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Glueball masses in (2 + 1)-dimensional anistropic weakly-coupled Yang-Mills theory

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The confinement problem has been solved in the anisotropic (2 + 1)-dimensional SU(N) Yang-Mills theory at weak coupling. In this paper, we find the low-lying spectrum for $N = 2$. The lightest excitations are pairs of fundamental particles of the (1 + 1)-dimensional SU(2) × SU(2) principal chiral sigma model bound in a linear potential, with a specified matching condition where the particles overlap. This matching condition can be determined from the exactly-known S-matrix for the sigma model.

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I. INTRODUCTION

In recent papers, some new techniques have been developed for calculating quantities in a (2 + 1)-dimensional SU(N) gauge theories [1–3]. These techniques exploit the fact that in an anisotropic limit of small coupling, the gauge theory becomes a collection of completely-integrable quantum field theories. These integrable systems are decoupled, save for a constraint which is necessary for complete gauge invariance. In the case of $N = 2$, is possible to perturb away from integrability, using exactly-known off-shell matrix elements of the integrable theory.

The model we consider is not spatial-rotation invariant, but has features one expects of real (3 + 1)-dimensional QCD; it is asymptotically free and confines quarks at weak coupling. Thus the limit of no regularization is accessible.

One can formally remove the regulator in strong-coupling expansions of (2 + 1)-dimensional gauge theories; the vacuum state in this expansion yields a string tension and a mass gap which have formal continuum limits. This can be done in a Hamiltonian lattice formalism [4], or with an ingenious choice of degrees of freedom and point-splitting regularization [5]. This leaves open the question of whether these expressions can be trusted at weak coupling (more discussion of this issue can be found in Ref. [2]), and one would like to rule out a deconfinement transition, or very different dependence of physical quantities on the coupling (as in compact QED [6]). A proposal has been made for the vacuum state [7], in the formulation of Ref. [5] which seems to give correct values for some glueball masses [8], but this proposal evidently requires more mathematical justification.

In this paper, we will work out the masses of the lightest glueballs for the case of gauge group SU(2). Our method would work in principle for SU(N) gauge theories, and our reason for choosing $N = 2$ is that the analysis is simplest for that case.

The basic connection between the gauge theory and integrable systems is most easily seen in axial gauge [1]. We made simple estimates, for the string tensions in the x^1- and x^2-directions (called the vertical and horizontal string tension, respectively), for small g_0'. The result for the horizontal string tension was confirmed for gauge group SU(2), and additional corrections in g_0' were found [2], through the use of exact form factors for the currents of the sigma model. String tensions for higher representations can also be worked out, and adjoint sources are not confined [3].

Careful derivations of the connection between the gauge theory and integrable systems use the Kogut-Susskind lattice formalism [1,2]. A shorter derivation was given in Ref. [9], which we summarize again here. The formalism is essentially that of "deconstruction" [10].

The Yang-Mills action is $\int d^3\mathcal{L}$, where the Lagrangian is $\mathcal{L} = \frac{1}{2e'^2}\,\mathrm{Tr}F_{01}^2 + \frac{1}{2e^2}\,\mathrm{Tr}F_{02}^2 - \frac{1}{2e^2}\,\mathrm{Tr}F_{12}^2$, and where A_0, A_1 and A_2 are SU(N)-Lie-algebra-valued components of the gauge field, and the field strength is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$. This action is invariant under the gauge transformation $A_\mu(x) \to i g(x)^{-1}[\partial_\mu - iA_\mu(x)]g(x)$, where $g(x)$ is an SU(N)-valued scalar field. We take $e' \neq e$, thereby losing rotation invariance.

We discretize the 2-direction, so that the x^2 takes on the values $x^2 = a, 2a, 3a \ldots$, where a is a lattice spacing. All fields are considered functions of $x = (x^0, x^1, x^2)$. We define the unit vector $\hat{2} = (0, 0, 1)$. We replace $A_2(x)$ by a field $U(x)$ lying in SU(N), via $U(x) \approx \exp -iaA_2(x)$. There is a natural discrete covariant-derivative operator: $\mathcal{D}_\mu \mathfrak{U}(x) = \partial_\mu \mathfrak{U}(x) - iA_\mu(x)\mathfrak{U}(x) + i\mathfrak{U}(x)A_\mu(x + \hat{2}a)$, $\mu = 0, 1$, for any $N \times N$ complex matrix field $\mathfrak{U}(x)$. The action is $S = \int dx^0 \int dx^1 \sum_{x^2} a\mathcal{L}$ where

$$\mathcal{L} = \frac{1}{2(g_0')^2 a}\,\mathrm{Tr}F_{01}^2 + \frac{1}{2g_0^2}\,\mathrm{Tr}[\mathcal{D}_0 U(x)]^\dagger \mathcal{D}_0 U(x) - \frac{1}{2g_0^2}\,\mathrm{Tr}[\mathcal{D}_1 U(x)]^\dagger \mathcal{D}_1 U(x), \tag{1.1}$$

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and where $g_0^2 = e_0^2 a$ and $(g_0')^2 = e'^2 a$. The Lagrangian (1.1) is invariant under the gauge transformation: $A_\mu(x) \to ig(x)^{-1}[\partial_\mu - iA_\mu(x)]g(x)$ and $U(x) \to g(x)^{-1}U(x)g(x + \hat{2}a)$ where again, $g(x) \in \mathrm{SU}(N)$ and μ is restricted to 0 or 1. The bare coupling constants g_0 and g_0' are dimensionless. We recover from (1.1) the anisotropic continuum action in the limit $a \to 0$. The sigma model field is $U(x^0, x^1, x^2)$, and each discrete x^2 corresponds to a different sigma model. The system (1.1) is a collection of parallel $(1+1)$-dimensional $\mathrm{SU}(N) \times \mathrm{SU}(N)$ sigma models, each of which couples to the auxiliary fields A_0, A_1. The sigma-model self-interaction is the dimensionless number g_0.

We briefly mention how the anisotropic regime is different from the standard $(2+1)$-dimensional Yang-Mills theory. The point where the regulator can be removed in the theory is $g_0' = g_0 = 0$. This point can be reached in our treatment, but only if

$$(g_0')^2 \ll \frac{1}{g_0} e^{-4\pi/(g_0^2 N)}. \tag{1.2}$$

The left-hand side and ride-hand side are proportional to the two energy scales in the theory (the latter comes from the two-loop beta function of the sigma model). Thus our method cannot accommodate fixing the ratio g_0'/g_0, which is natural in standard perturbation theory [11]. This is why the mass gap is not of order e, e' and the string tension is not of order e^2, $(e')^2$.

We now discuss the Hamiltonian in the axial gauge $A_1 = 0$. The left-handed and right-handed currents are, $j_\mu^L(x)_b = i\,\mathrm{Tr}\,t_b \partial_\mu U(x) U(x)^\dagger$ and $j_\mu^R(x)_b = i\,\mathrm{Tr}\,t_b U(x)^\dagger \partial_\mu U(x)$, respectively, where $\mu = 0, 1$. The Hamiltonian obtained from (1.1) is $H_0 + H_1$, where

$$H_0 = \sum_{x^2} \int dx^1 \frac{1}{2g_0^2} \{[j_0^L(x)_b]^2 + [j_1^L(x)_b]^2\}, \tag{1.3}$$

and

$$\begin{aligned} H_1 = {} & \sum_{x^2} \int dx^1 \frac{(g_0')^2 a^2}{4} \partial_1 \Phi(x^1, x^2) \partial_1 \Phi(x^1, x^2) \\ & - \left(\frac{g_0'}{g_0}\right)^2 \sum_{x^2=0}^{L^2-a} \int dx^1 [j_0^L(x^1, x^2)\Phi(x^1, x^2) \\ & - j_0^R(x^1, x^2)\Phi(x^1, x^2 + a)] + (g_0')^2 q_b \Phi(u^1, u^2)_b \\ & - (g_0')^2 q_b' \Phi(v^1, v^2)_b, \end{aligned} \tag{1.4}$$

where $-\Phi_b = A_{0b}$ is the temporal gauge field, and where in the last term we have inserted two color charges—a quark with charge q at site u and an antiquark with charge q' at site v. Some gauge invariance remains after the axial-gauge fixing, namely, that for each x^2

$$\left\{\int dx^1 [j_0^L(x^1, x^2)_b - j_0^R(x^1, x^2 - a)_b] - g_0^2 Q(x^2)_b\right\}\Psi = 0, \tag{1.5}$$

where $Q(x^2)_b$ is the total color charge from quarks at x^2 and Ψ is any physical state. To derive the constraint (1.5) more precisely, we started with open boundary conditions in the 1-direction and periodic boundary conditions in the 2-direction, meaning that the two-dimensional space is a cylinder [1,2].

From (1.4) we see that the left-handed charge of the sigma model at x^2 is coupled to the electrostatic potential Φ, at x^2. The right-handed charge of the sigma model is coupled to the electrostatic potential at $x^2 + a$. The excitations of H_0, which we call Fadeev-Zamoldochikov or FZ particles, behave like solitons, though they do not correspond to classical configurations. Some of these FZ particles are elementary and others are bound states of the elementary FZ particles. An elementary FZ particle has an adjoint charge and mass m_1. An elementary one-FZ-particle state is a superposition of color-dipole states, with a quark (antiquark) charge at x^1, x^2 and an antiquark (quark) charge at x^1, $x^2 + a$. The interaction H_1 produces a linear potential between color charges with the same value of x^2. Residual gauge invariance (1.5) requires that at each value of x^2, the total color charge is zero. If there are no quarks, the total right-handed charge of FZ particles in the sigma model at $x^2 - a$ is equal to the total left-handed charge of FZ particles in the sigma model at x^2.

The particles of the principal chiral sigma model carry a quantum number r, with the values $r = 1, \ldots, N-1$ [12]. Each particle of label r has an antiparticle of the same mass, with label $N - r$. The masses are given by

$$\begin{aligned} m_r &= m_1 \frac{\sin\frac{r\pi}{N}}{\sin\frac{\pi}{N}}, \\ m_1 &= K\Lambda(g_0^2 N)^{-1/2} e^{-4\pi/g_0^2 N} + \text{nonuniversal corrections}, \end{aligned} \tag{1.6}$$

where K is a nonuniversal constant and Λ is the ultraviolet cutoff of the sigma model.

Lorentz invariance in each x^0, x^1 plane is manifest. For this reason, the linear potential is not the only effect of H_1. The interaction creates and destroys pairs of elementary FZ particles. This effect is quite small, however, provided that g_0' is small enough. Specifically, this means that the square of the $1+1$ string tension in the x^1-direction coming from H_1 is small compared to the square of the mass of fundamental FZ particle; this is just the condition (1.2). The effect is important, however, in that it is responsible for the correction to the horizontal string discussed in the next paragraph in Eq. (1.8).

Simple arguments readily show that at leading order in g_0', the vertical and horizontal string tensions are given by

$$\sigma_V = \frac{m_1}{a}, \qquad \sigma_H = \frac{(g_0')^2}{2a^2} C_N, \tag{1.7}$$

respectively, where C_N is the smallest eigenvalue of the Casimir of $\mathrm{SU}(N)$. These naive results for the string tension have further corrections in g_0', which were determined for the horizontal string tension for SU(2) [2]:

$$\sigma_H = \frac{3}{2}\left(\frac{g_0'}{a}\right)^2\left[1 + \frac{4}{3}\frac{0.7296}{K^2\pi^2}\frac{(g_0')^2}{g_0^2}e^{4\pi/g_0^2}\right]^{-1}. \quad (1.8)$$

The leading term agrees with (1.7). This calculation was done using the exact form factor for sigma model currents obtained by Karowski and Weisz [13]. The form factor can also be used to find corrections of order $(g_0')^2$ to the vertical string tension; this problem should be solved soon.

Another recent application of exact form factors to the (2 + 1)-dimensional SU(2) gauge theory is Ref. [14], in which form factors of the two-dimensional Ising model [15] are used to find the profile of the electric string, near the high-temperature deconfining transition, assuming the Svetitsky-Yaffe conjecture [16].

A picture of a physical state for $N = 2$ is in Fig. 1. This figure is inaccurate in that the "ring" of particles is extremely broad in the x^2-direction, compared to the x^1-direction (because $\sigma_H \ll \sigma_V$). For $N > 2$, there are more complicated ways strings can join particles.

The lightest glueballs are pairs of FZ particles with the same value of x^2. For small enough g_0', the very lightest state has a mass well-approximated by $2m_1$. The purpose of this paper is to find the leading corrections in $(g_0')^2$ to this result.

In the next section we will discuss the wave function of an unbound pair of FZ particles. We find that this is described by phase shift for the color-singlet sector. In Sec. III, we determine the bound-state spectrum. The problem we solve is very similar to that of two particle-states of the two-dimensional Ising model with an external magnetic field [17] (for a good summary of this problem, see Ref. [18]); the only genuine difference is the presence of a matching condition where the particles overlap. This matching condition comes from the phase shift of the scattering problem.

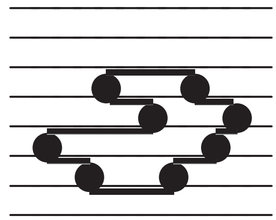

FIG. 1. A glueball state is a collection of heavy particles, held weakly together by strings. The horizontal coordinate is x^1 and the vertical coordinate is x^2.

II. SCATTERING STATES OF FZ PARTICLES

The lightest glueball state, as discussed above, is a pair of FZ particles located at the points (x^1, x^2) and (y^1, x^2) and bound in a linear potential. Residual gauge invariance (1.5), demands that the state be a color singlet. We begin by examining free states of two particles.

The state of the SU(2) × SU(2) ≃ $O(4)$ nonlinear sigma model with a particles of momenta p_1 and p_2 and quantum numbers j_1 and j_2 (which take the values 1, 2, 3, 4) is described by the wave function

$$\psi_{p_1p_2}(x^1, y^1)_{j_1,j_2} = \begin{cases} e^{ip_1x^1+ip_2y^1}A_{j_1,j_2}, & x^1 < y^1 \\ e^{ip_2x^1+ip_1y^1}\sum_{k_1,k_2=1}^4 S_{j_1j_2}^{k_1k_2}(p_1, p_2)A_{k_2,k_1}, & x^1 > y^1 \end{cases}, \quad (2.1)$$

where $A_{j_1j_2}$ is an arbitrary set of complex numbers and $S_{j_1j_2}^{k_1k_2}(p_1, p_2)$ is the two-particle S-matrix. We have not yet imposed (1.5).

The wave function (2.1) is written in a form where the $O(4)$ symmetry is manifest. It is straightforward to write it in a form where the left $SU(2)_L$ and the right $SU(2)_R$ symmetries are manifest, by writing

$$\psi_{p_1p_2}(x^1, y^1)_{a,\bar{b}}^{\bar{c},d} = \sum_{j_1,j_2}\frac{1}{\sqrt{2}}(\delta_{ac}^{j_14} - i\sigma_{ac}^{j_1})\frac{1}{\sqrt{2}}(\delta_{bd}^{j_24} - i\sigma_{bd}^{j_2})^* \times \psi_{p_1p_2}(x^1, y^1)_{j_1,j_2} \quad (2.2)$$

describing a pair of color dipoles, one with quantum numbers a, $\bar{b}$ and the other with quantum numbers $\bar{c}$, d, where σ^j, $j = 1, 2, 3$ denotes the Pauli matrices.

To impose the physical state condition (1.5) on (2.2), we set $a = b$ and $c = d$ and sum over these colors. The projected wave function is, up to an overall constant,

$$\psi_{p_1p_2}(x^1, y^1) = \begin{cases} e^{ip_1x^1+ip_2y^1}, & x^1 < y^1 \\ e^{ip_2x^1+ip_1y^1}S_0(p_1, p_2), & x^1 > y^1 \end{cases}, \quad (2.3)$$

where $S_0(p_1, p_2)$ is the singlet projection of the $O(4)$ S-matrix. This S-matrix was first obtained by Zamolodchikov and Zamolodchikov [19]. A useful form is given in Ref. [13]:

$$S_0(p_1, p_2) = S_0(\theta) = -\frac{\pi - i\theta}{\pi + i\theta}\exp i\int_0^\infty \frac{d\xi}{\xi}\frac{1 - e^{-\xi}}{1 + e^{\xi}}\sin\frac{\xi\theta}{\pi}, \quad (2.4)$$

where the relative rapidity θ is given by $\theta = \theta_2 - \theta_1$, $p_1 = m\sinh\theta_1$, $p_2 = m\sinh\theta_2$ and where we denote the particle mass m_1, given by (1.6), by m (because there is only one mass for the case of $N = 2$). This result is derived in the appendix of Ref. [2].

The singlet S-matrix is just a phase shift $\phi(\theta)$: $S_0(\theta) = \exp i\phi(\theta)$. The phase shift has a simple form in the lowenergy, nonrelativistic limit, $|p_1 - p_2| \ll m$. In this limit, $\theta \approx |p_1 - p_2|/m$. The integral on the right-hand side of (2.4) can be done by Taylor expanding in $|p_1 - p_2|/m$ yielding

$$\phi(\theta) = \phi(p_1, p_2) = \pi - \frac{3 - 2\ln 2}{\pi m}|p_1 - p_2| + O\left(\frac{|p-r|^2}{m^2}\right). \quad (2.5)$$

III. THE LOW-LYING GLUEBALL SPECTRUM

Let us now consider the states of a bound pair of FZ particles in the potential $V(x^1, y^1) = 2\sigma_H|x^1 - y^1|$ (the reason for the factor of 2 is that the particles are joined by a pair of strings). We keep our nonrelativistic approximation, used to find (2.5). For our problem the horizontal string tension times the size of a typical bound state is small compared to the mass, by (1.2). This justifies the nonrelativistic approximation for low-lying states. The mass of a low-lying glueball is given by

$$M = 2m + E,$$

where E is the energy eigenstate of the two-particle problem.

Let us introduce center-of-mass coordinates, $X = (x^1 + y^1)/2$ and $x = y^1 - x^1$. The reduced mass of the system is $m/2$. We can factor out a phase depending on X, leaving us only with a wave function depending on x. The Schrödinger equation we wish to consider is

$$-\frac{1}{m}\frac{d^2\psi}{dx^2} + 2\sigma_H|x|\psi = E\psi$$

with a matching condition at $x = 0$ between the wave function $\psi(x)$ at $x > 0$ and the wave function at $x < 0$.

Our result (2.4) for the unbound two-particle state, with phase shift (2.5) tells us that for $x^1 \approx y^1$, where the effect of the potential can be ignored, the bound-state wave function in the center-of-mass frame will be of the form

$$\psi(x) = \begin{cases} \cos(px + \omega), & x < 0 \\ \cos[-px + \omega - \phi(p)], & x > 0 \end{cases}, \quad (3.1)$$

for some angle ω, where $p = p_1 - p_2$ and $\phi(p) = \pi - \frac{3-2\ln 2}{\pi m}|p| + O(|p|^2/m^2)$. The value of p near $x = 0$ is given by $p = (mE)^{1/2}$, where E is the energy eigenvalue of the state. This is the matching condition between the wave function for $x > 0$ and for $x < 0$.

The wave function for $x < 0$ an Airy function. So is the wave function for $x > 0$. We therefore obtain the approximate WKB form

$$\psi(x) = \begin{cases} C\left(x + \frac{E}{2\sigma_H}\right)^{-1/4} \cos\left[\frac{2}{3}(2m\sigma_H)^{1/2}\left(x + \frac{E}{2\sigma_H}\right)^{3/2} - \frac{\pi}{4}\right], & x < 0 \\ C'\left(\frac{E}{2\sigma_H} - x\right)^{-1/4} \cos\left[-\frac{2}{3}(2m\sigma_H)^{1/2}\left(\frac{E}{2\sigma_H} - x\right)^{3/2} + \frac{\pi}{4}\right], & x > 0 \end{cases}, \quad (3.2)$$

for some constants C and C'. The expression (3.2) can be made to agree with (3.1) for small x, provided the generalization of the Bohr-Sommerfeld quantization condition

$$\frac{2(m)^{1/2}}{3\sigma_H}E_n^{3/2} + \frac{3 - 2\ln 2}{\pi m^{1/2}}E_n^{1/2} - \left(n + \frac{1}{2}\right)\pi = 0, \quad n = 0, 1, 2, \ldots, \quad (3.3)$$

is satisfied by $E = E_n$. The only new feature in this semiclassical formula is the second term, produced by the phase shift.

There is a unique real solution of the cubic equation (3.3) for a given integer $n \geq 0$, by virtue of the fact that $3 - 2\ln 2 = 1.613\,706 > 0$. The low-lying glueball masses are therefore given by

$$M_n = 2m + E_n = 2m + \left[\epsilon_n^{1/3} - \frac{3(3 - 2\ln 2)\sigma_H}{4\pi m}\epsilon_n^{-1/3}\right]^2, \quad (3.4)$$

where

$$\epsilon_n = \frac{3\pi\sigma_H(n + \frac{1}{2})}{4m^{1/2}} + \left\{\left[\frac{3\pi\sigma_H}{4m^{1/2}(n + \frac{1}{2})}\right]^2 + \frac{1}{8}\left[\frac{3(3 - 2\ln 2)\sigma_H}{2\pi m}\right]^3\right\}^{1/2}. \quad (3.5)$$

IV. CONCLUSIONS

We have identified the low-lying glueballs of the anisotropic Yang-Mills theory in (2 + 1) dimensions as bound pairs of the fundamental massive particles of the principal chiral nonlinear sigma model. We found a matching condition for the bound-state wave function at the origin, which when combined with elementary methods yields the spectrum of the lightest states.

There are important implications of the two-particle bound-state problem we have not considered here. For example, the existence of these bound states implies that there are small corrections to the form factors used in [2]. This, in turn, will give a further correction to the horizontal string tension. Such corrections to form factors in theories close to integrability were first discussed by Delfino, Mussardo and Simonetti [20]. The bound-state energies proliferate between $2m$ and $4m$, as $g_0' \to 0$. Our method breaks down as the bound-state mass reaches $4m$, because the bound state develops an instability towards fission into a pair of two-particle bound states. This is analogous to the situation for the Ising model in a field [17,18] as we stated earlier. It should be worthwhile to understand the relativistic corrections to the bound-state formula, along the lines of the work of Fonseca and Zamolodchikov [21].

A similar calculation is possible for SU(N). It should be possible to study the bound-state spectrum for any value of N. An interesting feature is that the phase shift should vanish as $N \to \infty$, with $g_0^2 N$ fixed, meaning that the wave function would be continuous where FZ particles overlap.

It would interesting to study the scattering of a glueball by an external particle. If the scattering is sufficiently short range, the FZ particles could be liberated from the glueball, after which hadronization would ensue.

It may be possible to extend the results of this paper, and Refs. [1,2] to the standard (2 + 1)-dimensional isotropic Yang-Mills theory with $g_0' = g_0$. The strategy we have in mind is an anisotropic renormalization procedure. At the start is a standard field theory with an isotropic cutoff. By anisotropically integrating out high-momentum degrees of freedom with an isotropic cutoff, the isotropic theory should flow to an anisotropic theory with a small momentum cutoff in the x^2-direction and a large momentum cutoff in the x^1 direction. If the renormalized couplings satisfy the condition (1.2), we could apply our techniques. A check of such a method would be approximate rotational invariance of the string tension. This would give an analytic first-principles method of solving the isotropic gauge theory with fixed dimensionful coupling constant e, and no cutoff. The only other analytic weak-coupling argument for a mass gap and confinement in (2 + 1)-dimensions, namely, that of orbit-space distance estimates, discussed by Feynman [22], by Karabali and Nair in the second of Refs. [5], and by Semenoff and the author [23] is suggestive, but has not yielded definite results yet.[1]

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[1]See also Ref. [24] for a general discussion of distance in orbit space.

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