Z_3 symmetry and neutrino mixing in type II seesaw models

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A new economical way to understand neutrino mixings in the seesaw framework is proposed. We argue that its origin can be understood within the seesaw framework by a hidden condition on the mass matrix of heavy right-handed neutrinos under the transformation of the Abelian finite group Z_3 on the flavor basis. Ignoring CP phases, we show that it can lead to the generic form of the effective light neutrino mass matrix from which the Harrison-Perkins-Scott mixing matrix appears naturally, as well as an experimentally allowed nonzero $\sin\theta_{13}$. Two examples are given to illustrate that the mass matrix based on our proposal is in good agreement with the current experimental data.

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The observed neutrino oscillations have long been considered to be the first convincing evidence of new physics beyond the standard model (SM). Although an enormous amount of effort has been made, there are still many unsolved problems in neutrino physics. Besides the smallness of neutrino masses, another outstanding problem is the origin of the observed neutrino mixings. As it is natural to expect that symmetries can explain neutrino mixings, introducing one or more new symmetries becomes the most common approach to resolve this problem. In particular, discrete symmetries including S_3 (e.g. [1]), S_4 (e.g. [2], or [3], as a recent work), and A_4 (e.g. [4]) etc. have been discussed extensively in the literature (for more related works, please see recent reviews [5,6] and references therein). In this approach, the mechanism generating neutrino masses usually does not interfere with the mechanism giving rise to neutrino mixings. In particular, in the seesaw framework, M_R, the mass matrix of heavy right-handed neutrinos, is usually required by symmetries or assumed to be proportional to the identity matrix, i.e. $M_R \propto I$, as in [2,3], respectively. In this way, M_R is only responsible for the smallness of neutrino masses, and thus some other physics have to be introduced to account for neutrino mixings.

However, in this paper, we show explicitly that an economic understanding of both neutrino masses and mixings can be achieved with the assumption that neutrino flavor mixings occur only in the heavy right-handed neutrino sector. Based on the cyclic group Z_3, we propose a new, simple but nontrivial way which will lead to an M_R in a cyclic permuted form, from which the phenomenologically acceptable neutrino mass matrix can be obtained in the type II seesaw model. Interestingly, the effective light neutrino mass matrix obtained in this way is the most generic one that can be obtained from the Harrison-Perkins-Scott (HPS) tribimaximal mixing matrix with an additional pure 1–3 rotation when vanishing CP phases are assumed. Our model in which the only source of lepton flavor mixing is M_R, different from other models including those mentioned above and those adopted in many phenomenological works, provides a new possibility for future model building and phenomenological studies.

We begin with a very brief review of the current status of neutrino mixings and the type II seesaw model. The global fit of current experimental data shows that, unlike the mixing angles in the quark sector, two of the three mixing angles are large and one of them might be maximal. As a matter of fact, $30° < \theta_{\rm sol} < 38°$, $36° < \theta_{\rm atm} < 54°$, and $\theta_{\rm CHOOZ} < 10°$ at the 99% confidence level [7]. Understanding this peculiar property is also an interesting theoretical issue. In fact, these mixing angles can be well described by the HPS mixing matrix [8] where $\sin^2\theta_{\rm sol} = \frac{1}{3}$, $\sin^2\theta_{\rm atm} = \frac{1}{2}$, and $\theta_{\rm CHOOZ} = 0$. The HPS mixing matrix can be considered as the lowest order approximation. Efforts on revealing its origin may help one to understand not only the neutrino physics, but also the physics beyond the SM, such as the new symmetries at high energy scales. In this paper we will propose a new way to understand the appearance of the HPS mixing matrix in the seesaw framework, as mentioned above.

The seesaw mechanism [9], which is the leading candidate for the neutrino mass generating mechanism, can provide a simple and natural way to understand neutrino masses. Although by itself the seesaw mechanism cannot explain the observed neutrino mixings, below we will show that it can play an important role in the generation of neutrino mixings.

In many models with right-handed neutrinos, e.g. SO(10) or $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ based models, the effective light neutrino mass matrix is given by the type II seesaw relation

$$M_\nu = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T, \tag{1}$$

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where M_L, M_R are the Majorana mass matrices for left-handed and right-handed neutrinos and M_ν^D is the Dirac mass matrix. Given that the neutrino mass is generated by the type II seesaw model, as shown in (1), the observed neutrino mixing can provide important information about the structure of M_ν and thus the physics behind M_L, M_R, and M_ν^D.

In this paper we make the natural assumptions that there are three Majorana neutrinos and consider the case where the flavor symmetry is only broken in the M_R sector. Currently none of the CP violating phases has been observed. In the following discussion, we assume vanishing CP phases and focus on the mixing pattern. The case with nonvanishing CP phases will be discussed elsewhere.

It is natural to expect that symmetries can lead to a specific neutrino mass matrix. This idea has been pursued in many works. In particular, discrete symmetries including S_3 (e.g. [1]), S_4 (e.g. [2,3]), A_4 (e.g. [4]), etc. have been discussed extensively in the literature (for a recent review, see [5] and references therein). Appropriate flavor symmetries can also lead to desired neutrino mixing. Without the CP violating phases, there are six free parameters in M_R in the second term of (1), the effective neutrino mass matrix in the type I seesaw model. In this paper, we propose a hidden condition on M_R under the transformation of the Abelian finite group Z_3 on the flavor basis, which will reduce the independent parameters down to three. We then use the resultant mass matrix to explain the observed mixing pattern.

First consider a finite group G. Each element U_i of G satisfies $U_i^{n_i} = 1$ for some nonzero integer n_i. Under an unitary transformation of G on the flavor basis $\nu_f = (\nu_e, \nu_\mu, \nu_\tau)$, we propose that for each U_i that belongs to G, the mass matrix M_R in the new basis satisfies

$$U_i M_R U_i^T = U_i' M_R. \tag{2}$$

We show below that U_i' is strongly constrained and any choice of U_i' satisfying the constraint will further restrict the possible form of M_R. In particular, we show that, if the finite group G is chosen to be Z_3, $U_i' = U_i^2$ will lead to a phenomenologically interesting M_ν and thus provide a possible origin of the observed neutrino mixing angles.

To see that U_i' cannot be arbitrary, consider that

$$\begin{aligned} M_R &= (U_i)^{n_i} M_R (U_i^T)^{n_i} = (U_i)^{n_i-1} U_i' M_R (U_i^T)^{n_i-1} \\ &= (U_i)^{n_i-1} U_i' (U_i^\dagger U_i') M_R (U_i^T)^{n_i-2} = \ldots \\ &= (U_i)^{n_i-1} U_i' (U_i^\dagger U_i')^{n_i-1} M_R = (U_i^\dagger U_i')^{n_i} M_R. \end{aligned} \tag{3}$$

From (3) we find that (2) requires $(U_i^\dagger U_i')^{n_i} = 1$. Consequently, we obtain $U_i' = e^{i2\pi m/n_i} U_i^k$ with m some integer and $k = 0, 1, \ldots, n_i - 1$. Note that $m = 0$ when U_i and M_R are real. Moreover, k can be different for different group elements U_i. Based on simplicity, we assume k is universal for all group elements. It is obvious that $(U_i^\dagger U_i')^{n_i} = 1$ is only a necessary condition for (2) to be satisfied. Given U_i', (2) will restrict the form of M_R. In general, different choices of k will lead to different M_R. We show this below in the case where the finite group G is the cyclic group Z_3.

The group Z_3 contains only three elements; thus $n_i \le 3$. Therefore, the only possible choices for U_i' are $U_i' = I$, or $U_i' = U_i$, or $U_i' = U_i^2$. The first choice, demanding M_R to be invariant under Z_3 on the flavor basis, leads to an unrealistic mass matrix with $\nu_e - \nu_\mu - \nu_\tau$ symmetry. Another choice that one might think interesting is the case where $U_i' = U_i$. One of the necessary conditions in this case requires M_R to be noninvertible so that at least one of the mass eigenvalues is zero. For the cyclic group Z_3, the symmetric mass matrix M_R turns out to be democratic in this case and there is only one nonzero eigenvalue. We will not pursue these in this paper.

In the following we focus on the case $U_i' = U_i^2$. M_R built in this way will give interesting phenomenology. In fact, the resultant M_R can be expressed as linear combinations of elements in one of the two cosets of Z_3 in the non-Abelian symmetric group S_3. Our bottom-up approach ends up with the proposal that, under some finite group G, $M_R U_i^T = U_i M_R$, $\forall U_i \in G$.

To be explicit, consider the following three dimensional unitary representation of $S_3 = \{I_i | i = 1 \sim 6\}$:

$$I_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad I_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$I_3 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad I_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$I_5 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad I_6 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

The four nontrivial subgroups $\{I_1, I_2\}$, $\{I_1, I_3\}$, $\{I_1, I_4\}$, and $Z_3 = \{I_1, I_5, I_6\}$ are all Abelian. Different from the other three subgroups, the cyclic group Z_3 is the only nontrivial invariant subgroup of S_3. $\{I_1, I_5, I_6\}$ form a regular representation of Z_3. It is straightforward to solve that the mass matrix M_R which satisfies (2) with $U_i' = U_i^2$ has the following form:

$$M_R = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix} = aI_2 + bI_3 + cI_4. \tag{4}$$

Note that $\{I_2, I_3, I_4\}$ is a coset of Z_3 in S_3.

Before proceeding to the discussion of the seesaw mechanism, we would like to point out another interesting feature of (2). In fact, before considering the constraints from symmetry, if one uses a novel mechanism to generate the most general mass matrix which does not necessarily to be symmetric, the nontrivial fact is that, with our proposal

$(U_i' = U_i^2)$, the mass matrix will still be in the form of (4) under the Z_3 group. This, however, is not true for the case $U_i' = I$ or $U_i' = U_i$. Starting with the most general M with nine parameters, for the case $U_i' = I$ one gets

$$M = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} = aI_1 + bI_5 + cI_6,$$

while for the case $U_i' = U_i$ one gets

$$M = \begin{pmatrix} a & a & a \\ b & b & b \\ c & c & c \end{pmatrix}.$$

But in the symmetric case, these two matrices will become a $\nu_e - \nu_\mu - \nu_\tau$ symmetric one and a democratic one, respectively, as discussed above. That is, the requirement for M to be symmetric will further reduce the number of free parameters in M. On the other hand, unlike the above two cases, without any assumption on M_R, starting from our simple proposal $M_R U_i^T = U_i M_R \ \forall U_i \in Z_3$ and the most general M_R with nine parameters, one still arrives at the unique form of M_R as given by (4).

Assuming that $M_L = m_0 I_1$ and $M_\nu^D = m_d I_1$ which are invariant trivially under Z_3, from (1) it can be shown that the effective neutrino mass can be written as

$$M_\nu = mI_1 + m_d^2 \begin{pmatrix} B+C & -B & -C \\ -B & A+B & -A \\ -C & -A & C+A \end{pmatrix} \quad (5)$$

where $m = m_0 - m_d^2(A+B+C)$, and

$$A = \frac{a^2 - bc}{R}, \qquad B = \frac{b^2 - ac}{R}, \qquad C = \frac{c^2 - ab}{R} \quad (6)$$

with $R = a^3 + b^3 + c^3 - 3abc$.

This particular form of mass matrix can be diagonalized by the tribimaximal mixing [10] followed by a pure 1–3 rotation. It is worth mentioning that any real symmetric mass matrix which is diagonalized by the tribimaximal mixing followed by a pure 1–3 rotation can always be written in the form of (5). Therefore what we derive here is a novel way to understand the phenomenological Majorana neutrino mass matrix with vanishing CP phases that one can construct from the current neutrino data. Note that, if $m_0 = 0$, then two of the three neutrino masses obtained from (5) will be identical; thus to obtain correct mass square differences a nonzero M_L is necessary.

Note that the form of M_ν in (5) is coincident with the one in the Friedberg-Lee (FL) model [11] in which a new symmetry, i.e. the invariance of the neutrino mass terms under the transformation

$$\nu_e \to \nu_e + z, \qquad \nu_\mu \to \nu_\mu + z, \qquad \nu_\tau \to \nu_\tau + z,$$

is proposed to explain the observed neutrino mixings. Although more work is necessary in order to understand the origin of this symmetry and its breaking mechanism leading to the first term in the right-hand side of (5), Friedberg and Lee's work provides an illuminating example showing that neutrino physics is a great arena for exploring new physics, which is also what we pursue here. Although sharing the same motivation to explain neutrino data, ideas presented in this paper and the physics discussed here are very different. For example, what Friedberg and Lee discussed are Dirac neutrinos, but here we consider Majorana neutrinos. Moreover, based on Z_3 symmetry and the seesaw mechanism, we provide a simple but new way which can lead to not only the desired neutrino mass matrix, but also the small neutrino masses.

Before we proceed, let us discuss another way to implement Z_3 symmetry via its one dimensional representation rather than the regular representation discussed above. Consider the Z_3 transformation which is realized in the following way:

$$\nu_{1R} \to \nu_{1R}, \qquad \nu_{2R} \to e^{i4\pi/3}\nu_{2R}, \qquad \nu_{3R} \to e^{i2\pi/3}\nu_{3R} \quad (7)$$

and

$$\phi_1 \to e^{i4\pi/3}\phi_1, \qquad \phi_2 \to \phi_2, \qquad \phi_3 \to e^{i2\pi/3}\phi_3$$

where ϕ_i are gauge singlet scalar fields. The invariant Majorana mass terms are

$$(\overline{(\nu_{1R})^C}, \overline{(\nu_{2R})^C}, \overline{(\nu_{3R})^C}) \begin{pmatrix} \phi_2 & \phi_3 & \phi_1 \\ \phi_3 & \phi_1 & \phi_2 \\ \phi_1 & \phi_2 & \phi_3 \end{pmatrix} \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}. \quad (8)$$

The vacuum expectation values (VEVs) of ϕ_i will lead to the same mass matrix as the one given in (4). This is equivalent to constructing the following mass term:

$$\overline{(\nu_{iR})^C} \phi_{\mathrm{ij}} \nu_{jR}$$

where $\phi_{\mathrm{ij}} = \phi_{(i+j)\ \mathrm{mod}3}$. Note that, to obtain a phenomenological mass matrix via the type II seesaw relation (1), appropriate M_L and M_ν^D have to be constructed. It is easy to find that a $M_\nu^D \propto I_1$ can be obtained by the same Z_3 charge assignment to ν_{iL} as in (7). It may then lead to an M_L similar to that of M_R, as given by (8). If the pattern of M_L is exactly the same as (4), then the final neutrino mass matrix will also be in the form of (4) from which the correct neutrino masses cannot be generated. In (8) all the couplings are set to be unity for simplicity. This may not be the case for M_L and a proper choice of relevant couplings can also lead to an M_ν approximately given by (5).

We show above that the desired mass matrix can be obtained via Z_3 symmetry. Although more work is necessary to build a complete model and, in particular, appropriate assignment of the charges of gauge symmetries are needed, here we concentrate on possible consequences of Z_3 in the neutrino sector and assume that other symmetries will not spoil our discussion. For example, we require any U(1) symmetry or other symmetries, if they exist, will not forbid the required mass terms under discussion. In addition, in our discussion the three mass matrices relevant to the type II seesaw model, i.e. M_L, M_ν^D, and M_R, are taken to be independent. There might be some relations between them in certain models. For example, in SO(10) models a discrete parity symmetry may lead to $M_L \propto M_R$. However, it may be broken as well. Moreover, the discussion here does not depend on any specific models, although it is interesting to study how to embed our proposal to the existing models such as SO(10), which is beyond the scope of this paper. Therefore, we take them to be independent for generality.

From (6), we have $A + B + C = 1/(a + b + c)$, and

$$a = \frac{A^2 - BC}{R'}, \qquad b = \frac{B^2 - AC}{R'}, \qquad c = \frac{C^2 - AB}{R'}, \tag{9}$$

where $R' = A^3 + B^3 + C^3 - 3ABC$. Now, from any set of A, B, and C which satisfies experimental data, the corresponding a, b, and c can be found by (9). For heavy right-handed neutrinos, $m \simeq m_0$.

Under tribimaximal rotation, we have

$$(U_0)^T M_\nu U_0 = mI_1 + m_d^2 \begin{pmatrix} \frac{3(B+C)}{2} & 0 & \frac{\sqrt{3}}{2}(-B + C) \\ 0 & 0 & 0 \\ \frac{\sqrt{3}}{2}(-B + C) & 0 & \frac{1}{2}(4A + B + C) \end{pmatrix}$$

where

$$U_0 = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ -1 & \sqrt{2} & -\sqrt{3} \end{pmatrix} \tag{10}$$

is the tribimaximal mixing matrix. From the above expression, one can find that the matrix element U_{13} given by $\sin\theta_{13}$ in the standard parametrization is given by

$$|\sin\theta_{13}| = \frac{\sqrt{2}|B - C|}{\sqrt{(R'')^2 + 3(B - C)^2}} \tag{11}$$

where

$$R'' = -2A + B + C - 2\sqrt{A^2 + B^2 + C^2 - AB - AC - BC}.$$

The current experimental bound on $\sin\theta_{13}$ is $\sin^2\theta_{13} \leq 0.040$ at 3σ C.L. (please see the latest arXiv version of [12]). This can be satisfied if

$$\left(\frac{B - C}{A - C}\right)^2 = \left(\frac{b - c}{a - c}\right)^2 \ll 1.$$

Without loss of generality, assume $A > C$. The neutrino masses are found to be

$$m_1 \simeq m + \tfrac{3}{2}m_d^2(B + C), \qquad m_2 = m,$$
$$m_3 \simeq m + 2m_d^2 A + \tfrac{1}{2}m_d^2(B + C).$$

One can examine that appropriately chosen a, b, and c can satisfy the current experimental data. As an example,

$$m = 0.01 \text{ eV}, \qquad m_d = 100 \text{ GeV},$$
$$a = 4.7 \times 10^{14} \text{ GeV}, \qquad b = 5.7 \times 10^{13} \text{ GeV},$$
$$c = 3.0 \times 10^{13} \text{ GeV}$$

will lead to

$$|U_{13}| = 0.022,$$

and

$$\Delta m_{21}^2 = m_2^2 - m_1^2 = 7.9 \times 10^{-5} \text{ eV}^2,$$
$$\Delta m_{31}^2 = m_3^2 - m_1^2 = 2.6 \times 10^{-3} \text{ eV}^2,$$

which are in good agreement with the current neutrino experimental data, i.e.

$$7.1 \times 10^{-5} \text{ eV}^2 < \Delta m_{21}^2 < 8.9 \times 10^{-5} \text{ eV}^2,$$
$$2.0 \times 10^{-3} \text{ eV}^2 < \Delta m_{31}^2 < 3.2 \times 10^{-3} \text{ eV}^2$$

at 3σ C.L. [12].

In addition, one can show that this model can account for the case of nearly degenerate neutrinos. As an example,

$$m = 0.25 \text{ eV}, \qquad m_d = 15 \text{ GeV},$$
$$a = 8.93 \times 10^{13} \text{ GeV}, \qquad b = 2.92 \times 10^{12} \text{ GeV},$$
$$c = 9.01 \times 10^{11} \text{ GeV}$$

will lead to the same squared mass differences as given above and $|U_{13}| = 0.008$.

In conclusion, we argue that Z_3 symmetry can lead to observed neutrino mixing. With our proposal that $M_R U_i^T = U_i M_R$, $\forall U_i \in Z_3$, M_R must be in a cyclic permuted form, as shown in (4). This will lead to tribimaximal mixing followed by an additional 1–3 rotation, capable of explaining the current data. Another way to reach (5) is based on the invariance of the mass terms under Z_3 transformations, similar to the usual Z_2 R-parity transformations. In the seesaw framework, this will lead to a possible explanation of both the smallness of neutrino masses and the origin of the neutrino mixing. It can be easily shown that $\theta_{13} = 0$ requires $b = c$ in (4), i.e., the $\nu_\mu - \nu_\tau$ symmetry [13]. Therefore, from the naturalness principle, the smallness of θ_{13} is presumably protected by the symmetry. Based on our work, the yet unknown $\nu_\mu - \nu_\tau$ symmetry breaking mechanism leading to the smallness of $\sin\theta_{13}$, and other possible phenomena including lepton flavor violations, is worth further studies in the future.

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