Rare decay $\pi^0 \to e^+e^-$: Theory confronts KTeV data

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Within the dispersive approach to the amplitude of the rare decay $\pi^0 \to e^+e^-$ the nontrivial dynamics is contained only in the subtraction constant. We express this constant, in the leading order in $(m_e/\Lambda)^2$ perturbative series, in terms of the inverse moment of the pion transition form factor given in symmetric kinematics. By using the CELLO and CLEO data on the pion transition form factor given in asymmetric kinematics, the lower bound of the decay branching ratio is found. The restrictions following from QCD allow us to make a quantitative prediction for the branching $B(\pi^0 \to e^+e^-) = (6.2 \pm 0.1) \cdot 10^{-8}$ which is 3σ below the recent KTeV measurement. We confirm our prediction by using the quark models and phenomenological approaches based on the vector meson dominance. The decays $\eta \to l^+l^-$ are also discussed.

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Experimental measurements of neutral pseudoscalar meson decays into lepton pairs and the comparison with theoretical predictions offer an interesting way to study long-distance dynamics in the standard model. Recently, the KTeV E799-II experiment at Fermilab has observed $\pi^0 \to e^+e^-$ events using $K_L \to 3\pi$ decay as a source of tagged neutral pions [1]. The branching ratio of the pion decay into an electron-positron pair was determined to be equal to

$$B^{\rm KTeV}(\pi^0 \to e^+e^-, x_D > 0.95) = (6.44 \pm 0.25 \pm 0.22) \cdot 10^{-8}, \quad (1)$$

where $x_D \equiv (m_{e^+e^-}/m_{\pi_0})^2$ is the Dalitz variable. By extrapolating the full radiative tail beyond $x_D > 0.95$ and scaling the result back up by the overall radiative correction [2,3] to find the lowest-order rate for $\pi^0 \to e^+e^-$, the KTeV Collaboration obtained

$$B^{\rm KTeV}_{\rm no-rad}(\pi^0 \to e^+e^-) = (7.49 \pm 0.29 \pm 0.25) \cdot 10^{-8}. \quad (2)$$

The rare decay $\pi^0 \to e^+e^-$ has been studied theoretically over the years, starting with the first prediction of the rate by Drell [4]. Since no spinless current coupling of quarks to leptons exists, the decay is described in the lowest order of QED as a one-loop process via the two-photon intermediate state, as shown in Fig. 1. A factor of $2(m_e/m_\pi)^2$ corresponding to the approximate helicity conservation of the interaction and two orders of α suppress the decay with respect to the $\pi^0 \to \gamma\gamma$ decay, leading to an expected branching ratio of about 10^{-7}. In the standard model, contributions from the weak interaction to this process are many orders of magnitude smaller and can be neglected. The interaction of leptons and quarks with leptoquarks is a possible mechanism for the pion decay from physics beyond the standard model. The confrontation of the theory and experiment will have some influence on the problem of strong sector contribution to the muon anomalous magnetic moment $g - 2$ [5,6].

To lowest order in QED, the normalized branching ratio is given by

$$R(\pi^0 \to e^+e^-) = \frac{B(\pi^0 \to e^+e^-)}{B(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha}{\pi}\frac{m_e}{m_\pi}\right)^2 \beta_e(m_\pi^2)|\mathcal{A}(m_\pi^2)|^2, \quad (3)$$

where $\beta_e(q^2) = \sqrt{1 - 4\frac{m_e^2}{q^2}}$, $B(\pi^0 \to \gamma\gamma) = 0.988$ [7]. In this article, we describe the process $\pi^0 \to e^+e^-$ by the diagram shown in Fig. 1, where $F_{\pi\gamma^*\gamma^*}$ is the form factor of the transition $\pi^0 \to \gamma^*\gamma^*$ with off-shell photons. The reduced amplitude $\mathcal{A}$ can be written as

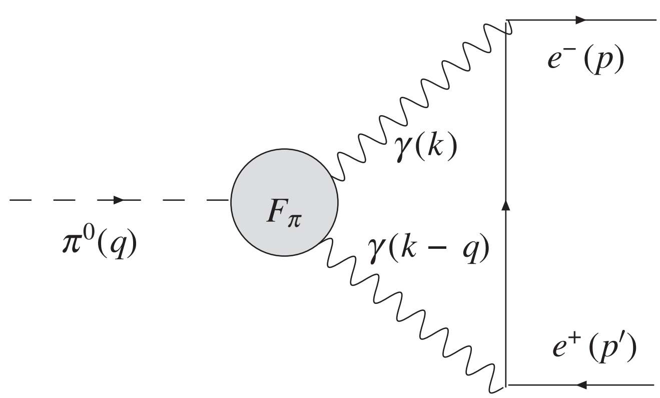

FIG. 1. Triangle diagram for $\pi^0 \to e^+e^-$ process with a pion $\pi^0 \to \gamma^*\gamma^*$ form factor in the vertex.

$$\mathcal{A}(q^2) = \frac{2i}{q^2}\int\frac{d^4k}{\pi^2}\frac{q^2k^2-(qk)^2}{(k^2+i\varepsilon)((k-q)^2+i\varepsilon)((k-p)^2-m_e^2+i\varepsilon)}F_{\pi\gamma^*\gamma^*}(-k^2,-(k-q)^2), \tag{4}$$

where $q^2 = m_\pi^2$, $p^2 = m_e^2$. We put the sign minus in the arguments of the form factor explicitly to emphasize that Eq. (4) is written in the Minkowski space. The form factor is normalized as $F_{\pi\gamma^*\gamma^*}(0,0) = 1$ and falls down quite rapidly in the Euclidean region of momenta to provide the ultraviolet convergence of the integral. A number of model calculations of the amplitude $\mathcal{A}(q^2)$ was performed [4,8–13] by employing different shapes of the form factor $F_{\pi\gamma^*\gamma^*}$. We discuss some of them below.

The aim of the present paper is to calculate the branching ratio $B(\pi^0\to e^+e^-)$ and estimate the uncertainties by using the available experimental and theoretical information on the pion transition form factor. In particular, the important constraints follow from the results obtained by the CELLO and CLEO collaborations and restrictions set by QCD.

First, we derive a suitable representation for the amplitude in Eq. (4) which would help us to perform a straightforward analysis by using the available information on the pion transition form factor. To do this, we employ the dispersive approach to the calculation of the amplitude developed in many papers (see, e.g. [12] and references therein). The imaginary part of the amplitude in Eq. (4)

$$\mathrm{Im}\mathcal{A}(q^2) = \frac{\pi}{2\beta_e(q^2)}\ln(y_e(q^2)),\qquad y_e(q^2) = \frac{1-\beta_e(q^2)}{1+\beta_e(q^2)}, \tag{5}$$

comes from the contribution of real photons in the intermediate state and is model independent since $F_{\pi\gamma^*\gamma^*}(0,0) = 1$. Using $|\mathcal{A}|^2 \geq (\mathrm{Im}\mathcal{A})^2$ and neglecting radiative corrections one can get the well-known unitary bound for the branching ratio in Eq. (3) [8]

$$B(\pi^0\to e^+e^-) \geq B^{\mathrm{unitary}}(\pi^0\to e^+e^-) = 4.69\cdot 10^{-8}. \tag{6}$$

A once-subtracted dispersion relation for the amplitude in Eq. (4) is written as [12]

$$\mathcal{A}(q^2) = \mathcal{A}(q^2=0) + \frac{q^2}{\pi}\int_0^\infty ds\frac{\mathrm{Im}\mathcal{A}(s)}{s(s-q^2)}. \tag{7}$$

The second term in Eq. (7) takes into account strong q^2 dependence of the amplitude around the point $q^2 = 0$ occurring due to the branch cut coming from the two-photon intermediate state. Integrating Eq. (7) one arrives for $q^2 \geq 4m_e^2$ at [14–16]

$$\mathrm{Re}\mathcal{A}(q^2) = \mathcal{A}(q^2=0) + \frac{1}{\beta_e(q^2)}\left[\frac{1}{4}\ln^2(y_e(q^2)) + \frac{\pi^2}{12} + \mathrm{Li}_2(-y_e(q^2))\right], \tag{8}$$

where $\mathrm{Li}_2(z) = -\int_0^z(dt/t)\ln(1-t)$ is the dilogarithm function.[1] For the pion in the leading order in $(m_e/m_\pi)^2$, one gets

$$\mathrm{Re}\,\mathcal{A}(m_\pi^2) = \mathcal{A}(q^2=0) + \ln^2\left(\frac{m_e}{m_\pi}\right) + \frac{\pi^2}{12}. \tag{9}$$

Thus, the nontrivial dynamics is only contained in the subtraction constant $\mathcal{A}(q^2=0)$. We evaluate this quantity in the following way [10]. We use the double Mellin representation for the pion transition form factor reducing the integral in Eq. (4) to the convolution of propagatorlike expressions. Then we perform the loop integration by using the standard Feynman α representation. Finally, we are able to expand the integral over the ratios of the electron and pion masses to the characteristic scale of the pion form factor $\Lambda \propto m_\rho$ by closing the Mellin contours in the appropriate manner and take the leading term of expansion. We arrive at the following representation:

$$\mathcal{A}(q^2=0) = 3\ln\left(\frac{m_e}{\mu}\right) + \chi_P(\mu), \tag{10}$$

where the constant $\chi_P(\mu)$ is defined by

$$\begin{aligned}\chi_P(\mu) &= -\frac{5}{4} + \frac{3}{2}\int_0^\infty dt\ln\left(\frac{t}{\mu^2}\right)\frac{\partial F_{\pi\gamma^*\gamma^*}(t,t)}{\partial t}\\ &= -\frac{5}{4} - \frac{3}{2}\left[\int_0^{\mu^2} dt\frac{F_{\pi\gamma^*\gamma^*}(t,t)-1}{t} + \int_{\mu^2}^\infty dt\frac{F_{\pi\gamma^*\gamma^*}(t,t)}{t}\right],\end{aligned} \tag{11}$$

with $F_{\pi\gamma^*\gamma^*}(t,t)$ being the physical pion transition form factor given in symmetric kinematics for spacelike photon momenta $t = Q^2 = -q^2 > 0$. One has to note that the logarithmic dependence on the scale μ appearing in Eq. (10) as a result of the decomposition of the integral over the dimensional variable t into two parts is compensated by the scale dependence of the low-energy constant

[1] For completeness we give explicit expressions for the amplitude $\tilde{\mathcal{A}}(q^2) = \mathcal{A}(q^2) - \mathcal{A}(0)$ for different regions of q^2: $\mathrm{Re}\tilde{\mathcal{A}}(q^2) = \frac{1}{\beta(q^2)}[\mathrm{Li}_2(-y(q^2)) + \frac{\pi^2}{3} + \frac{1}{4}\ln^2(-y(q^2))]$, $\mathrm{Im}\tilde{\mathcal{A}}(q^2) = 0$, for $q^2 \leq 0$; and $\mathrm{Re}\tilde{\mathcal{A}}(q^2) = -\frac{1}{\tilde{\beta}(q^2)}\mathrm{Cl}_2(-2\theta)$, $\mathrm{Im}\tilde{\mathcal{A}}(q^2) = -\frac{\pi}{\tilde{\beta}(q^2)}\,\mathrm{arctg}[\tilde{\beta}(q^2)]$, for $0 \leq q^2 \leq 4m^2$. Here $\beta(q^2) = \sqrt{1-4m^2/q^2}$, $\tilde{\beta}(q^2) = \sqrt{4m^2/q^2-1}$, $\theta = \mathrm{arctg}[1/\tilde{\beta}(q^2)]$, and $\mathrm{Cl}_2(z) = -\int_0^z dt\ln|2\sin(t/2)|$ is the Clausen's integral.

$\chi_P(\mu)$ displayed in Eq. (11). The obtained representation defines the unknown subtraction constant in dispersion formula, Eq. (7), via the pion transition form factor in a simple and transparent way. It is consistent with the result of the chiral perturbation theory [14–18] where $\chi_P(\mu)$ is the unknown low-energy constant. The result of Eq. (10) also agrees with the conclusions made in [12] where the inequalities $m_e \ll m_\pi \ll \Lambda$ were exploited.

In order to estimate the integral in Eq. (11), one needs to define the pion transition form factor in symmetric kinematics for spacelike photon momenta. Since it is not known from the first principles, we will adapt the available experimental data to perform such estimates. Let us first use the fact that $F_{\pi\gamma^*\gamma^*}(t,t) < F_{\pi\gamma^*\gamma^*}(t,0)$ for $t > 0$ in order to obtain the lower bound of the integral in Eq. (11). For this purpose, we take the experimental results from the CELLO [19] and CLEO [20] Collaborations for the pion transition form factor in asymmetric kinematics for spacelike photon momentum which is well parametrized by the monopole form

$$F^{\rm CLEO}_{\pi\gamma^*\gamma^*}(t,0) = \frac{1}{1 + t/s_0^{\rm CLEO}}, \qquad s_0^{\rm CLEO} = (776 \pm 22\ {\rm MeV})^2. \tag{12}$$

Note that $s_0^{\rm CLEO} \approx m_\rho^2$, as predicted by simple vector meson dominance models (VMD) [8,9,12], and is not far from the asymptotic prediction of operator product expansion (OPE) QCD [21]: $s_0^{\rm OPE} = 8\pi^2 f_\pi^2 = (821\ {\rm MeV})^2$. For this type of form factor one finds from Eqs. (10) and (11) that

$$\mathcal{A}(q^2 = 0) > -\frac{3}{2}\ln\left(\frac{s_0^{\rm CLEO}}{m_e^2}\right) - \frac{5}{4} = -23.2 \pm 0.1. \tag{13}$$

Thus, for the branching ratio we are able to establish the important lower bound which considerably improves the unitary bound given by Eq. (6)

$$B(\pi^0 \to e^+e^-) > B^{\rm CLEO}(\pi^0 \to e^+e^-) = (5.84 \pm 0.02)\cdot 10^{-8}. \tag{14}$$

Now let us proceed further in this manner and assume that the monopole form is also a good parametrization for the form factor in symmetric kinematics

$$F_{\pi\gamma^*\gamma^*}(t,t) = \frac{1}{1 + t/s_1}. \tag{15}$$

The scale s_1 can be fixed from the relation for the slopes of the form factors in symmetric and asymmetric kinematics at low t [22],

$$-\frac{\partial F_{\pi\gamma^*\gamma^*}(t,t)}{\partial t}\bigg|_{t=0} = -2\frac{\partial F_{\pi\gamma^*\gamma^*}(t,0)}{\partial t}\bigg|_{t=0}, \tag{16}$$

that gives $s_1 = s_0/2$. Note that similar reduction of the scale is predicted also by OPE QCD from the large momentum behavior of the form factors: $s_1^{\rm OPE} = s_0^{\rm OPE}/3$ [21].

Thus, the estimate for the amplitude in the limit $q^2 \to 0$ can be obtained from Eq. (13) by shifting the lower bound by a positive number which belongs to the interval $[3\ln(2)/2, 3\ln(3)/2]$. One finds

$$\mathcal{A}(q^2 = 0) = -\frac{3}{2}\ln\left(\frac{s_1}{m_e^2}\right) - \frac{5}{4} = -21.9 \pm 0.3, \tag{17}$$

that corresponds to the value $\chi_P(m_\rho) = 0.1 \pm 0.3$ of the low-energy constant χ_P taken at the scale $\mu = m_\rho$. Using the obtained prediction for the subtraction constant, one can evaluate the branching ratio

$$B(\pi^0 \to e^+e^-) = (6.23 \pm 0.09)\cdot 10^{-8}. \tag{18}$$

This is 3 standard deviations lower than the KTeV result given by Eq. (2). One can convert the experimental data to obtain the restriction on the scale parameter s_1 in Eq. (17). Then for the amplitude at $q^2 = 0$ estimated from the experimental data for $B^{\rm KTeV}_{\text{no-rad}}(\pi^0 \to e^+e^-)$ one finds

$$\mathcal{A}^{\rm KTeV}(q^2 = 0) = -18.6 \pm 0.9. \tag{19}$$

Since the amplitude in Eq. (17) depends logarithmically on the scale parameter s_1, one needs to reduce the value of s_1 by a factor of larger than 4. However, it obviously contradicts the experimental data for the slope parameters discussed above.

Let us compare our estimates with the results obtained in other approaches. We start with the QCD sum rule approach [23]. There, the pion form factor of the transition process $\gamma^*\gamma^* \to \pi^0$ was found in the form

$$F^{\rm QCDsr}_{\pi\gamma^*\gamma^*}(t,t) = 2\int_0^{s_0^{\rm QCDsr}} ds \int_0^1 dx \frac{x(1-x)t^2}{[x(1-x)s + t]^3} + \text{v.c.}, \tag{20}$$

where v.c. are small corrections from the vacuum condensates and $s_0^{\rm QCDsr}$ is the so-called dual interval parameter taken as $s_0^{\rm QCDsr} = s_0^{\rm OPE} = 0.7\ {\rm GeV}^2$ in the original work [23]. By using the relation for the form factor slopes in Eq. (16) and the expression for the radii given by

$$\langle r^2\rangle^{\rm QCDsr}_{\pi^0\gamma^*\gamma^*} = -6\frac{\partial F_{\pi\gamma^*\gamma^*}(t,t)}{\partial t}\bigg|_{t=0} = \frac{12}{s_0^{\rm QCDsr}}, \tag{21}$$

one can identify the duality parameter with the CLEO parameter

$$s_0^{\rm QCDsr} = s_0^{\rm CLEO}. \tag{22}$$

Then by using Eq. (11) one finds

$$\mathcal{A}^{\rm QCDsr}(q^2 = 0) = -\frac{3}{2}\ln\left(\frac{s_0^{\rm CLEO}}{m_e^2}\right) + \frac{1}{4} = -21.7 \pm 0.1, \tag{23}$$

that corresponds to the rescaling $s_1^{\rm QCDsr} = (s_0^{\rm QCDsr}/e)$ and is well suited to the interval in Eq. (17). The corresponding branching ratio is shown in Table I.

TABLE I. Values of the quantity $\mathcal{A}(q^2=0)$ and the branching ratio $B(\pi^0 \to e^+e^-)$ obtained in our approach [see, Eq. (17)] and compared with various phenomenological models and the KTeV experimental result.

	CLEO + OPE	QCDsr	gVMD	QM [12]	NχQM	NQM [10]	Experiment [1]
$-\mathcal{A}(q^2=0)$	21.9 ± 0.3	21.7 ± 0.1	21.9	23.4 ± 0.5	22.1 ± 0.5	24.5	18.6 ± 0.9
$B(\pi^0 \to e^+e^-) \times 10^8$	6.23 ± 0.09	6.21 ± 0.05	6.2	5.8 ± 0.2	6.1 ± 0.2	5.38	7.49 ± 0.38

Next we consider the parametrization of the pion form factor motivated by the generalized VMD [24]:

$$F^{\mathrm{gVMD}}_{\pi\gamma^*\gamma^*}(s,t) = \frac{4\pi^2 f_\pi^2}{3} \times \frac{(s+t)st - h_2 st + h_5(s+t) + M_V^4 M_{V1}^4 h_7}{(M_V^2+s)(M_V^2+t)(M_{V1}^2+s)(M_{V1}^2+t)} \tag{24}$$

with the parameters $M_V = 769$ MeV, $M_{V1} = 1465$ MeV, $h_2 = -10$ GeV^2, $h_5 = 6.93$ GeV^4, $h_7 = 3/(4\pi^2 f_\pi^2)$. This parametrization satisfies the above mentioned constraints on the pion transition form factor. At zero virtualities, the form factor is normalized by the axial anomaly. The relation in Eq. (16) is valid, which yields for the radius

$$\begin{aligned}\langle r^2\rangle^{\mathrm{gVMD}}_{\pi\gamma\gamma^*} &= -6\frac{\partial F(t,0)}{\partial t}\bigg|_{t=0} \\ &= 6\left(\frac{1}{M_V^2} + \frac{1}{M_{V1}^2} - \frac{h_5}{M_V^2 M_{V1}^2}\frac{3}{4\pi^2 f_\pi^2}\right) \\ &= 0.39\ \mathrm{fm}^2,\end{aligned} \tag{25}$$

which is close to PDG average $\langle r^2\rangle^{\mathrm{PDG}}_{\pi\gamma\gamma^*} = 0.407 \pm 0.051$ fm^2 [7]. Note also that numerically the second and third terms in Eq. (25) almost cancel each other. The form factor $F^{\mathrm{gVMD}}_{\pi\gamma^*\gamma^*}(s,t)$ has also correct OPE QCD motivated behavior at large virtualities [21]

$$F^{\mathrm{OPE}}_{\pi\gamma^*\gamma^*}(t,0)|_{t\to\infty} = \frac{4\pi^2 f_\pi^2}{3}\frac{h_5}{M_V^2 M_{V1}^2}\frac{1}{t}, \tag{26}$$

$$F^{\mathrm{OPE}}_{\pi\gamma^*\gamma^*}(t,t)|_{t\to\infty} = \frac{8\pi^2 f_\pi^2}{3}\frac{1}{t}. \tag{27}$$

The constant h_5 was fixed in [24] from a fit of CLEO data [20]; the constant h_2 defining the next-to-leading order power correction to the form factor at large t was estimated in [25] using the results of QCD sum rules [26]. In the asymptotic OPE QCD limit, one has $h_5/(M_V^2 M_{V1}^2) \to 6$.

For the generalized VMD form factor we estimate the subtraction constant

$$\begin{aligned}\mathcal{A}^{\mathrm{gVMD}}(q^2=0) &= -3\ln\left(\frac{M_V}{m_e}\right) + \frac{1}{4} + \frac{3r}{(r-1)^2} - \frac{3}{2}\frac{3r-1}{(r-1)^3}\ln r \\ &\quad + \frac{4\pi^2 f_\pi^2}{M_V^2(r-1)^3}\left(\frac{h_2}{2M_V^2}((r+1)\ln r - 2(r-1)) - \left(1+\frac{h_5}{M_{V1}^2 M_V^2}\right)(r^2 - 1 - 2r\ln r)\right) = -21.94,\end{aligned} \tag{28}$$

where $r = (M_{V1}/M_V)^2$. It agrees well with our prediction given by Eq. (17).

Let us now consider the amplitude $\pi^0 \to e^+e^-$ in the context of the constituent quark models. Within this model, the pion form factor is given by the quark-loop (triangle) diagram. Taking the constituent quark mass in the loop to be momentum independent, the result for the form factor in symmetric kinematics is given by [27]

$$\begin{aligned}F^{\mathrm{QM}}_{\pi\gamma^*\gamma^*}(t,t) &= \frac{2M_q^2}{\beta_q(t)t}\ln\left(\frac{\beta_q(t)+1}{\beta_q(t)-1}\right), \\ \beta_q(t) &= \sqrt{1+4\frac{M_q^2}{t}}.\end{aligned} \tag{29}$$

Substituting it into Eq. (11), one can get

$$\mathcal{A}_{\mathrm{QM}}(q^2=0) = 3\ln\left(\frac{m_e}{M_q}\right) - \frac{17}{4}, \tag{30}$$

in accordance with [12]. For the constituent quark mass in the interval $M_q = 300 \pm 50$ MeV one finds

$$\mathcal{A}_{\mathrm{QM}}(q^2=0) = -(23.4 \pm 0.5), \tag{31}$$

which is in agreement with the above estimates but contradicts the experimental result. In order to fit the KTeV value in Eq. (19), one needs to take the quark mass $M_q < 100$ MeV, which is an unacceptable region for the constituent quark mass.

By using the results for the pion transition form factor [28] obtained in the nonlocal quark model based on the instanton picture of QCD vacuum [29], one finds numerically from Eq. (10) that

$$\mathcal{A}_{\chi\mathrm{QM}}(q^2=0) = -(22.1 \pm 0.5), \tag{32}$$

which is consistent with Eq. (17). Within the instanton model, the quark mass is momentum dependent and for simplicity is taken in the Gaussian form

TABLE II. Values of the branchings $B(P \to l^+l^-)$ obtained in our approach and compared with the available experimental results.

B	Unitary bound	CLEO bound	CLEO + OPE	Experiment
$B(\pi^0 \to e^+e^-) \times 10^8$	≥ 4.69	$\geq 5.85 \pm 0.03$	6.23 ± 0.09	7.49 ± 0.38 [1]
$B(\eta \to \mu^+\mu^-) \times 10^6$	≥ 4.36	$\leq 6.23 \pm 0.12$	5.11 ± 0.20	5.8 ± 0.8 [7,32]
$B(\eta \to e^+e^-) \times 10^9$	≥ 1.78	$\geq 4.33 \pm 0.02$	4.60 ± 0.06	...

$$M(k) = M_q \exp(-k^2/\Lambda_{\chi\text{QM}}^2), \tag{33}$$

with $M_q = 300 \pm 50$ MeV and $\Lambda_{\chi\text{QM}} \sim 1$ GeV fixed by fitting the pion decay constant f_π.

A similar result was obtained in [10] in the nonlocal quark model (NQM) [30,31] (see, Table I). This model is based on the assumption that the quark propagator is described by the entire function without any singularities in the momentum space. Such an assumption guarantees the quark confinement in a sense that eliminates the threshold cuts corresponding to free quark production. A unified and uniformly accurate description of broad range of physical observables is obtained within the framework of this model; see, e.g. [31].

One has to note that the nonlocal quark model with momentum-dependent mass has an important advantage as compared to the mass-independent (local) quark model. The first model has correct large momentum behavior given by Eq. (27), while the latter has an extra $\log(t)$ term in the asymptotic, as follows from Eq. (29). For this reason, the local model is not suitable for fitting the CLEO data on the pion transition form factor.

The $\eta \to l^+l^-$ decay can be analyzed in a similar manner. As in the pion case, the CLEO Collaboration has parametrized the data for the η meson in the monopole form [20]:

$$F^{\text{CLEO}}_{\eta\gamma^*\gamma^*}(t, 0) = \frac{1}{1 + t/s^{\text{CLEO}}_{0\eta}}, \qquad s^{\text{CLEO}}_{0\eta} = (774 \pm 29\ \text{MeV})^2, \tag{34}$$

which is very close to the relevant pion parameter. Then, following the pion case (with evident substitutions), one finds the bounds for the $q^2 \to 0$ limit of the amplitude $\eta \to \mu^+\mu^-$ as

$$\mathcal{A}_\eta(q^2 = 0) > -\frac{3}{2}\ln\left(\frac{s^{\text{CLEO}}_{0\eta}}{m_\mu^2}\right) - \frac{5}{4} = -(7.2 \pm 0.1), \tag{35}$$

and for $\eta \to e^+e^-$ one gets again Eq. (13). The obtained estimates allow one to find the bounds for the branching ratios

$$B(\eta \to \mu^+\mu^-) < (6.23 \pm 0.12) \cdot 10^{-6}, \qquad B(\eta \to e^+e^-) > (4.33 \pm 0.02) \cdot 10^{-9}. \tag{36}$$

Note that for the decay $\eta \to \mu^+\mu^-$ we get the upper limit for the branching. This is because the real part of the amplitude for this process taken at the physical point $q^2 = m_\eta^2$ for the parameter $s^{\text{CLEO}}_{0\eta}$ remains negative and a positive shift due to the change of the scale $s_{0\eta} \to s_{1\eta}$ reduces the absolute value of the real part of the amplitude $|\text{Re}\mathcal{A}(m_\eta^2)|$. At the same time, considering the decays of π^0 and η into an electron-positron pair, the evolution to physical point (8) makes the real part of the amplitude to be positive for the parameter s_0^{CLEO} and the absolute value of the real part of the amplitude increases in changing the scales of the meson form factors. The predictions for the decays $\eta \to l^+l^-$ obtained by reducing the scale $s^{\text{CLEO}}_{0\eta} \to s_{1\eta}$ for the case of the η-meson transition form factor are given in Table II.

In conclusion, we have derived in the leading order in $(m_e/\Lambda)^2$ the representation for the amplitude of the rare $\pi^0 \to e^+e^-$ process in the limit $q^2 \to 0$. It is given in terms of the inverse moment of the transition pion form factor in symmetric kinematics for spacelike photon momenta. By using data of the CELLO and CLEO Collaborations on the pion-photon transition form factor in the obtained representation, we found the new lower bound for the decay branching ratio which essentially improves the well-known unitary bound. Further constraints follow from the results of OPE QCD correlating the pion transition form factor in different kinematics as the change of characteristic scales. These considerations allow us to reconstruct the full decay amplitude and make predictions for the decay branching. A similar procedure is also applied to the decays $\eta \to l^+l^-$. We compared our predictions with the results obtained in various phenomenological approaches and found that all of them are in agreement with our results. However, the obtained prediction for the branching ratio $\pi^0 \to e^+e^-$ is 3σ below the recent KTeV measurement.

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