Enhanced nonlocal power corrections to the $\bar{B} \to X_s \gamma$ decay rate

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A new class of enhanced nonperturbative corrections to the inclusive $\bar{B} \to X_s \gamma$ decay rate is identified, which contribute first at order Λ/m_b in the heavy-quark expansion and cannot be described using a local operator product expansion. Instead, these effects are described in terms of hadronic matrix elements of nonlocal operators with component fields separated by lightlike distances. They contribute to the high-energy part of the photon-energy spectrum but do not reduce to local operators when an integral over energy is taken to obtain the total inclusive decay rate. The dominant corrections depend on the flavor of the B-meson spectator quark and are described by trilocal four-quark operators. Their contribution is estimated using the vacuum insertion approximation. The corresponding uncertainty in the total decay rate is found to be at the few percent level. This new effect accounts for the leading contribution to the rate difference between B^- and $\bar{B}^0$ mesons.

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I. INTRODUCTION

Precision studies of inclusive B-meson decays are a cornerstone of quark flavor physics. Detailed measurements of various kinematical distributions in the semileptonic decays $\bar{B} \to X l \bar{\nu}$, when combined with elaborate theoretical calculations, provide the currently most precise measurements of the elements $|V_{cb}|$ and $|V_{ub}|$ of the quark mixing matrix (see [1] for a comprehensive recent analysis of charmless inclusive decays). Studies of the rare decays $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_s l^+ l^-$ allow sensitive tests of the flavor sector and provide constraints on extensions of the standard model.

The theoretical description of inclusive B-meson decay rates is based on the operator product expansion (OPE) [2,3], by which total decay rates can be expressed in terms of forward B-meson matrix elements of local operators. Only two nontrivial matrix elements appear up to order $(\Lambda/m_b)^2$ in the expansion, one of which can be extracted from spectroscopy. The OPE breaks down when one tries to calculate differential inclusive decay distributions near phase-space boundaries. A twist expansion involving forward matrix elements of nonlocal light-cone operators (so-called shape functions) is then required to properly account for nonperturbative effects [4,5]. Recently, these nonlocal structures have been analyzed systematically beyond the leading order in Λ/m_b [6–8]. It is generally believed that the nonlocal operators reduce to local ones when the differential decay distributions are integrated over all of phase space. Here we show that this is not always the case.

A precise control of hadronic power corrections is particularly important in the case of the inclusive radiative decay $\bar{B} \to X_s \gamma$, which is the prototype of all flavor-changing neutral current processes. A significant effort is currently underway to complete the calculation of the leading-power (in Λ/m_b) contribution to the decay rate at next-to-next-to-leading order in renormalization-group improved perturbation theory. This leaves nonperturbative power corrections as the potentially largest source of theoretical uncertainty.

It is well known that in $\bar{B} \to X_s \gamma$ decay the OPE faces some limitations, which result from the fact that the photon has a partonic substructure. For instance, there exists a contribution to the total decay rate involving the interference of the $b \to s\gamma$ transition amplitude mediated by the electromagnetic dipole operator $Q_{7\gamma}$ with the charm-penguin amplitude ($b \to c\bar{c}s$ followed by $c\bar{c} \to \gamma g$) mediated by the current-current operator Q_1 (see [9] for the definition of the operators in the effective weak Hamiltonian). When the charm-quark is treated as a heavy quark ($m_c \sim m_b$), this contribution can be expanded in local operators [10–13], and it is believed to be a good approximation to keep only the first term in this expansion. Its contribution to the total $\bar{B} \to X_s \gamma$ decay rate can be written as

$$\frac{\Delta\Gamma}{\Gamma_{77}} = -\frac{C_1}{C_{7\gamma}} \frac{\lambda_2}{9m_c^2} \approx 0.03,$$

where $\lambda_2 = \frac{1}{4}(m_{B^*}^2 - m_B^2)$, and

$$\Gamma_{77} = \frac{G_F^2 \alpha}{32\pi^4} |V_{tb} V_{ts}^*|^2 m_b^5 |C_{7\gamma}|^2$$

is the leading-order contribution to the decay rate from the electromagnetic dipole operator. The ratio $\Delta\Gamma/\Gamma_{77}$ therefore provides an estimate of the relative magnitude of the nonperturbative effect. We stress that when the scaling $m_c^2 \sim \Lambda m_b$ is adopted instead of $m_c \sim m_b$, then the charm-loop contribution must be described by the matrix element of a nonlocal operator [11–14].

It has been noted in [11] that the OPE for $\bar{B} \to X_s \gamma$ decay breaks down when one includes diagrams from operators other than $Q_{7\gamma}$, in which the photon couples to light quarks. An example of such a contribution, resulting from the decay $b \to sg$ mediated by the chromomagnetic dipole operator Q_{8g} followed by photon emission from the light partons, was studied in [15]. It was argued that the corresponding effect can be estimated in terms of the parton fragmentation functions of a quark or gluon into a photon. Since this contribution is numerically very small, it has not received much further attention in the literature. A more careful analysis reveals that the correct interpretation of this effect is in terms of a subleading shape function [14].

In this paper, we identify and analyze a novel class of nonlocal power corrections to the $\bar{B} \to X_s \gamma$ decay rate, which were not considered before. We argue that they can affect the total decay rate at the few percent level, and that they give the dominant flavor-specific contribution to the rate difference between charged and neutral B mesons. The presence of this effect leads to a dominant and irreducible source of theoretical uncertainty in the prediction for the total $\bar{B} \to X_s \gamma$ branching fraction.

II. NONLOCAL POWER CORRECTIONS

Power corrections to the high-energy part of the $\bar{B} \to X_s \gamma$ photon spectrum can be systematically parametrized in terms of subleading shape functions defined in terms of forward B-meson matrix elements of nonlocal light-cone string operators [6–8]. Some of these operators—the ones considered so far in the literature—reduce to local operators when one considers the total decay rate (i.e., the integral over the photon spectrum); however, a detailed analysis shows that several of them do not [14]. Representative diagrams giving rise to such operators are depicted in Fig. 1. The graphs show different contributions to the discontinuity of the hadronic tensor $W^{\mu\nu}$, which determines the $\bar{B} \to X_s \gamma$ photon-energy spectrum via the optical theorem. The total decay rate is obtained by an integration over the photon energy. In this paper we focus on the two graphs shown in the first row (and two mirror graphs, in which the order of the weak vertices is interchanged). They describe the interference of the $b \to s\gamma$ transition amplitude mediated by the electromagnetic dipole operator $Q_{7\gamma}$ with the $b \to sg$ amplitude mediated by the chromomagnetic dipole operator Q_{8g} followed by the fragmentation of the gluon into an energetic photon and a soft quark-antiquark pair. While other diagrams, such as the first graph in the second row in the figure, give rise to four-quark operators containing strange quarks, the graphs in the first row produce all light-quark flavors. We expect

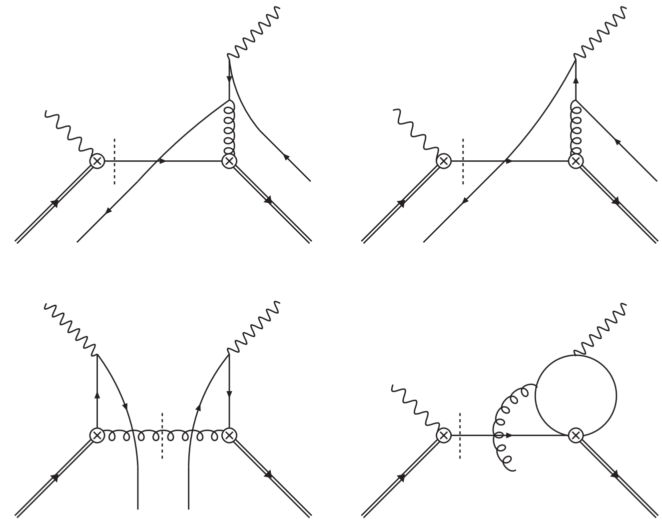

FIG. 1. Diagrams representing enhanced nonlocal power corrections to the $\bar{B} \to X_s \gamma$ photon spectrum, which do not reduce to local power corrections to the total decay rate. The double lines represent heavy-quark fields h_v. The vertical dashed lines indicate cuts of the relevant propagators.

that the resulting four-quark operators will have a larger overlap with the B-meson states and thus give rise to the dominant power corrections. For simplicity, we also do not consider loop-suppressed effects such as the second graph in the second row of the figure. This diagram would match onto a nonlocal operator containing a soft gluon field, which mixes with the operators we consider under renormalization.

The top two diagrams in Fig. 1 affect the $\bar{B} \to X_s \gamma$ photon spectrum in the high-energy region, $E_\gamma \approx m_b/2$, where it is most accessible to experiment. This is enforced by the fact that the amplitude mediated by the insertion of Q_{8g} interferes with the two-body decay amplitude mediated by the insertion of $Q_{7\gamma}$. The effect can therefore not be eliminated using kinematical cuts. We find that the corresponding contribution to the total decay rate can be parametrized in terms of forward B-meson matrix elements of trilocal light-cone operators (with $C_F = 4/3$ for $N_c = 3$ colors):

$$\Delta\Gamma = -\Gamma_{77} \frac{C_{8g}}{C_{7\gamma}} \frac{4\pi\alpha_s}{N_c m_b} \int_{-\infty}^{0} ds \int_{-\infty}^{0} dt \times \langle \bar{B} | C_F(O_1 + O_2) - (T_1 + T_2) | \bar{B} \rangle, \qquad (1)$$

where we have used that the Wilson coefficients C_i are real in the standard model. The relevant factorization scale to use in this result is of order $\mu^2 \sim m_b \Lambda$. We use a mass-independent normalization of meson states, such that $\langle \bar{B} | \bar{h}_v h_v | \bar{B} \rangle = 2$. The four-quark operators are defined as

$$O_1 = \sum_q e_q \bar{h}_v(0)\Gamma_R q(t\bar{n})\bar{q}(s\bar{n})\Gamma_R h_v(0),$$

$$O_2 = \sum_q \frac{e_q}{2} \bar{h}_v(0)\Gamma_R \gamma_{\perp\alpha} q(t\bar{n})\bar{q}(s\bar{n})\gamma_\perp^\alpha \Gamma_R h_v(0),$$

$$T_1 = \sum_q e_q \bar{h}_v(0)\Gamma_R t_a q(t\bar{n})\bar{q}(s\bar{n})\Gamma_R t_a h_v(0),$$

$$T_2 = \sum_q \frac{e_q}{2} \bar{h}_v(0)\Gamma_R \gamma_{\perp\alpha} t_a q(t\bar{n})\bar{q}(s\bar{n})\gamma_\perp^\alpha \Gamma_R t_a h_v(0),$$

where e_q is the electric charge of the soft light quark in units of e, $\Gamma_R = \bar{\not n}(1+\gamma_5)/2$ is a right-handed Dirac structure, and t_a are the generators of color SU(3). Here h_v are the two-component heavy-quark fields defined in heavy-quark effective theory, while q are soft light-quark fields ($q = u, d, s$). The light-quark fields are located on the light cone defined by the direction of the emitted photon with momentum $q^\mu = E_\gamma \bar{n}^\mu$ (with $\bar{n}^2 = 0$). The nonlocal operators are made gauge invariant by the insertion of soft Wilson lines $S_{\bar{n}}$ in the $\bar{n}$-direction. The Wilson lines are absent in light-cone gauge $\bar{n}\cdot A = 0$, which we adopt implicitly to simplify the notation.

Evaluating the hadronic matrix elements in (1) using a systematic nonperturbative approach is a very challenging task. In particular, lattice QCD is unable to handle operators with component fields separated by lightlike distances. Naive dimensional analysis suggests that $\Delta\Gamma/\Gamma_{77} \sim (C_{8g}/C_{7\gamma})\pi\alpha_s(\Lambda/m_b)$, which could easily amount to a 5% correction to the decay rate. In more traditional applications of the OPE to inclusive B-meson decays, four-quark operators contribute at order $(\Lambda/m_b)^3$ in the heavy-quark expansion. The nonlocal operators in (1) lead to enhanced power corrections of order Λ/m_b, because the two "vertical" propagators in Fig. 1 have virtualities of order $m_b\Lambda$ and so introduce two powers of soft scales in the denominator. This mechanism was first studied in [16]. The existence of such enhanced power corrections in the total decay rate may seem puzzling at first sight. Consider the diagram in Fig. 2, which represents the contribution to the total rate derived from the first graph in Fig. 1. Before taking the cut indicated by the vertical dashed line the diagram only receives contributions from

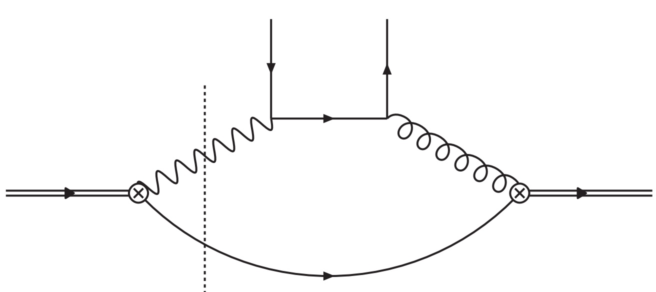

FIG. 2. Diagram representing an enhanced nonlocal power correction to the total $\bar{B}\to X_s\gamma$ decay rate.

hard loop momenta $p^\mu \sim m_b$, and it would thus seem appropriate to shrink all propagators to a point. In this case the diagram would contribute at order $(\Lambda/m_b)^3$ in the heavy-quark expansion, and this scaling would appear to be preserved when one takes the discontinuity of the diagram, apparently contradicting our conclusion. The loophole is that the contribution to the total $\bar{B}\to X_s\gamma$ decay rate is not given by the discontinuity of the loop graph, which would correspond to the sum of all three possible cuts, but instead it is given by the single cut shown in the figure.

In order to obtain at least some model estimate of the magnitude of the effect in (1) we adopt the vacuum insertion approximation (VIA), in which the vacuum state $|0\rangle\langle 0|$ is inserted between the two light-quark fields inside the four-quark operators. This is a crude approximation, which however appears to work well in the analysis of b-hadron lifetimes [17,18]. The approximation has thus been checked for local four-quark operator evaluated between B-meson states. Also, it can be justified using large-N_c counting rules. Applications of the VIA to nonlocal operators can be found in [8,19].

In the present case, the matrix elements of the operators O_2 and $T_{1,2}$ vanish in the VIA, either due to the color-octet structure of the quark bilinears ($T_{1,2}$) or due to the fact that there is no external perpendicular Lorentz vector available (O_2 and T_2). The matrix element of O_1 can be expressed in terms of the leading light-cone distribution amplitude of the B meson in position space [20,21]. We obtain

$$\langle\bar{B}|O_1|\bar{B}\rangle_{\rm VIA} = e_q \frac{f_B^2 m_B}{4}\tilde{\phi}_+^B(s)[\tilde{\phi}_+^B(t)]^*,$$

where e_q now refers to the charge of the light spectator quark in the B meson. The integral over the position-space distribution amplitude can be evaluated to yield

$$-i\int_{-\infty}^0 ds\,\tilde{\phi}_+^B(s) = \int_0^\infty \frac{d\omega}{\omega}\phi_+^B(\omega) = \frac{1}{\lambda_B},$$

where $\phi_+^B(\omega)$ is the light-cone distribution amplitude in momentum space, and λ_B^{-1} is the common notation for the first inverse moment of this quantity [9]. Numerical estimates of λ_B are very uncertain, but typically fall in the range between 0.25 and 0.75 GeV [9,20–24]. In the VIA, our estimate of the spectator-dependent, nonlocal power corrections then takes the final form

$$\frac{\Delta\Gamma_{\rm VIA}}{\Gamma_{77}} = -\frac{e_q C_{8g}}{C_{7\gamma}}\frac{\pi\alpha_s}{2}\left(1-\frac{1}{N_c^2}\right)\frac{f_B^2 m_B}{\lambda_B^2 m_b}. \qquad (2)$$

Recalling that $f_B \sim 1/\sqrt{m_B}$ in the heavy-quark limit, we indeed recover the scaling behavior anticipated above. Numerically, with $\mu \sim 1.5$ GeV as a typical factorization scale and $f_B \approx 0.215$ GeV for the B-meson decay constant (see [25] for a recent determination using unquenched lattice QCD), we obtain

$$\frac{\Delta\Gamma_{\rm VIA}}{\Gamma_{77}} \approx -0.26 e_q \left(\frac{f_B}{\lambda_B}\right)^2 \approx -0.05 e_q \left(\frac{\lambda_B}{0.5\ \mathrm{GeV}}\right)^{-2},$$

where $e_q = 2/3$ for decays of B^- mesons, while $e_q = -1/3$ for decays of $\bar{B}^0$ mesons. For the range of λ_B values quoted above, the effect is between -2% and -19% times e_q. Taking Γ_{77} as an estimate of the total decay rate at leading power, this implies that the enhanced power corrections to the total, flavor-averaged $\bar{B} \to X_s\gamma$ decay rate are expected (in the VIA) to be between -0.3% and -3%, while these effects induce a flavor-dependent rate asymmetry

$$\frac{\Gamma(B^- \to X_s\gamma) - \Gamma(\bar{B}^0 \to X_s\gamma)}{\Gamma(\bar{B} \to X_s\gamma)} \approx -0.05 \left(\frac{\lambda_B}{0.5\ \mathrm{GeV}}\right)^{-2}, \qquad (3)$$

which could amount to an effect between -2% and -19%.

When considering these estimates one should keep in mind that the VIA can at best provide a very simple model of the effect of the nonlocal four-quark operators in (1). Conservatively, we can therefore not exclude that the type of enhanced power corrections identified in this paper could contribute to the total $\bar{B} \to X_s\gamma$ decay rate at the 5% level. The magnitude of the flavor-specific effects studied above could be probed by a measurement of the flavor asymmetry (3); but there are other four-quark operator contributions with flavor structure $\bar{b}s\bar{s}b$ (see e.g. the bottom left diagram in Fig. 1), whose matrix elements vanish in the VIA but could still be significant in real QCD. Their contributions are flavor-blind and hence not tested by (3).

The *BABAR* collaboration has measured the flavor-dependent rate asymmetry in Eq. (3), finding the value $(1.2 \pm 11.6 \pm 1.8 \pm 4.8)\%$, where the errors are statistical, systematic, and due to the production ratio $\bar{B}^0/B^-$, respectively [26]. The dominant error is statistical and therefore likely to decrease when more data is collected.

III. CONCLUSIONS

We have identified a new class of enhanced power corrections to the total inclusive $\bar{B} \to X_s\gamma$ decay rate, which cannot be parametrized in terms of matrix elements of local operators. These effects are nevertheless "calculable" in the sense that they can be expressed in terms of subleading shape functions. At tree level, the corresponding operators are trilocal four-quark operators. While local four-quark operators contribute at order $(\Lambda/m_b)^3$ in the heavy-quark expansion of the total decay rate, the effects we have explored are enhanced by the nonlocal structure of the operators and promoted to the level of Λ/m_b corrections. We have identified and estimated what we believe are the dominant corrections of this type, namely, those that match the flavor quantum numbers of the external B-meson states.

Our results imply that a local operator product expansion for the inclusive $\bar{B} \to X_s\gamma$ decay rate does not exist. Even at first order in Λ/m_b there are hadronic effects that can only be accounted for in terms of nonlocal operators. The precise impact of these power corrections will be notoriously difficult to estimate using our present command of nonperturbative QCD on the light cone. While a naive estimate using the vacuum insertion approximation suggests that the effects are at the few percent level, we conclude that they are nevertheless a source of significant hadronic uncertainty in the calculation of partial or total $\bar{B} \to X_s\gamma$ decay rates. After the perturbative analysis of the decay rate will have been completed, the enhanced nonlocal power corrections will remain as the dominant source of theoretical uncertainty. A measurement of the flavor-dependent asymmetry (3) could help to corroborate our numerical estimates of such corrections.

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