Cerenkov radiation from moving straight strings

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We study Cerenkov radiation from moving straight strings which slide with respect to each other in such a way that the projected intersection point moves faster than light. To calculate this effect we develop classical perturbation theory for the system of Nambu-Goto strings interacting with the dilaton, two-form, and gravity fields. In the first order, one encounters divergent self-action terms which are eliminated by classical renormalization of the string tension. Cerenkov radiation arises in the second order. It is generated by an effective source which contains contributions localized on the string world sheets and bulk contributions quadratic in the first-order fields. In the ultrarelativistic limit radiation exhibits angular peaking on the Cerenkov cone in the forward direction of the fast string in the rest frame of another. The radiation spectrum then extends up to high frequencies proportional to the square of the Lorentz factor of the relative velocity. Gravitational radiation is absent since the $1 + 2$ space-time transverse to the straight string does not allow for gravitons. A rough estimate of the Cerenkov radiation in the cosmological cosmic strings network is presented.

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I. INTRODUCTION

During the past few years the hypothesis of cosmic strings received new impetus from superstring theory. Although perturbative superstrings in ten dimensions are too heavy to be admitted as cosmic strings, it was realized that there are various possibilities for geometry of string compactifications to accommodate four-dimensional strings with much lower tensions. Copious production of cosmic strings is typical [1,2] for the brane inflation scenario [3], in which the period of inflation is associated with the collision of branes. This scenario provides an acceptable model of inflation and predicts creation of cosmic superstrings consistent with the current cosmic microwave background (CMB) data. These strings typically have lower tensions than the usual grand unified theory (GUT) cosmic strings and thus they are not the main players in the formation of cosmic structures, but their observational signatures could provide a direct confirmation of the string theory. This stimulated a detailed study of creation and evolution of the cosmic superstring network within the KKLMMT model [4], racetrack inflation [5], and other particular scenarios [6–8] (for recent reviews see [9–13]). One particular feature of the warped IIB compactifications involved in these considerations is the prediction of two types of cosmic strings: F-strings (fundamental) and D-strings (Dirichlet) with different tensions μ_F, μ_D [14–17]. F- and D-strings may also form the so-called p, q composites [18] and provide for triple junctions [19] with new exotic observational predictions. Typical values of the dimensionless parameter $G\mu$ are in the interval $(10^{-11}$–$10^{-7})$. Another new feature is that cosmic superstrings generically have lower reconnection probabilities lying in the range $P \sim (10^{-3}$–$1)$. This changes the cosmological evolution of the string network [20–24] leading, in particular, to enhancement of the fraction of straight strings.

The evolution and possible observational signatures of the strings crucially depend on radiation processes. In fact, it was recognized long ago that oscillating loops of cosmic strings generate large output of gravitational waves [25–28] at the level accessible for current and future detectors. Global strings produce massless axions [28–33] which become massive at a later stage of expansion, creating an observational constraint on the axion mass. In the models containing the massless dilaton (like cosmic superstrings), the dilaton radiation from strings may also constrain the string tension parameter $G\mu$ [34–36].

The main mechanisms of radiation in the string network which have been explored so far were radiation from smoothly oscillating string loops and radiation from kinks and cusps formed on them. It was tacitly assumed that straight unexcited long strings do not radiate. Meanwhile, interaction of long strings via massless fields gives rise to another radiation mechanism of Cerenkov nature. When two straight nonexcited Nambu-Goto strings interacting at a distance via the dilaton, two-form, and gravity slide with respect to each other, they get deformed in the vicinity of the point of their minimal separation. This point can propagate with a faster-than light velocity, provided the inclination angle between the strings is sufficiently small. (For the strictly parallel strings this velocity is infinite for any relative string speed.) In this case, the propagating deformation, together with the associated field tensions, becomes the source of Cerenkov radiation, which is the effect of the second order in the interaction of the string with massless fields. This mechanism was suggested in [37] for gravitationally interacting strings, but it turned out that, although the effect is kinematically allowed, the corresponding amplitude is zero on the mass shell of the graviton. The reason is that the $1 + 2$ space-time orthogonal to the straight string does not allow for gravitons. (However, gravitons *are* produced at the quantum level in the string recombination and annihilation [38].) But the Cerenkov mechanism works for other possible string massless excitations leading, e.g., to electromagnetic radiation from superconducting strings [39] or emission of axions. The latter effect was recently studied in detail in the flat space-time [40]. It was found that, although being of the second order in the axion coupling constant, it still gives a considerable contribution to the total cosmological axion production by the string network.

Cerenkov radiation from straight strings is similar to bremsstrahlung of point charges in electrodynamics. Moreover, the system of parallel strings interacting via a two-form field in D space-time dimensions is exactly equivalent to the system of point charges interacting with the vector field in $(D-1)$-dimensional space-time. A distinction of the Cerenkov mechanism of dissipation in the cosmic string network from conventional radiation via formation of loops can be understood as follows. Formation of loops from initially disconnected straight strings is effected via direct *contact* interaction of intersecting strings. In our case the *long-range* interaction of strings via massless fields which can be potentially radiated underlies the formation of the superluminal radiation source. Cerenkov radiation is a higher-order effect and thus potentially smaller. But it works for a wider set of initial data in the string network (nonzero impact parameters of colliding strings) than that corresponding to intercommuting strings. Also, as we will show here, Cerenkov radiation is strongly enhanced in the case of relativistic velocities. So it can still be nonsmall in the cosmological setting. The detailed cosmological applications remain outside the scope of the present paper, but we give the rough estimates of the dilaton and two-form field backgrounds generated via the Cerenkov mechanism in the evolving string network.

In this paper we consider Cerenkov radiation of moving straight strings interacting with three massless fields: dilaton, antisymmetric second rank tensor (NS-NS or RR two-forms), and gravity. To facilitate construction of the solution of the coupled system of the Nambu-Goto and the field equations in the second order, we introduce a diagrammatic representation similar to Feynman graphs. In the first-order approximation one encounters divergencies which are cured by classical renormalization of the string tension. Then the effective sources of the dilaton and two-form radiation are constructed and the radiation rates are calculated.

The plan of the paper is as follows. In Sec. II we give the general setting of the problem and introduce graphic representations for classical vertices corresponding to interactions of strings with the dilaton, two-form, and gravity fields and to interactions between the fields. Section III contains the formulation of the perturbation approach and an explicit construction of the second-order equations. In Sec. IV we reproduce in our framework the main features of classical renormalization for strings (in the lowest order of perturbation theory): elimination of scalar and two-form divergences via renormalization of the string tension, and the absence of gravitational divergency. In Sec. V the first-order deformations of the string world sheets due to interaction via the dilaton, two-form, and linearized gravity are considered. The main calculation is presented in Sec. VI where we construct radiation generating currents by computing the source terms in the wave equations of the second order. These include the world-sheet local terms and the bulk terms which correspond to multiple second-order graphs including all relevant elementary vertices. Radiation rates for the dilaton and the two-form fields are computed in Sec. VII and analyzed in detail in the case of ultrarelativistic velocities. Finally, in Sec. VIII we present rough cosmological estimates and summarize our results in Sec. IX.

II. ACTION AND EQUATIONS OF MOTION

We consider a pair $n = 1, 2$ of straight Nambu-Goto strings described by the world sheets

$$x^\mu = X_n^\mu(\sigma_n^a), \qquad \mu = 0, 1, 2, 3,$$
$$\sigma_a = (\tau, \sigma), \qquad a = 0, 1. \tag{1}$$

Strings interact via the gravitational $g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$, the dilaton $\phi(x)$, and the two-form (axion) $B_{\mu\nu}(x)$. Using the Polyakov form for the string action, we present the total action in the form

$$S = -\sum \int \left\{ \frac{\mu}{2} X_a^\mu X_b^\nu g_{\mu\nu} \gamma^{ab} \sqrt{-\gamma} e^{2\alpha\phi} + 2\pi f X_a^\mu X_b^\nu \epsilon^{ab} B_{\mu\nu} \right\} d^2\sigma + \int \left\{ 2\partial_\mu \phi \partial_\nu \phi g_{\mu\nu} + \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho} e^{-4\alpha\phi} - \frac{R}{16\pi G} \right\} \sqrt{-g} d^4x, \tag{2}$$

where the sum is taken over $n = 1, 2$ (the corresponding index is omitted for brevity). Our signature choice is mostly minus for the space-time metric, and $(+, -)$ for the world sheets. The Levi-Cività symbol is $\epsilon^{01} = -\epsilon^{10} = 1$, γ_{ab} is the metric on the world sheet, and the lower Latin indices mean partial derivatives with respect to the world-sheet coordinates $X_a^\mu \equiv \partial_a X^\mu$. The action contains four parameters: the string tension μ, the Newton constant G, the dilaton coupling α, and the two-form coupling f. The antisymmetric three-form field strength is defined as

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}. \tag{3}$$

Variation of the action (2) with respect to X^μ leads to the equations of motion for strings,

$$\partial_a(\mu X_b^\nu g_{\mu\nu} \gamma^{ab} \sqrt{-\gamma} e^{2\alpha\phi} + 4\pi f X_b^\nu \epsilon^{ab} B_{\mu\nu}) = X_a^\lambda X_b^\nu \partial_\mu \left(\frac{\mu}{2} g_{\lambda\nu} \gamma^{ab} \sqrt{-\gamma} e^{2\alpha\phi} + 2\pi f \epsilon^{ab} B_{\lambda\nu} \right). \tag{4}$$

The coordinate derivatives on the right-hand side apply to the metric, dilaton, and two-form fields; their values are taken on the world sheet. The derivatives with respect to the world-sheet coordinates σ^a on the left-hand side apply both to the world-sheet quantities $X^\nu(\sigma^a)$, γ_{ab} and to the metric, dilaton, and two-form fields, e.g., for the metric

$$\partial_a g_{\mu\nu} = X_a^\lambda \partial_\lambda g_{\mu\nu}, \tag{5}$$

and similarly for ϕ, $B_{\mu\nu}$. In this formalism γ_{ab} is an independent variable; variation of the action with respect to γ_{ab} gives the constraint equation

$$(X_a^\mu X_b^\nu - \tfrac{1}{2}\gamma_{ab}\gamma^{cd}X_c^\mu X_d^\nu)g_{\mu\nu} = 0, \tag{6}$$

whose solution defines γ_{ab} as the induced metric on the world sheet:

$$\gamma_{ab} = X_a^\mu X_b^\nu g_{\mu\nu}. \tag{7}$$

Consider now the field equations. Variation over ϕ gives the dilaton equation

$$\partial_\mu(g^{\mu\nu}\partial_\nu\phi\sqrt{-g}) + \frac{\alpha}{6}H^2 e^{-4\alpha\phi} = -\sum\frac{\mu\alpha}{4}\int X_a^\mu X_b^\nu g_{\mu\nu}\gamma^{ab}\sqrt{-\gamma}e^{2\alpha\phi}\delta^4(x - X(\sigma,\tau))d^2\sigma, \tag{8}$$

where the sum in the source term is taken over the contribution of two strings. The equation for the two-form field reads

$$\partial_\mu(H^{\mu\nu\lambda}e^{-4\alpha\phi}\sqrt{-g}) = -\sum 2\pi f\int X_a^\nu X_b^\lambda \epsilon^{ab} \times \delta^4(x - X(\sigma,\tau))d^2\sigma. \tag{9}$$

We also have the Bianchi identity

$$\nabla_{[\mu}H_{\alpha\beta\gamma]} = 0, \tag{10}$$

where alternation over indices has to be performed and the derivative can be equivalently treated as a covariant derivation with respect to $g_{\mu\nu}$ or a partial derivative.

Finally, for the metric we have the Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = 8\pi G(\overset{\phi}{T}_{\mu\nu} + \overset{B}{T}_{\mu\nu} + \overset{st}{T}_{\mu\nu}), \tag{11}$$

where the source terms read

$$\overset{st}{T}{}^{\mu\nu} = \sum\mu\int X_a^\mu X_b^\nu\gamma^{ab}\sqrt{-\gamma}e^{2\alpha\phi}\frac{\delta^4(x - X(\sigma,\tau))}{\sqrt{-g}}d^2\sigma, \tag{12}$$

$$\overset{\phi}{T}{}^{\mu\nu} = 4(\partial^\mu\phi\partial^\nu\phi - \tfrac{1}{2}g^{\mu\nu}(\nabla\phi)^2), \tag{13}$$

$$\overset{B}{T}{}^{\mu\nu} = (H^\mu_{\alpha\beta}H^{\nu\alpha\beta} - \tfrac{1}{6}H^2 g^{\mu\nu})e^{-4\alpha\phi}. \tag{14}$$

The total system of equations consists of two equations for strings of the type (4) and the field equations (8), (9), and (11). It describes classically the motion of the strings mutually interacting via the dilaton, two-form, and gravitational fields, and evolution of the generated fields. This system can be solved iteratively in terms of the coupling constants α, f, G. To facilitate this construction, it is convenient to introduce a classical analog of the Feynman graphs, denoting the string by a bold solid line, the dilaton by a solid line, the two-form by a dashed line, and the graviton by wavy lines (Fig. 1). Simple analysis of the equations reveals that we have vertices involving linear and nonlinear string-field interactions, as well as three-leg and multileg interactions of fields between themselves.

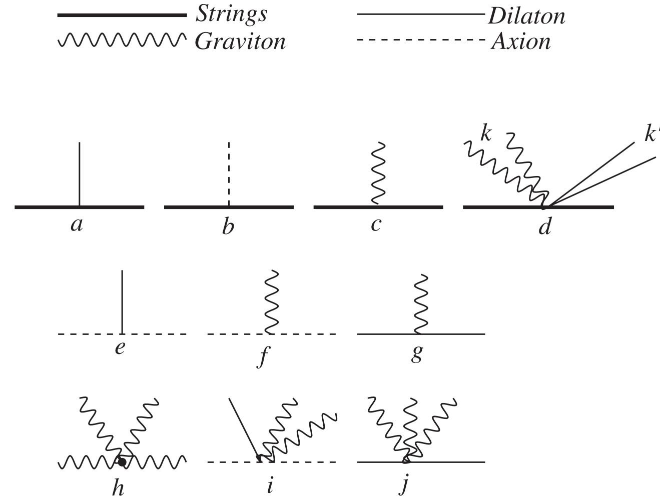

FIG. 1. Vertices associated with the action (2): graphs a, b, c are string-dilaton, string–two-form and string-graviton vertices of the lowest order; graph d depicts higher-order string-dilaton-graviton vertices (for all $k \geq 1$, $k' \geq 1$); graphs e, f, g are lowest-order field interactions present in the action (2); and graphs h, i, j are multigraviton vertices accompanying the lowest-order ones.

III. PERTURBATION THEORY

We construct an iterative solution of the coupled string-field system expanding the string world-sheet mapping functions $X_n^\mu(\sigma^a)$, $n = 1, 2$, and the field variables in terms of the coupling constants α, f, G:

$$X^\mu = \overset{0}{X}{}^\mu + \overset{1}{X}{}^\mu + \ldots, \qquad \phi = \overset{1}{\phi} + \overset{2}{\phi} + \ldots,$$
$$B_{\mu\nu} = \overset{1}{B}_{\mu\nu} + \overset{2}{B}_{\mu\nu} + \ldots, \qquad h_{\mu\nu} = \overset{1}{h}_{\mu\nu} + \overset{2}{h}_{\mu\nu} + \ldots. \tag{15}$$

Here the expansions of X_n^μ start from zero order, while those of the field variables start from the first-order terms; that is, we assume that there are no background dilaton, two-form, or gravitational fields. Zero-order mapping functions describe the straight infinite uniformly moving strings,

$$\overset{0}{X}{}^{\mu}_{n} = d^{\mu}_{n} + u^{\mu}_{n}\tau + \Sigma^{\mu}_{n}\sigma. \tag{16}$$

Here Σ^{μ}_{n} is the unit spacelike constant vectors along the strings, and u^{μ}_{n} are the unit timelike constant vectors—-four-velocities of the strings. The corresponding three-dimensional velocities are orthogonal to the strings. The constant vectors d^{μ}_{n} can be regarded as impact parameters for two strings with respect to the chosen frame. In the zero order the space-time metric is flat, and the corresponding world-sheet metrics are also Minkowskian $\eta_{ab} = \mathrm{diag}(1, -1)$ in view of the normalization assumed,

$$(\Sigma_n\Sigma_n) = \eta_{\mu\nu}\Sigma^{\mu}_{n}\Sigma^{\nu}_{n} = -1, \qquad (u_n u_n) = 1, \qquad (\Sigma_n u_n) = 0. \tag{17}$$

It is convenient the choose the Lorentz frame in which the first string is at rest and is stretched along the z axis:

$$u^{\mu}_{1} = [1, 0, 0, 0], \qquad \Sigma^{\mu}_{1} = [0, 0, 0, 1]. \tag{18}$$

The second string is assumed to move in the plane x^2, x^3 with the velocity v orthogonal to the string itself:

$$u^{\mu}_{2} = \gamma[1, 0, -v\cos\alpha, v\sin\alpha], \qquad \Sigma^{\mu}_{2} = [0, 0, \sin\alpha, \cos\alpha], \tag{19}$$

where $\gamma = (1 - v^2)^{-1/2}$. In such a configuration the strings never intersect each other, always remaining in the parallel planes. Apart from the orthogonality conditions (17), four other scalar products are

$$(u_1, u_2) = \gamma, \qquad (\Sigma_1, \Sigma_2) = -\cos\alpha, \qquad (u_1, \Sigma_2) = 0, \qquad (u_2, \Sigma_1) = -v\gamma\sin\alpha; \tag{20}$$

note that u_1 and Σ_2 are orthogonal. We also choose both impact parameters d^{μ}_{n} to be orthogonal to u^{μ}_{n} and Σ^{μ}_{n} and aligned with the axis x^1, the distance between the planes being $d = d_2 - d_1$. The angle of inclination α of the second string with respect to the first one can be written in a Lorentz-invariant form,

$$\alpha = \arccos(-\Sigma_1\Sigma_2). \tag{21}$$

Similarly, the invariant expression for the relative velocity of the strings is

$$v = (1 - (u_1u_2)^{-2})^{1/2}. \tag{22}$$

With this parametrization of the unperturbed world sheets, the projected intersection point (the point of the minimal separation between the strings) moves with the velocity

$$v_p = \frac{v}{\sin\alpha} = (u_1u_2)^{-1}\left(\frac{(u_1u_2)^2 - 1}{1 - (\Sigma_1\Sigma_2)^2}\right)^{1/2} \tag{23}$$

along the x^3 axis. This motion is not associated with propagation of any signal, so the velocity v_p may be arbitrary, in particular, superluminal $v_p > 1$. The case of parallel strings corresponds to $v_p = \infty$.

Note that the above introduced parameters (21) and (22) are not invariant under reparametrizations of the world sheets [40]. The quantity which is invariant under the volume preserving reparametrizations is

$$\kappa = \det(X^{\mu}_{1a}X^{\nu}_{2b}\eta_{\mu\nu}) = \gamma\cos\alpha. \tag{24}$$

The superluminal regime corresponds to $\kappa > 1$.

The expansions (15) are substituted into the system of equations (4), (8), (9), and (11) which has to be solved iteratively. The zero-order differential operator in the dilaton equation (4) is the flat-space D'Alembert operator $\Box = -\eta^{\mu\nu}\partial_\mu\partial_\nu$. Similarly, choosing the Lorentz gauge for the two-form and the metric perturbations

$$\partial_\mu B^{\mu\nu} = 0, \qquad \partial_\mu \psi^{\mu\nu} = 0, \qquad \psi^{\mu\nu} = h^{\mu\nu} - \tfrac{1}{2}\eta^{\mu\nu}h, \tag{25}$$

where $h = h^{\mu}_{\mu}$, we get the linear D'Alembert equations for the first-order two-form and the gravitational field as well. Because of linearity of the field equations, the first-order dilaton, two-form, and metric perturbations can be presented as the sums of the separate contributions due to two strings,

$$\overset{1}{\phi} = \overset{1}{\phi}_1 + \overset{1}{\phi}_2, \qquad \overset{1}{B}{}^{\mu\nu} = \overset{1}{B}{}^{\mu\nu}_1 + \overset{1}{B}{}^{\mu\nu}_2, \qquad \overset{1}{h}{}^{\mu\nu} = \overset{1}{h}{}^{\mu\nu}_1 + \overset{1}{h}{}^{\mu\nu}_2. \tag{26}$$

Here each term with $n = 1, 2$ satisfies the individual D'Alembert equation with the source labeled by the same index n:

$$\Box\overset{1}{\phi}_n = 4\pi\overset{0}{J}_n, \tag{27}$$

$$\Box\overset{1}{B}{}^{\mu\nu}_n = 4\pi\overset{0}{J}{}^{\mu\nu}_n, \tag{28}$$

$$\Box\overset{1}{h}{}^{\mu\nu}_n = 4\pi\overset{0}{\tau}{}^{\mu\nu}_n. \tag{29}$$

The coupling constants are included into the source terms, while zero indices in the sources indicate that they are computed using the zero-order approximations for the string mapping functions. The source terms thus read

$$\overset{0}{J}_n = \frac{\alpha\mu}{8\pi}\int \delta^4(x - \overset{0}{X}_n(\tau, \sigma))d^2\sigma, \tag{30}$$

$$\overset{0}{J}{}^{\mu\nu}_n = \frac{f}{2}\int V^{\mu\nu}_n\delta^4(x - \overset{0}{X}(\tau, \sigma))d^2\sigma, \tag{31}$$

$$\overset{0}{J}{}^{\mu\nu}_n = 4G\mu_n\int W^{\mu\nu}_n\delta^4(x - \overset{0}{X}_n(\tau, \sigma))d^2\sigma, \tag{32}$$

where the following antisymmetric and symmetric tensors are introduced:

$$V^{\mu\nu}_n = \epsilon^{ab}\overset{0}{X}{}^{\mu}_{bn}\overset{0}{X}{}^{\nu}_{bn} = u^{\mu}_n\Sigma^{\nu}_n - u^{\nu}_n\Sigma^{\mu}_n, \tag{33}$$

$$U_n^{\mu\nu} = \eta^{ab}\overset{0}{X}{}^{\mu}_{an}\overset{0}{X}{}^{\nu}_{bn} = u_n^\mu u_n^\nu - \Sigma_n^\mu \Sigma_n^\nu, \quad U_n^{\mu\nu}\eta_{\mu\nu} = U_n = 2, \tag{34}$$

$$W_n^{\mu\nu} = U_n^{\mu\nu} - \tfrac{1}{2}\eta^{\mu\nu}U_n. \tag{35}$$

Now consider the perturbations of the string world sheets induced metrics (index n is omitted)

$$\gamma_{ab} = \eta_{ab} + \overset{1}{\gamma}_{ab}, \tag{36}$$

where in the general gauge the first-order correction reads

$$\overset{1}{\gamma}_{ab} = \overset{0}{X}{}^{\mu}_{a}\overset{0}{X}{}^{\nu}_{b}\overset{1}{h}_{\mu\nu} + 2\overset{0}{X}{}^{\mu}_{a}\overset{1}{X}{}^{\nu}_{b}\eta_{\mu\nu}. \tag{37}$$

However, to be able to disentangle the higher-order perturbed equations one has to get rid of the second term in (37) proportional to $\overset{1}{X}{}^{\nu}$. Using the space-time and the world-sheet diffeomorphism invariance, we can impose the gauge condition

$$\overset{0}{X}{}^{\mu}_{a}\overset{1}{X}{}^{\nu}_{b}\eta_{\mu\nu} = 0, \tag{38}$$

in which case the perturbed induced metric will contain only the zero-order string mapping functions:

$$\overset{1}{\gamma}_{ab} = \overset{0}{X}{}^{\mu}_{a}\overset{0}{X}{}^{\nu}_{b}\overset{1}{h}_{\mu\nu}, \qquad \overset{1}{\gamma} = U^{\mu\nu}\overset{1}{h}_{\mu\nu}. \tag{39}$$

The first-order perturbations of the mapping functions $\overset{1}{X}{}^{\mu}$ describing deformations of the flat world sheets satisfy the equations following from Eq. (4):

$$\begin{aligned}&\mu\eta^{ab}\partial_a\overset{1}{X}{}^{\nu}_{b}\eta_{\mu\nu} + \mu\partial_a\left\{\overset{0}{X}{}^{\nu}_{b}\left[\left(\overset{1}{h}_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu}\overset{1}{\gamma}\right)\eta^{ab}\right.\right.\\ &\left.\left. - \eta_{\mu\nu}\overset{1}{\gamma}{}^{ab}\right] + 2\alpha\eta_{\mu\nu}\eta^{ab}\phi\right\} + 4\pi f\epsilon^{ab}\overset{0}{X}{}^{\nu}_{b}\partial_a\overset{1}{B}{}^{\mu\nu}\\ &- 2\alpha\mu\partial_\mu\overset{1}{\phi} - \frac{\mu}{2}U^{\lambda\nu}\partial_\mu\overset{1}{h}_{\lambda\nu} - 2\pi f V^{\lambda\nu}\partial_\mu\overset{1}{B}_{\lambda\nu} = 0,\end{aligned} \tag{40}$$

where raising of the world-sheet indices is performed by the Minkowski metric:

$$\overset{1}{\gamma}{}^{ab} = \overset{1}{\gamma}_{cd}\eta^{ac}\eta^{bd}, \qquad \overset{1}{\gamma} = \overset{1}{\gamma}_{cd}\eta^{cd}. \tag{41}$$

Differentiating the field variables along the world sheet according to the rules

$$\partial_a\overset{1}{\phi} = \overset{0}{X}{}^{\lambda}\partial_\lambda\overset{1}{\phi}, \qquad \partial_a\overset{1}{B}_{\mu\nu} = \overset{0}{X}{}^{\lambda}_{a}\partial_\lambda\overset{1}{B}_{\mu\nu}, \qquad \partial_a\overset{1}{h}_{\mu\nu} = \overset{0}{X}{}^{\lambda}_{a}\partial_\lambda\overset{1}{h}_{\mu\nu}, \tag{42}$$

we can rewrite the above equation as

$$\mu\eta^{ab}\partial_a\partial_b\overset{1}{X}{}^{\mu} = F^\mu_{(\phi)} + F^\mu_{(B)} + F^\mu_{(h)}, \tag{43}$$

where the forces due to the dilaton, two-form, and graviton are introduced:

$$F^\mu_{(\phi)} = \alpha\mu(U\partial^\mu\overset{1}{\phi} - 2U^{\mu\nu}\partial_\nu\overset{1}{\phi}), \tag{44}$$

$$F^\mu_{(B)} = 2\pi f V^{\nu\lambda}\overset{1}{H}{}^{\mu}{}_{\nu\lambda}, \tag{45}$$

$$\begin{aligned}F^\mu_{(h)} = \mu[&\overset{1}{h}_{\lambda\tau,\nu}(U^{\mu\lambda}U^{\nu\tau} - \tfrac{1}{2}U^{\mu\nu}U^{\lambda\tau})\\ &- U_{\nu\lambda}(\overset{1}{h}{}^{\mu\nu,\lambda} - \tfrac{1}{2}\overset{1}{h}{}^{\nu\lambda,\mu})].\end{aligned} \tag{46}$$

In these equations the indices labeling the strings are not shown, but it is understood that for each string we have to take into account on the right-hand side both the self-force terms arising from the fields due to the same string and the mutual interaction terms coming from the partner string.

Consider now the second-order field equations. Expanding the equations (8)–(11) to the next order and imposing again the Lorentz gauge for the two-form and the linearized gravity, one obtains the D'Alembert equations for the second-order fields with the source terms involving contributions due to deformations of the string world sheets (local terms) as well as the quadratic combinations of the first-order fields (bulk terms). The dilaton equation, taking into account (39), will read

$$\begin{aligned}\Box\overset{2}{\phi} &= 4\pi\overset{1}{J}\\ &= \frac{\mu\alpha}{4}\sum\int[U^{\mu\nu}(\overset{1}{h}_{\mu\nu} + 2\eta_{\mu\nu}\alpha\overset{1}{\phi}) - 4\overset{1}{X}{}^{\mu}\partial_\mu]\\ &\quad\times\delta^4[x - \overset{0}{X}(\tau,\sigma)]d^2\sigma + \frac{\alpha}{6}\overset{1}{H}{}^{\mu\nu\lambda}\overset{1}{H}_{\mu\nu\lambda}\\ &\quad - \partial_\mu\left[\left(\overset{1}{h}{}^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\overset{1}{h}\right)\partial_\nu\overset{1}{\phi}\right],\end{aligned} \tag{47}$$

where the partial derivative operator acts on the delta function. Note that the first-order terms in the second line are multiplied by the dilaton coupling constant and thus give the same order quantities as the products of two first-order field quantities.

The equation for the second-order two-form field is

$$\begin{aligned}\Box\overset{2}{B}{}^{\mu\nu} &= 4\pi\overset{1}{J}{}^{\mu\nu}\\ &= 2\pi f\sum\int(2\epsilon^{ab}\overset{0}{X}{}^{[\mu}_{a}\overset{1}{X}{}^{\nu]}_{b}\delta^4[x - \overset{0}{X}(\tau,\sigma)]\\ &\quad - V^{\mu\nu}\overset{1}{X}{}^{\lambda}\partial_\lambda\delta^4[x - \overset{0}{X}(\tau,\sigma)])d^2\sigma\\ &\quad + \partial_\lambda\left[\left(\frac{1}{2}\overset{1}{h} - 4\alpha\overset{1}{\phi}\right)\overset{1}{H}{}^{\lambda\mu\nu}\right],\end{aligned} \tag{48}$$

where alternation over indices includes the factor $1/2$.

The situation is slightly more complicated for the graviton. To obtain an equation for the second-order gravitational perturbation, one has to take into account the quadratic terms in the Einstein equations. In the gauge

$$\partial_\mu\psi^{\mu\nu} = 0$$

one has the following expansion of the Einstein tensor up to the second order:

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = \tfrac{1}{2}\Box\psi_{\mu\nu} + \tfrac{1}{2}S_{\mu\nu}(\overset{1}{h}_{\lambda\tau}), \tag{49}$$

where $\psi_{\mu\nu}$ contains the first and the second-order quantities, and the quadratic term reads

$$\begin{aligned} S_{\mu\nu} = {}& [\partial^\beta h^\alpha_\mu(\partial_\alpha h_{\nu\beta} - \partial_b h_{\nu\alpha}) - \tfrac{1}{2}\partial_\mu h^{\alpha\beta}\partial_\nu h_{\alpha\beta} \\ & - \tfrac{1}{2}h_{\mu\nu}\partial_\alpha\partial^\alpha h + h^{\alpha\beta}(\partial_\nu\partial_\beta h_{\mu\alpha} + \partial_\mu\partial_\beta h_{\nu\alpha} \\ & - \partial_\mu\partial_\nu h_{\alpha\beta} - \partial_\alpha\partial_\beta h_{\mu\nu}) + \tfrac{1}{2}\eta_{\mu\nu}(2h^{\alpha\beta}\partial^\lambda\partial_\lambda h_{\alpha\beta} \\ & - \partial_\lambda h_{\alpha\beta}\partial_\beta h^{\alpha\lambda} + \tfrac{3}{2}\partial_\lambda h_{\alpha\beta}\partial^\lambda h^{\alpha\beta})]. \end{aligned} \tag{50}$$

Extracting the second-order terms, we obtain from Eq. (11)

$$\Box\overset{2}{\psi}_{\mu\nu} = 16\pi G\overset{1}{\tau}_{\mu\nu}, \tag{51}$$

where the source term reads

$$\begin{aligned} \overset{1}{\tau}_{\mu\nu} = {}& \frac{\mu}{2}\sum\int(2\overset{0}{X}{}^{a}_{(\mu} - U_{\mu\nu}\overset{1}{X}{}^{\lambda}\partial_\lambda)\delta^4(x - \overset{0}{X}(\tau,\sigma))d^2\sigma \\ & + \frac{\mu}{4}\sum\int[(U_{\mu\nu}U^{\lambda\tau} - 2U^\lambda_\mu U^\tau_\nu)\overset{1}{h}_{\lambda\tau} \\ & + U_{\mu\nu}(4\alpha\overset{1}{\phi} - \overset{1}{h})]\delta^4(x - \overset{0}{X}(\tau,\sigma))d^2\sigma \\ & + G^{-1}S_{\mu\nu}(\overset{1}{h}_{\lambda\tau}) + 2\partial_\mu\overset{1}{\phi}\partial_\nu\overset{1}{\phi} - \eta_{\mu\nu}\partial_\lambda\overset{1}{\phi}\partial^\lambda\overset{1}{\phi} \\ & + \frac{1}{2}\overset{1}{H}_{\mu\lambda\tau} - \frac{1}{12}\eta_{\mu\nu}\overset{1}{H}_{\nu\lambda\tau}\overset{1}{H}{}^{\nu\lambda\tau}. \end{aligned} \tag{52}$$

All source terms in the second-order field equations have similar structure. Note the presence of the derivatives from delta functions in the string world-sheet contributions.

It is worth noting that the second-order dilaton, two-form, and gravity fields already cannot be presented as a sum of terms due to separate strings, but rather look as generated by the collective sources. These sources contain contributions not only from the perturbed world sheets, but also the bulk contributions which are not associated with separate strings. As we will see later, these sources may have superluminal nature, in which case the Cerenkov radiation will appear.

IV. RENORMALIZATION

The action of the proper fields upon the source string is described by the self-action terms in Eq. (43) corresponding to the graphs shown in Fig. 2. The dilaton and the two-form lead to divergences while the contribution from the linearized gravity vanishes. Divergent terms can be absorbed by renormalization of the string tension parameter μ [41,42]. We consider renormalization in the first order of the perturbation theory. Linearizing the string part of the action (2), one can split it into the sum

$$S = S_{st} + S_\phi + S_B + S_h, \tag{53}$$

where S_{st} is the Polyakov action with the bare tension parameter,

$$S_{st} = -\frac{\mu_0}{2}\int X^\mu_a X^\nu_b\eta_{\mu\nu}\eta^{ab}d^2\sigma, \tag{54}$$

and three other terms describe the interaction with the dilaton, the two-form, and the linearized gravity:

$$S_\phi = -\mu_0\alpha\int\phi X^\mu_a X^\nu_b\eta_{\mu\nu}\eta^{ab}d^2\sigma, \tag{55}$$

$$S_B = -2\pi f\int B_{\mu\nu}X^\mu_c X^\nu_d\epsilon^{cd}d^2\sigma, \tag{56}$$

$$S_h = -\frac{\mu_0}{2}\int X^\mu_a X^\nu_b\eta^{ab}h_{\mu\nu}d^2\sigma. \tag{57}$$

Since we are working in the lowest order of the perturbation theory, the mapping functions X^μ here are quantities of zero order. In obtaining the last formula we used the following expansion:

$$\gamma^{ab}\sqrt{-\gamma} = \eta^{ab} + h_{\mu\nu}(\tfrac{1}{2}X^\mu_c X^{\nu c}\eta^{ab} - X^{\mu a}X^{\nu b}) + \cdots. \tag{58}$$

Consider the first-order dilaton field on the world sheet of the source string

$$\phi(\tau,\sigma) = \frac{\alpha\mu_0}{8\pi^2}\int\frac{\mathrm{e}^{iq_\mu(d^\mu - X^\mu(\tau,\sigma))}\delta(qu)\delta(q\Sigma)}{q^2}d^4q, \tag{59}$$

where $qu = q_\mu u^\mu$, $q\Sigma = q_\mu\Sigma^\mu$ are the flat-space scalar products, and $q^2 = q_\mu q^\mu$. Because of the delta functions, the scalar product in the exponential is constant:

$$q_\mu X^\mu(\tau,\sigma) = (qd) + (qu)\tau + (q\Sigma)\sigma = (qd) = \text{const}, \tag{60}$$

so the integrand does not depend on σ and τ. The integral diverges as

$$I = -\int\frac{\delta(qu)\delta(q\Sigma)}{q^2}d^4q = 2\pi\int_0^\infty\frac{dq_\perp}{q_\perp} \tag{61}$$

where we used the frame (18) and introduced polar coordinates in the 2-plane orthogonal to u^μ and Σ^μ:

$$q^2 = (qu)^2 - (q\Sigma)^2 - q_1^2 - q_2^2, \qquad q_1^2 + q_2^2 = q_\perp^2. \tag{62}$$

The integral logarithmically diverges at both ends. With the infrared (IR) and ultraviolet (UV) cutoff parameters

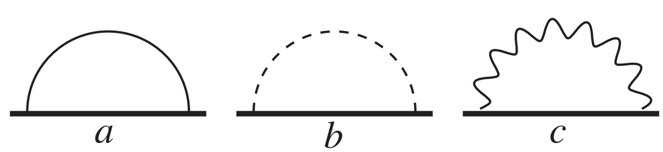

FIG. 2. Graphs describing self-interaction due to dilaton (a), two-form (b), and linearized gravity (c). The contributions from a and b are divergent and have different signs. The contribution of c is zero.

$q_\perp^{\min}$, $q_\perp^{\max}$, one can write

$$I = 2\pi \ln\frac{q_\perp^{\max}}{q_\perp^{\min}}. \tag{63}$$

Substituting this into Eq. (59) and further into (55) we find the regularized dilaton part of the action:

$$S_\phi^{\rm reg} = \frac{\mu_0^2\alpha^2}{4\pi}\ln\frac{q_\perp^{\max}}{q_\perp^{\min}}\int X_a^\mu X_b^\nu \eta_{\mu\nu}\eta^{ab}d^2\sigma. \tag{64}$$

Since the functional is the same as the bare action functional S_{st}, one can absorb divergencies by renormalization of μ_0.

Similarly, the first-order two-form field on the world sheet of the source string reads

$$B^{\mu\nu}(\tau,\sigma) = \frac{f}{2\pi}\int \frac{{\rm e}^{iq_\mu(d^\mu - X^\mu(\tau,\sigma))}V^{\mu\nu}\delta(qu)\delta(q\Sigma)}{q^2}d^4q. \tag{65}$$

This integral also diverges as (63). In view of the relation

$$V^{\mu\nu}V_\nu^\lambda = -U^{\mu\nu}, \tag{66}$$

one can see that the action (56) also has the functional form of (54), namely,

$$S_B^{\rm reg} = -2\pi f^2 \ln\frac{q_\perp^{\max}}{q_\perp^{\min}}\int X_a^\mu X_b^\nu \eta_{\mu\nu}\eta^{ab}d^2\sigma. \tag{67}$$

Finally, for the graviton $h_{\mu\nu}(\tau,\sigma)$ on the world sheet we have the divergent integral

$$h_{\mu\nu}(\tau,\sigma) = \frac{4G\mu}{\pi}\int\frac{{\rm e}^{iq_\mu(d^\mu - X^\mu(\tau,\sigma))}W_{\mu\nu}\delta(qu)\delta(q\Sigma)}{q^2}d^4q = -8G\mu W_{\mu\nu}\ln\frac{q_\perp^{\max}}{q_\perp^{\min}}, \tag{68}$$

where $W_{\mu\nu}$ is given by Eq. (35). However, substituting this into (57) one obtains zero in view of the identity

$$W_{\mu\nu}U^{\mu\nu} = 0. \tag{69}$$

Therefore, gravitational interaction of the strings does not lead to classical divergences in the lowest order of the perturbation theory. This result conforms with previous results [43,44] obtained with different tools.

Collecting the above formulas, we see that to remove self-interaction divergences one has merely to replace the tension parameter in the action (54) as follows:

$$\mu_0 - \left(\frac{\mu_0^2\alpha^2}{2\pi} - 4\pi f^2\right)\ln\frac{q_\perp^{\max}}{q_\perp^{\min}} = \mu. \tag{70}$$

Divergences due to the dilaton and the two-form have opposite signs. This reflects the fact that the scalar interaction is attractive, while interaction via the two-form is repulsive. If the Bogomolny-Prasad-Sommerfield (BPS) relation between the dilaton and the two-form couplings is satisfied,

$$\frac{\mu^2\alpha^2}{2\pi} = 4\pi f^2, \tag{71}$$

the divergent terms mutually cancel, and there is no renormalization at all, $\mu = \mu_0$ (for earlier work on this subject see Refs. [41–46]). Note that our dilaton coupling constant has dimension of length; the usual dimensionless constant $\bar\alpha$ (quantity of the order of unity) is related to it as

$$\alpha^2 = G\bar\alpha^2. \tag{72}$$

It has to be noted that renormalization can be performed in a simple way only at the linearized level. When all nonlinearities are taken into account, classical renormalizability of the bosonic string theory interacting with gravity is lost. In this paper we will be restricted by the second-order calculation of radiation which involves only on-shell second-order quantities. These are unaffected by the higher-order renormalization effects. So, in what follows, we will use the value for the string tension renormalized in the first order, and thus omit all self-interaction terms.

V. PERTURBATIONS OF THE STRING WORLD SHEETS

Now consider the first-order perturbations of the mapping functions $X_n^\mu(\tau,\sigma)$, $n = 1, 2$ caused by mutual interactions. We have to substitute into each string equation of motion (4) the first-order fields generated by another string. It is convenient to use the following index convention: $n = 1, 2$; $n' = 1, 2$; $n \neq n'$. The total perturbation thus splits into three terms due to the dilaton, two-form, and graviton exchange:

$$\overset{1}{X}{}_n^\mu = \overset{\phi}{X}{}_n^\mu + \overset{B}{X}{}_n^\mu + \overset{h}{X}{}_n^\mu, \tag{73}$$

as shown in Fig. 3. Let us start with the dilaton exchange contribution [Fig. 3(a)]. The corresponding world-sheet perturbation $\overset{1}{X}{}_n^\mu(\tau,\sigma)$ is the solution of the two-dimensional D'Alembert equation [following from Eqs. (43) and (44)]:

$$(\partial_\tau^2 - \partial_\sigma^2)\overset{\phi}{X}{}_n^\mu = -2\alpha[(\overset{\phi}{U}{}^{\mu\nu}\partial_\nu - \partial^\mu)\overset{1}{\phi}_n(x)]_{x=\overset{0}{X}_n(\tau,\sigma)}, \tag{74}$$

where the dilaton field generated by the string n' is taken on the world sheet of the string n:

$$\partial_\mu\overset{1}{\phi}_n\Big|_{x=\overset{0}{X}_n(\tau,\sigma)} = \frac{\alpha\mu}{8\pi^2 i}\int\frac{{\rm e}^{iq_\lambda(d_{n'}^\lambda - X_n^\lambda(\tau,\sigma))}\delta(qu_{n'})\delta(q\Sigma_{n'})}{q^2 + 2i\epsilon q^0}\times q_\mu d^4q. \tag{75}$$

Substituting (75) into (74) one can obtain the solution by dividing the right-hand side by the two-dimensional D'Alembert operator as follows:

$$X^{\mu}_{n}\overset{\phi}{X}{}^{\mu}_{n}(\tau,\sigma) = i\frac{\alpha^2\mu}{(2\pi)^2}\int \frac{\Delta_{n'}[q^{\mu}+\Sigma^{\mu}_{n}(q\Sigma_n)-u^{\mu}_{n}(qu_n)]\mathrm{e}^{-iq_{\lambda}\overset{0}{X}{}^{\lambda}_{n}(\tau,\sigma)}}{q^2[(qu_n)^2-(q\Sigma_n)^2]}d^4q, \tag{76}$$

where

$$\Delta_{n'} = \mathrm{e}^{iqd_{n'}}\delta(qu_{n'})\delta(q\Sigma_{n'}). \tag{77}$$

Note that the delta functions in the integrand have support outside both the light cone $q^2 = 0$ and the surface $(qu_n)^2 = (q\Sigma_n)^2$ (except for the trivial point $q^{\mu} = 0$), so the integral is finite.

Consider now the two-form interaction [Fig. 3(b)]. We have to solve the equation

$$(\partial_\tau^2 - \partial_\sigma^2)\overset{B}{X}{}^{\mu}_{n} = 2\pi f V^{\nu\lambda}_{n}\overset{1}{H}{}^{\mu}{}_{\nu\lambda}|_{x=\overset{0}{X}_n(\tau,\sigma)}, \tag{78}$$

where the three-form is

$$\overset{1}{H}{}^{\mu}{}_{\nu\lambda}|_{x=\overset{0}{X}_n(\tau,\sigma)} = \frac{f}{2\pi i}\int \frac{\Delta_{n'}q^{\{\mu}V^{\nu\lambda\}}_{n'}e^{-iq_{\lambda}\overset{0}{X}{}^{\lambda}_{n}(\tau,\sigma)}}{q^2+2i\epsilon q^0}d^4q, \tag{79}$$

where curly brackets {} denote the cyclic permutation of indices. Again, dividing by the operator $(\partial_\tau^2 - \partial_\sigma^2)$ one obtains

$$\overset{B}{X}{}^{\mu}_{n}(\tau,\sigma) = i\frac{f^2}{\mu}\int \frac{\Delta_{n'}V_{n\nu\lambda}q^{\{\mu}V^{\nu\lambda\}}_{n'}\mathrm{e}^{-iq_{\lambda}\overset{0}{X}{}^{\lambda}_{n}(\tau,\sigma)}}{q^2[(qu_n)^2-(q\Sigma_n)^2]}d^4q. \tag{80}$$

Similarly, the gravitational contribution is described by the equation

$$(\partial_\tau^2 - \partial_\sigma^2)\overset{h}{X}{}^{\mu}_{n} = \frac{\mu}{2}U^{\alpha\beta}_{n}[\partial^{\mu}\overset{1}{h}_{\alpha\beta} - 2\partial_{\alpha}\overset{1}{h}{}^{\mu}_{\beta} - U^{\mu\nu}_{n}(\partial_{\nu}\overset{1}{h}_{\alpha\beta} - 2\partial_{\beta}\overset{1}{h}_{\alpha\nu})]_{x=\overset{0}{X}_n(\tau,\sigma)}, \tag{81}$$

where the variation $\overset{1}{h}_{\mu\nu}$ is generated by the partner string n':

$$\overset{1}{h}_{\mu\nu}|_{x=\overset{0}{X}_n(\tau,\sigma)} = \frac{4\mu G}{\pi}\int \frac{\Delta_{n'}W_{n'\mu\nu}e^{-iq_{\lambda}\overset{0}{X}{}^{\lambda}_{n}(\tau,\sigma)}}{q^2+2i\epsilon q^0}d^4q. \tag{82}$$

Solving this equation, one finds

$$\overset{h}{X}{}^{\mu}_{n}(\tau,\sigma) = i\frac{2\mu G}{\pi}\int \frac{\Delta_{n'}U_{n\alpha\beta}[q^{\mu}W^{\alpha\beta}_{n'} - 2W^{\mu\alpha}_{n'}q^{\beta} - U^{\mu\nu}_{n}(q_{\nu}W^{\alpha\beta}_{n'} - 2q^{\alpha}W^{\beta}_{n'\nu})]\mathrm{e}^{-iq_{\lambda}\overset{0}{X}{}^{\lambda}_{n}(\tau,\sigma)}}{q^2[(qu_1)^2-(q\Sigma_1)^2]}d^4q. \tag{83}$$

It can be checked that the gauge condition (38) imposed in the beginning of the calculation holds indeed for each of the three separate contributions to the perturbed mapping functions.

VI. EFFECTIVE SOURCES OF RADIATION

The first-order fields $\overset{1}{\phi}$, $\overset{1}{B}{}^{\mu\nu}$, $\overset{1}{h}{}^{\mu\nu}$ do not contain the radiative parts. Consider, e.g., the Fourier transform of the dilaton:

$$\phi(k) = \int \phi(x)\mathrm{e}^{ikx}d^4x. \tag{84}$$

The retarded and advanced solutions of the first-order wave equation (27) will read

$$\overset{1}{\phi}{}^{\pm}_{n}(k) = 2\pi^2\mu\alpha\frac{\delta(ku_n)\delta(k\Sigma_n)}{k^2 \pm 2i\epsilon k^0}\mathrm{e}^{ikd_n}. \tag{85}$$

The radiative part

$$\overset{1}{\phi}{}^{\mathrm{rad}}_{n}(k) = \tfrac{1}{2}(\overset{1}{\phi}{}^{+}_{n}(k)\overset{1}{\phi}{}^{-}_{n}) = -i\pi^3\mu\alpha\delta(ku_n)\delta(k\Sigma_n)\delta(k^2)\mathrm{e}^{ikd_n} \tag{86}$$

is the distribution having support only at the trivial point $k^{\mu} = 0$ in the momentum space. Thus, to investigate radiation, we have to pass to the second order of the perturbation theory. The problem reduces to the construction of the source terms in the wave equations of the second order.

A. Dilaton

Consider the second-order dilaton equation (47) in more detail. The current on the right-hand side contains the contributions localized on the string world sheets [the upper line in (47)] and the bulk contributions coming from the products of the first-order dilaton, two-form, and graviton fields (the lower line). The former contains the sum over the strings which can be understood as follows. One has to take the perturbations of the world-sheet mapping functions X^{μ}_{n} for each string $n = 1, 2$ due to the first-order field generated by the partner string $n = 2$,

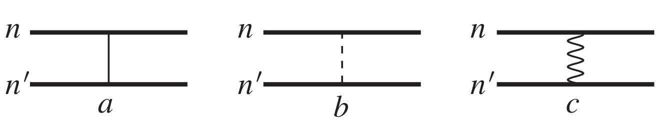

FIG. 3. Deformations of the string world sheets due to interactions via the dilaton (a), two-form (b), and linearized gravity (c). $n = 1, 2$; $n' = 1, 2$; $n \neq n'$.

1, respectively. These contributions are depicted by the graphs a_n, b_n, c_n, $n = 1, 2$ in Fig. 4. The external dilaton leg corresponds to the emitted dilaton with the momentum k^μ (in our classical treatment to the Fourier transform of the current); thus the terms in the sum in (47) with $n = 1$ are given by the graphs a_1, b_1, c_1 and those with $n = 2$ by the graphs a_2, b_2, c_2. Other upper line terms must be treated in a similar way: one has to take $U_n^{\mu\nu}$ for each $n = 1, 2$ and multiply by the graviton and dilaton perturbations caused by the other string $n = 2, 1$. These are depicted by the graphs d_n, e_n, $n = 1, 2$. In this way the self-action of the fields upon the string will be excluded.

On the contrary, the terms in the lower line of Eq. (47) are nonlocal and not pairwise. Here the self-action terms also have to be excluded, so we take in the quadratic terms only the products of the first-order fields generated by different strings.

Consider first the contributions described by graphs a_1, b_1, c_1 (dilaton emission from the first string line). The corresponding current reads

$$\overset{st}{J}_1(x) = -\frac{\mu\alpha}{8\pi}\int \overset{1}{X}{}_1^{\mu}\,\partial_\mu \delta^4[x - \overset{0}{X}_1(\tau, \sigma)]d^2\sigma, \qquad (87)$$

where the perturbation of the mapping function of the first string $\overset{1}{X}{}_1^{\mu}$ is caused by the first-order dilaton, two-form, and graviton fields generated by the second string. Substituting the corresponding terms into (87) we obtain, after some rearrangements,

$$\overset{1}{X}{}_1^{\mu}(\tau, \sigma) = i\int \frac{Q_1^\mu \delta(qu_2)\delta(q\Sigma_2)\mathrm{e}^{iq(d_2 - d_1 - u_1\tau - \Sigma_1\sigma)}}{q^2[(qu_1)^2 - (q\Sigma_1)^2]} d^4q. \qquad (88)$$

The vector Q_1^μ is the sum of three terms according to the decomposition described above:

$$Q_1^\mu = \frac{\alpha^2\mu}{(2\pi)^2}D_1^\mu + \frac{2f^2}{\mu}Y_1^\mu + \frac{2\mu G}{\pi}Z_1^\mu, \qquad (89)$$

where the dilaton exchange contribution is

$$D_1^\mu = q^\mu + \Sigma_1^\mu(q\Sigma_1) - u_1^\mu(qu_1), \qquad (90)$$

the two-form contribution is

$$\begin{aligned} Y_1^\mu = {}& q^\mu[(u_1u_2)(\Sigma_1\Sigma_2) - (\Sigma_1u_2)(u_1\Sigma_2)] \\ &+ \Sigma_2^\mu[(qu_1)(u_2\Sigma_1) - (\Sigma_1 q)(u_1u_2)] \\ &+ u_2^\mu[(u_1\Sigma_2)(\Sigma_1 q) - (qu_1)(\Sigma_1\Sigma_2)], \end{aligned} \qquad (91)$$

and the graviton contribution is

$$\begin{aligned} Z_1^\mu = {}& q^\mu[(u_1u_2)^2 + (\Sigma_1\Sigma_2)^2 - (u_1\Sigma_2)^2 - (u_2\Sigma_1)^2 - 2] \\ &+ [u_1^\mu(qu_1) - \Sigma_1^\mu(q\Sigma_1)][(u_1u_2)^2 - (\Sigma_1\Sigma_2)^2 \\ &- (u_1\Sigma_2)^2 + (u_2\Sigma_1)^2 + 2] - 2[u_1^\mu(q\Sigma_1) \\ &+ \Sigma_1^\mu(qu_1)][(u_1u_2)(u_2\Sigma_1) - (\Sigma_1\Sigma_2)(u_1\Sigma_2)] \\ &- 2u_2^\mu[(u_1u_2)(qu_1) - (\Sigma_1u_2)(q\Sigma_1)] \\ &+ 2\Sigma_2^\mu[(u_1\Sigma_2)(qu_1) - (\Sigma_1\Sigma_2)(q\Sigma_1)]. \end{aligned} \qquad (92)$$

Consider the Fourier transform

$$J(k) = \int J(x)\mathrm{e}^{ikx}d^4x \qquad (93)$$

with $k^2 = 0$ (on-shell condition for the massless particle). Substituting the above expressions and integrating over the world sheet of the first string, one obtains two more delta functions:

$$\int \mathrm{e}^{i(k-q)(u_1\tau - \Sigma_1\sigma)}d\tau d\sigma = (2\pi)^2\delta[(k-q)u_1]\delta[(k-q)\Sigma_1], \qquad (94)$$

so we will totally have the product of four delta functions in the integrand:

$$\Lambda_1(q, k) = \delta(qu_1)\delta(q\Sigma_1)\delta[(k-q)u_2]\delta[(k-q)\Sigma_2]. \qquad (95)$$

Now consider the contribution $J_2(k)$ coming from the second string (the graphs a_2, b_2, c_2 in Fig. 4). Obviously it can be obtained from the previous result by interchanging indices $1 \leftrightarrow 2$ elsewhere. In this case we will get the

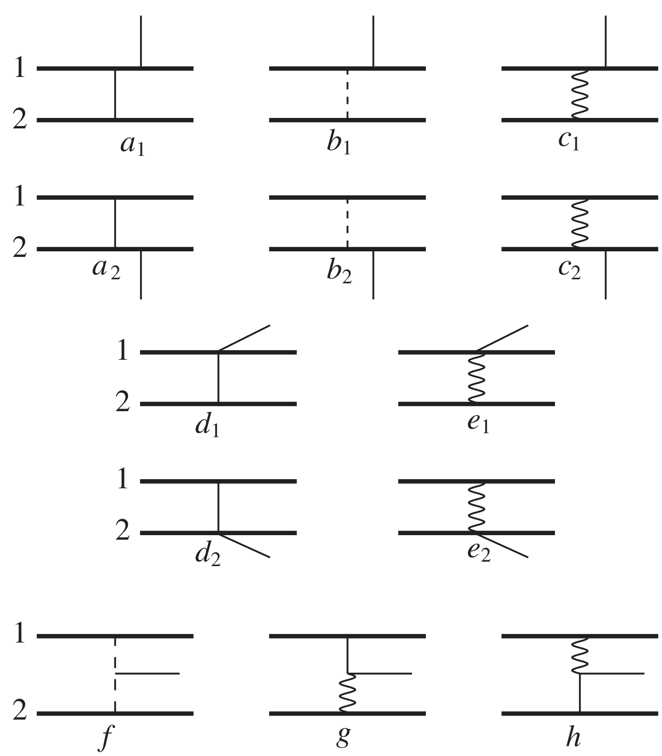

FIG. 4. The diagrams contributing to dilaton radiation in the second order of the perturbation theory: a_1, b_1, c_1 correspond to deformation of the first string, and a_2, b_2, c_2 to deformation of the second one; graphs d, e stand for contact terms. Diagram f corresponds to the product of (first-order) two-form fields generated by the first and the second string; graphs g, h correspond to mixed graviton-dilaton contributions.

product of the δ functions in the form

$$\Lambda_2(q,k) = \delta(qu_2)\delta(q\Sigma_2)\delta[(k-q)u_1]\delta[(k-q)\Sigma_1]. \tag{96}$$

It is convenient to cast this second integral into the same form as the previous one. For this it is sufficient to shift the integration variable in $J_2(k)$ as follows: $q^\mu \to k^\mu - q^\mu$. Since $\Lambda_1(q,k) = \Lambda_2(k-q,k)$, we will get the same product of the δ functions, so finally we can present the total contribution of the first six graphs as follows:

$$\overset{st}{J}(k) = \overset{st}{J}_1(k) + \overset{st}{J}_2(k) = \int \Pi(q,k)\left(\frac{\overset{st}{\Theta}_1(q)}{q^2} + \frac{\overset{st}{\Theta}_2(k-q)}{(k-q)^2}\right) d^4q, \tag{97}$$

where

$$\Pi(q,k) = \mathrm{e}^{i(d_1k + q(d_2-d_1))}\Lambda_1(q,k), \tag{98}$$

and

$$\overset{st}{\Theta}_1(q) = \frac{\alpha}{[(q\Sigma_1)^2 - (qu_1)^2]}\left[\frac{\alpha^2\mu^2}{8\pi}(kD_1) + \pi f^2(kY_1) + G\mu^2(kZ_1)\right]. \tag{99}$$

To get the function $\overset{st}{\Theta}_2$ from $\overset{st}{\Theta}_1$ one merely has to interchange the indices 1, 2 labeling vectors in the scalar products, changing simultaneously $q^\mu \to (k-q)^\mu$.

The contributions of the next four graphs, d_1, d_2, e_1, e_2, are computed similarly. The resulting "contact" term in the source can be presented again in the form (97) with equal contributions from two strings,

$$\overset{ct}{\Theta}_1(q) = \overset{ct}{\Theta}_2(q) = \frac{\alpha^3\mu^2}{8\pi} + G\mu^2\alpha[(u_1u_2)^2 + (\Sigma_1\Sigma_2)^2 - (u_1\Sigma_2)^2 - (u_2\Sigma_1)^2 - 2]. \tag{100}$$

Now consider the bulk terms [bulk terms in (47)] which are due to nonlinear field interactions—dilaton–two-form and dilaton-graviton. Their contribution is illustrated by graphs f and e_n, the second being pairwise. In quantum theory terms they can be interpreted as the coalescence of two virtual axions into the on-shell dilaton, and the coalescence of the off-shell dilaton and graviton into the on-shell dilaton. We have to compute the Fourier transform of the bulk current,

$$\overset{b}{J}(x) = \frac{\alpha}{24\pi}\overset{1}{H}_{\mu\nu\lambda}\overset{1}{H}{}^{\mu\nu\lambda} + \frac{1}{4\pi}\partial_\mu\left[\left(\overset{1}{h}{}^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\overset{1}{h}\right)\partial_\nu\overset{1}{\phi}\right]. \tag{101}$$

Here, in the products $\overset{1}{H}_{\mu\nu\lambda}\overset{1}{H}{}^{\mu\nu\lambda}$ and $\overset{1}{\phi}\,\overset{1}{h}_{\mu\nu}$, we have to substitute one field generated by the first string and another by the second one.

In spite of the different appearance of the bulk terms as compared to the world-sheet terms, one can cast them into the unique form (97) as well. Consider the corresponding transformations for the first term in (101) depicted by the graph f in Fig. 4. The first-order quantities to be substituted here read

$$\overset{1}{H}{}_n^{\mu\nu\lambda}(x) = \frac{f}{2i\pi}\int \frac{\Delta_n q^{\{\mu}V_n^{\nu\lambda\}}\mathrm{e}^{-iqx}}{q^2} d^4q. \tag{102}$$

(We can omit the ϵ-term in the denominator indicating the position of the pole since the resulting integral does not depend on its shift from the real axis.) Now let us calculate the contraction

$$\int \mathrm{e}^{ikx}\overset{1}{H}_{\mu\nu\lambda}\overset{1}{H}{}^{\mu\nu\lambda} d^4x = -(2\pi f)^2 \int \frac{\Delta_1(q)\Delta_2(q')q_{\{\mu}V_{1\nu\lambda\}}q'^{\{\mu}V_2^{\nu\lambda\}}\mathrm{e}^{i(k-q-q')x}}{q^2q'^2} d^4q = -(2\pi f)^2\int \frac{\Delta_1(q)\Delta_2(k-q)q_{\{\mu}V_{1\nu\lambda\}}(k-q)^{\{\mu}V_2^{\nu\lambda\}}\mathrm{e}^{i(k-q-q')x}}{q^2(k-q)^2} d^4q. \tag{103}$$

We are interested in the on-shell value of k^μ, i.e. $k^2 = 0$. In this case one can write

$$\frac{1}{q^2(k-q)^2} = \left(\frac{1}{(q-k)^2} - \frac{1}{q^2}\right)\frac{1}{2kq}. \tag{104}$$

Using this relation one can cast the above contraction into the form (97) with the convention that the argument q^μ is used for the $n=1$ terms, and $(k-q)^\mu$ for the $n=2$ terms.

Performing similar calculations for the graphs g_1, g_2 and combining with the above, we obtain the total bulk term in the form (97) with $\Theta = \overset{b}{\Theta}$:

$$\overset{b}{\Theta}_1 = \pi f^2\alpha\left[\frac{(Y_1k)}{(kq)} - (u_1u_2)(\Sigma_1\Sigma_2) + (\Sigma_1u_2)(u_1\Sigma_2)\right] + \mu^2\alpha G\frac{(ku_2)^2 - (k\Sigma_2)^2 + (ku_1)^2 - (k\Sigma_1)^2}{(kq)}, \tag{105}$$

with the rule of getting $\overset{b}{\Theta}_2$ from $\overset{b}{\Theta}_1$ the same as before. In obtaining this expression we have used the delta functions in the integrand, fixing $(qu_1) = (q\Sigma_1) = 0$ and $(qu_2) = (ku_2)$, $(q\Sigma_1) = (k\Sigma_1)$.

B. Two-form

The source term in Eq. (8) for the second-order two-form field can be presented as the sum of the ten graphs shown in Fig. 5. Here we have string contributions corresponding to exchange by the dilaton, two-form, and graviton, the bulk terms describing coalescence of virtual quanta to the on-shell axion, but no contact term because of the absence of multileg vertices associated with the two-form. The string terms a_n, b_n, c_n are computed as the Fourier transform of the currents

$$\overset{st}{J}{}^{\mu\nu}_n(x) = f\int\Big[\overset{0}{X}{}^{\mu}_{na}\overset{1}{X}{}^{\nu}_{nb}\epsilon^{ab} - \frac{1}{2}V^{\mu\nu}_n\overset{1}{X}{}^{\lambda}_n\partial_\lambda\Big] \times \delta^4[x - X_n(\sigma,\tau)]d^2\sigma, \tag{106}$$

where perturbations of the mapping functions X^μ_n, $n = 1, 2$ are generated by the partner string $n = 2, 1$, respectively. Using the same rearrangements as before, we obtain

$$\overset{st}{J}{}^{\mu\nu}(k) = \overset{st}{J}{}^{\mu\nu}_1(k) + \overset{st}{J}{}^{\mu\nu}_2(k) = \int \Pi(q,k)\bigg(\frac{\overset{st}{\Theta}{}^{\mu\nu}_1(q)}{q^2} + \frac{\overset{st}{\Theta}{}^{\mu\nu}_2(k-q)}{(k-q)^2}\bigg)d^4q, \tag{107}$$

where

$$\overset{st}{\Theta}{}^{\mu\nu}_1(q) = -\frac{1}{[(q\Sigma_1)^2 - (qu_1)^2]}\bigg[8\pi^2\frac{f^3}{\mu}\Big((q\Sigma_1)u_1^{[\mu}Y_1^{\nu]} - (qu_1)\Sigma_1^{[\mu}Y_1^{\nu]} - \frac{1}{2}V_1^{\mu\nu}(Y_1k)\Big) + \frac{1}{2}\alpha^2 f\mu\Big((q\Sigma_1)u_1^{[\mu}D_1^{\nu]} - (qu_1)\Sigma_1^{[\mu}D_1^{\nu]} - \frac{1}{2}V_1^{\mu\nu}(D_1k)\Big) + 8\pi f\mu G((q\Sigma_1)u_1^{[\mu}Z_1^{\nu]} + (qu_1)\Sigma_1^{[\mu}\Big(Z_1^{\nu]} - \frac{1}{2}V_1^{\mu\nu}(Z_1k)\Big)\bigg], \tag{108}$$

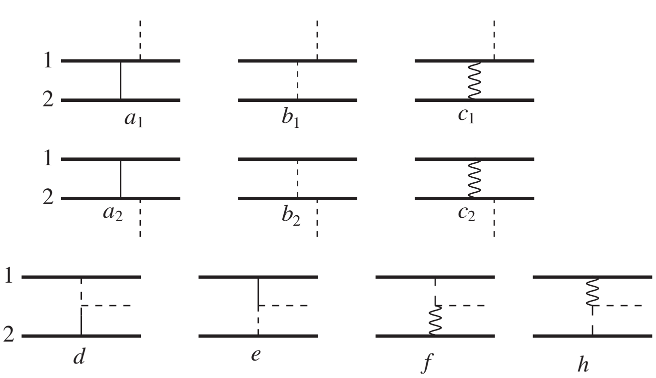

FIG. 5. The second-order amplitudes of the two-form emission. Graphs a, b, c show contributions from deformations of the string world sheets, and graphs d, e, f, h the quadratic bulk terms.

and to get the second string term we must interchange indices $1 \leftrightarrow 2$ and momenta $q^\mu \leftrightarrow k^\mu - q^\mu$.

The bulk terms d_n, e_n are the Fourier transform of the bulk current

$$\overset{f}{J}(x)_{\mu\nu} = \frac{1}{4\pi}\partial_\lambda\Big[\overset{1}{H}{}^{\lambda}{}_{\mu\nu}\Big(4\alpha\overset{1}{\phi} - \frac{1}{2}\overset{1}{h}\Big)\Big], \tag{109}$$

where again we have to take the products of fields generated by different strings. The result can be cast into the form (107) with

$$\overset{b}{\Theta}{}^{\mu\nu}_1 = -\frac{8\pi f\mu G}{(kq)}[V_1^{\mu\nu}(kq) + (q^{[\mu}u_1^{\nu]})(\Sigma_1 q) - (q^{[\mu}\Sigma_1^{\nu]})(u_1q) - V_2^{\mu\nu}(kq) + (q^{[\mu}u_2^{\nu]})(\Sigma_2k) - (q^{[\mu}\Sigma_2^{\nu]})(u_2k))] - \frac{f\alpha^2\mu}{2(kq)}[V_1^{\mu\nu}(kq)) - (q^{[\mu}u_1^{\nu]})(\Sigma_1q) + (q^{[\mu}\Sigma_1^{\nu]})(u_1q) - V_2^{\mu\nu}(kq) - (q^{[\mu}u_2^{\nu]})(\Sigma_2k) + (q^{[\mu})\Sigma_2^{\nu]}(u_2k)], \tag{110}$$

with the same rule for the second string term as in (108).

C. Graviton

The source (52) in the second-order graviton equation can be treated along the same lines. It includes contributions of the 13 graphs shown in Fig. 6. It turns out, however,

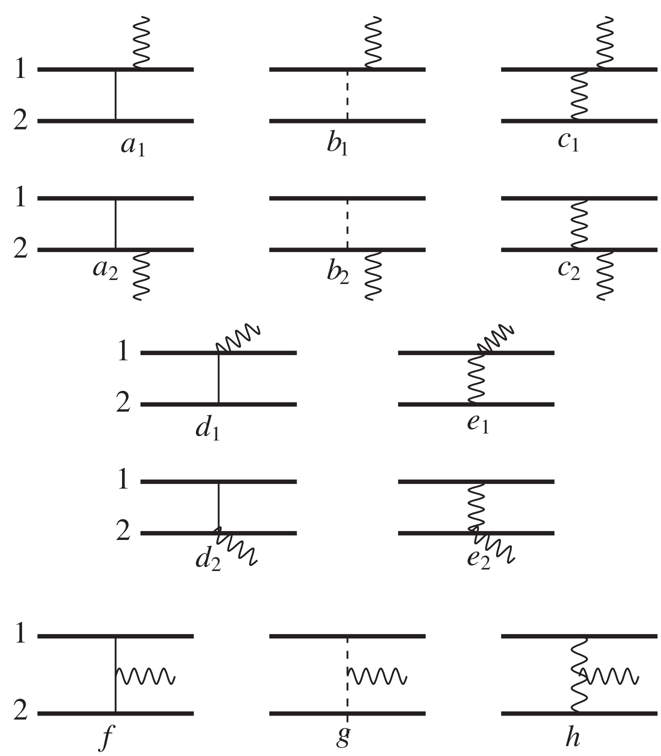

FIG. 6. The second-order amplitudes of the graviton emission. The sum of all graphs is zero on the graviton mass shell.

that the projection of the total on-shell ($k^2 = 0$) current onto the graviton transverse polarization states gives zero. The reason lies in the dimensionality of the space transverse to the string: the configuration of the parallel strings is reduced to that of point particles in the $1 + 2$ theory, where there are no on-shell transverse gravitational degrees of freedom. As it was shown in [37], the transformation to the parallel string configuration is always possible for a superluminal moving intersecting string. So there is no gravitational radiation in this case and we do not give the details of the calculation here.

VII. RADIATION

Total radiation four-momentum loss can be presented in a standard way through the on-shell Fourier transform of the source current, the vector $k^\mu = (\omega, \mathbf{k})$, $\omega = |\mathbf{k}|$ playing the role of the radiation four-momentum. In the case of the dilaton one obtains the following explicitly Lorentz-covariant expression:

$$P^\mu_{(\phi)} = \frac{16}{\pi} \int k^\mu \frac{k^0}{|k^0|} |J_{(\phi)}(k)|^2 \delta(k^2) d^4k, \tag{111}$$

and similarly for the two-form,

$$P^\mu_{(B)} = \frac{1}{\pi} \int k^\mu \frac{k^0}{|k^0|} |J_{\alpha\beta}(k)|^2 \delta(k^2) d^4k. \tag{112}$$

Alternatively, the latter quantity can be presented as a square of the polarization projection of the current. Indeed, in three space dimensions the two-form field propagating along the wave vector $\mathbf{k}$ has a unique polarization state,

$$e_{ij} = \frac{1}{\sqrt{2}} (e^\theta_i e^\varphi_j - e^\varphi_i e^\theta_j), \qquad i, j = 1, 2, 3, \tag{113}$$

where $\mathbf{e}^\theta$ and $\mathbf{e}^\varphi$ are two unit vectors orthogonal to $\mathbf{k}$ and to each other:

$$\mathbf{e}^\varphi \cdot \mathbf{e}^\theta = 0, \qquad \mathbf{k} \cdot \mathbf{e}^\varphi = \mathbf{k} \cdot \mathbf{e}^\theta = 0. \tag{114}$$

Using antisymmetry and transversality of the two-form current $k^\mu J_{\mu\nu}(k) = 0$, and the completeness condition

$$e^\theta_i e^\theta_j + e^\varphi_i e^\varphi_j = \delta_{ij} - k_i k_j/\omega^2, \tag{115}$$

one finds

$$P^\mu_{(B)} = \frac{1}{\pi} \int k^\mu \frac{k^0}{|k^0|} |J^{ij}_{(B)}(k) e_{ij}|^2 \delta(k^2) d^4k. \tag{116}$$

Integrating over k^0, we finally obtain

$$P^\mu_{(\phi)} = \frac{16}{\pi} \int \frac{k^\mu}{|\mathbf{k}|} |J_{(\phi)}(\mathbf{k})|^2 d^3k, \tag{117}$$

$$P^\mu_{(B)} = \frac{1}{\pi} \int \frac{k^\mu}{|\mathbf{k}|} |J_{(B)}(\mathbf{k})|^2 d^3k, \tag{118}$$

where

$$J_{(B)}(\mathbf{k}) = J^{ij}_{(B)}(k) e_{ij} \tag{119}$$

with $k^0 = |\mathbf{k}|$. In what follows we shall use the parametrization of three-vectors by the spherical angles: $\mathbf{k} = \omega[\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta]$, $\mathbf{e}^\varphi = [-\sin\varphi, \cos\varphi, 0]$, $\mathbf{e}^\theta = [\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta]$.

A. Cerenkov condition

With our conventions, the radiation amplitudes associated with the first string contain the integral

$$I_1 = \int \frac{\delta(qu_2)\delta(q\Sigma_2)\delta[(k-q)u_1]\delta[(k-q)\Sigma_1] f(q) e^{iqd}}{q^2} d^4q, \tag{120}$$

where $d^\mu = d^\mu_2 - d^\mu_1$, and $f(q)$ is some regular function of q. In the Lorentz frame where $u^\mu_1 = (1, 0, 0, 0)$, $\Sigma^\mu_1 = (0, 0, 0, 1)$, $u^\mu_2 = \gamma(1, 0, -v\cos\alpha, v\sin\alpha)$, $\Sigma^\mu_2 = (0, 0, \sin\alpha, \cos\alpha)$, one can integrate over q^0 using

$$\delta[(k-q)u_1] = \delta(q^0 - \omega). \tag{121}$$

Two other δ functions,

$$\delta(qu_2) = \gamma^{-1}\delta(\omega + q_y v\cos\alpha - q_z v\sin\alpha), \tag{122}$$

$$\delta(q\Sigma_2) = \gamma^{-1}\delta(q_y\sin\alpha + q_z v\cos\alpha), \tag{123}$$

can be used to fix the values of q_y, q_z:

$$q_y = -\frac{\omega\cos\alpha}{v}, \qquad q_z = \frac{\omega\sin\alpha}{v}. \tag{124}$$

The remaining δ function

$$\delta[(k-q)\Sigma_1] = \delta(k_z - q_z) = \delta\left(k_z - \frac{\omega\sin\alpha}{v}\right) \tag{125}$$

no longer depends on q, but rather restricts the value of the wave vector of radiation k_z:

$$k_z = \frac{\omega\sin\alpha}{v} = v_p. \tag{126}$$

This is Cerenkov's condition for an emitted massless wave. Indeed, in our frame an effective source of radiation is moving along the z axis with the velocity v_p; thus the quantity

$$\cos\theta = \frac{k_z}{\omega} = \frac{1}{v_p} \tag{127}$$

defines Cerenkov's angle of emission if $v_p > 1$, i.e. the source is superluminal. Thus, radiation arises if the string relative velocity v and the inclination angle α satisfy the Cerenkov condition

$$\sin\alpha < v. \tag{128}$$

Given v, this condition will always be satisfied for sufficiently small α. In particular, it is identically fulfilled for $\alpha = 0$, i.e. for parallel strings. Moreover, it can be shown that, if $v_p > 1$, the Lorentz frame and the world-sheet coordinates τ_n, σ_n always exist such that two strings look parallel [37]. In the frame where the first string is at rest and stretched along the z axis, an effective source moves in the z direction, and radiation will be emitted along the Cerenkov cone around the z axis with an angle

$$\theta_C = \arccos\frac{v}{\sin\alpha}. \tag{129}$$

The remaining integral over q_x in (120) can be evaluated using contour integration. With our choice of coordinates, the scalar product $q^\mu d_\mu = -q_x d$, so we have the integral over q_x,

$$I_1 = \int \frac{e^{-iq_x d} f(q_x) dq_x}{q_x^2 + p^2} = \frac{\pi}{p} f(-ip) e^{-\kappa d}, \tag{130}$$

where

$$p = \sqrt{q_y^2 + q_z^2 - q_0^2} = \frac{\omega}{\gamma v}. \tag{131}$$

For the second term in (97) containing the pole $(k-q)^{-2}$, one obtains the same fixed values of q^0, q_y, q_z, but the integration over q_x gives the value

$$q_x = k_x + \frac{\omega\xi}{iv}, \qquad \xi = \cos\alpha + v\sin\theta\sin\phi. \tag{132}$$

Summarizing the above results, we obtain

$$\int\left(\frac{\Theta_1(q)}{q^2} + \frac{\Theta_2(k-q)}{(k-q)^2}\right) e^{ikd_1 + iqd_2} d^4q = [E_1\Theta_1(q_1) + (\gamma\xi)^{-2} E_2 \Theta_2(k - q_2)] \times \delta(\cos\theta - \cos\theta_C), \tag{133}$$

where

$$E_1 = e^{ikd_1 - \omega d/(v\gamma)}, \qquad E_2 = e^{ikd_2 - \omega d\xi/v}, \tag{134}$$

and the following complex vectors are introduced:

$$q_1^\mu = \frac{\omega}{v}[v, -i/\gamma, -\cos\alpha, \sin\alpha], \tag{135}$$

$$k^\mu - q_2^\mu = \frac{\omega\xi}{v}[0, i, 1, 0]. \tag{136}$$

The presence of the delta function on the right-hand side of (133) means that the total radiation loss is infinite. This could be expected since we deal with the stationary motion of an infinitely long string. So it is natural to consider the radiation loss per unit length of the string at rest. Redefining the currents as

$$J(\mathbf{k}) = I(\mathbf{k})\delta(\cos\theta - \cos\theta_C), \tag{137}$$

and using the identity

$$\delta^2(\cos\theta - \cos\theta_C) = \frac{L\omega}{2\pi}\delta(\cos\theta - \cos\theta_C), \tag{138}$$

where L is the normalization length, we find

$$\mathcal{P}^\mu_{(\phi)} = L^{-1} P^\mu_{(\phi)} = \frac{8}{\pi^2}\int k^\mu |I_{(\phi)}(\mathbf{k})|^2 \delta(\cos\theta - \cos\theta_C) d^3k, \tag{139}$$

$$\mathcal{P}^\mu_{(B)} = L^{-1} P^\mu_{(B)} = \frac{1}{2\pi^2}\int k^\mu |I_{(B)}(\mathbf{k})|^2 \delta(\cos\theta - \cos\theta_C) d^3k. \tag{140}$$

B. Relativistic peaking and spectrum enhancement

According to (133), there is a frequency cutoff due to exponential factors (134). Actually, the amplitude is the superposition of two terms which can be associated with contributions of two strings (this can not be taken literally: in the second order of the perturbation theory the superposition principle does not hold). Two terms exhibit different frequency cutoffs. The first string term has a cutoff

$$\omega \lesssim \frac{v\gamma}{d}, \tag{141}$$

which does not depend on the radiation angle, while the second one exhibits a φ-dependent cutoff:

$$\omega \lesssim \frac{v}{d\xi} = \frac{v}{d(\cos\alpha + v\sin\theta\sin\varphi)}. \tag{142}$$

This means that the angular distribution of radiation on the Cerenkov cone is anisotropic. This feature becomes especially pronounced in the ultrarelativistic case.

In view of the identity

$$\cos^2\alpha = v^2\sin^2\theta + \gamma^{-2}, \tag{143}$$

which holds on the radiation cone, in the ultrarelativistic limit $\gamma \to \infty$ the quantity ξ has a sharp minimum at $\varphi = -\pi/2$ corresponding to the direction of the moving string in the rest frame of the first string:

$$\xi \approx \frac{1}{2\gamma^2\cos\alpha}(1 + \beta^2\gamma^2\cos\alpha^2), \tag{144}$$

where $\beta = \pi/2 + \varphi \ll 1$. Because of the factor ξ^{-2} in the second term in (119) the Cerenkov radiation is peaked around the direction $\varphi = -\pi/2$ within the narrow angular region

$$\beta \lesssim \gamma^{-1}. \tag{145}$$

Moreover, the frequency range associated with the second terms is substantially larger in the ultrarelativistic limit than that associated with the first term. Indeed, if $\kappa = \gamma\cos\alpha \gg 1$, one has

$$\xi \approx \frac{1}{2\gamma\kappa}(1 + \kappa^2\beta^2), \tag{146}$$

so the frequency range extends up to the frequency

$$\omega \lesssim \frac{\gamma\kappa}{d} \tag{147}$$

in the angular region (145). Therefore, radiation exhibits relativistic peaking in the forward direction in the same way as radiation of the ultrarelativistic particle. This could be expected, since relativistic peaking has a purely kinematical nature.

C. Dilaton radiation

Collecting the world-sheet contributions to radiation amplitudes we obtain, after integration over q,

$$\overset{st}{I}(k) + \overset{ct}{I}(k) = \frac{\alpha}{\omega^2}\left\{\frac{\alpha^2\mu^2}{8}\left(E_1\left[1 - \frac{i\cos\phi + \kappa\sin\phi}{\gamma v\sin\theta}\right] + \frac{E_2}{\gamma^2\xi^2}\left[\frac{ie^{-i\varphi}}{\gamma v\sin\theta} + 2\xi\gamma\right]\right) + \pi^2 f^2\left(E_1\frac{\sin\phi + i\kappa\cos\varphi}{\gamma v\sin\theta} - \frac{E_2}{\gamma^2\xi^2}\frac{\gamma v\sin\theta + i\kappa e^{-i\varphi}}{\gamma v\sin\theta}\right) + \mu^2 G\pi\left(E_1[(\kappa\sin\varphi - i\cos\phi) + \gamma v\sin\theta] + \frac{E_2}{\gamma^2\xi^2}ie^{i\varphi}\right)\gamma v\sin\theta\right\}. \tag{148}$$

Here the first term corresponds to the dilaton exchange, the second term to the two-form, and the last term to the graviton exchange. The bulk contribution after integration reads

$$\overset{b}{I}(k) = \frac{\pi^2 f^2\alpha}{\gamma\xi}[E_1(\gamma\kappa\xi - 1 - i\gamma v\sin\theta\cos\varphi) + iE_2\gamma v\sin\theta e^{i\phi}] + \frac{G\mu^2\pi}{\gamma\xi}[E_1(\gamma v\sin\theta + \kappa\sin\phi - i\cos\phi) + E_2 ie^{i\phi}]2\gamma\kappa v\sin\theta. \tag{150}$$

Consider first the Cerenkov threshold $v = \sin\alpha$, when the radiation cone shrinks to $\theta_C = 0$. This corresponds to $\kappa = 1$, $\xi = 1/\gamma$. Since in this limit $kd = 0$, the exponents become equal, $E_1 = E_2$, and the bulk term vanishes. The graviton contribution to (148) vanishes too, while the dilaton and two-form contributions differ only by the coefficients. The total radiation amplitude at the threshold will be given by

$$I_{\rm thr}(k) = \frac{\alpha}{8\omega^2}E_1(3\alpha^2\mu^2 - 8\pi^2 f^2). \tag{151}$$

Integrating the expression (139) for $\mu = 0$ (the energy loss rate) over the angles in $d^3k = \omega^2 d\omega d\cos\theta d\varphi$, we obtain the infrared-divergent integral over frequencies. Introducing the inverse correlation distance Δ as an infrared cutoff parameter, one finds

$$\mathcal{P}_{\rm thr} = \frac{\alpha^2}{4\pi}(3\alpha^2\mu^2 - 8\pi^2 f^2)^2\int_{\Delta^{-1}}^{\infty}\frac{d\omega}{\omega}\exp\left(-2\frac{\omega d}{v\gamma}\right). \tag{152}$$

In the BPS limit (71) one has

$$\mathcal{P}_{\rm thr} = \frac{\mu^4\alpha^6}{\pi}\int_{\Delta^{-1}}^{\infty}\frac{d\omega}{\omega}\exp\left(-2\frac{\omega d}{v\gamma}\right). \tag{153}$$

Integrating this over frequencies we obtain the total radiation rate in terms of the integral exponential function (see the Appendix):

$$\mathcal{P}_{\rm thr} = \frac{\mu^4\alpha^6}{\pi}{\rm Ei}\left(1, \frac{2d}{v\gamma\Delta}\right). \tag{154}$$

For small impact parameters, $d \ll v\gamma\Delta$, this expression can be approximated by the logarithm

$$\mathcal{P} \approx \frac{\mu^4\alpha^6}{\pi}\ln\left(\frac{v\gamma\Delta}{2de^C}\right), \tag{155}$$

where C is the Euler constant, $e^C = 1.781\,072\,418$. For large impact parameters, $d \gg v\gamma\Delta$, radiation exponentially falls off:

$${\rm Ei}\left(1, \frac{2d}{v\gamma\Delta}\right) \approx \frac{2d}{v\gamma\Delta}\exp\left(-\frac{2d}{v\gamma\Delta}\right). \tag{156}$$

For $\theta \neq 0$ the radiation amplitudes are more complicated. Note that in our reference frame the first string is at rest while the second one is moving. For this reason the radiation amplitudes are not symmetric with respect to two strings. As it was observed in the previous section, in the ultrarelativistic case the frequency range associated with the factor E_2 is much larger than that with E_1, and in this limit the E_2 terms are dominant. Collecting the dominant terms, we obtain for the total amplitude of the dilaton emission in the ultrarelativistic case

$$I(\mathbf{k}) = \frac{\alpha E_2}{\gamma^2\xi^2\omega^2}\left\{\frac{\alpha^2\mu^2}{8}\left(\frac{ie^{-i\phi}}{\gamma v\sin\theta} + 2\gamma\xi\right) - \pi^2 f^2\left(1 + \frac{i\cos\alpha e^{-i\phi}}{v\sin\theta} - i\xi\gamma^2 v\sin\theta e^{i\phi}\right) + \mu^2 G\pi i\gamma v\sin\theta(1 + 2\kappa\gamma\xi)e^{i\phi}\right\}. \tag{157}$$

Substituting this into (139) and taking into account the angular peaking near $\phi = -\pi/2$, we obtain the spectral-angular distribution of radiation per unit length in the vicinity of this direction:

$$\frac{d\mathcal{P}}{d\omega d\beta} = \frac{32\alpha^2\kappa^2}{\omega}\frac{(\Omega_1 + \Omega_2\kappa^2\beta^2)^2}{(1 + \kappa^2\beta^2)^4} \times \exp\left(-\frac{\omega d(1 + \kappa^2\beta^2)}{\gamma\kappa}\right), \tag{158}$$

where

$$\Omega_1 = 4G\mu^2\kappa^2 + \pi f^2\kappa,$$
$$\Omega_2 = 2G\mu^2\kappa^2 + \pi f^2\kappa + \frac{\alpha^2\mu^2}{4\pi}. \qquad (159)$$

Dividing by ω we will also get the number of emitted dilatons,

$$\frac{d\mathcal{N}}{d\omega d\beta} = \frac{1}{\omega}\frac{d\mathcal{P}}{d\omega d\beta}. \qquad (160)$$

Note that the graviton, axion, and dilaton exchange terms entering into the above expressions for the parameters $\Omega_{1,2}$ exhibit different behavior in the invariant Lorentz factor κ, the dominant for large κ being the gravitational term.

The spectrum exhibits an infrared divergence, and in the forward direction $\beta = 0$ it extends up to $\omega \sim \omega_{\max}$, where

$$\omega_{\max} = \frac{\gamma\kappa}{d}. \qquad (161)$$

Integrating over frequencies with the infrared cutoff Δ^{-1}, we obtain the angular distribution of radiation:

$$\frac{d\mathcal{P}}{d\beta} = 32\alpha^2\kappa^2\frac{(\Omega_1 + \Omega_2\kappa^2\beta^2)^2}{(1+\kappa^2\beta^2)^4}\mathrm{Ei}\left(1, \frac{d(1+\beta^2\gamma\kappa)}{\gamma\kappa\Delta}\right). \qquad (162)$$

Since the integral exponential function decays exponentially for large values of the argument, the total radiation is peaked around $\beta = 0$ within the angle

$$\beta \lesssim \sqrt{\gamma\kappa}. \qquad (163)$$

One can also obtain the spectral distribution of radiation by extending the integration domain over β in (158) to the full axis in view of the exponential decay of the integrand:

$$\frac{d\mathcal{P}}{d\omega} = \omega\frac{d\mathcal{N}}{d\omega}\frac{2\pi\alpha^2 d}{3\gamma}[\Omega_1^2F_1(z) + 2\Omega_1\Omega_2F_2(z) + \Omega_2^2F_3(z)], \qquad (164)$$

where $z = \frac{\omega d}{\gamma\kappa}$ and the functions F_i are expressed in terms of the probability integral (A4):

$$F_1(z) = (8z^2 - 16z + 30)\frac{e^{-z}}{\sqrt{\pi z}} - \left(8z^2 - 12z + 18 - \frac{15}{z}\right)\mathrm{erfc}(\sqrt{z}),$$
$$F_2(z) = (-8z^2 - 8z + 6)\frac{e^{-z}}{\sqrt{\pi z}} + \left(8z^2 + 12z - 6 + \frac{3}{z}\right)\mathrm{erfc}(\sqrt{z}),$$
$$F_3(z) = (8z^2 + 32z + 6)\frac{e^{-z}}{\sqrt{\pi z}} - \left(8z^2 + 36z + 18 - \frac{3}{z}\right)\mathrm{erfc}(\sqrt{z}). \qquad (165)$$

For small frequencies, $\omega \ll \gamma\kappa/d$, these functions grows as

$$F_1(z) \sim \frac{15}{z}, \qquad F_2(z) \sim \frac{3}{z}, \qquad F_3(z) \sim \frac{3}{z}, \qquad (166)$$

while for large z they exponentially decay:

$$F_1(z) \sim \frac{48e^{-z}}{\sqrt{\pi z^3}}, \qquad F_2(z) \sim -\frac{57e^{-z}}{2\sqrt{\pi z^5}},$$
$$F_3(z) \sim \frac{105e^{-z}}{2\sqrt{\pi z^5}}. \qquad (167)$$

To regularize the infrared divergence one has to introduce the cutoff length Δ. After integration over frequencies ω from Δ^{-1} to infinity we obtain the total dilaton radiation rate in the ultrarelativistic limit,

$$\mathcal{P} = \frac{2\pi\alpha^2\kappa}{3}[(\Omega_1^2f_1(y) + 2\Omega_1\Omega_2f_2(y) + \Omega_2^2f_3(y))], \qquad (168)$$

where three new functions are introduced:

$$f_1(y) = 5f(y) + \mathrm{erfc}(\sqrt{y})\left(\frac{8}{3}y^3 - 6y^2 + 18y + \frac{37}{2}\right) - \frac{e^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 - \frac{22}{3}y + 23\right), \qquad (169)$$

$$f_2(y) = f(y) - \mathrm{erfc}(\sqrt{y})\left(\frac{8}{3}y^3 + 6y^2 - 6y - \frac{5}{2}\right) + \frac{e^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 + \frac{14}{3}y - 7\right), \qquad (170)$$

$$f_3(y) = f(y) + \mathrm{erfc}(\sqrt{y})\left(\frac{8}{3}y^3 + 18y^2 + 18y + \frac{1}{2}\right) - \frac{e^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 + \frac{50}{3}y + 11\right), \qquad (171)$$

where

$$y = \frac{d}{\gamma\kappa\Delta}, \qquad (172)$$

and the function $f(y)$ is expressed through the generalized hypergeometric function (A8),

$$f(y) = 12\sqrt{\frac{y}{\pi}}{}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -y\right) - 3\ln(4ye^C). \qquad (173)$$

For small y these functions grow as

$$f_1(y) \sim 15\ln\frac{1}{y}, \qquad f_{2,3}(y) \sim 3\ln\frac{1}{y}, \qquad (174)$$

while for large y they tend to zero in view of the asymptotic relation (A11).

Similar expressions can be obtained for the total number of dilatons:

$$\mathcal{N} = \frac{2\pi\alpha^2 d}{3\gamma}(\Omega_1^2\mathcal{F}_1(y) + 2\Omega_1\Omega_2\mathcal{F}_2(y) + \Omega_2^2\mathcal{F}_3(y)), \tag{175}$$

where three more functions are introduced:

$$\mathcal{F}_1(y) = \mathrm{erfc}(\sqrt{y})\left(4y^2 - 12y - 39 + \frac{15}{y}\right) - \frac{\mathrm{e}^{-y}}{\pi\sqrt{y}}(4y^2 - 14y - 30) - 6f(y), \tag{176}$$

$$\mathcal{F}_2(y) = \mathrm{erfc}(\sqrt{y})\left(-4y^2 - 12y - 9 + \frac{3}{y}\right) + \frac{\mathrm{e}^{-y}}{\pi\sqrt{y}}(4y^2 + 10y + 6) - 2f(y), \tag{177}$$

$$\mathcal{F}_3(y) = \mathrm{erfc}(\sqrt{y})\left(4y^2 + 36y + 9 + \frac{3}{y}\right) - \frac{\mathrm{e}^{-y}}{\pi\sqrt{y}}(4y^2 + 34y - 6) - 6f(y), \tag{178}$$

tending to zero at infinity, and

$$\mathcal{F}_1 \sim \frac{15}{y}, \qquad \mathcal{F}_{2,3} \sim \frac{3}{y} \tag{179}$$

for small y.

As we have noted, in the ultrarelativistic limit the dominant contribution to the radiation amplitude comes from the graviton exchange term. Leaving only this contribution we find for the radiation rate

$$\mathcal{P}^{(\phi)} = \tfrac{8}{3}\pi G^2\alpha^2\mu^4\kappa^5 g(y), \tag{180}$$

where

$$g(y) = 25f(y) + \mathrm{erfc}(\sqrt{y})\left(\frac{8}{3}y^3 - 30y^2 + 114y + \frac{169}{2}\right) - \frac{\mathrm{e}^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 - \frac{94}{3}y + 131\right). \tag{181}$$

Similarly, for the dilaton number,

$$\mathcal{N}^{(\phi)} = \frac{8d}{3\gamma}\pi G^2\alpha^2\mu^4\kappa^4\mathcal{G}(y), \tag{182}$$

$$\mathcal{G}(y) = \mathrm{erfc}(\sqrt{y})\left(+4y^2 - 60y - 183 + \frac{75}{y}\right) - \frac{\mathrm{e}^{-y}}{\pi\sqrt{y}}(4y^2 - 62y - 150) - 38f(y). \tag{183}$$

The corresponding numerical curves are shown in Figs. 7 and 8. For small y one has

$$\mathcal{P}^{(\phi)} = 200\pi G^3\bar{\alpha}^2\mu^4\kappa^5\ln\left(\frac{\gamma\kappa\Delta}{d}\right), \tag{184}$$

$$\mathcal{N}^{(\phi)} = 200\pi G^3\bar{\alpha}^2\mu^4\kappa^5\Delta, \tag{185}$$

where we introduced the dimensionless dilaton coupling constant. These quantities rapidly grow with the increasing Lorentz factor of the collision γ (recall that $\kappa = \gamma\cos\alpha$ where α is the angle between the strings). Thus, Cerenkov radiation is greatly enhanced for ultrarelativistic velocities.

D. Two-form radiation

After integration over the momentum q, the string contribution to the two-form radiation amplitude (107) takes the form

$$\overset{st}{I}(k) = \frac{4\sqrt{2}\pi^3 f^3}{\mu\gamma v\sin\theta}\left(E_1(\cos\varphi - i\kappa\sin\varphi) + \frac{E_2}{\gamma^2\xi^2}[(iv\xi\gamma^2\sin\theta - \mathrm{e}^{-i\varphi})\right) + \frac{\alpha^2 f\mu\pi\sqrt{2}}{2\gamma v\sin\theta}(E_1(i\sin\varphi - \kappa\cos\varphi) + \frac{E_2}{\gamma^2\xi^2}(\kappa\mathrm{e}^{-i\varphi} - iv\gamma\sin\theta - 2\gamma^3\xi v^2\sin^2\theta\cos\varphi)) + 4\sqrt{2}\pi^2 f\mu G\gamma v\sin\theta\left(E_1(\kappa\cos\varphi + i\sin\varphi) - \frac{E_2}{\gamma^2\xi^2}[i\gamma v\sin\theta + \kappa\mathrm{e}^{i\varphi} - 2\cos\varphi\gamma\xi]\right). \tag{186}$$

The bulk amplitude (107) will read

$$\overset{b}{I}(k) = \left(\frac{\alpha^2 f\mu\pi\sqrt{2}}{2} + 4\sqrt{2}\pi^2 f\mu G\gamma v\sin\theta\right)\times\left(\frac{E_1}{\gamma\xi}[(1 - 2\gamma\xi)\gamma v\sin\theta\cos\varphi - i] + \frac{E_2}{\xi}(2v\sin\theta\cos\varphi - i\xi)\right). \tag{187}$$

Consider first the Cerenkov threshold $v = \sin\alpha$. In the above expressions the graviton exchange term in the string term vanishes, and the bulk term is zero. The remaining string amplitudes due to the dilaton and the two-form exchange simplify as follows:

$$I_{\mathrm{thr}}(k) = \frac{i\pi f}{\sqrt{2}\mu\omega^2}E_1(8\pi^2 f^2 - 3\alpha^2\mu^2). \tag{188}$$

Substituting this into (140) and integrating over the angles, we obtain the spectral distribution of the two-form radiation on the Cerenkov threshold:

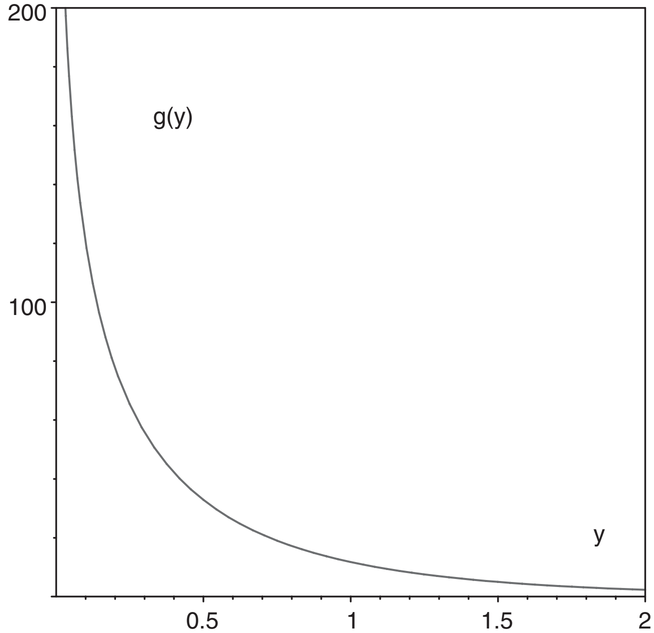

FIG. 7. Plot for $g(y)$.

$$\mathcal{P}_{\rm thr} = \frac{\pi f^2}{2\mu^2}(8\pi^2 f^2 - 3\alpha^2\mu^2)^2 \int_{\Delta^{-1}}^{\infty} \frac{d\omega}{\omega} \exp\left(-2\frac{\omega d}{v\gamma}\right), \quad (189)$$

where Δ is the infrared cutoff frequency. After integration over frequencies one finds

$$\mathcal{P}_{\rm thr} = \frac{4(2\pi)^5 f^6}{\mu^2} \mathrm{Ei}\left(1, \frac{2d}{v\gamma\Delta}\right). \quad (190)$$

For small impact parameters, $d \ll v\gamma\Delta$, this expression can be approximated by

$$\mathcal{P}_{\rm thr} \approx \frac{4(2\pi)^5 f^6}{\mu^2} \ln\left(\frac{v\gamma\Delta}{2d e^C}\right), \quad (191)$$

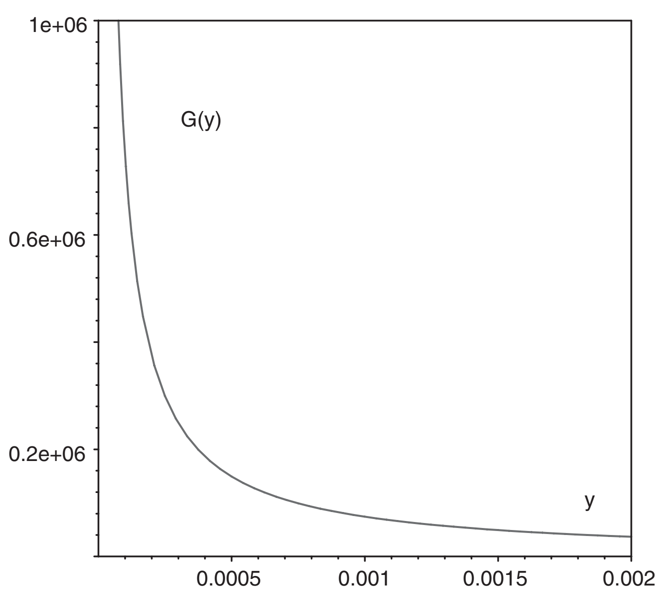

FIG. 8. Plot for $G(y)$.

and for $d \gg v\gamma\Delta$ one can use

$$\mathrm{Ei}\left(1, \frac{2d}{v\gamma\Delta}\right) \approx \frac{2d}{v\gamma\Delta} \exp\left(-\frac{2d}{v\gamma\Delta}\right). \quad (192)$$

Now consider the case of the arbitrary Cerenkov angles in the ultrarelativistic limit. Leaving only the relativistic string contribution, we obtain

$$I(\mathbf{k}) = \frac{f E_2}{\gamma^2 \xi^2 \omega^2}\left\{\frac{4\sqrt{2}\pi^3 f^2}{\mu\gamma v \sin\theta}(i\gamma^2 \xi v \sin\theta - i e^{-i\phi}) + \frac{\pi\mu\alpha^2}{\sqrt{2}}\left(\frac{\kappa e^{-i\phi} + \gamma v \sin\theta}{\gamma v \sin\theta} - i\gamma^2\xi^2\right) - 4\sqrt{2}\pi^2 G\mu\left[i(1-\kappa^2) + \kappa\gamma v \sin\theta e^{i\phi}) - i\gamma^2\xi^2\right]\right\}. \quad (193)$$

Substituting this into (140) we obtain the spectral-angular distribution in the vicinity of $\phi = \pi/2$,

$$\frac{d\mathcal{P}}{d\omega d\beta} = \omega \frac{d\mathcal{N}}{d\omega d\beta} = \frac{64\pi^4\kappa^2 f^2}{\omega} \times \frac{\chi_B^2(1-\kappa^2\beta^2)^2 + 4\kappa^4\beta^2(\chi_h\kappa^2 - \chi_D)^2}{(1+\kappa^2\beta^2)^4} \times \exp\left(-\frac{\omega d(1+\kappa^2\beta^2)}{\gamma\kappa}\right), \quad (194)$$

where

$$\chi_D = \frac{\mu\alpha^2}{8\pi^2}, \qquad \chi_B = \frac{f^2}{\mu}, \qquad \chi_h = \frac{G\mu}{\pi}. \quad (195)$$

Integrating over the angles we get

$$\frac{d\mathcal{P}}{d\omega} = \frac{16\pi^5 d}{3\gamma}[F_4(z)\chi_B^2 + F_2(z)(\chi_h\kappa^2 - \chi_D)^2], \quad (196)$$

$$\frac{d\mathcal{N}}{d\omega} = \frac{16\pi^5 d^2 f^2}{3\gamma^2\kappa z}[F_4(z)\chi_B^2 + F_2(z)(\chi_h\kappa^2 - \chi_D)^2], \quad (197)$$

where

$$F_4 = (8z^2 + 8z + 6)\frac{e^{-z}}{\sqrt{\pi z}} - (8z^2 + 12z + 6 - 3/z)\mathrm{erfc}(\sqrt{z}), \quad (198)$$

and F_2 is given by (165). For small and large frequencies one has

$$F_4(z) \sim \frac{3}{z}, \qquad F_4(z) \sim \frac{12 e^{-z}}{\sqrt{\pi z^3}}, \quad (199)$$

respectively. Finally, integrating over frequencies we find the total two-form radiation rate and the number of axions:

$$\mathcal{P}^{(B)} = \frac{16\pi^5\kappa f^2}{3}(\chi_B^2 f_4(y) + (\chi_h\kappa^2 - \chi_D)^2 f_2(y)), \quad (200)$$

$$\mathcal{N}^{(B)} = \frac{16\pi^5 d f^2}{3\gamma}(\chi_B^2 \mathcal{F}_4(y) + (\chi_h \kappa^2 - \chi_D)^2 \mathcal{F}_2(y)), \tag{201}$$

where

$$f_4(y) = f(y) + \mathrm{erfc}(\sqrt{y})\left(\frac{8}{3}y^3 + 6y^2 + 5y + \frac{7}{2}\right) - \frac{e^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 + \frac{14}{3}y + 5\right), \tag{202}$$

$$\mathcal{F}_4(y) = \mathrm{erfc}(\sqrt{y})\left(4y^2 + 12y - 3 + \frac{3}{y}\right) - \frac{e^{-y}}{\pi\sqrt{y}}(4y^2 + 10y - 6) - 2f(y). \tag{203}$$

For small y

$$\mathcal{F}_4 \sim \frac{3}{y}, \qquad f_4(y) \sim 3\ln\frac{1}{y}, \tag{204}$$

while for large y both functions tend to zero.

The result of the calculation of the axion Cerenkov radiation in the flat space-time [40] is reproduced putting $\chi_D = \chi_h = 0$. It reads

$$\mathcal{P}_0^{(B)} = \frac{16\pi^5 \kappa f^6}{3\mu^2} f_4(y). \tag{205}$$

In our case the dominant contribution comes form the graviton exchange. In the BPS limit one has

$$\chi_D = \chi_B, \qquad \chi_h = \frac{8}{\bar{\alpha}^2}\chi_B, \tag{206}$$

where $\bar{\alpha}$ is the dimensionless dilaton coupling constant. For large κ the leading term is

$$\mathcal{P}^{(B)} = \frac{2^{10}\pi^5 \kappa^5 f^6}{3\bar{\alpha}^4 \mu^2} f_2(y). \tag{207}$$

In the most interesting case of small y, when $f_2 \simeq f_4$, so the ratio

$$\frac{\mathcal{P}^{(B)}}{\mathcal{P}_0^{(B)}} = \frac{64\kappa^4}{\bar{\alpha}^4} \tag{208}$$

can be large, e.g. for $\bar{\alpha} = 1$ and $\kappa = 5$, it is equal to 4×10^7.

The dominant term for the number of axions is

$$\mathcal{N} = \frac{2^{10}\pi^5 \kappa^4 f^6 d}{3\gamma\bar{\alpha}^4 \mu^2} \mathcal{F}_2(y). \tag{209}$$

The numerical curves $f_2(y)$, $\mathcal{F}_2(y)$ are shown in Figs. 9 and 10. For small y one has

$$\mathcal{P}^{(B)} = \frac{2^{10}\pi^5 \kappa^5 f^6}{\bar{\alpha}^4 \mu^2} \ln\left(\frac{\gamma\kappa\Delta}{d}\right), \tag{210}$$

$$\mathcal{N} = \frac{2^{10}\pi^5 \kappa^5 f^6 \Delta}{\bar{\alpha}^4 \mu^2}. \tag{211}$$

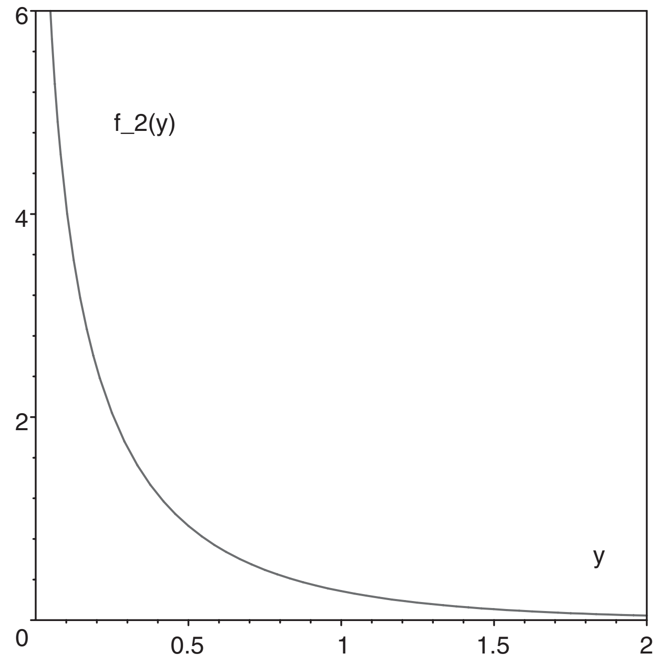

FIG. 9. Plot for $f_2(y)$.

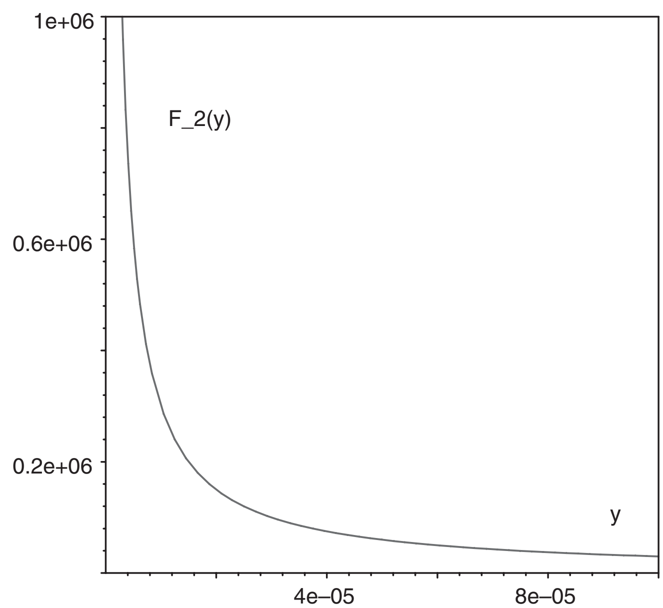

FIG. 10. Plot for $\mathcal{F}_2(y)$.

VIII. COSMOLOGICAL ESTIMATES

Cosmic strings are formed as a network of long strings of the size comparable to the horizon size. Colliding strings intercommute and form closed loops. At some stage the scale-invariant string network is formed consisting of long strings and loops which move freely with relativistic velocities. Evolution of cosmic superstring networks was recently discussed in [20–24]. In many respects cosmic superstrings are similar to the gauge theory cosmic strings, with some distinctions, however. In particular, for gauge theory strings the probability of the formation of loops P is of the order of unity, whereas for F-strings $P \sim 10^{-3}$ and for D-strings $P \sim 10^{-1}$. The cosmic superstring network has a scaling solution, and the characteristic scale is proportional to the square root of the reconnection probability. A typical separation between two long strings is comparable to the horizon size t (we use the standard cosmological units), $\zeta(t) \simeq \sqrt{P}t$. The results of the numerical simulation show that the network of long strings reaches an energy density

$$\rho_s = \frac{\mu}{Pt^2}. \tag{212}$$

Let us estimate the energy loss of long strings due to Cerenkov radiation of dilatons and axions. Consider an ensemble of randomly oriented straight strings moving chaotically in space. Let us choose one target string between them and introduce the Lorentz frame where it is at rest. Other strings will have different orientations and velocities, and we can characterize them very roughly as moving in three orthogonal directions with equal probability. Since the dependence of the Cerenkov radiation on the inclination angle α is smooth, we can use for an estimate the particular result obtained for parallel strings ($\alpha = 0$) introducing an effective fraction ν of "almost" parallel strings and taking into account the effect of the angular spread. Assuming N to be the number of strings in the normalization volume $V = L^3$, we have to integrate the radiation energy released, $\mathcal{P}$, in the collision with the impact parameter $d = x$ over the plane perpendicular to the target string with the measure $N/L^2 \cdot 2\pi x dx$. To estimate the radiation power per unit time we then have to divide the integrand by the impact parameter. Multiplying this quantity by the total number of strings N to get the radiation energy released per unit time within the normalization volume, we obtain for the Cerenkov luminosity

$$Q_C = \int_0^L \mathcal{P}\nu \frac{N}{L^2}\frac{N}{V} 2\pi dx. \tag{213}$$

For BPS strings we use as $\mathcal{P}$ the leading relativistic terms (180) and (207). Taking into account that the string number density is related to the energy density (212) via

$$\frac{N}{V} = \frac{\rho_s}{\mu L}, \tag{214}$$

and assuming for a rough estimate $L \sim \Delta \sim t$, where t is cosmological time, we obtain

$$Q_C^{(\phi)} \simeq \frac{16}{3}\pi^2 G^3 \bar{\alpha}^2 \mu^4 \gamma^7 \nu S_1(w) \frac{1}{Pt^3}, \tag{215}$$

$$Q_C^{(B)} \simeq \frac{2^{11}\pi^6\gamma^7 f^6 \nu}{3\bar{\alpha}^4\mu^2} S_2(w) \frac{1}{Pt^3}, \tag{216}$$

where

$$S_1(w) = \int_0^w g(y)dy, \quad S_2(w) = \int_0^w f_2(y)dy, \quad w = \frac{L}{\gamma^2\Delta}. \tag{217}$$

The exact values of these integrals are given in the Appendix and shown in Figs. 11 and 12. Note that the realistic value of γ is of the order of unity, so applying our formulas obtained in the limit $\gamma \gg 1$ is only an order of magnitude estimate.

Now we can calculate the energy density ε_C of Cerenkov radiation as a function of time in the radiation dominated universe. The energy density of massless fields scales with the Hubble parameter H as H^{-4}, so we have to solve the equation

$$\frac{d\varepsilon_C}{dt} = -4H\varepsilon_C + Q_C, \tag{218}$$

where $H = \frac{1}{2t}$. Thus we obtain for the energy density of the Cerenkov dilaton and two-form radiation

$$\varepsilon_C^{(\phi)} \simeq \frac{16}{3}\pi^2 G^3 \bar{\alpha}^2 \mu^4 \gamma^7 \nu S_1(w) \frac{\ln(t/t_0)}{Pt^2}, \tag{219}$$

$$\varepsilon_C^{(B)} \simeq \frac{2^{11}\pi^6\gamma^7 f^6 \nu}{3\bar{\alpha}^4\mu^2} S_2(w) \frac{\ln(t/t_0)}{Pt^2}, \tag{220}$$

where t_0 is the initial time of the long string formation. Finally, using the approximate formulas (A14) and (A15) valid in the relativistic case, we find

$$\varepsilon_C^{(\phi)} \simeq 800\pi^2 G^3 \bar{\alpha}^2 \mu^4 \gamma^5 \ln\gamma \nu \frac{\ln(t/t_0)}{Pt^2}, \tag{221}$$

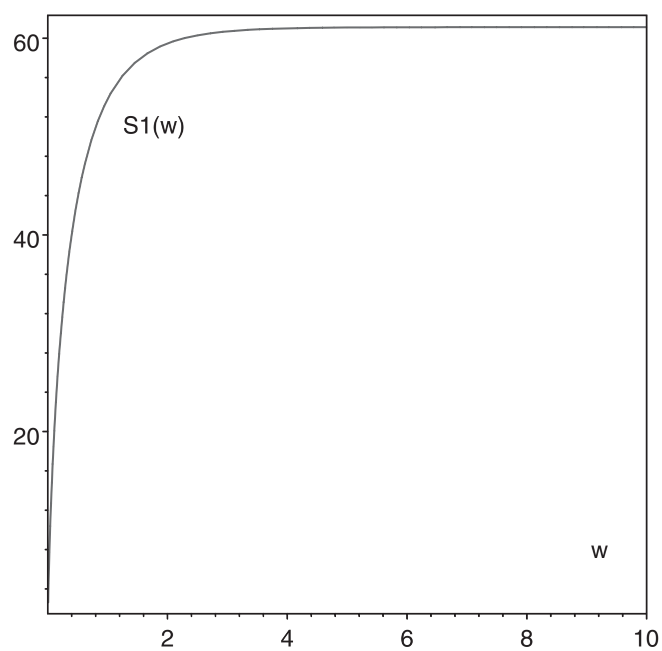

FIG. 11. Plot for $S_1(w)$, $w = L/(\gamma^2\Delta)$.

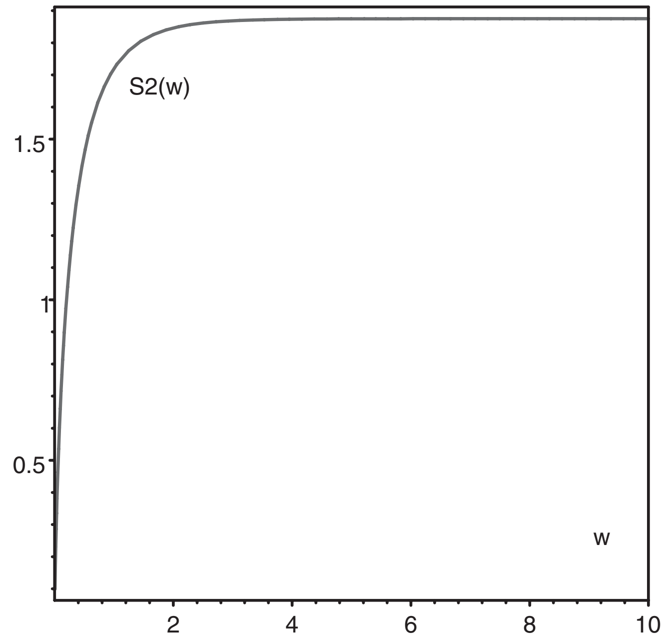

FIG. 12. Plot for $S_2(w)$.

$$\varepsilon_C^{(B)} \simeq \frac{2^{12}\pi^6 f^6 \gamma^5 \ln\gamma\nu}{\bar{\alpha}^4\mu^2}\frac{\ln(t/t_0)}{Pt^2}. \qquad (222)$$

Comparing the last expression for axions with the flat-space result [40], we observe an enhancement due to the higher power of the Lorentz factor and a bigger numerical coefficient. This difference is due to the fact that here the main contribution to the first-order interaction between the strings comes from gravitational force which is proportional to energy. In view of the previous analysis [40] we can conclude that Cerenkov radiation from long strings is a non-negligible effect in the cosmic superstring network. More detailed analysis will be given elsewhere.

IX. CONCLUSION

In this paper we have studied in detail Cerenkov radiation from moving straight strings interacting with the dilaton, two-form, and gravity fields. Formation of the faster-than-light sources in the system of randomly oriented moving straight strings is rather generic. As we have shown, these sources have a collective nature and arise due to deformations of the string world sheets caused by their interactions via massless fields. These deformations propagate with superluminal velocities if the inclination angle is sufficiently small; for parallel strings the source velocity is infinite. Radiation wave vectors lie on the Cerenkov cone in the same way as in the case of Cerenkov radiation of point charges in dielectric media.

One interesting feature related to dimensionality of a string compared to a point charge is the absence of gravitational radiation in four space-time dimensions. This is related to the fact that the space transverse to the straight string is two dimensional, so the emitted massless fields must live in 1 + 2 dimensional space-time rather than in four dimensional. As it is well known, gravity in 1 + 2 dimensions does not contain free gravitons; this is why one can expect gravitational radiation from straight strings to vanish. In higher dimensions this objection does not work, so Cerenkov gravitational radiation can be expected in space-time dimensions higher than four. In four dimensions the Cerenkov mechanism works for the dilaton and the two-form field which is equivalent to a pseudoscalar.

To avoid complications due to the possibility of "physical" string intersections (leading to the well-studied processes of intercommutation and formation of loops), we consider the "collision" of strings moving in parallel planes. At each instant of time there exists a point of minimal separation between the strings, and it is this point which may propagate with the superluminal velocity. When interaction between the strings via the dilaton, two-form, and gravity fields is taken into account, strings get deformed in the vicinity of this point; these deformations contribute to an effective radiation source. Another contribution comes from tensions associated with the first-order fields which give rise to these deformations. The string deformations give contributions localized on the world sheets, while the field stresses give bulk contributions. Both have the same order of magnitude.

Cerenkov radiation from strings has some peculiar features in the highly relativistic region. We have shown that in this case radiation exhibits strong beaming on the Cerenkov cone in the direction of the fast string in the rest frame of the target string. The main radiation frequency is proportional to the inverse impact parameter, but in the ultrarelativistic case the spectrum is enhanced to high frequencies proportional to the square of the Lorentz factor of the collision. It is shown that in this limit gravitational interaction between the strings dominates and gives the main contribution to the effective sources of the dilaton and two-form radiation.

Cerenkov's mechanism can be regarded as an analog of the bremsstrahlung of point charges in electrodynamics, which gives the main contribution to radiation in plasma. In the string case, however, there is another radiation mechanism due to the existence of the internal string dynamics: radiation from oscillating loops. This effect is of the first order in couplings between the string and massless fields. Cerenkov radiation arises only in the second order in these coupling, so presumably it is less important. But as we have shown here, it has a stronger dependence on the Lorentz factor of the string collision, so it must become dominant for highly relativistic strings. Also, in the cosmic string network it is a pairwise effect which gives the contribution to the radiation loss proportional to the square of the density of strings. Our rough cosmological estimates indicate that Cerenkov radiation is indeed a non-negligible effect in the cosmic string context.

ACKNOWLEDGMENTS

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APPENDIX

Here we collect properties of some special functions used in the main text.

1. Integral exponential function

The definition:

$$\mathrm{Ei}\,(1, z) = \int_z^\infty \frac{\mathrm{e}^t dt}{t}. \tag{A1}$$

The series expansion:

$$\mathrm{Ei}\,(1, z) = -C - \ln z + z - z^2/2 + z^3/18 - \ldots, \tag{A2}$$

where C is the Euler constant, $\mathrm{e}^C = 1.781\,072\,418$.

The asymptotic expansion:

$$\mathrm{Ei}\,(1, z) = \frac{\mathrm{e}^{-z}}{z}\left(1 - \frac{1}{z} + \frac{2}{z^2} + \ldots.\right). \tag{A3}$$

2. Probability integral

The definition:

$$\mathrm{erf}\,(z) = \frac{2}{\sqrt{\pi}} \int_0^z \mathrm{e}^{-t^2} dt. \tag{A4}$$

The series expansion:

$$\mathrm{erf}\,(z) = \frac{2z}{\sqrt{\pi}}(1 - z^2/3 + z^4/10 - \ldots). \tag{A5}$$

The asymptotic expansion:

$$\mathrm{erfc}\,(z) \equiv 1 - \mathrm{erf}(z) = \frac{\mathrm{e}^{-z^2}}{\sqrt{\pi} z}\left(1 - \frac{1}{2z^2} + \frac{3}{4z^2} - \ldots\right). \tag{A6}$$

In Sec. VII we used the following indefinite integral:

$$\int \mathrm{erf}(\sqrt{z})\frac{dz}{z} = 4\sqrt{\frac{z}{\pi}}{}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -z\right), \tag{A7}$$

where

$${}_2F_2(\alpha_1, \alpha_2; \beta_1, \beta_2; x) = \sum_{k=0}^\infty \frac{\alpha_{1k}, \alpha_{2k}}{\beta_{1k}, \beta_{2k}} \frac{x^k}{k!}, \qquad \alpha_k = \alpha(\alpha + 1)\cdots(\alpha + k), \ldots \tag{A8}$$

is the generalized hypergeometric function. To obtain an asymptotic behavior of the latter for $z \to \infty$, we use the following identity:

$$\int_0^z \ln w \frac{\exp(-w)}{\sqrt{\pi w}} dw = \ln z \,\mathrm{erf}(\sqrt{z}) - 4\sqrt{\frac{z}{\pi}}{}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -z\right), \tag{A9}$$

which can be easily proved by integrating by parts. Then taking into account the integral

$$\int_0^\infty \ln w \frac{\exp(-w)}{\sqrt{\pi w}} dw = -C - 2\ln 2, \tag{A10}$$

we find that for $z \to \infty$

$$4\sqrt{\frac{z}{\pi}}{}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -z\right) \approx \ln(4z\mathrm{e}^C). \tag{A11}$$

Integration of the functions $g(y)$ defined in (181) and $f_2(y)$ defined in (170) over y can be performed analytically:

$$\begin{aligned} S_1(w) &= \int_0^w g(y)dy \\ &= 300\sqrt{\frac{w}{\pi}}\left[{}_2F_2\left(-\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{3}{2}; -w\right) - {}_2F_2\left(-\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; -w\right)\right] + 75w(1 - \ln(4w\mathrm{e}^C)) \\ &\quad + \left(\frac{169}{2}w + \frac{2}{3}w^4 + 57w^2 - 10w^3 - \frac{189}{8}\right)\mathrm{erfc}(\sqrt{w}) \\ &\quad - \frac{1}{12}\frac{\mathrm{e}^{-y}\sqrt{y}}{\sqrt{\pi}}(8w^3 - 124w^2 + 750w + 567) + \frac{189}{8}, \end{aligned} \tag{A12}$$

$$\begin{aligned} S_2(w) &= \int_0^w f_2(y)dy \\ &= 12\sqrt{\frac{w}{\pi}}\left[{}_2F_2\left(-\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{3}{2}; -w\right) - {}_2F_2\left(-\frac{1}{2}, -\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; -w\right)\right] + 3w(1 - \ln(4w\mathrm{e}^C)) \\ &\quad + \frac{1}{24}(60w - 16w^4 + 72w^2 - 48w^3 - 9)\mathrm{erfc}(\sqrt{w}) \\ &\quad + \frac{1}{24}\frac{\mathrm{e}^{-y}\sqrt{y}}{\sqrt{\pi}}(16w^3 + 40w^2 - 84w - 18) + \frac{3}{8}. \end{aligned} \tag{A13}$$

For small arguments the leading terms are

$$S_1(w) \simeq -75w\ln w, \tag{A14}$$

$$S_2(w) \simeq -3w\ln w. \tag{A15}$$

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