

Lorentz violation and perpetual motion

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We show that any Lorentz-violating theory with two or more propagation speeds is in conflict with the generalized second law of black hole thermodynamics. We do this by identifying a classical energy-extraction method, analogous to the Penrose process, which would decrease the black hole entropy. Although the usual definitions of black hole entropy are ambiguous in this context, we require only very mild assumptions about its dependence on the mass. This extends the result found by Dubovsky and Sibiriyakov, which uses the Hawking effect and applies only if the fields with different propagation speeds interact just through gravity. We also point out instabilities that could interfere with their black hole *perpetuum mobile*, but argue that these can be neglected if the black hole mass is sufficiently large.

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I. INTRODUCTION

Is Lorentz symmetry an exact symmetry in nature, or is it only approximate? In order to address this question, several models have been proposed in which some dynamical fields break Lorentz symmetry. In such models different fields can have different maximal speeds of vacuum propagation, as measured in a preferred reference frame at each point. When gravity is included, black hole solutions can exist [1–4] with multiple, nested horizons, one for each maximal speed of propagation in the theory. Each horizon traps the corresponding species of field excitations. Only the innermost horizon is a true event horizon, which traps all information inside of it.

Are the laws of thermodynamics obeyed by black hole systems in Lorentz-violating theories, as they are in standard general relativity? An ominous new feature is that the multiple horizons will generally have different surface gravities and therefore different temperatures. This conflicts with the “zeroth law,” by which a system in thermal equilibrium has a single temperature. Consequently, the entropy cannot be determined via the usual relation $dS = dE/T$. Also, the usual identification of the entropy with horizon area becomes ambiguous since there are multiple horizons. Moreover, the entropy might not even be proportional to *any* area. Perhaps related to these problems is the failure of the Noether charge algorithm [5] for identifying the entropy of a stationary black hole when applied to Einstein-aether theory [6] and, by extension, other theories of gravity with a dynamical preferred frame.

Nevertheless, Dubovsky and Sibiriyakov (DS) [7] were able to investigate the status of the second law in a Lorentz-violating gravity theory by considering a process in which the macroscopic state of the black hole is held fixed. Their analysis is presented in the context of the ghost condensate theory [8] but, as they suggest, it should apply more generally to any Lorentz-violating gravity theory with multiple maximal speeds. DS describe a *perpetuum mobile* that pumps heat from a colder to a hotter reservoir, by taking

advantage of the Hawking effect and the differing temperatures of the nested horizons. They consider a static black hole, and two fields A and B that travel at different maximal speeds c_A and $c_B \sim c_A$, with $c_B > c_A$. (We will use units in which $c_{A,B} \sim 1$.) Via the Hawking effect, the A and B horizons thermally radiate the corresponding species of particle, with $T_B > T_A$ since the Hawking temperature scales inversely with the horizon radius in the ghost condensate theory. DS assume the A and B fields have no interaction except through gravity. To construct the device, they place A and B shells surrounding the black hole that interact only with A and B fields, respectively. They then show that it is possible to choose the temperatures of the shells such that

$$T_{B,\text{Hawking}} > T_{B,\text{Shell}} > T_{A,\text{Shell}} > T_{A,\text{Hawking}} \quad (1)$$

and such that the energy fluxes balance one another so that the black hole stays the same size. Energy flows from the colder A shell into the black hole, and from the black hole to the hotter B shell. The second law thus appears to be violated.

DS consider three possible ways this conclusion might be evaded [7]. We quote:

- (i) “The presented description of the Hawking radiation in the ghost condensate is correct, but there is some subtle way in which a low energy effective theory forces our *perpetuum mobile* to change its state so that the entropy actually increases.
- (ii) The derivation of the Hawking radiation using only low energy theory is incorrect.
- (iii) The presented description of the Hawking radiation in the ghost condensate is correct, and the violation of the second law of thermodynamics within a low energy effective theory is a physical effect. According to the discussion in the Introduction this means that the UV completion of the ghost condensate, if it exists at all, has very unusual properties.”

DS put forth some arguments against the first two possibilities, but do not claim to have ruled them out conclusively. Our interpretation of what DS mean by (iii) (considering related remarks made in their paper) is as follows: the device works as described, but another version of the second law remains valid if, due to nonlocality or unbounded propagation speed in the UV completion, the notion of a causally hidden black hole region is eliminated. In this case, the entropy increase inside the black hole can influence the outside state, and the total entropy, inside and out, is nondecreasing.

This last point highlights the need to distinguish two different versions of the “second law”: the ordinary second law (OSL) and the generalized second law (GSL) [9]. The OSL refers to the total entropy, as counted both inside and outside black holes, whereas the GSL replaces the inside entropy by a special “black hole entropy” S_{bh} determined by the macroscopic geometry alone. The validity of the OSL for quantum fields in curved spacetime on a complete spacelike foliation is unaffected by the presence of black holes. It should thus be valid in Lorentz-violating theories. Therefore exotic properties of a UV completion are not required to uphold the OSL. Only the validity of the GSL is in question.

The GSL states that the generalized entropy $S_{\text{bh}} + S_{\text{outside}}$ cannot decrease. It is not obvious here which region is the “outside” for defining S_{outside} . One might suppose it should be the outside of the innermost causal horizon, but for the purposes of this paper it will not be necessary to specify exactly which region is the “outside.” In general relativity, S_{bh} is one quarter the horizon area in Planck units. In the Lorentz-violating case, DS did not need to specify S_{bh} , since the macroscopic properties of their black hole were held fixed. In section IVA we shall make only some weak assumptions about the form of S_{bh} .

II. SUMMARY OF OUR RESULTS

It is surprising that a thought experiment with black holes could reveal such an unexpected and drastic consequence of Lorentz violation. We thus set out to find a flaw in the proposed *perpetuum mobile*. However, rather than finding a flaw, we found only further support for the GSL violation.

We first consider two processes not discussed by DS that could potentially destabilize the device: (i) gravitationally mediated equilibration of A and B species in each shell, and (ii) classical or quantum instability of the ergoregion between the A and B horizons. We shall argue that none of these phenomena can save the GSL in all circumstances.

Next we present a *classical* process by which energy can be extracted from the black hole much quicker than any instability we are aware of, lowering the black hole entropy and thus violating the GSL. This process sidesteps the use of Hawking radiation, and permits direct interaction be-

tween the A and B fields. It should also be possible to violate the GSL by dumping heat into the black hole and then using this classical process to extract the corresponding energy without entropy, thus lowering the outside entropy without a net change of the black hole entropy.

That the microstates of the heat energy cannot engender gravitational ripples outside with equal entropy has generally been assumed in discussions of black hole thermodynamics. This assumption is plausible and might be established using a multipole expansion of the source, together with assumed quantization of graviton number. We assume it without proof here, along with the extension to include perturbations of the Lorentz-violating fields.

Throughout this paper we assume that the black hole mass and radius are related by $R \sim GM$, as in general relativity, the ghost condensate theory [2], and Einstein-aether theory [4].

III. DESTABILIZING PROCESSES

In this section we discuss the rate of various processes that could potentially destabilize the black hole *perpetuum mobile*. We will argue that they can be ignored for systems in which the gravitational coupling is sufficiently weak.

A. Equilibration of species

It was stipulated by DS that the A and B fields do not interact directly with each other. Since, however, they both interact with gravity, they must at least have gravitationally mediated interactions. This implies that in true equilibrium the A and B species must be thermally populated in each shell. But then the device malfunctions, since the colder A shell absorbs heat from the hotter B shell and B horizon. Nevertheless, it could operate for long enough to violate the GSL, if the equilibration were slow enough compared to the heat pump rate.

Rather than attempt here to estimate the actual equilibration rate, we instead employ a simple scaling argument. The gravity-mediated equilibration rate can be decreased by “turning down” the gravitational constant. Meanwhile the heat pump rate can be held fixed by scaling M so that $R \sim GM$, and therefore the Hawking temperatures and absorption and emission cross sections, remain fixed. Thus, for sufficiently weak gravitational coupling, the GSL can be violated before equilibration ensues.

Since G is not dimensionless, turning it down must be equivalent to leaving it fixed while scaling the system parameters. If we replace R by λR , and divide the shell temperatures by λ to match the scaling of the Hawking temperatures, the DS entropy pump rate will scale as $1/\lambda$, since it then depends only on unchanged dimensionless parameters and the radius λR . On the other hand, the entropy production due to gravity-mediated species equilibration scales with an additional factor of $1/\lambda^4$. This is

because the gravitational coupling between two particles scales with the particle energies, which in turn each scale with the temperature as $1/\lambda$, and the amplitude is squared to obtain the rate. So by increasing R and decreasing the shell temperatures the equilibration rate can be made much slower than the pump rate.

B. Ergoregion instability

An ergoregion is a place where the asymptotic time translation Killing vector of a spacetime becomes space-like, allowing negative energy states to exist. The Hawking effect is an instability brought about by the existence of an ergoregion hidden behind a horizon. But if an ergoregion exists *outside* a horizon then other instabilities can arise. A rotating black hole, for example, exhibits superradiant scattering—the amplification of classically scattered fields—and therefore is unstable to quantum spontaneous emission [10–12]. Both effects result in a transfer of the body’s rotational energy to outgoing field modes.

Could spontaneous ergoregion decay occur also for the *perpetuum mobile*? Since the slower, A field will possess an ergoregion that lies outside the faster, B horizon, processes can occur in which negative energy A particles that fall across the inner horizon are generated along with positive energy B particles that escape to infinity. These could in principle compete with the Hawking flux. Any such process, however, must be gravitationally mediated if A and B particles do not interact directly. Therefore, as argued above, the rate of these processes can be suppressed below that of the Hawking flux by turning down the gravitational constant. If instead direct A - B interactions exist with dimensionless coupling, then ergoregion decay would scale with R in the same way as does Hawking radiation, potentially interfering with the *perpetuum mobile* unless the coupling is sufficiently weak.

One might worry about exponentially growing instabilities. These are known to occur if the positive energy radiation returns coherently to the ergoregion, or the negative energy radiation remains in the ergoregion. Either way, emission of further radiation can be stimulated. For example, if a rotating black hole is surrounded by a mirror, the outgoing positive energy modes can be reflected back to the ergoregion creating a “black hole bomb” [13]. (The same thing can happen with the mirror replaced by anti-de Sitter boundary conditions [14].) Alternatively, a rotating star with an ergoregion but no horizon is unstable because the negative energy radiation piles up in the ergoregion [15]. The *perpetuum mobile* could perhaps be similarly unstable due to a gravity-mediated process in which negative energy A -modes are stimulated along with positive energy B -modes which are coherently reflected off the B shell. However, to be unstable the total amplitude for this process must exceed a critical value. For sufficiently small gravitational coupling or shell reflectivity, no instability will occur.

IV. CLASSICAL VIOLATIONS OF THE SECOND LAW

We now turn from the *perpetuum mobile* of DS to a purely classical process that leads to GSL violation. It makes no use of the Hawking effect, vitiating the need to verify that effect in this Lorentz-violating context. Instead, it takes advantage of the A ergoregion in a way analogous to the Penrose process in the ergoregion of a rotating black hole [16].

A. Mass and entropy

We begin by discussing the connection between extracting energy from the black hole and lowering its entropy. As discussed in the Introduction, it is not yet clear how the black hole entropy should be defined in a Lorentz-violating theory. Therefore, we will make only the following mild assumptions about the entropy $S(M)$ of black holes of mass M and size $R \sim GM$:

- (1) When $M_1 \gg M_2$, then $S(M_1) \gg S(M_2)$.
- (2) By choosing M sufficiently large, the entropy of any radiation emitted by the hole over a time R can be made to be an arbitrarily small fraction of $S(M)$.

Note that Hawking radiation, and ergoregion decay with dimensionless A - B interaction, both produce entropy at a rate scaling as $1/R$, since R is the only relevant length scale. To satisfy both assumptions, it is therefore sufficient that $S(M)$ increase with M at least as fast as M^α for some $\alpha > 0$.

We will show that a process exists that reduces the energy of a black hole by an amount proportional to M over a time of order R , without any incidental entropy increase outside the black hole. By repeating this process one can shrink the black hole down to a much smaller size in a time proportional to R . The first of the above assumptions implies the final black hole then has much smaller entropy, and the second assumption implies that if one starts and ends with a sufficiently large black hole, any radiated entropy is negligible. Thus the process violates the GSL.

B. Classical energy extraction from black holes

We now discuss the spacetime structure, which gives rise to the conservation laws governing the energy extraction process. Let g_{ab} be the metric felt by the A field. The assumed Lorentz violation involves a “preferred” unit timelike vector u^a that has unit norm with respect to g_{ab} :

$$g_{ab}u^a u^b = 1. \quad (2)$$

The metric \tilde{g}_{ab} felt by the B field is given (up to an arbitrary conformal factor) by

$$\tilde{g}_{ab} = u_a u_b + \frac{c_A^2}{c_B^2} (g_{ab} - u_a u_b), \quad (3)$$

where the index on u_a is lowered using g_{ab} . We are con-

sidering a black hole spacetime in which g_{ab} , \tilde{g}_{ab} , and u^a are all spherically symmetric and static, with asymptotically timelike Killing field ξ^a , and we assume $c_B > c_A$. For each metric, ξ^a is timelike outside and spacelike inside the corresponding horizon.

The 4-momentum covector p_a of a particle is naturally defined in a metric-independent way as the gradient of the Hamilton-Jacobi principal function associated with the particle. This 4-momentum is locally conserved. The Killing energy \mathcal{E} is defined by $\mathcal{E} = p_a \xi^a$. The mass shell conditions depend on the metric; for example, massless A particles satisfy $g^{ab} p_a p_b = 0$ while massless B particles satisfy $\tilde{g}^{ab} p_a p_b = 0$, where $\tilde{g}^{ab} = u^a u^b + (c_B^2/c_A^2)(g^{ab} - u^a u^b)$ is the inverse of \tilde{g}_{ab} . The energy E and 3-momentum \vec{p}_a in the preferred frame are defined by $p_a = E u_a + \vec{p}_a$, where $\vec{p}_a u^a = 0$. Massless A particles then satisfy $E^2 = |g^{ab} \vec{p}_a \vec{p}_b|$, while massless B particles satisfy $E^2 = (c_B^2/c_A^2) |g^{ab} \vec{p}_a \vec{p}_b|$. Hence the B null covector cone lies *within* the A null covector cone.

Now let a system Σ composed of A and B particles fall through the A horizon, meeting at a point x in the ergoregion outside the B horizon. Henceforth we refer to this zone as simply the “ergoregion.” We will assume that the energy in Σ is sufficiently small that it does not appreciably disturb the black hole. We also require Σ to be well localized compared to the size of the ergoregion so that it can be treated as a “point particle.” These conditions can both be satisfied if the energy in the system is less than the black hole mass times some fixed small constant k that depends on the particular theory. For example if c_A and c_B are very close, then the ergoregion is very thin and k must be correspondingly smaller.

We arrange Σ so that at the meeting point x its net 4-momentum P_a is radial and outward pointing, lying outside the B -metric momentum-space null cone as depicted in Fig. 1.

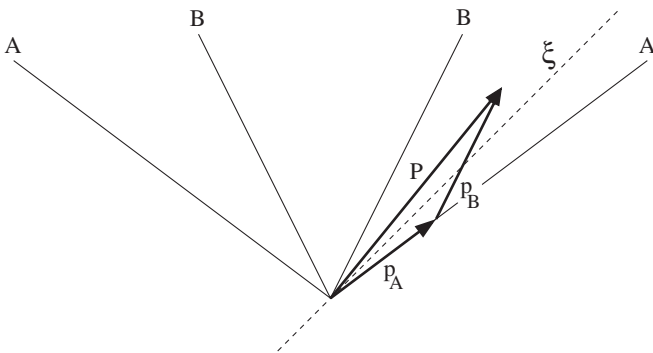


FIG. 1. The radial 4-momentum covector space at the point x in the ergoregion ($p_a m^a = 0$ for all vectors m^a tangent to the symmetry sphere through x). The momenta with vanishing Killing energy $\mathcal{E} = p_a \xi^a$ lie along the dashed line. \mathcal{E} is negative for p_A , since x lies in the A ergoregion, while it is positive for all momenta on the B null cone. The total 4-momentum P has positive \mathcal{E} , hence it points above the dashed line.

Further the system should have positive Killing energy so it can have come from outside the outer horizon. A system containing just one massive A particle, for example, can satisfy these conditions, if dropped in from just outside the A horizon with 4-velocity sufficiently close to the outgoing A null ray. However, since we want to arrange for ejection of a B particle in a classical process we should start with at least one B particle in the system. One possible scenario is that the A and B components fall in together in a gravitationally bound configuration, or they could just be arranged to meet in the ergoregion and interact there. The net 4-momentum can still satisfy the required conditions if the A 4-momentum dominates.

After Σ has fallen into the ergoregion, we imagine it splits at x , where it has 4-momentum P , into two separate components, one consisting of outgoing massless A particles with 4-momentum p_A , and the other outgoing massless B particles with 4-momentum p_B . The total 4-momentum covector can be conserved in such a process, as illustrated in Fig. 1. The A 's then fall across the B horizon carrying negative Killing energy, while the B 's escape outwards across the A horizon. Since the Killing energy is conserved, these carry out more energy than originally fell in, so the black hole mass decreases. The mass decrease scales with the energy of Σ , whose upper bound is kM , so the mass can be decreased by some fraction $k'M$.

So far we have only imposed 4-momentum conservation; we have not addressed what kind of interaction could result in the final A component of the system having negative Killing energy. Since the initial A component falls into the ergoregion with positive Killing energy, some Killing energy transfer to the B component must be effected. This transfer requires an interaction, but A and B always interact at least gravitationally. Seeing as the conservation laws permit the process, and an interaction capable of mediating it exists, we shall presume that it can be achieved. The black hole size R sets the time scale for the process, since the particles need only travel this distance, and the energy transfer occurs on a much smaller length scale.

The outgoing B particles need not carry any entropy at all. This is because the system may be prepared in a pure state, and the whole splitting process can occur via classical deterministic evolution, which does not generate any entanglement entropy between the A and B components. This process therefore reduces the mass of the black hole without creating any compensating matter entropy outside of the black hole. Given our assumptions above, by repeating this process many times, the GSL can be violated.

V. DISCUSSION

We have identified possible destabilizing mechanisms that might have interfered with the black hole *perpetuum mobile* devised by Dubovsky and Sibiryakov, but found

that they can be neglected for sufficiently large black holes. Furthermore, we devised a classical energy extraction process, which strengthens the case for GSL violation in Lorentz-violating theories. Unlike the DS *perpetuum mobile*, it does not rely on the Hawking effect, and the entropy decrease occurs much more quickly. Most importantly, it can operate even if the A and B species have direct interactions. Thus the GSL violation is shown to occur for a much broader class of Lorentz-violating theories with multiple speeds, not only those with limited interactions.

Is this violation of the GSL necessarily unacceptable? It would certainly seem unlikely to have led to any observable consequences, given our current state of astrophysical observation. Moreover, there is no *a priori* reason why the GSL should hold, considering the fact that the outside of a black hole is not a closed system. From this perspective it is perhaps more surprising that the GSL holds for Lorentz symmetric systems, than that it might fail for the rest.

On the other hand, if the GSL does *not* hold, then the apparently deep connection between black holes and thermodynamics would have been a coincidental false lead, not arising from fundamental principles. This is true even if the difference in speeds were very small so it would take a very long time to execute a violation of the GSL.

GSL violation might be avoided if the UV completion of the Lorentz-violating theory eliminates the notion of a causally hidden black hole region. Then the outside would include the black hole interior, leaving no “black hole” contribution to the generalized entropy, thus reducing the GSL to the OSL. But this eliminates the essence of black hole thermodynamics. So it appears that the only way to save black hole thermodynamics is to reject the sort of Lorentz violation considered here (and likely any other sort involving Lorentz-violating dispersion).

In retrospect, it is perhaps not so mysterious that the validity of black hole thermodynamics is tied to Lorentz symmetry. After all, at the root of the thermality of the Hawking effect lies the Unruh effect: the vacuum is a thermal state with respect to the boost Hamiltonian when restricted to the Rindler wedge [17]. This in turn can only hold for the vacuum of interacting fields if they share a common Lorentz symmetry.

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- [1] D. Giannios, Phys. Rev. D **71**, 103511 (2005).
 - [2] S. Mukohyama, Phys. Rev. D **71**, 104019 (2005).
 - [3] E. Babichev, V.F. Mukhanov, and A. Vikman, J. High Energy Phys. 09 (2006) 061.
 - [4] C. Eling and T. Jacobson, Classical Quantum Gravity **23**, 5643 (2006).
 - [5] R.M. Wald, Phys. Rev. D **48**, R3427 (1993); V. Iyer and R.M. Wald, Phys. Rev. D **50**, 846 (1994).
 - [6] B.Z. Foster, Phys. Rev. D **73**, 024005 (2006).
 - [7] S.L. Dubovsky and S.M. Sibiryakov, Phys. Lett. B **638**, 509 (2006).
 - [8] N. Arkani-Hamed, H.C. Cheng, M.A. Luty, and S. Mukohyama, J. High Energy Phys. 05 (2004) 074.
 - [9] J.D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
 - [10] Y.B. Zel'dovich, Zh. Eksp. Teor. Fiz. **62**, 2076 (1972) [Sov. Phys. JETP **35**, 1085 (1972)].
 - [11] A. A. Starobinsky, Zh. Eksp. Teor. Fiz. **64**, 48 (1973) [Sov. Phys. JETP **37**, 28 (1973)].
 - [12] W.G. Unruh, Phys. Rev. D **10**, 3194 (1974).
 - [13] W.H. Press and S.A. Teukolsky, Nature (London) **238**, 211 (1972); V. Cardoso, O.J.C. Dias, J.P.S. Lemos, and S. Yoshida, Phys. Rev. D **70**, 044039 (2004); **70**, 049903(E) (2004).
 - [14] V. Cardoso and O.J.C. Dias, Phys. Rev. D **70**, 084011 (2004).
 - [15] J.L. Friedman, Commun. Math. Phys. **63**, 243 (1978); N. Comins and B.F. Schutz, Proc. R. Soc. A **364**, 211 (1978).
 - [16] R. Penrose, Riv. Nuovo Cimento **1**, 252 (1969); Gen. Relativ. Gravit. **34**, 1141 (2002); R. Penrose and R.M. Floyd, Nature (London) **229**, 177 (1971).
 - [17] G.L. Sewell, Phys. Lett. **79A**, 23 (1980); Ann. Phys. (N.Y.) **141**, 201 (1982).