Decays of the X(3872) and the $\chi_{c1}(2P)$ charmonium state

Ce Meng[1] and Kuang-Ta Chao[1,2]

[1]*Department of Physics, Peking University, Beijing 100871, People's Republic of China*

[2]*China Center of Advanced Science and Technology (World Laboratory), Beijing 100080, People's Republic of China*

(Received 11 April 2007; published 5 June 2007)

We reexamine the rescattering mechanism for the X(3872), as a candidate for the 2P charmonium state χ_{c1}(2P), decaying to $J/\psi\rho(\omega)$ through exchanging $D^{(*)}$ mesons between intermediate states $D(\bar{D})$ and $\bar{D}^*(D^*)$. We evaluate the dispersive part, as well as the absorptive one, of the rescattering amplitude and find that the contribution from the dispersive part is dominant even when X(3872) lies above the threshold of the neutral channel $\mathrm{th}_n = m_{D^0} + m_{D^{*0}}$. We predict $R_{\rho/\omega} \simeq 1$ for the m_X region scanned by experiments. Meanwhile, we also estimate the rate of $X \to D^0\bar{D}^0\pi^0$. Our results favor a charmonium interpretation of X(3872) when it lies slightly below the threshold of $D^0\bar{D}^{*0}$. Furthermore, we evaluated the width of $X \to J/\psi\rho$ with the help of a phenomenological effective coupling constant g_X, and find the total width of X(3872) to be in the range of 1–2 MeV.

DOI: 10.1103/PhysRevD.75.114002 PACS numbers: 14.40.Gx, 13.25.Gv, 13.75.Lb

I. INTRODUCTION

In recent years there have been a number of exciting discoveries of new hadron states (for a recent review, see, e.g. [1]). These discoveries are enriching and also challenging our knowledge for the hadron spectroscopy, and the underlining theory for strong interactions. Among these new states, the X(3872) may be the most mysterious one, which was first discovered by the Belle collaboration [2] in the invariant mass spectrum of $J/\psi\pi^+\pi^-$ in the decay $B^+ \to J/\psi\pi^+\pi^-K^+$, and confirmed soon by *BABAR* [3], CDF [4], and D0 [5] collaborations. The world average mass is $m_X = (3871.2 \pm 0.5)$ MeV and the width is $\Gamma_X <$ 2.3 MeV at 90% C.L. [6], which is consistent with the detector resolution. The dipion mass distribution in $J/\psi\pi^+\pi^-$ seems to favor a ρ resonance for the dipion structure. This implies the C parity of X(3872) is even, which is finally confirmed by the measurement of X(3872) $\to \gamma J/\psi$ [7,8]. The angular distribution analysis by Belle [9] favors $J^{PC} = 1^{++}$. Analogous analysis [10] and the analysis for the dipion mass spectrum [11] by the CDF collaboration allow $J^{PC} = 1^{++}$ and $J^{PC} = 2^{-+}$ as well. The recent observation of the near-threshold decay $X \to D^0\bar{D}^0\pi^0$ [12] (with a slightly higher mass of about 3875 MeV) by Belle may favor $J^{PC} = 1^{++}$ but can not rule out $J^{PC} = 2^{-+}$. Moreover, Belle also see the subthreshold decay $X \to \omega J/\psi$ in $B^+ \to J/\psi\pi^+\pi^-\pi^0K^+$ [7]. So far, for the X(3872) four decay modes have been observed with the following fractions [6,7,12]

$$\mathcal{B}(B^\pm \to K^\pm X) \times \mathcal{B}(X \to \pi^+\pi^-J/\psi) = (1.14 \pm 0.20) \times 10^{-5}, \quad (1)$$

$$\frac{\mathcal{B}(X \to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X \to J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4 \pm 0.3, \quad (2)$$

$$\mathcal{B}(B \to KX) \times \mathcal{B}(X \to D^0\bar{D}^0\pi^0) = (1.22 \pm 0.31^{+0.23}_{-0.30}) \times 10^{-4}, \quad (3)$$

$$\mathcal{B}(X \to \gamma J/\psi)/\mathcal{B}(X \to \pi^+\pi^-J/\psi) = 0.14 \pm 0.05, \quad (4)$$

where the experimental value for $X \to \gamma J/\psi$ is taken from the Belle measurement [7], while the observed value of about 0.25 by *BABAR* is somewhat larger [8].

For convenience, we define the following ratios and their values can be deduced from (1)–(3):

$$R_{\rho/\omega} = \frac{\Gamma_{\psi\rho}}{\Gamma_{\psi\omega}} = 1.0 \pm 0.5, \quad (5)$$

$$R_{\rho/DD\pi} = \frac{\Gamma_{\psi\rho}}{\Gamma_{DD\pi}} = 0.10 \pm 0.05, \quad (6)$$

where Γ_i denotes the width of decay $X \to i$ with $i = \psi\rho$, $\psi\omega$, and $D^0\bar{D}^0\pi^0$, respectively.

Because of the closeness of m_X to the threshold $M_{D^0\bar{D}^{*0}} = 3871.81 \pm 0.36$ MeV [13], many authors identify the X(3872) with a molecule of $D^0\bar{D}^{*0}$ + c.c. in S wave [14], a loosely bound state of charmed mesons. This is certainly a very attractive interpretation, which also gives a natural explanation of the J^{PC} of X(3872), and predicts $R_{\rho/\omega} \approx 1$ (see Ref. [15]) as well. Thus, the molecule becomes the most popular interpretation for the X(3872). However, it seems to be difficult for the molecule models to account for the large production rates of X(3872) at B factories and the Tevatron [16] unless $\mathcal{B}(X \to J/\psi\rho)$ is large [17], which, however, seems to be in contradiction with (6). Furthermore, the molecule model predicted the decay into $J/\psi\rho$ to be much superior to that into $D^0\bar{D}^0\pi^0$, but this seems not to be supported by the experiment. Moreover, the molecule model predicted that the production rate of X(3872) in $B^+ \to XK^+$ is much larger than that in $B^0 \to XK^0$ [17], but the Belle data show that the rate of $B^0 \to XK^0$ with $X \to D^0\bar{D}^0\pi^0$ is approximately equal to that of $B^+ \to XK^+$ though the errors for the measurements are large [12]. So, it might be useful to try other possible interpretations for the X(3872).

Motivated by the large production rates in B decays and in $p\bar{p}$ collisions at the Tevatron, we suggested that the X(3872) be a $J^{PC} = 1^{++}$ $\chi_{c1}(2P)$ charmonium-dominated state [18]. This possibility has also been suggested by Suzuki with detailed discussions on its decay properties [19]. In this charmonium picture, the large rate of $B \to$ X(3872)K, which is comparable to (not much less than) $B \to \chi_{c1}(1P)K$, and the similarity in production between X(3872) and $\psi(2S)$ observed by the CDF and D0 collaborations at the Tevatron can be well understood [18]. Leaving the mass problem [20–23] alone, the most difficult problem of this assignment is how to explain the observed large isospin violating effect expressed by $R_{\rho/\omega} \approx 1$, since the state $\chi_{c1}(2P)$ is an isospin scalar. Suzuki [19] estimates that $R_{\rho/\omega} \approx 1/2$ in a semiquantitative way in which both $J/\psi\rho$ and $J/\psi\omega$ are produced through the $D(D^*)$ exchange between $D\bar{D}^*$ pair, and the large isospin violation can be accounted for by the mass difference between neutral and charged $D\bar{D}^*$ thresholds and the large difference between the phase spaces of $X \to J/\psi\rho$ and $X \to J/\psi\omega$. Recently, the ratio $R_{\rho/DD\pi}$ was studied in a similar but more quantitative way [24], and was predicted to be $R_{\rho/DD\pi} \approx 10^{-6} - 10^{-4}$, which is far smaller than the experimental data in (6).

The estimation of $R_{\rho/DD\pi}$ given in Ref. [24] is based only on the imaginary part of the amplitude $\mathcal{A}(X \to D^0\bar{D}^{*0} + \text{c.c.} \to J/\psi\rho)$. The quantity $|\text{Im}\mathcal{A}|^2$ can be understood as the probability of finding the final state $J/\psi\rho$ through rescattering of the *real* $D^0\bar{D}^{*0}$ + c.c. pair per unit of final-state phase space. Then Im$\mathcal{A}$ is proportional to the phase-space factor of $X \to D^0\bar{D}^{*0}$ + c.c., which is small or even zero, since the mass of X is taken to be very close but above the $D^0\bar{D}^{*0}$ threshold. As a consequence, the value of $R_{\rho/DD\pi}$ given in Ref. [24] is small. Furthermore, since the charged channel $D^+\bar{D}^{*-}$ + c.c. is forbidden by phase space, this mechanism will predict almost equal amplitudes for $J/\psi\rho$ and $J/\psi\omega$, and then result in a large $R_{\rho/\omega}$ of order 10 (or even larger) due to the difference between $J/\psi\rho$ and $J/\psi\omega$ phase spaces (see, e.g. [19]).

On the other hand, in contrast to the imaginary part, the real part of the rescattering amplitude, which represents the effects of virtual intermediate states $D\bar{D}^*$, may be dominant in this case, since it does not suffer from the phase-space suppression for producing a real $D\bar{D}^*$ pair. Moreover, the real part may also give a moderate value of $R_{\rho/\omega}$, since both the neutral channel $D^0\bar{D}^{*0}$ + c.c. and the charged channel $D^+\bar{D}^{*-}$ + c.c. can contribute through these virtual effects.

In this paper, we reexplore the rescattering mechanism and evaluate the real part of the amplitude as well as the imaginary one. With a reasonable choice for the phenomenological parameters, we find that the experimental data in (5) and (6) can be explained quite well if X(3872) is a 2P charmonium state $\chi_{c1}(2P)$ lying below the threshold of $D^0\bar{D}^{*0}$.

II. THE MODEL

A. $X \to J/\psi\rho(\omega)$

In the rescattering mechanism, the decay $X \to J/\psi\rho(\omega)$ can arise from the exchange of a $D^{(*)}$ meson between $D(\bar{D})$ and $\bar{D}^*(D^*)$. The Feynman diagrams for $X \to D^0\bar{D}^{*0}$ + c.c. $\to J/\psi\rho(\omega)$ are shown in Fig. 1, and those involving charged intermediate states can be easily obtained through replacements of $D^0\bar{D}^{*0}$ by D^+D^{*-} and $\bar{D}^0D^{*0}$ by D^-D^{*+} in Fig. 1. Since m_X is very close to the threshold $M_{D\bar{D}^*}$ and the X(3872) couples to $D\bar{D}^*$ in an S wave, we assume that the rescattering contributions are dominated by the intermediate states $D\bar{D}^*$, and those arising from higher exited D meson states are neglected.

Assume that X(3872) is the χ_{c1}(2P) state, and all the vertexes in Fig. 1 are determined by the effective Lagrangians, which are constructed based on the chiral and heavy quark spin symmetries and parity conservation (for a review, see Ref. [25]). These Lagrangians read [26,27] (for convenience here we use the same notations

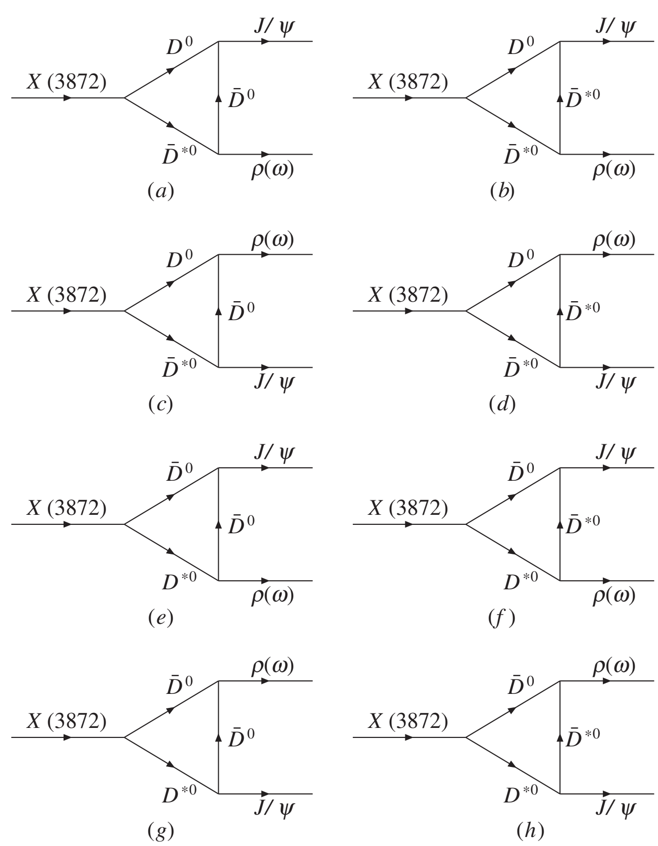

FIG. 1. The decay diagrams for X(3872) $\to \bar{D}^{*0}D^0$ + c.c. $\to J/\psi\rho(\omega)$.

and symbols as in Ref. [24]):

$$\mathcal{L}_X = g_X X^\mu (D D_\mu^{*\dagger} - D^\dagger D_\mu^*), \tag{7a}$$

$$\mathcal{L}_{\psi DD} = i g_{\psi DD} \psi_\mu (\partial^\mu D D^\dagger - D \partial^\mu D^\dagger), \tag{7b}$$

$$\mathcal{L}_{\psi D^* D} = -g_{\psi D^* D} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu \psi_\nu (\partial_\alpha D_\beta^* D^\dagger + D \partial_\alpha D_\beta^{*\dagger}), \tag{7c}$$

$$\mathcal{L}_{\psi D^* D^*} = -i g_{\psi D^* D^*} \{\psi^\mu (\partial_\mu D^{*\nu} D_\nu^{*\dagger} - D^{*\nu} \partial_\mu D_\nu^{*\dagger}) + \psi^\nu D^{*\mu} \partial_\mu D_\nu^{*\dagger} - \psi_\nu \partial_\mu D^{*\nu} D^{*\mu\dagger}\}, \tag{7d}$$

$$\mathcal{L}_{DDV} = -i g_{DDV} D_i^\dagger \overleftrightarrow{\partial}_\mu D^j (\mathbb{V}^\mu)^i_j, \tag{7e}$$

$$\mathcal{L}_{D^* DV} = -2 f_{D^* DV} \varepsilon_{\mu\nu\alpha\beta} (\partial^\mu \mathbb{V}^\nu)^i_j (D_i^\dagger \overleftrightarrow{\partial}^\alpha D^{*\beta j} - D_i^{*\beta\dagger} \overleftrightarrow{\partial}^\alpha D^j), \tag{7f}$$

$$\mathcal{L}_{D^* D^* V} = +i g_{D^* D^* V} D_i^{*\nu\dagger} \overleftrightarrow{\partial}_\mu D_\nu^{*j} (\mathbb{V}^\mu)^i_j + 4 i f_{D^* D^* V} D_{i\mu}^{*\dagger} (\partial^\mu \mathbb{V}^\nu - \partial^\nu \mathbb{V}^\mu)^i_j D_\nu^{*j}, \tag{7g}$$

where the indexes i, j in (7e)–(7g) represent the flavors of light quarks, i.e. $D^{(*)} = (\bar{D}^{(*)0}, D^{(*)-}, D_s^{(*)-})^T$, and they are hidden in (7a)–(7d). $\mathbb{V}$ is the 3×3 matrix for the nonet vector meson,

$$\mathbb{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{8}$$

We assume that chiral symmetry is preserved in (7). That is, the coupling constants are blind to the flavor and there are no isospin violations at the Lagrangian level. All the coupling constants will be determined in the next section. However, it is necessary to emphasize here that the determinations will not account for the off-shell effect of the exchanged $D(D^*)$ meson, of which the virtuality cannot be ignored. As shown in Ref. [24], such effects can be accounted for by introducing, e.g., the monopole form factors for off-shell vertexes. Let q denote the momentum transferred and m_i the mass of exchanged meson, the form factor can be written as [24,26]

$$\mathcal{F}(m_i, q^2) = \left(\frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2}\right), \tag{9}$$

and the cutoff Λ can be parametrized as

$$\Lambda(m_i) = m_i + \alpha \Lambda_{\mathrm{QCD}}. \tag{10}$$

We are now in a position to compute the diagrams in Fig. 1. If the X(3872) lies above the $D^0\bar{D}^{*0}$ threshold, in the process $X(p_X, \epsilon_X) \to D^0(p_1) + \bar{D}^{*0}(p_2, \epsilon_2) \to J/\psi(p_3, \epsilon_3) + \rho(\omega)(p_4, \epsilon_4)$, where the momenta p and polarization vectors ϵ are denoted explicitly for the mesons, we can calculate the absorptive part (imaginary part) of Fig. 1(a) and find it to be given by

$$\mathbf{Abs}(a_n) = \frac{|\vec{p}_1|}{32\pi^2 m_X} \int d\Omega \mathcal{A}(X \to D^0 \bar{D}^{*0}) \times \mathcal{A}_a(D^0 \bar{D}^{*0} \to J/\psi \rho(\omega)), \tag{11}$$

where $\vec{p}_1$ is the 3-momentum of the on-shell D^0 meson in the rest frame of X(3872) and the subindex "n" denotes the contribution coming from the neutral channel (for the charged channel we use "c"). Analogous expressions can be found for diagrams 1(b)–1(d), and the absorptive parts of diagrams 1(e)–1(h) are the same as those of diagrams 1(a)–1(d), respectively. Explicitly, the absorptive parts of diagrams 1(a)–1(d) are given by

$$\mathbf{Abs}(a_n) = -\frac{|\vec{p}_1|}{32\pi^2 m_X} \int d\Omega (4\sqrt{2} g_X g_{\psi DD} f_{D^* DV}) \frac{\mathcal{F}^2(m_1, q^2)}{q^2 - m_1^2} (p_1 \cdot \epsilon_3^*) \varepsilon_{\mu\nu\alpha\beta} p_4^\mu \epsilon_4^{*\nu} p_2^\alpha \epsilon_X^\beta,$$

$$\mathbf{Abs}(b_n) = \frac{|\vec{p}_1|}{32\pi^2 m_X} \int d\Omega (\sqrt{2} g_X g_{\psi D^* D} g_{D^* D^* V}) \frac{\mathcal{F}^2(m_2, q^2)}{q^2 - m_2^2} \varepsilon_{\mu\nu\alpha\beta} p_3^\mu \epsilon_3^{*\nu} p_1^\alpha \Big\{ (p_2 \cdot \epsilon_4^*) \Big[\epsilon_X^\beta - \frac{p_2 \cdot \epsilon_X}{m_2^2} p_2^\beta \Big] + 2r\Big[p_4 \cdot \epsilon_X - \frac{(p_2 \cdot \epsilon_X)(p_2 \cdot p_4)}{m_2^2} \Big] \epsilon_4^{*\beta} - 2r \Big[\epsilon_4^* \cdot \epsilon_X - \frac{(p_2 \cdot \epsilon_X)(p_2 \cdot \epsilon_4^*)}{m_2^2} \Big] p_4^\beta \Big\},$$

$$\mathbf{Abs}(c_n) = \frac{|\vec{p}_1|}{32\pi^2 m_X} \int d\Omega (\sqrt{2} g_X g_{\psi D^* D} g_{DDV}) \frac{\mathcal{F}^2(m_1, q'^2)}{q'^2 - m_1^2} (p_1 \cdot \epsilon_4^*) \varepsilon_{\mu\nu\alpha\beta} p_3^\mu \epsilon_3^{*\nu} p_2^\alpha \epsilon_X^\beta,$$

$$\mathbf{Abs}(d_n) = -\frac{|\vec{p}_1|}{32\pi^2 m_X} \int d\Omega (2\sqrt{2} g_X g_{\psi D^* D^*} f_{D^* DV}) \frac{\mathcal{F}^2(m_2, q'^2)}{q'^2 - m_2^2} \varepsilon_{\mu\nu\alpha\beta} p_4^\mu \epsilon_4^{*\nu} p_1^\alpha \Big\{ 2(p_2 \cdot \epsilon_3^*) \epsilon_X^\beta + \Big[p_3 \cdot \epsilon_X - \frac{(p_2 \cdot \epsilon_X)(p_2 \cdot p_3)}{m_2^2} \Big] \epsilon_3^{*\beta} - \Big[(\epsilon_3^* \cdot \epsilon_X) + \frac{(p_2 \cdot \epsilon_X)(p_2 \cdot \epsilon_3^*)}{m_2^2} \Big] p_3^\beta \Big\}, \tag{12}$$

where the ratio $r = f_{D^*DV}/g_{D^*DV}$ and the momentum transferred $q = p_3 - p_1$, $q' = p_4 - p_1$. The imaginary parts of the charged channel amplitudes are the same as (12) for $X \to J/\psi\omega$ and of opposite signs for $X \to J/\psi\rho$. The total absorptive part of the neutral (charged) channel amplitude can be obtained by a simple summation and read

$$\mathbf{Abs}_{n(c)} = 2[\mathbf{Abs}(a_{n(c)}) + \mathbf{Abs}(b_{n(c)}) + \mathbf{Abs}(c_{n(c)}) + \mathbf{Abs}(d_{n(c)})], \tag{13}$$

where the factor "2" comes from the equality of contributions from diagrams 1(a)–1(d) and 1(e)–1(h).

The amplitudes in (12) are almost equal to those given in Ref. [24] except that some minor errors in Ref. [24] have been corrected. All these amplitudes are proportional to the phase-space factor

$$\frac{|\vec{p}_1|}{m_X} = \frac{\sqrt{(m_X^2 - (m_1 + m_2)^2)(m_X^2 - (m_1 - m_2)^2)}}{2m_X^2} \simeq \sqrt{\frac{m_X - (m_1 + m_2)}{2m_X}}, \tag{14}$$

which is very small even if X(3872) is above the $D^0\bar{D}^{*0}$ threshold.

In the case that X(3872) lies below the $D^0\bar{D}^{*0}$ threshold, the absorptive part (imaginary part) vanishes, and the dispersive part (real part) of the rescattering amplitudes will play the role in the decay.

The dispersive part of the rescattering amplitude can be obtained from $\mathbf{Abs}_n$ and $\mathbf{Abs}_c$ via the dispersion relation [19,26]

$$\mathbf{D\,is}(m_X^2) = \frac{1}{\pi}\left(\int_{\mathrm{th}_n^2}^{\infty} \frac{\mathbf{Abs}_n(s')}{s' - m_X^2} ds' + \int_{\mathrm{th}_c^2}^{\infty} \frac{\mathbf{Abs}_c(s')}{s' - m_X^2} ds'\right), \tag{15}$$

where $\mathrm{th}_n = m_{D^0} + m_{D^{*0}}$ and $\mathrm{th}_c = m_{D^\pm} + m_{D^{*\mp}}$ are the thresholds of neutral and charged channels, respectively, and the contributions arising from higher channels are neglected in (15) as we have mentioned before. Unlike the absorptive part, the dispersive contribution suffers from the large uncertainties arising from the complicated integrations in (15). Since the absorptive parts $\mathbf{Abs}_{n(c)}(s)$ fall off as s increases, it is reasonable to choose a cutoff for the integration to make a numerical estimation. Following Ref. [19], we choose the cutoff around $s_{\max} = 4m_{D^{*0}}^2$, which can shut the windows of higher channels automatically.

It is worth emphasizing again that for $X \to J/\psi\omega$ the contributions from neutral and charged channels are nearly equal and share the same sign, while for $X \to J/\psi\rho$ they almost cancel each other. This is not surprising since the explicit chiral symmetry is maintained in the effective Lagrangians in (7). So if we neglect the absorptive part, the isospin violation, which is mainly due to the difference between th_n and th_c and that between the thresholds of $J/\psi\rho$ and $J/\psi\omega$, seems to be too small to account for the experimental data in (5). However, the large difference between the phase spaces of $X \to J/\psi\rho$ and $X \to J/\psi\omega$ due to the large width of ρ resonance may result in a favorable prediction for $R_{\rho/\omega}$. To achieve this, we smear the width $\Gamma^0_{\psi\rho(\omega)}(t)$, which is obtained through the rescattering amplitude in the narrow width approximation (NWA), over the variable $t = m_4^2$ by the Breit-Wigner distribution as

$$\Gamma(X \to \rho(\pi^+\pi^-)J/\psi) = \frac{1}{\pi}\int_{m_{2\pi}}^{(m_X - m_3)^2} \frac{\Gamma^0_{\psi\rho}(t) m_\rho \Gamma_\rho}{(t - m_\rho^2)^2 + m_\rho^2\Gamma_\rho^2} dt,$$
$$\Gamma(X \to \omega(\pi^+\pi^-\pi^0)J/\psi) = \frac{1}{\pi}\int_{m_{3\pi}}^{(m_X - m_3)^2} \frac{\Gamma^0_{\psi\omega}(t) m_\omega \Gamma_\omega}{(t - m_\omega^2)^2 + m_\omega^2\Gamma_\omega^2} dt, \tag{16}$$

where $m_{2\pi}$ and $m_{3\pi}$ are the experimental cutoffs on the $(\pi^+\pi^-)$ and the $(\pi^+\pi^-\pi^0)$ invariant masses, respectively, and $\Gamma_{\rho(\omega)}$ denotes the total width of $\rho(\omega)$.

B. $X \to D^0\bar{D}^0\pi^0$

If X(3872) lies above the $D^0\bar{D}^{*0}$ threshold, the width of $X \to D^0\bar{D}^0\pi^0$ can be given by

$$\Gamma(X \to D^0\bar{D}^0\pi^0) = 2\Gamma(X \to D^0\bar{D}^{*0})\mathrm{Br}(\bar{D}^{*0} \to \bar{D}^0\pi^0), \tag{17}$$

where the branching ratio $\mathrm{Br}(\bar{D}^{*0} \to \bar{D}^0\pi^0)$ is known [6] and the width $\Gamma(X \to D^0\bar{D}^{*0})$ can be easily obtained from $\mathcal{L}_X$ in the NWA:

$$\Gamma(X \to D^0\bar{D}^{*0}) = \frac{g_X^2|\vec{p}_1|}{24\pi m_X^2}\left(3 + \frac{|\vec{p}_1|^2}{m_{D^{*0}}^2}\right) \simeq \frac{g_X^2|\vec{p}_1|}{8\pi m_X^2}, \tag{18}$$

where the 3-momentum $\vec{p}_1$ is the same as in (11).

On the other hand, when m_X is below the threshold th_n, it can decay to $D^0\bar{D}^0\pi^0$ through virtual $D^{*0}(\bar{D}^{*0})$ as illustrated in Fig. 2 [28]. Here, we need another effective Lagrangian to describe the $D^0\bar{D}^{*0}\pi^0$ coupling [25,28]:

$$\mathcal{L}_{D^*D\pi} = i\frac{g_{D^*D\pi}}{\sqrt{2}}(D^{*0}_\mu\partial^\mu\pi^0\bar{D}^0 - D^0\partial^\mu\pi^0\bar{D}^{*0}_\mu). \tag{19}$$

Then the amplitude for $X(p_X, \epsilon_X) \to D^0\bar{D}^0\pi^0(k_3)$ reads

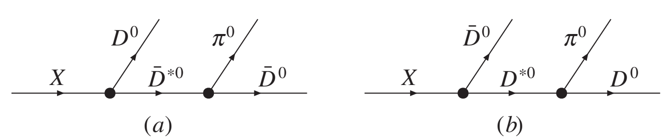

FIG. 2. The diagrams for X(3872) $\to D^0\bar{D}^0\pi$.

$$
\begin{aligned}
i\mathcal{M} &= i(\mathcal{M}_a + \mathcal{M}_b) \\
&= \frac{i\sqrt{2}g_X g_{D^*D\pi}}{q^2 - m_{D^{*0}}^2 + im_{D^{*0}}\Gamma(D^{*0})}\left[\frac{(q\cdot k_3)(q\cdot \epsilon_X)}{m_{D^{*0}}^2} \right. \\
&\quad \left. - (k_3\cdot\epsilon_X)\right], \qquad (20)
\end{aligned}
$$

where $q = p_X - p_1$ is the momentum transferred and $\Gamma(D^{*0})$ is the total width of D^{*0}. It can be verified that the amplitude $\mathcal{M}$ generates the same width as that given in (17) in the limit $m_X - \mathrm{th}_n \gg \Gamma(D^{*0})$. However, the validity of (18) and (20) are questionable in the near-threshold region where $|m_X - \mathrm{th}_n| \approx \Gamma(D^{*0})$ since the perturbation calculations are known to be invalid in this region.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Parameter determinations

Since the numerical results are indeed sensitive to some of the parameters introduced above, we need to explain how we determine these parameters.

The coupling constants in Eqs. (7e)–(7g) are universal for ρ and ω. They can be related to the standard parameters in the so-called heavy meson chiral Lagrangian [25] through the relations [24,26]

$$g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}}, \qquad f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}}.$$

The values of g_V, β, and λ used here are the same as in Ref. [24] and are listed in Table I. Similarly, the coupling constant in (19) can be related to the well-known parameter g [25,28] through the relation $g_{D^*D\pi} = 2\sqrt{m_D m_{D^*}}g/f_\pi$, where f_π is the decay constant of π and the value of g can be determined by the measurement of the width of D^{*+} [28]. As a byproduct, we can estimate the total width $\Gamma(D^{*0}) \approx 0.07$ MeV by using the value of g listed in Table I together with the branching ratio $\mathrm{Br}(D^{*0} \to D^0\pi^0) = (61.9 \pm 2.9)\%$ [6].

The coupling constant $g_{\psi DD}$ in (7b) can be estimated by the vector meson dominance (VMD) mechanism [27,29]. Considering the matrix element $\langle D|\bar{c}\gamma_\mu c|D\rangle$, it can be represented in Fig. 3 in the VMD mechanism, where the circle vertex is determined by Eq. (7b) and the box vertex is related to the J/ψ decay constant f_ψ through the matrix element $\langle 0|\bar{c}\gamma_\mu c|J/\psi(p,\epsilon)\rangle = f_\psi m_\psi \epsilon_\mu$. Then at the normalization point, where the initial and final D mesons have the same 4-velocities, we can determine $g_{\psi DD} = m_\psi/f_\psi \simeq$ 8, which is consistent with the prediction of QCD sum rules [30]. Other coupling constants in (7c) and (7d) can be estimated through heavy quark symmetry relations: $g_{\psi D^*D^*} = m_D g_{\psi D^*D} = g_{\psi DD}$ [27]. One should notice that the use of VMD here does not mean that all higher resonances give contributions far smaller than those from J/ψ, but it lies on the argument that these contributions tend to cancel [27,31]. For example, the analogous effective coupling constant governing $\psi(3770) \to D\bar{D}$ decay is about 3 times larger than $g_{\psi DD}$ [28].

TABLE I. Parameters used in the calculations.

g_V	β	λ	g	$g_{\psi DD}$
5.9	0.9	0.56 GeV^{-1}	0.6	8
$g_{\psi D^*D}$	$g_{\psi D^*D^*}$	f_π	Λ_{QCD}	α
4.3 GeV^{-1}	8	132 MeV	220 MeV	4

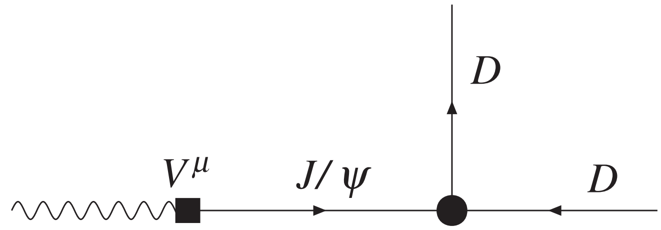

FIG. 3. The diagram for calculating the matrix element $\langle D|\bar{c}\gamma_\mu c|D\rangle$ in the VMD mechanism.

The coupling g_X is not involved in the ratios $R_{\rho/\omega}$ and $R_{\rho/DD\pi}$. However, g_X is important for determining the decay widths and clarifying the properties of X(3872). Assuming that X(3872) is a pure charmonium 2P state $\chi_{c1}(2\mathrm{P})$, we parametrize $g_X = 2\sqrt{2m_{D^0}m_{D^{*0}}m_X}g_1(2\mathrm{P})$, where $g_1(2\mathrm{P})$ is the coupling constant governing the interactions of 2P charmonium states with $D^{(*)}\bar{D}^{(*)}$ [28]. In Ref. [32], the 1P partner of $g_1(2\mathrm{P})$ is estimated in a similar way to that for $g_{\psi DD}$. One only needs to replace the vector current $V^\mu = \bar{c}\gamma_\mu c$ by the scalar one $S = \bar{c}c$, and the J/ψ by the χ_{c0} in Fig. 3, and the result is [32]

$$g_1(1\mathrm{P}) = \sqrt{\frac{m_{\chi_{c0}}}{3}}\frac{1}{f_{\chi_{c0}}}, \qquad (21)$$

where the decay constant $f_{\chi_{c0}}$ is defined by $\langle 0|\bar{c}c|\chi_{c0}(p)\rangle = f_{\chi_{c0}}m_{\chi_{c0}}$. Using $f_{\chi_{c0}} = (510 \pm 40)$ MeV estimated by the sum rule analysis [32], one can get $g_1(1\mathrm{P}) \approx 2.1\ \mathrm{GeV}^{-1/2}$. As we have mentioned, for the charm systems $g_1(2\mathrm{P})$ should be of the same order as $g_1(1\mathrm{P})$. Then, the value of g_X can be estimated through $g_X \approx 2\sqrt{2m_{D^0}m_{D^{*0}}m_X}g_1(1\mathrm{P}) \approx 23$ GeV. On the other hand, the effective interactions between charmonium and $D^{(*)}\bar{D}^{(*)}$ can also be estimated by the quark pair creation models [33]. From available calculations in Refs. [20,21] together with Eq. (17), we can deduce the effective coupling at the hadronic level $g_X \simeq 8$–15 GeV when $\delta m_X = m_X - \mathrm{th}_n$ varying from 80 MeV to 0.5 MeV. Based on the two estimates mentioned above, we will choose $g_X = 20$ GeV in our calculations. This should be a reasonable choice for the coupling which describes the $\chi_{c1}(2\mathrm{P})$ decay to $D\bar{D}^*$.

Since the virtuality of the exchanged meson in Fig. 1 is always larger than 1 GeV^2, the amplitudes in (12) are sensitive to α when $\alpha < 3$. The authors of Ref. [24] choose $\alpha = 0.5$–3.0. In Ref. [32], it is argued that the value of Λ in (9) can be around 3 GeV, which corresponds to $\alpha \approx 5$. In our calculations we choose $\alpha = 4$.

For the charm meson masses we take $m_{D^0} = 1864.847 \pm 0.178$ MeV [13] and $m_{D^{*0}} - m_{D^0} = 142.12 \pm 0.07$ MeV [6], so the threshold $\mathrm{th}_n = 3871.81$ MeV. For other mass and width parameters, we refer them to PDG2006 [6]. The cutoffs on dipion and tripion invariant masses in (15) are taken to be the same as in the Belle experiments [2,7]:

$$m_{2\pi} = 400 \text{ MeV}, \qquad m_{3\pi} = 750 \text{ MeV}. \qquad (22)$$

B. Numerical analysis

Our numerical results for the m_X-dependence of $R_{\rho/\omega}$ and $R_{\rho/DD\pi}$ are illustrated in Fig. 4. In the below-threshold region where $m_X = 3870.8$–3871.8 MeV, Eq. (20) is used to deduce the width $\Gamma_{DD\pi}$. For the region above the threshold th_n with $m_X = 3871.9$–3874.5 MeV, we use Eqs. (17) and (18) to calculate $\Gamma_{DD\pi}$. The contribution from the absorptive part of the rescattering amplitude is not involved in Fig. 4, since it is numerically far smaller than that from the dispersive part even after the phase-space smearing in (16). For comparison, we also choose the naive cutoffs $m_{2\pi} = 2m_\pi$ and $m_{3\pi} = 3m_\pi$ to evaluate the integrations in (16), and the results are shown in Figs. 4(b) and 4(d). As usual, we use the central values of the parameters given in the last subsection.

From Fig. 4(a) and 4(c), one can see that in both regions of m_X,

$$R_{\rho/\omega} = 1.0 \pm 0.3, \qquad (23)$$

which is consistent with Eq. (5). This result is sensitive to the cutoff $m_{3\pi}$. For example, if we choose the cutoffs given in Figs. 4(b) and 4(d), the width $\Gamma(X \to J/\psi\omega)$ will be enlarged by a factor of 4, and the corresponding value of $R_{\rho/\omega}$ is smaller than 0.4.

Our prediction of $R_{\rho/\omega}$ in (23) is a bit larger than that given in Ref. [19]. It is not due to a larger isospin violation in the dispersive part of the rescattering amplitude. In fact, here the isospin violation in the dispersive part is only about 10%, which is smaller than that expected in Ref. [19] by a factor of 2. The difference is mainly due to the fact that the momentum factors in vertexes $J/\psi D^{(*)}D^{(*)}$ and $D^{(*)}D^{(*)}\rho(\omega)$ in Fig. 1 are not considered in the semiquantitative estimation in Ref. [19]. In fact, the rescattering $D\bar{D}^* \to J/\psi\rho(\omega)$ is a D-wave process, so that the phase-space smearing in (16) is more significant than it is customarily expected.

Furthermore, one can see from Fig. 4(a) that the ratio $R_{\rho/DD\pi}$ is roughly consistent with Eq. (6) except for the very near-threshold region where $m_X = 3871.6$–3871.8 MeV. However, in this region, the width

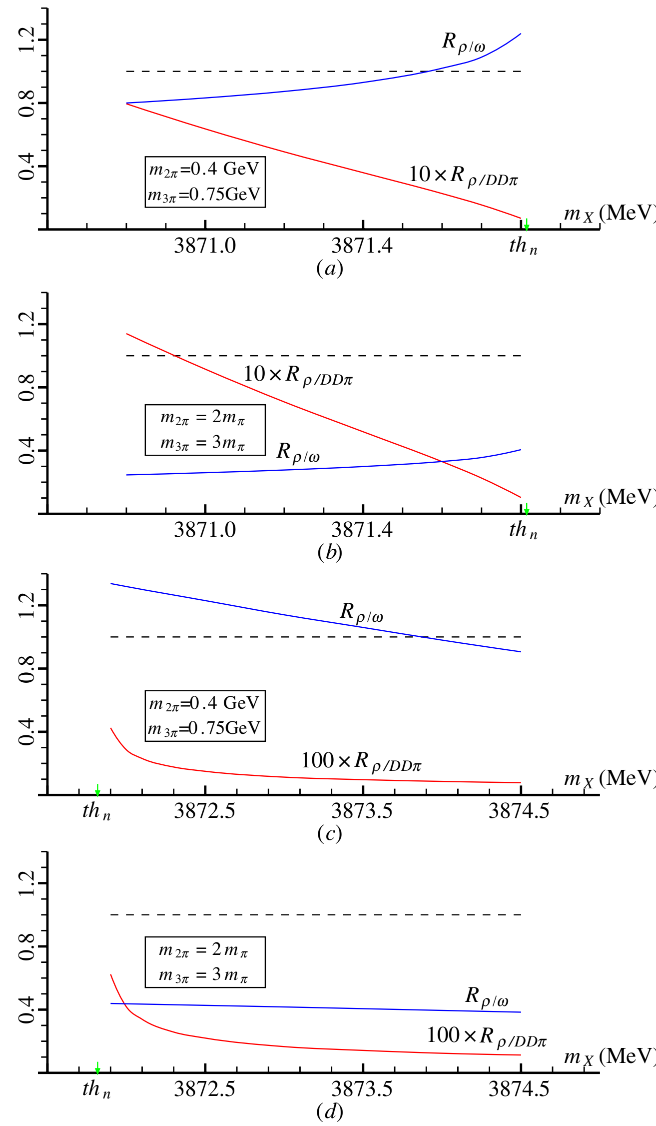

FIG. 4 (color online). The m_X-dependence of $R_{\rho/\omega}$ and $R_{\rho/DD\pi}$. (a,b) for the region $m_X = 3870.8$–3871.8 MeV and (c,d) for $m_X = 3871.9$–3874.5 MeV.

of $X \to D^0\bar{D}^0\pi^0$, which is obtained from Eq. (20), is questionable. Roughly speaking, the charmonium picture of X(3872) is not in serious contradiction with experimental data in (6) if the X(3872) is slightly below the $D^0\bar{D}^{*0}$ threshold, i.e., $m_X < \mathrm{th}_n$.

In the region where $m_X > \mathrm{th}_n$, the pure charmonium picture is disfavored since the prediction of $R_{\rho/DD\pi}$ in Fig. 4(c) is about 2 orders of magnitude smaller than the experimental data in (6). This is due to a rapid increase of the decay rate of $X \to D^0\bar{D}^{*0}$ + c.c. as the mass of X exceeds the $D^0\bar{D}^{*0}$ threshold.

We evaluate the width $\Gamma_{\psi\rho}$ by using $g_X = 20$ GeV, and the result is shown in Fig. 5. One can see that $\Gamma_{\psi\rho} = 35$–70 KeV. Then from the obtained ratios in Fig. 4 at the mass slightly below th_n (say around 3871.2 MeV) we

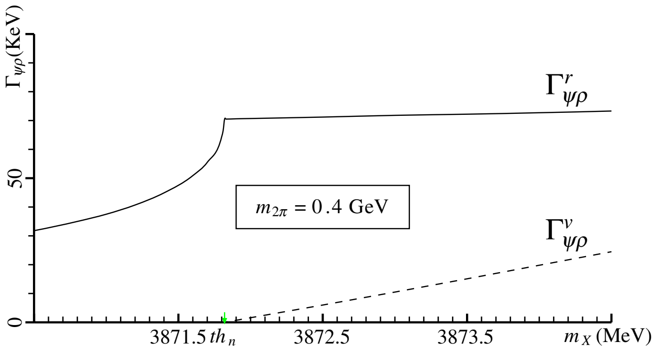

FIG. 5 (color online). The width of $X \to J/\psi\rho$. $\Gamma^r_{\psi\rho}$ arises from the real part of the rescattering amplitude and $\Gamma^v_{\psi\rho}$ from the imaginary part. Here $g_X = 20$ GeV is used.

have $\Gamma(X \to \psi\omega) = 25$–$100$ KeV and $\Gamma(X \to D^0\bar{D}^0\pi^0) = 250$–$1000$ KeV. The width of $X \to D^0\bar{D}^0\gamma$ can be estimated by a similar model shown in Fig. 2 and the value is no more than 300 KeV. Another potential decay mode of the X(3872) is the inclusive light hadron (LH) decay. We can use the available measurement of the width of the 1P state χ_{c1}(1P) [6] to roughly estimate that $\Gamma(X \to \text{LHs}) \simeq \Gamma(\chi_{c1} \to \text{LHs}) \simeq 600$ KeV. The E1 transition width of $\chi_{c1}(2\text{P}) \to \psi(2S)\gamma$ could be in the range 50–80 KeV; and the hadronic transition width of $\chi_{c1}(2\text{P}) \to \chi_{c1}(1\text{P})\pi\pi$ could be 10–100 KeV. Summing up all the estimated decay widths given above, we find that the total width of X(3872) is about 1–2 MeV, which is consistent with the experimental upper limit $\Gamma_X < 2.3$ MeV. Meanwhile, the branching ratio $\text{Br}(X \to J/\psi\rho) = (2$–$7)\%$, which can match the request of the large production rates at B factories and at the Tevatron [16,18,19].

Finally, it is worthwhile to mention that the E1 transition width for $\chi_{c1}(2\text{P}) \to \gamma J/\psi$ can be estimated to be as small as 7–11 KeV with relativistic corrections taken into account [20,23]. Although this $2\text{P} - 1S$ E1 transition rate is sensitive to the model details duo to the node structure of charmonium wave functions, there seems no difficulty in principle to explain the ratio (4).

IV. SUMMARY

In summary, we reexamine the rescattering mechanism for X(3872), as a candidate for the 2P charmonium state χ_{c1}(2P), decaying to $J/\psi\rho(\omega)$ through exchanging $D^{(*)}$ mesons between intermediate states D and $\bar{D}^*$ (or between $\bar{D}$ and D^*). We evaluate the dispersive part, as well as the absorptive one, of the rescattering amplitude, and find that the contribution from the dispersive part is dominant even when X(3872) lies above the threshold of the neutral channel $\text{th}_n = m_{D^0} + m_{D^{*0}}$. We predict $R_{\rho/\omega} \simeq 1$ for the m_X region scanned by experiments. The prediction for $R_{\rho/DD\pi}$ favors $m_X < \text{th}_n$ and disfavors $m_X > \text{th}_n$, since in the latter case the prediction is 2 orders of magnitude smaller than the experimental data due to a much too large decay width into real $D^0\bar{D}^{*0}$ mesons. Whereas when $m_X < \text{th}_n$ the X can decay to $D^0\bar{D}^0\pi^0$ only through a virtual $\bar{D}^{*0}$ and a D^0, and therefore the decay width of $X \to D^0\bar{D}^0\pi^0$ becomes much milder. Furthermore, we evaluated the width of $X \to J/\psi\rho$ with the help of a phenomenological effective coupling constant g_X, which can be estimated from two different ways related to P-wave charmonium decaying into two charmed mesons. We find that the total width of the X(3872) is in the range of 1–2 MeV, and the theoretical results for the four decay channels are roughly consistent with experimental ratios (5) and (6), as well as (4). The remaining problem is how to accept the low mass of X(3872) as the candidate of a χ_{c1}(2P)-dominated state. As shown in e.g. Table I of Ref. [20], the mass splitting between χ_{c1}(2P) and χ_{c2}(2P) is predicted to be about 30 MeV in potential models without the coupled channel effects. Including the mass shifts due to coupled channel effects one finds that the mass of χ_{c1}(2P) could be further lowered by about 30 MeV relative to that of χ_{c2}(2P) and results in a mass difference between χ_{c2}(2P) and χ_{c1}(2P) of about 60 MeV (detailed discussions will be presented in Ref. [23]). The Z(3930) meson observed by Belle has been identified as the χ_{c2}(2P) charmonium [34], and if the above estimated mass splitting makes sense, then the mass of χ_{c1}(2P) will be around 3872 MeV. We will leave the mass issue to be discussed elsewhere. Finally, based on the obtained results, we tend to conclude that a χ_{c1}(2P)-dominated state could be compatible with the observed decays, production in the B decay and at the Fermilab Tevatron, and even the mass of the X(3872). Therefore, aside from the molecule and other interpretations, the χ_{c1}(2P) charmonium-dominated state could still be a possible assignment for the X(3872).

ACKNOWLEDGMENTS

We wish to thank B. Zhang, H. Q. Zheng, and S. L. Zhu for helpful discussions. We also thank G. Bauer for a useful comment on the experimental determination of the X(3872) quantum numbers. This work was supported in part by the National Natural Science Foundation of China (No. 10421503, No. 10675003), the Key Grant Project of Chinese Ministry of Education (No. 305001), and the Research Found for Doctorial Program of Higher Education of China.

[1] E. Swanson, Phys. Rep. **429**, 243 (2006) and references therein.
[2] S. K. Choi *et al.* (Belle Collaboraion), Phys. Rev. Lett. **91**, 262001 (2003).
[3] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **71**, 071103 (2005).
[4] D. Acosta *et al.* (CDFII Collaboration), Phys. Rev. Lett. **93**, 072001 (2004).
[5] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **93**, 162002 (2004).
[6] W. M. Yao *et al.*, J. Phys. G **33**, 1 (2006).
[7] K. Abe *et al.* (Belle Collaboraion), arXiv:hep-ex/0505037.
[8] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **74**, 071101 (2006).
[9] K. Abe *et al.* (Belle Collaboraion), arXiv:hep-ex/0505038.
[10] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **98**, 132002 (2007).
[11] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **96**, 102002 (2006).
[12] G. Gokhroo *et al.* (Belle Collaboration), Phys. Rev. Lett. **97**, 162002 (2006).
[13] C. Cawlfield *et al.* (CLEO collaboration), Phys. Rev. Lett. **98**, 092002 (2007).
[14] N. A. Tornqvist, Phys. Lett. B **590**, 209 (2004); F. Close and P. Page, Phys. Lett. B **578**, 119 (2004); C. Y. Wong, Phys. Rev. C **69**, 055202 (2004); M. B. Voloshin, Phys. Lett. B **604**, 69 (2004); M. T. AlFiky *et al.*, Phys. Lett. B **640**, 238 (2006); S. Fleming *et al.*, arXiv:hep-ph/0703168.
[15] E. S. Swanson, Phys. Lett. B **588**, 189 (2004); **598**, 197 (2004).
[16] G. Bauer, Int. J. Mod. Phys. A **21**, 959 (2006).
[17] E. Braaten, M. Kusunoki, and S. Nussinov, Phys. Rev. Lett. **93**, 162001 (2004); E. Braaten and M. Kusunoki, Phys. Rev. D **71**, 074005 (2005).
[18] C. Meng, Y. J. Gao, and K. T. Chao, arXiv:hep-ph/0506222.
[19] M. Suzuki, Phys. Rev. D **72**, 114013 (2005).
[20] T. Barnes and S. Godfrey, Phys. Rev. D **69**, 054008 (2004); T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[21] E. J. Eichten, K. Lane, and C. Quigg, Phys. Rev. D **69**, 094019 (2004).
[22] Y. Chen *et al.* (CLQCD collaboration), arXiV:hep-lat/0701021.
[23] B. Q. Li and K. T. Chao (unpublished).
[24] X. Liu, B. Zhang, and S. L. Zhu, Phys. Lett. B **645**, 185 (2007).
[25] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[26] H. Y. Cheng, C. K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005).
[27] A. Deandrea, G. Nardulli, and A. D. Polosa, Phys. Rev. D **68**, 034002 (2003).
[28] P. Colangelo, F. De Fazio, and S. Nicotri, arXiv:hep-ph/0701052.
[29] N. N. Achasov and A. A. Kozhevnikov, Phys. Rev. D **49**, 275 (1994).
[30] R. D. Matheus *et al.*, Phys. Lett. B **541**, 265 (2002).
[31] P. Cea and G. Nardulli, Phys. Rev. D **34**, 1863 (1986).
[32] P. Colangelo, F. De Fazio, and T. N. Pham, Phys. Lett. B **542**, 71 (2002); Phys. Rev. D **69**, 054023 (2004).
[33] A. Le Yaouanc *et al.*, Phys. Rev. D **8**, 2223 (1973); E. Eichten *et al.*, Phys. Rev. D **17**, 3090 (1978).
[34] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 082003 (2006).