard scattering are determined from the kinematics of H_3 and H_4, only a sum of their transverse momenta is determined. Hence a convolution over the transverse-momentum densities is needed. As before, there is no SSA at this order.

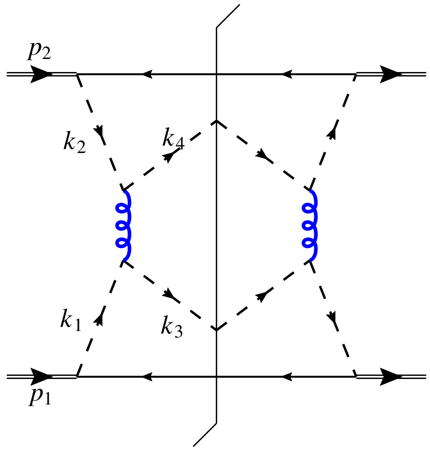

FIG. 6 (color online). Lowest-order graph in model for hadroproduction of hadrons of high transverse momentum. The initial-state particles are color-singlet Dirac particles. The spectator lines are for Dirac quark fields of charges g_1 and g_2, and the active partons are for scalar diquark fields. The exchanged gluon line is thickened to denote the hard scattering.

The graphs giving the lowest-order SSA are shown in Fig. 7. They have an extra gluon exchanged between the spectator line in the polarized hadron and one of the active partons. As with the DY process, a sum over cuts of graphs with a spectator-spectator interaction cancels, while exchanges purely between active partons give no SSA. An exchange purely between the spectator ($p_1 - k_1$) and the active parton line (k_1) in the polarized hadron has no relevant intermediate on-shell state and therefore does not contribute to the SSA.

So only the three graphs shown in Fig. 7 contribute to the SSA, together with their conjugate graphs. Exactly as in our discussion of SIDIS and DY, the graphs are the same to the leading power except for eikonal factors from the parton lines connecting the upper end of the gluon to the hard scattering. The appropriate replacements for these lines are

$$\frac{-g_2(2k_4^\mu - l^\mu)}{(k_4 - l)^2 - m_{\phi 2}^2 + i\epsilon} \mapsto ig_2\pi\delta^\mu_- \delta(l^+), \tag{4.1}$$

$$\frac{-g_2(2k_2^\mu + l^\mu)}{(k_2 + l)^2 - m_{\phi 2}^2 + i\epsilon} \mapsto ig_2\pi\delta^\mu_- \delta(l^+), \tag{4.2}$$

$$\frac{-g_1(2k_3^\mu - l^\mu)}{(k_3 - l)^2 - m_{\phi 1}^2 + i\epsilon} \mapsto ig_1\pi\delta^\mu_- \delta(l^+), \tag{4.3}$$

for a total of $i\pi(g_1 + 2g_2)\delta(l^+)$.

The g_1 term corresponds to a gluon coupling to a future-pointing Wilson line in the operator definition of the parton density, the same as for SIDIS. However, it is impossible for the contribution proportional to g_2 to be represented in terms of a Wilson line connecting the two parton fields for the distribution of partons of type ϕ_1 in the hadron H_1. This is simply because the coupling for any such Wilson line has to correspond to the color charge of the parton, i.e., g_1, and not g_2. The full Wilson line, or some generalization, is needed because exchanges of multiple gluons also contribute. The quantity we are looking at is definitely associated with the hadron H_1 rather than being in some kind of exotic soft factor, since the attachment of the lower end of the gluon line to the spectator parton is necessary for the nonzero SSA; the triviality of this observation is a special feature of our particular model.

The contribution to the SSA is therefore nonuniversal and does not correspond to a parton density. That is, factorization is broken.

Of course, the fact that the contribution is obtained from an eikonalized line indicates that it can be obtained from some kind of representation in terms of Wilson line operators. But the matrix element is for some more complicated and nonuniversal object [1–4] that cannot be treated as a parton density. It is allied to the objects used by Balitsky

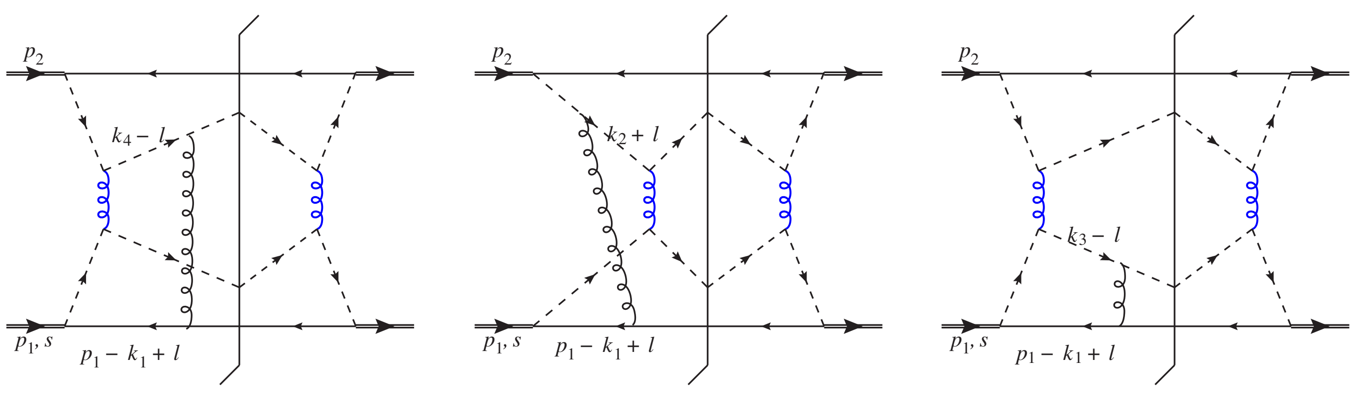

FIG. 7 (color online). One-gluon exchange in model for hadroproduction of hadrons of high transverse momentum. The specification of the process is as in Fig. 6. Only graphs that contribute to the SSA are shown. Hermitian conjugates of these graphs also contribute, with an equal value.

[15] to discuss scattering at high energy and small angles. The eikonalization indicates that substantial simplifications are possible. But that situation would go well beyond normal factorization.

V. DISCUSSION

We should first emphasize that there is a large overlap between the present paper and the work in Refs. [1–4]. What is not so clear from the earlier work is whether factorization in any standard sense continues to hold in the process (1.1). For example, in [1], we read "We have assumed factorization to hold in this treatment of TMD effects although it is, at the present, certainly not clear whether such a factorization holds for hadron-hadron scattering processes with explicitly TMD correlators."

Our primary result is to show by a counterexample that hard-scattering k_T factorization with universal parton densities fails for the production of high p_T hadrons in hadron-hadron collisions, when a pair of measured hadrons is close to back-to-back azimuthally. The overall issue is that in a gauge theory arbitrary exchanges of gauge fields between different collinear groups ("jets") can occur without any power suppression. To obtain factorization it is necessary to show that the sum over these exchanges can be absorbed into the definitions of the parton densities and fragmentation functions, assisted by certain cancellations. A full proof will be quite general, applying to a general gauge theory and to many reactions. So one particular counterexample is sufficient to show that such a proof does not exist; we can then choose the counterexample for maximum clarity and simplicity.

Even for those cases where factorization does hold, the need to make suitable definitions of the parton densities, etc., so as to absorb the effects of the gluon exchanges indicates that the parton densities, etc., can always be regarded as *effective* densities [16]. The primary practical issue is whether they are universal, i.e., the same for all reactions. In a certain sense, the well-known scale dependence of the densities is a kind of nonuniversality: different parton densities are needed when the scale of the hard scattering is given a large increment. But there is an evolution equation for the scale dependence, and this applies to an individual parton density. No details or specification of the hard scattering is needed to treat the evolution equation, either to derive it or to apply it. We should therefore refer in this case to "modified universality," not to nonuniversality. Similarly the reversal of the sign of the Sivers function between SIDIS and DY processes is a case of modified universality.

At the upper end of the exchanged gluon in our counterexample, the interactions can be treated in the eikonal approximation. This is very similar to other discussions of partons passing through the gluon field of another hadron. A selection of relevant papers is [15,17–19]. Much of that work concerns the small x region, diffractive scattering, etc., whereas our counterexample applies in the fully conventional region where normal parton-model concepts are generally considered as fully applicable, i.e., parton fractional momenta are moderate and the scale of the hard scattering is comparable to the center-of-mass energy rather than being much less.

Of course, interesting simplifications do occur, so that useful quantitative estimates can surely be obtained for the nonfactorizing effects. However the methods are rather different than those for conventional factorization. References [15,17–19] indicate that the effects of the eikonalized interactions are substantial, so that the numerical effects of nonfactorization should be significant; in the present paper we did not estimate the numerical size of the nonfactorization.

The gluon exchanges in our counterexample are clearly tied to the target hadron at their lower end. But the coupling at the upper end concerns some parton other than the one coming out of the lower hadron. The noncanceling terms are sufficiently tied to the color flow at the hard interaction that they are not universal in any normal sense. This is the clearest indication of nonuniversality.

The reader should not be misled by specific features of our counterexample into supposing that the failure of factorization is correspondingly restricted. These features include: the use of an SSA, the particular features of the model, and the particular order of perturbation theory. The use of the SSA is simply a way of getting the maximal conceptual sensitivity to problems in constructing a proof of factorization. For an unpolarized cross section, we would need an extra gluon to be exchanged in order for the nonfactorization issues to arise, from graphs such as those in Fig. 8. Evidently, to demonstrate nonfactorization explicitly in this case, the number of graphs would be larger than in our example, and the explicit calculations would be much more lengthy. Standard power-counting arguments show that the contribution of this and related graphs is of leading power. It is very important to deter-

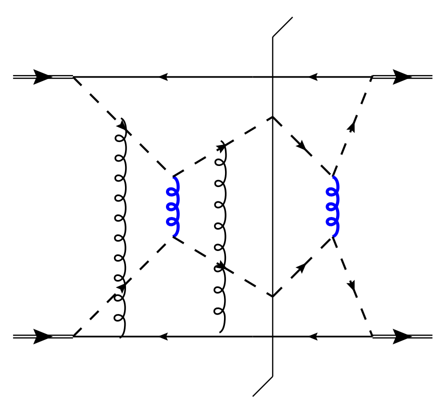

FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.

mine whether or not the sum of the potentially nonfactorizing contributions actually does or does not cancel in the unpolarized cross section.

Similarly, the choice of quantum numbers of the parton fields, of the Abelian gauge group, and of the quantum numbers for the detected particles was simply to provide maximum transparency and simplicity to the counterexample.

The fact that nonfactorization can only occur with at least two extra gluons in an unpolarized cross section might suggest that the nonfactorization is at high order in the strong coupling and therefore substantially suppressed. However the region of interest is at low virtuality for the gluons, so that the appropriate coupling is for a low momentum scale, where QCD is a strongly coupled theory.

Even so, the number of extra gluons needed implies that the effects of nonfactorization will only appear in quite high order in conventional perturbative QCD calculations. Normally one performs calculations with on-shell massless quarks and gluons, and extracts collinear divergences that are grouped with parton densities and fragmentation functions; any remaining divergences cancel between graphs. Nonfactorization in the hadronic cross section corresponds to uncanceled divergences in quark-gluon calculations. The lowest order in which the mechanisms we have discussed could possible give an uncanceled divergence in unpolarized partonic cross sections is next-to-next-to-next-to-leading-order (NNNLO), as in Fig. 9. The region for the uncanceled divergence is where the lower gluon is collinear to the lower incoming quark, and two of the exchanged gluons are soft. This graph is at least one order beyond all standard perturbative QCD calculations.

Because our calculations directly concern cross sections that use transverse-momentum-dependent parton densities, a certain amount of care is needed in interpreting the results. The natural direction for the Wilson lines is lightlike, as from Eq. (3.8). However lightlike Wilson lines give divergences in transverse-momentum-dependent densities

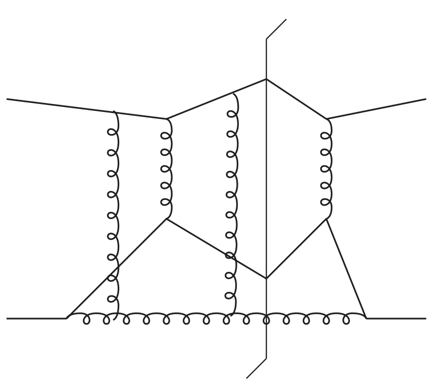

FIG. 9. In a conventional perturbative QCD calculation for an unpolarized partonic cross section, nonfactorization by the mechanisms discussed in this paper would first appear in graphs of this order.

[7]. These are due to rapidity divergences [20] in integrals over gluon momentum; they cancel [7] in conventional parton densities only because of an integral over all transverse momentum in integrated parton densities. The solution adopted by Collins, Soper, and Sterman [7] (CSS) was to define parton densities without Wilson lines but in a nonlightlike axial gauge. The gauge-fixing vector introduces a cutoff on gluon rapidity, and then an evolution equation with respect to the cutoff was derived. The nonperturbative functions involved in this CSS evolution equation have been measured (e.g., [21]) in fits to DY cross sections, and would be an essential ingredient in testing nonfactorization.

However, there are some unsatisfactory features of the use of axial gauges, which are made particularly evident in polarized cross sections. This includes complications concerning gauge links at infinity [22], when a Wilson line formalism is used. A much better definition is to use a nonlightlike Wilson line. This again obeys an equation of the CSS form. It is also possible to use a subtractive formalism [20,23] with lightlike Wilson lines but with generalized renormalization factors involving vacuum expectation values of Wilson lines, which also implement a rapidity cutoff, and lead to a CSS equation.

To test the predicted nonfactorization, we simply need predictions of high-p_T hadrons in hadron-hadron collisions, made on the basis of fits to parton densities in DIS and DY and to fragmentation functions in e^+e^- and SIDIS [24]. Probing the SSA would be particularly interesting, and such measurements are underway at Relativistic Heavy Ion Collider (RHIC) [25,26]. The same physics is probed in the transverse shape of jets, and would be worth investigating.

Our counterexample applies in a kinematic region where the normal intuitive ideas of the parton model appear quite appropriate, even with a generalization to k_T factorization. Therefore it forces us to question under what conditions factorization is actually valid and to what extent it has actually been demonstrated. It cannot be assumed that naive extensions of apparently established results are applicable beyond the cases to which the actual proofs explicitly apply.

For hadron-hadron collisions, factorization has been proved [5,6] for the Drell-Yan process integrated over transverse momentum or at large transverse momentum (of order Q). These proofs apply in the presence of gluon exchanges of the kind that we discuss in the present paper. But these papers do not go beyond this, to the production of hadrons. Because factorization is important to all aspects of hadron-collider phenomenology, it is critical to solve this problem for the hadroproduction of high-p_T hadrons. Given our counterexample to k_T factorization, a proof of factorization can only succeed in a situation where conventional collinear factorization is appropriate. For dihadron production this is when the hadron pair has itself large transverse momentum or when the pair's out-ofplane transverse momentum is integrated over a wide range.

In fact, Nayak, Qiu, and Sterman [27] have recently given strong arguments that collinear factorization does indeed hold in such situations. The graphs examined are similar to ours. They apply Ward identities to prove an eikonalization generalizing our specific calculations. Then they observe that a unitarity cancellation occurs of a kind endemic in factorization proofs [6,28]. This concerns graphs that are related by different placements of the final-state cut. In our model, one example is given by Figs. 2(a) and 2(b), and another is Fig. 7(a) and its conjugate. Such cancellations fail in our examples, because the final states of the related graphs have different transverse momentum, and the cross section is not sufficiently inclusive in transverse momentum.

Mechanisms that cause k_T factorization to fail in back-to-back hadron production also tend to cause resummation methods to fail. They will also tend to break factorization or cause large perturbative corrections when detailed distributions of final-state hadrons are examined. Since many such cases are implicit in the analysis of complicated multijet cross sections, and of jet shapes and the like, encountered in searches for new physics, further understanding is essential as are quantitative estimates of the effects. They can have a particularly important effect in the extrapolation to the LHC of quantitatively measured distributions at the Tevatron, for example, as embodied in Monte Carlo event generators. The methods of [15,17,18] will be important. Probably some of these effects have already been modeled in some approximation and in at least some Monte Carlo event generators, for example, by the soft color interaction model [29].

Troublesome though it may be for phenomenology, breaking of factorization should be viewed not as some kind of failure, but as an opportunity. Examination of the distribution of high-transverse-momentum hadrons in hadron-hadron collisions will lead to interesting nontrivial phenomena.

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