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# Probing deviations from tribimaximal mixing through ultrahigh energy neutrino signals

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We investigate deviation from the tribimaximal mixing in the case of ultrahigh energy neutrino using the ICECUBE detector. We consider the ratio of the number of muon tracks to the shower generated due to eletrons and hadrons. Our analysis shows that for tribimaximal mixing the ratio comes out around 4.05. Keeping $\theta_{12}$ and $\theta_{23}$ fixed at tribimaximal value, we have varied the angle $\theta_{13} = 3°, 6°, 9°$ and the value of the ratio gradually decreases. The variation of ratio lies within 8% to 18% from the tribimaximal mixing value and it is very difficult to detect such a small variation by the ICECUBE detector.

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## I. INTRODUCTION

Various experiments for solar and atmospheric neutrinos provide a range for the values of the solar mixing angle $\theta_{\odot} = \theta_{12}$ (the 1-2 mixing angle) [1] that corresponds to solar neutrino oscillations and also a range for the atmospheric mixing angle $\theta_{\rm atm} = \theta_{23}$ (the 2-3 mixing angle) [1] around their best-fit values. The tribimaximal mixing condition of neutrinos are given by $\sin\theta_{12} = \frac{1}{\sqrt{3}}$, $\sin\theta_{23} = \frac{1}{\sqrt{2}}$, and $\sin\theta_{13} = 0$ [2]. Possible deviations from tribimaximal mixing can be obtained by probing the ranges of $\theta_{12}$ and $\theta_{23}$ given by the experiments. Also the exact 13 mixing angle $\theta_{13}$ is not known except that the CHOOZ [3] gives an upper limit for $\theta_{13}(<9°)$. Probing the deviations of $\theta_{12}$ and $\theta_{23}$ for different values for $\theta_{13}$ is significant not only to understand the neutrino flavor oscillations in general but also for the purpose of model building for neutrino mass matrices.

In this work we explore the possibility for ultrahigh energy (UHE) neutrinos from distant gamma ray bursts (GRBs) for probing the signatures of these deviations of the values of the mixing angles from tribimaximal mixing as discussed above. One such proposition of using UHE neutrinos is described in a recent work by Xing [4]. Gamma ray bursts are short lived but intense bursts of gamma rays. During its occurrence it outshines all other luminous objects in the sky. Although the exact mechanism of GRBs could not be ascertained so far, the general wisdom is that it is powered by a central engine provided by a failed star or supernova that possibly turned into a black hole, and accretes mass at its surroundings. This infalling mass due to gravity bounces back from the surface of the black hole much the same way as the supernova explosion mechanism and a shock is generated that flows radially outwards with enormous amount of energies ( $\sim 10^{53}$ ergs). This highly energetic shock wave drives the mass outwards, in the form of a ''fireball'' that carries in it, protons, $\gamma$, etc. The pions are produced when the accelerated protons inside the fireball interacts with $\gamma$ through a cosmic beam dump process. UHE neutrinos are produced by the decay of these pions. Thus a generic cosmic accelerator accelerates the protons into very high energies which then beam dump on $\gamma$ in the fireball as at the cosmic microwave background (CMB), and ultrahigh energy neutrinos are produced.

The GRB neutrinos, due to their origin at astronomical distances from Earth, provide a very long baseline for the Earth-bound detectors for UHE neutrinos such as ICECUBE [5]. The oscillatory part of the neutrino flavor oscillation probabilities ($\sin^2(\Delta m^2[L/4E])$) averages out to $1/2$ because of this very long baseline $L$ ($\sim$ hundreds of Mpc) and the $\Delta m^2$ (mass square difference of two neutrinos) range obtained from solar and atmospheric neutrino experiments are $\Delta m^2_{21} \sim 10^{-4}$ eV$^2$ and $\Delta m^2_{32} \sim 10^{-3}$ eV$^2$, respectively ($L/\Delta m^2 \gg 1$). Thus for neutrino flavor oscillation, in this case, the effect of $\Delta m^2$ is washed out and governed only by the three mixing angles namely $\theta_{12} = \theta_{\odot}$, $\theta_{23} = \theta_{\rm atm}$, and $\theta_{13}$. The purpose of the present work is to probe whether or not the possible variations of $\theta_{12}$ and $\theta_{23}$ from their best-fit values can be ascertained by UHE from distant GRBs.

The GRB neutrinos, on arriving at Earth, undergo charged current (CC) and neutral current (NC) interactions with the Earth rock and the detector material. The CC interactions of $\nu_\mu$ produce secondary muons and the electrons produce electromagnetic shower ($\nu_\mu + N \rightarrow \mu + X$ and $\nu_e + N \rightarrow e + X$). The former will produce secondary muon tracks and can be detected by a track-signal produced by the Cerenkov light emitted by these muons during their passage through large underground water/ice Cerenkov detectors like ICECUBE. The ICECUBE is a 1 km$^3$ detector in south pole ice and can be considered to be immersed in the target material for the UHE neutrinos where the neutrino interactions are initiated. In case of $\nu_e$, the electrons from the $\nu_e N$ CC interactions shower quickly and can also be detected by the ICECUBE detector. The case of $\nu_\tau$ is somewhat complicated. The first CC interaction of $\nu_\tau$ ($\nu_\tau + N \rightarrow \tau + X$) produces a shower (''first bang'') along with a $\tau$ track. But the $\nu_\tau$ is regenerated (with diminished energy) by the decay of $\tau$ and in the process produces another hadronic or electromagnetic shower (''second bang''). The whole process is called a double bang event. In case the first bang could not be detected, then by possible detection of the second bang (with showers) the $\tau$ track can be reconstructed or identified and this scenario (the $\tau$ track and the second bang) is called the lollipop event. An inverted lollipop event is one where only the first bang ($\nu_\tau + N \to \tau + X$) is detected and the subsequent $\tau$ track is detected or reconstructed. As mentioned in Ref. [6], the detection of $\nu_\tau$ from their CC interaction mentioned above is not very efficient by a 1 km$^3$ detector since the double bang events can possibly be detected only for the $\nu_\tau$ energies between 1 PeV to 20 PeV beyond which the tau decay length is longer than the width of such a detector and at still higher energies the flux is too small for such detectors for their detection. Hence, in the present work we do not consider the events initiated by $\nu_\tau N$ CC interactions. However, for $\nu_\tau$ we consider the process that may yield events higher than the ‘‘double bang’’ events. We consider the decay channel of the $\tau$ lepton [7], obtained from charged current interactions of $\nu_\tau$, where muons are produced ($\nu_\tau \to \tau \to \bar{\nu}_\mu \mu \nu_\tau$) which can then be detected as muon tracks [8] in the ICECUBE detector. The NC interactions of all flavors however will produce the shower events at ICECUBE and they are considered in this investigation.

This paper is organized as follows. In Sec. II we describe the formalism for neutrino fluxes of the three species while reaching the Earth. The nature of the GRB flux taken for the present calculations is also discussed. The flux suffers flavor oscillations while traversing fromthe GRB site to the Earth. The oscillation probabilities are also calculated and the oscillated flux obtained on reaching the Earth is determined. They are given in Sec. II A. We also describe in this section the analytical expressions for the yield of secondary muons and shower events at the ice Cerenkov kilometer square detector like ICECUBE. This is given in Sec. II B. The actual calculations and results are discussed in Sec. III. Finally, in Sec. IV, some discussions and a summary are given.

## II. FORMALISM

### A. GRB neutrinos fluxes

The neutrino production in the GRB is initiated through the process of cosmological beam dump by which highly accelerated protons from the GRB interacts with $\gamma$ to produce pions which in turn decays to produce $\nu_\mu(\bar{\nu}_\mu)$ and $\nu_e(\bar{\nu}_e)$ much the same ways as atmospheric neutrinos are produced. They are produced in the proportion $2\nu_\mu{:}2\bar{\nu}_\mu{:}1\nu_e{:}1\bar{\nu}_e$ [9].

For the present calculation we consider the isotropic flux [10] resulting from the summation over the sources and as given in Gandhi *et al.* [11]. The isotropic GRB flux for $\nu_\mu + \bar{\nu}_\mu$ is given as

$$\mathcal{F}(E_\nu) = \frac{dN_{\nu_\mu+\bar{\nu}_\mu}}{dE_\nu} = \mathcal{N}\left(\frac{E_\nu}{1\ \mathrm{GeV}}\right)^{-n} \mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}\,\mathrm{GeV}^{-1}. \tag{1}$$

In the above,

$$\mathcal{N} = 4.0\times 10^{-13}, \qquad n=1 \quad \text{for } E_\nu < 10^5\ \text{GeV},$$
$$\mathcal{N} = 4.0\times 10^{-8}, \qquad n=2 \quad \text{for } E_\nu > 10^5\ \text{GeV}.$$

Thus,

$$\begin{aligned} \frac{dN_{\nu_\mu}}{dE_\nu} &= \phi_{\nu_\mu} = \frac{dN_{\bar{\nu}_\mu}}{dE_\nu} = \phi_{\bar{\nu}_\mu} = 0.5\mathcal{F}(E_\nu), \\ \frac{dN_{\nu_e}}{dE_\nu} &= \phi_{\nu_e} = \frac{dN_{\bar{\nu}_e}}{dE_\nu} = \phi_{\bar{\nu}_e} = 0.25\mathcal{F}(E_\nu). \end{aligned} \tag{2}$$

The neutrinos undergo flavor oscillation during their passage from the GRB to the Earth. Under three-flavor oscillation, the $\nu_e$ and $\nu_\mu$ originally created at the GRB will be oscillated to $\nu_\tau$. Thus after flavor oscillations, the $\nu_e$ fluxes ($F_{\nu_e}$), $\nu_\mu$ fluxes ($F_{\nu_\mu}$), $\nu_\tau$ fluxes ($F_{\nu_\tau}$) become

$$\begin{aligned} F_{\nu_e} &= P_{\nu_e\to\nu_e}\phi_{\nu_e} + P_{\nu_\mu\to\nu_e}\phi_{\nu_\mu}, \\ F_{\nu_\mu} &= P_{\nu_\mu\to\nu_\mu}\phi_{\nu_\mu} + P_{\nu_e\to\nu_\mu}\phi_{\nu_e}, \\ F_{\nu_\tau} &= P_{\nu_e\to\nu_\tau}\phi_{\nu_e} + P_{\nu_\mu\to\nu_\tau}\phi_{\nu_\mu}. \end{aligned} \tag{3}$$

The transition probability of a neutrino of flavor $\alpha$ to a flavor $\beta$ is given by

$$P_{\nu_\alpha\to\nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha_i}U_{\beta_i}U_{\alpha_j}U_{\beta_j}\sin^2\left(\frac{\pi L}{\lambda_{ij}}\right). \tag{4}$$

In the above oscillation length $\lambda_{ij}$ is given by

$$\lambda_{ij} = 2.47\ \mathrm{Km}\left(\frac{E}{\mathrm{GeV}}\right)\left(\frac{\mathrm{eV}^2}{\Delta m^2}\right). \tag{5}$$

Because of astronomical baseline $\Delta m^2 L/E \gg 1$, the oscillatory part becomes averaged to half. Thus,

$$\left\langle \sin^2\left(\frac{\pi L}{\lambda_{ij}}\right)\right\rangle = \frac{1}{2}. \tag{6}$$

Therefore

$$\begin{aligned} P_{\nu_\alpha\to\nu_\beta} &= \delta_{\alpha\beta} - 2\sum_{j>i} U_{\alpha_i}U_{\beta_i}U_{\alpha_j}U_{\beta_j} \\ &= \delta_{\alpha\beta} - \sum_i U_{\alpha_i}U_{\beta_i}\left[\sum_{j\neq i} U_{\alpha_j}U_{\beta_j}\right] \\ &= \sum_j |U_{\alpha_j}|^2|U_{\beta_j}|^2, \end{aligned} \tag{7}$$

where use has been made of the condition $\sum_i U_{\alpha_i}U_{\beta_i} = \delta_{\alpha\beta}$.

With Eq. (7), Eq. (3) can be rewritten in matrix form

$$\begin{pmatrix} F_{\nu_e} \\ F_{\nu_\mu} \\ F_{\nu_\tau} \end{pmatrix} = \begin{pmatrix} U_{e1}^2 & U_{e2}^2 & U_{e3}^2 \\ U_{\mu 1}^2 & U_{\mu 2}^2 & U_{\mu 3}^2 \\ U_{\tau 1}^2 & U_{\tau 2}^2 & U_{\tau 3}^2 \end{pmatrix} \begin{pmatrix} U_{e1}^2 & U_{\mu 1}^2 & U_{\tau 1}^2 \\ U_{e2}^2 & U_{\mu 2}^2 & U_{\tau 2}^2 \\ U_{e3}^2 & U_{\mu 2}^2 & U_{\tau 3}^2 \end{pmatrix} \times \begin{pmatrix} \phi_{\nu_e} \\ \phi_{\nu_\mu} \\ \phi_{\nu_\tau} \end{pmatrix} = \begin{pmatrix} U_{e1}^2 & U_{e2}^2 & U_{e3}^2 \\ U_{\mu 1}^2 & U_{\mu 2}^2 & U_{\mu 3}^2 \\ U_{\tau 1}^2 & U_{\tau 2}^2 & U_{\tau 3}^2 \end{pmatrix} \begin{pmatrix} U_{e1}^2 & U_{\mu 1}^2 & U_{\tau 1}^2 \\ U_{e2}^2 & U_{\mu 2}^2 & U_{\tau 2}^2 \\ U_{e3}^2 & U_{\mu 2}^2 & U_{\tau 3}^2 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \phi_{\nu_e}. \quad (8)$$

In Eq. (8) above, we have used the initial flux ratio from the GRB to be $\phi_{\nu_e}:\phi_{\nu_\mu}:\phi_{\nu_\tau} = 1:2:0$. From Eq. (8) it then follows that

$$\begin{aligned} F_{\nu_e} &= \{U_{e1}^2[1 + (U_{\mu 1}^2 - U_{\tau 1}^2)] + U_{e2}^2[1 + (U_{\mu 2}^2 - U_{\tau 2}^2)] \\ &\quad + U_{e3}^2[1 + (U_{\mu 3}^2 - U_{\tau 3}^2)]\}\phi_{\nu_e}, \\ F_{\nu_\mu} &= \{U_{\mu 1}^2[1 + (U_{\mu 1}^2 - U_{\tau 1}^2)] + U_{\mu 2}^2[1 + (U_{\mu 2}^2 - U_{\tau 2}^2)] \\ &\quad + U_{\mu 3}^2[1 + (U_{\mu 3}^2 - U_{\tau 3}^2)]\}\phi_{\nu_e}, \\ F_{\nu_\tau} &= \{U_{\tau 1}^2[1 + (U_{\mu 1}^2 - U_{\tau 1}^2)] + U_{\tau 2}^2[1 + (U_{\mu 2}^2 - U_{\tau 2}^2)] \\ &\quad + U_{\tau 3}^2[1 + (U_{\mu 3}^2 - U_{\tau 3}^2)]\}\phi_{\nu_e}. \end{aligned} \quad (9)$$

The Maki-Nakagawa-Sakata (MNS) mixing matrix $U$ for the three-flavor case is given as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{pmatrix}. \quad (10)$$

We are not considering any $CP$ violation here. Hence Eqs. (3)–(9) above also hold for antineutrinos.

### B. Detection of GRB neutrinos

The $\nu_\mu$'s from a GRB can be detected from the tracks of the secondary muons produced through the $\nu_\mu$ CC interactions.

The total number of secondary muons induced by GRB neutrinos at a detector of unit area is given by (following [9,12,13])

$$S = \int_{E_{\rm thr}}^{E_{\nu\,\rm max}} dE_\nu \frac{dN_\nu}{dE_\nu} P_{\rm surv}(E_\nu) P_\mu(E_\nu, E_{\rm thr}). \quad (11)$$

In the above, $P_{\rm surv}$ is the probability that a neutrino reaches the detector without being absorbed by the Earth. This is a function of the neutrino-nucleon interaction length in the Earth and the effective path length $X(\theta_z)$ (gm cm$^{-2}$) for the incident neutrino zenith angle $\theta_z$ ($\theta_z = 0$ for vertically downward entry with respect to the detector). This attenuation of neutrinos due to passage through the Earth is referred to as shadow factor. For an isotropic distribution of flux, this shadow factor (for upward going neutrinos) is given by

$$P_{\rm surv}(E_\nu) = \frac{1}{2\pi} \int_{-1}^{0} d\cos\theta \int d\phi \exp[-X(\theta_z)/L_{\rm int}], \quad (12)$$

where interaction length $L_{\rm int}$ is given by

$$L_{\rm int} = \frac{1}{\sigma^{\rm tot}(E_\nu) N_A}. \quad (13)$$

In the above $N_A (= 6.022 \times 10^{23}\ {\rm gm}^{-1})$ is the Avogadro number and $\sigma^{\rm tot} (= \sigma^{\rm CC} + \sigma^{\rm NC})$ is the total cross section. The effective path length $X(\theta_z)$ is calculated as

$$X(\theta_z) = \int \rho(r(\theta_z, \ell)) d\ell. \quad (14)$$

In Eq. (9), $\rho(r(\theta_z, \ell))$ is the matter density inside the Earth at a distance $r$ from the center of the Earth for neutrino path length $\ell$ entering into the Earth with a zenith angle $\theta_z$. The quantity $P_\mu(E_\nu, E_{\rm thr})$ in Eq. (6) is the probability that a secondary muon is produced by the CC interaction of $\nu_\mu$ and reaches the detector above the threshold energy $E_{\rm thr}$. This is then a function of $\nu_\mu N$ ($N$ represents nucleon)—CC interaction cross section $\sigma^{\rm CC}$ and the range of the muon inside the rock,

$$P_\mu(E_\nu, E_{\rm thr}) = N_A \sigma^{\rm CC} \langle R(E_\nu; E_{\rm thr}) \rangle. \quad (15)$$

In the above $\langle R(E_\nu; E_{\rm thr}) \rangle$ is the average muon range given by

$$\langle R(E_\nu; E_{\rm thr}) \rangle = \frac{1}{\sigma^{\rm CC}} \int_0^{1 - E_{\rm thr}/E_\nu} dy R(E_\nu(1-y), E_{\rm thr}) \times \frac{d\sigma^{\rm CC}(E_\nu, y)}{dy}, \quad (16)$$

where $y = (E_\nu - E_\mu)/E_\nu$ is the fraction of energy loss by a neutrino of energy $E_\nu$ in the charged current production of a secondary muon of energy $E_\mu$. Needless to say, a muon thus produced from a neutrino with energy $E_\nu$ can have the detectable energy range between $E_{\rm thr}$ and $E_\nu$. The range $R(E_\mu, E_{\rm thr})$ for a muon of energy $E_\mu$ is given as

$$R(E_\mu, E_{\rm thr}) = \int_{E_{\rm thr}}^{E_\mu} \frac{dE_\mu}{\langle dE_\mu/dX \rangle} \simeq \frac{1}{\beta} \ln\left(\frac{\alpha + \beta E_\mu}{\alpha + \beta E_{\rm thr}}\right). \quad (17)$$

The average lepton energy loss with energy $E_\mu$ per unit distance travelled is given by [12]

$$\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \beta E_\mu. \quad (18)$$

The values of $\alpha$ and $\beta$ used in the present calculations are

$$\alpha = \{2.033 + 0.077 \ln[E_\mu(\mathrm{GeV})]\} \times 10^{-3}\ \mathrm{GeV\,cm^2\,gm^{-1}},$$
$$\beta = \{2.033 + 0.077 \ln[E_\mu(\mathrm{GeV})]\} \times 10^{-6}\ \mathrm{cm^2\,gm^{-1}} \tag{19}$$

for $E_\mu \lesssim 10^6$ GeV [14] and

$$\alpha = 2.033 \times 10^{-3}\ \mathrm{GeV\,cm^2\,gm^{-1}},\quad \beta = 3.9 \times 10^{-6}\ \mathrm{cm^2\,gm^{-1}} \tag{20}$$

otherwise [15]. For muon events obtained from $\nu_\mu$ CC interactions, $\frac{dN_\nu}{dE_\nu}$ in Eq. (11) will be replaced by $F_{\nu_\mu}$ (Eq. (9)).

As discussed earlier, the events due to $\nu_\tau$ CC interactions are considered only for the process where the decay of the secondary $\tau$ lepton produces muon which then are detected by the muon track. The probability of the production of muons in the decay channel $\tau \to \bar{\nu}_\mu \mu \nu_\tau$ is 0.18 [7,8]. The generated muon carries a fraction 0.3 of energy of original $\nu_\tau$ (a fraction 0.75 of the energy of the $\nu_\tau$ is carried by a secondary $\tau$ lepton and a fraction of 0.4 of the $\tau$ lepton energy is carried by the muon [7,8,12]). For the detection of such muons, the Eqs. (10)–(16) are applicable with properly incorporating the muon energy described above. Needless to say, in this case, $\frac{dN_\nu}{dE_\nu}$ in Eq. (11) is to be replaced by $F_{\nu_\tau}$ (Eq. (9)).

For the case of showers, we do not have the advantage of a specific track and then the whole detector volume is to be considered. The event rate for the shower case is given by

$$N_{\mathrm{sh}} = \int dE_\nu \frac{dN_\nu}{dE_\nu} P_{\mathrm{surv}}(E_\nu) \times \int \frac{1}{\sigma^j} \frac{d\sigma^j}{dy} P_{\mathrm{int}}(E_\nu, y). \tag{21}$$

In the above, $\sigma^j = \sigma^{\mathrm{CC}}$ (for electromagnetic shower from $\nu_e$ charged current interactions) or $\sigma^{\mathrm{NC}}$ as the case may be. In the above $P_{\mathrm{int}}$ is the probability that a shower produced by the neutrino interactions will be detected and is given by

$$P_{\mathrm{int}} = \rho N_A \sigma^j L, \tag{22}$$

where $\rho$ is the density of the detector material and $L$ is the length of the detector ($L = 1$ Km for ICECUBE).

For each case of shower events, $\frac{dN_\nu}{dE_\nu}$ in Eq. (21) is to be replaced by $F_{\nu_e}$, $F_{\nu_\mu}$, or $F_{\nu_\tau}$ as the case may be.

## III. CALCULATIONS AND RESULTS

The secondary muon yield at a kilometer scale detector such as ICECUBE is calculated using Eqs. (6)–(20). The Earth matter density in Eq. (9) is taken from [9] following the preliminary Earth reference model (PREM). The interaction cross sections—both charged current and total—used in these equations are taken from the tabulated values (and the analytical form) given in Ref. [11]. In the present calculations $E_{\nu\,\mathrm{max}} = 10^{11}$ GeV and threshold energy $E_{\mathrm{thr}} = 1$ TeV are considered.

For our investigations, we first define a ratio $\mathcal{R}$ of the muon events (both from $\nu_\mu$ (and $\bar{\nu}_\mu$) and $\nu_\tau$ (and $\bar{\nu}_\tau$)) and the shower events. As described in the previous sections, the muon events are from $\nu_\mu$ (and $\bar{\nu}_\mu$) and $\nu_\tau$ (and $\bar{\nu}_\tau$), whereas the shower events include the electromagnetic shower initiated by the CC interaction of $\nu_e$ and NC interactions of neutrinos of all flavors. Therefore,

$$\mathcal{R} = \frac{T_\mu}{T_{\mathrm{sh}}}, \tag{23}$$

where

$$\begin{aligned} T_\mu &= S(\text{for } \nu_\mu) + S(\text{for } \nu_\tau),\\ T_{\mathrm{sh}} &= N_{\mathrm{sh}}(\text{for } \nu_e \text{ CC interaction})\\ &\quad + N_{\mathrm{sh}}(\text{for } \nu_e \text{ NC interaction})\\ &\quad + N_{\mathrm{sh}}(\text{for } \nu_\mu \text{ NC interaction})\\ &\quad + N_{\mathrm{sh}}(\text{for } \nu_\tau \text{ NC interaction}). \end{aligned} \tag{24}$$

The purpose of this work is to explore whether UHE neutrinos from the GRB will be able to distinguish any variation of $\theta_{12}$ and $\theta_{23}$ from their best-fit values. The tribimaximal mixing condition is denoted by the best-fit values of $\theta_{12}$ and $\theta_{23}$ for $\theta_{13} = 0°$. The best-fit value of $\theta_{12} = 35.2°$ and that of $\theta_{23} = 45°$. We first vary $\theta_{12}$ in the limit $30° \leq \theta_{12} \leq 38°$ and vary $\theta_{23}$ in the limit $38° \leq \theta_{12} \leq 54°$ with $\theta_{13} = 0$ and for each case calculate the ratio $\mathcal{R}$ using Eqs. (1)–(24). We find that $\mathcal{R}$ varies from 3.14 to 4.25. One readily sees that the variation in muon to shower ratio is not very significant. The flux and other uncertainties of the detector may wash away these small variations. $\mathcal{R}$ obtained from the tribimaximal condition given above is 4.05.

The same operation is repeated for three different values of $\theta_{13}$, namely $\theta_{13} = 3°$, $6°$, and $9°$ with similar results. The results are tabulated below.

We have also plotted the variation of $\mathcal{R}$ with $\theta_{12}$ and $\theta_{23}$ for four fixed values of $\theta_{13}$ as given in Table I. These are shown in Figs. 1(a)–1(d) for $\theta_{13} = 0°$, $3°$, $6°$, and $9°$, respectively.

As is evident from Table I and Fig. 1, the variation of muon tracks to shower ratio is not very significant with the

TABLE I. Maximum and minimum values of the ratio $\mathcal{R}$ for different values of mixing angles.

| $\theta_{13}$ | $\mathcal{R}_{\max}$ | $\mathcal{R}_{\min}$ | $\mathcal{R}$ at $\theta_{12} = 35.2°$, $\theta_{23} = 45°$ |
|---|---|---|---|
| 0° | 4.78 | 3.80 | 4.05 |
| 3° | 4.75 | 3.77 | 4.01 |
| 6° | 4.72 | 3.75 | 3.98 |
| 9° | 4.69 | 3.73 | 3.96 |

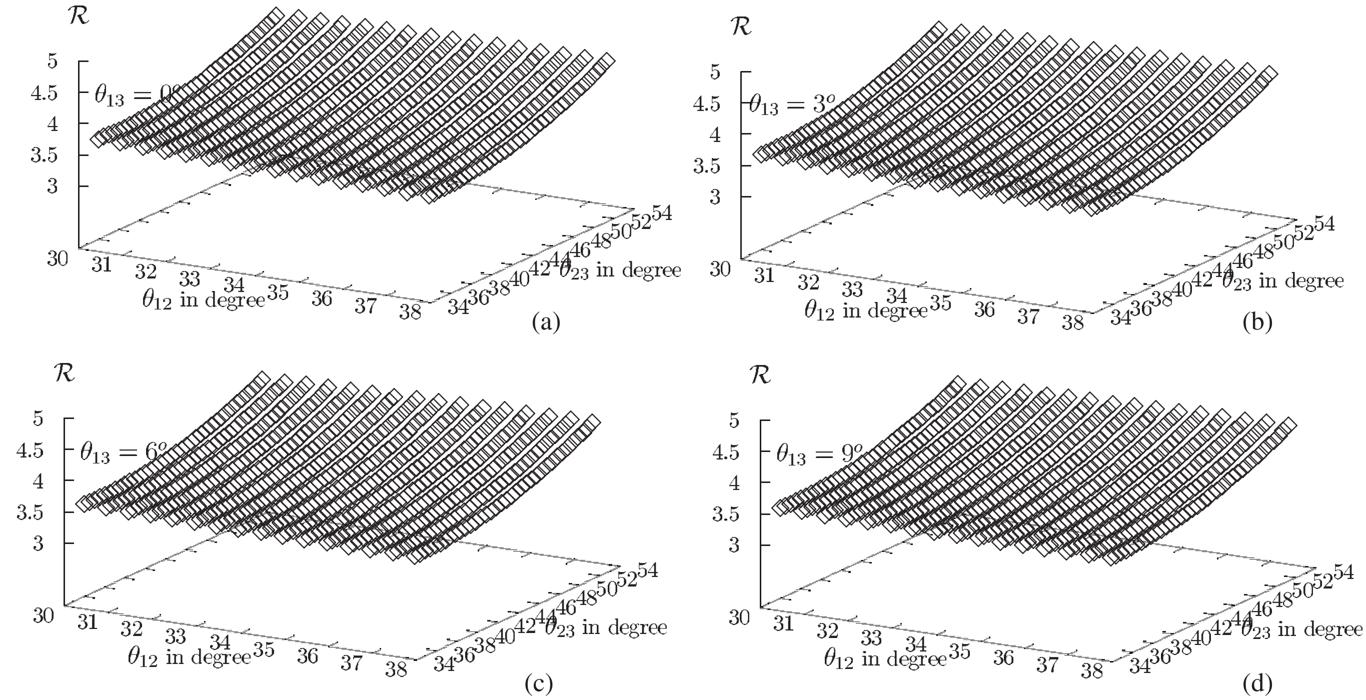


FIG. 1. Variation of $\mathcal{R}$ with $\theta_{12}$ and $\theta_{23}$ for (a) $\theta_{13} = 0°$, (b) $\theta_{13} = 3°$, (c) $\theta_{13} = 6°$, and (d) $\theta_{13} = 9°$. See text for details.

deviation from the best-fit values of the mixing angles. The ratio $\mathcal{R}$ varies up to only $\sim$18%. We have also calculated the muon track-signal for 1 yr of the ICECUBE run. For $\theta_{13} = 0$, this varies from $\sim$99 to $\sim$115, whereas the muon yield obtained for the tribimaximal mixing is 103. So the variation for deviation from the tribimaximal mixing condition is between 4%–11%. This variation is also not significant given the sources of uncertainty in the flux and the sensitivity of the ICECUBE detector. First, the flux itself can be uncertain by several factors. This can induce errors in the calculation of muon yield and shower rate. If the flux uncertainties are energy dependent, even the ratio $\mathcal{R}$ can also be affected. Also the simulation results for the ICECUBE detector by Ahrens *et al.* [16] shows the cosmic neutrino signal is well below the atmospheric neutrino background for the 1 yr data sample after applying suitable cuts (for the source flux $E_\nu^2 \times dN_\nu/dE_\nu = 10^{-7}\ \mathrm{cm}^2\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}\,\mathrm{GeV}$). The diffuse flux needed for a $5\sigma$ significance detection after 1 yr is well below the experimental limits [16,17]. There can also be systematic uncertainty arises out of optical module (OM) sensitivity which is affected by the refrozen ice around OM, optical properties of the surrounding ice, trapped air bubbles in the OM neighborhood, etc. An estimation of these uncertainties for a $E^{-2}$ signal is calculated to be around 20% [16]. Taking into account these uncertainties and the sensitivity limit, it is difficult by a detector like ICECUBE to detect the deviation ($\lesssim 18\%$), if any, from the tribimaximal mixing through the detection of UHE neutrinos from a GRB.

## IV. SUMMARY AND DISCUSSIONS

In summary, we investigate the deviation from the well-known tribimaximal mixing in the case of ultrahigh energy neutrinos from a gamma ray burst detected in a kilometer scale detector such as ICECUBE. We have calculated the ratio $\mathcal{R}$ of the muon track events and shower events (electromagnetic shower from charged current interactions of $\nu_e$ and hadronic showers from neutral current interactions of neutrinos of all flavors) for the tribimaximal mixing condition given by $\theta_{12} = 35.2°$, $\theta_{23} = 45.0°$, $\theta_{13} = 0°$. We then investigate the possible variation of $\mathcal{R}$ from the tribimaximal mixing condition by varying $\theta_{12}$ and $\theta_{23}$ within their experimentally obtained range for four different values of $\theta_{13}$ namely 0°, 3°, 6°, and 9°.

The isotropic flux of GRB neutrinos is obtained following Waxman-Bahcall [10] type parametrization of the flux and summation over the sources. The initial parametrization of neutrino flux can be written as

$$\frac{dN_\nu}{dE_\nu} = \begin{cases} \frac{A}{E_\nu E_\nu^b}, & E_\nu < E_\nu^b, \\ \frac{A}{E_\nu^2}, & E_\nu > E_\nu^b, \end{cases} \tag{25}$$

where $E_\nu^b$ is the spectral break energy ($\sim 10^5$ GeV) and is related to photon spectral break energy, Lorentz factor, etc.

The GRB neutrinos after reaching the Earth have to pass through the Earth rock (for upward going events) to reach the detector to produce muon tracks or shower. In the calculation therefore, the attenuation of neutrinos through the Earth (shadow factor) is estimated. The muons produced out of charged current interactions of neutrinos should also survive to enter the detector and produce tracks. Therefore, to estimate the muon track events, the energy loss of muons through the rock is also estimated. The average lepton energy loss rate (with lepton energy $E_\mu$) due to ionization and the losses due to bremsstrahlung, pair production, hadron production, etc. (catastrophic losses) is parametrized as

$$\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \beta E_\mu,$$

where $\beta$ describes the catastrophic loss which dominates over the ionization loss above a certain critical energy $\zeta = \alpha/\beta$. This induces a logarithmic dependence of the lepton energy loss.

The calculated ratio $\mathcal{R}$ varies between ~8% and ~18% for variation from the tribimaximal mixing scenario and for different values of $\theta_{13}$. Given the sensitivity of the ICECUBE detector in terms of detecting GRB neutrino flux and considering other uncertainties like that in estimating the flux itself, the atmospheric background, low signal yield, and the systematic uncertainties of the detector, it appears that ICECUBE with its present sensitivity will not be able to detect significantly such a small variation due to deviations from the tribimaximal mixing. Hence to detect such a small deviation, very precise measurement is called for. This requires more data (more years of run) and larger detector size for more statistics. The increase in detector size will not widen the deviation of the ratio significantly as the total area factor of the detector cancels out in the ratio (Eq. (23)) although the total number of both muon tracks and total shower yield increase significantly. For the case of shower, the whole detector volume is to be considered and from Eq. (22), there is indeed an $L$ dependence. This makes the deviation of the ratio wider, although very marginally, as we increase the detector dimension.

It is difficult to predict the detector dimension and/or the time of exposure that will be suitable for such a precision measurement discussed above. Detailed simulation studies taking into account factors like atmospheric neutrino background, photomultiplier tube efficiency, and other possible uncertainties like the one carried out in Ref. [16] is required for being able to comment on the detector parameters for such precise measurements.

We also want to mention in passing that we have repeated the same calculation for single GRBs with fixed redshift ($z$) values with similar results.

[1] A. Strumia and F. Vissani, arXiv:hep-ph/0606054.
[2] P. F. Harrison, D. H. Parkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002).
[3] M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998); **466**, 415 (1999).
[4] Z. Xing, Phys. Rev. D **74**, 013009 (2006).
[5] J. Ahrens *et al.* (IceCube Collaboration), in Proceedings of the 27th International Cosmic Ray Conference, Hamburg, Germany, 2001 (unpublished), p. 1237, http://icecube.wisc.edu.
[6] Z. Cao, M. A. Huang, P. Sokolsky, and Y Hu, J. Phys. G **31**, 571 (2005).
[7] S. I. Dutta, M. H. Reno, and I. Saecevic, Phys. Rev. D **62**, 123001 (2000).
[8] N. Gupta, Phys. Lett. B **541**, 16 (2002).
[9] R. Gandhi, C. Quigg, M. H. Reno, and I. Sarcevic, Astropart. Phys. **5**, 81 (1996); arXiv:hep-ph/9512364.
[10] E. Waxman and J. Bahcall, Phys. Rev. Lett. **78**, 2292 (1997); Phys. Rev. D **59**, 023002 (1998).
[11] R. Gandhi, C. Quigg, M. H. Reno, and I. Sarcevic, Phys. Rev. D **58**, 093009 (1998).
[12] T. K. Gaisser, *Cosmic Rays and Particle Physics* (Cambridge University Press, Cambridge, England, 1992); T. K. Gaisser, F. Halzen, and T. Stanev, Phys. Rep. **258**, 173 (1995).
[13] N. Gupta, Phys. Rev. D **65**, 113005 (2002).
[14] A. Dar, J. J. Lord, and R. J. Wilkes, Phys. Rev. D **33**, 303 (1986).
[15] D. Guetta, D. Hooper, J. A. Muniz, F. Halzen, and E. Reuveni, Astropart. Phys. **20**, 429 (2004).
[16] J. Ahrens *et al.*, Astropart. Phys. **20**, 507 (2004).
[17] J. Ahrens *et al.*, Phys. Rev. Lett. **90**, 251101 (2003).