Determining the CKM angle γ with B_c decays

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We consider the possibility of extracting the CKM angle γ with B_c decays. The modes $B_c^\pm \to (D^0)D_s^\pm \to (K^{*+}K^-)D_s^\pm$ and $B_c^\pm \to (\bar{D}^0)D_s^\pm \to (K^{*+}K^-)D_s^\pm$ are found to be well suited for the extraction of γ. Since a large number of B_c mesons are expected to be produced at the Large Hadron Collider, it would be very interesting to explore the determination of γ with these modes.

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It is strongly believed that the elusive Higgs boson, the missing entity in the otherwise immensely successful standard model (SM) of electroweak interactions, will be chased and most likely be found at the Large Hadron Collider (LHC), which is going to be started very soon. While a detailed understanding of the SM description might be accomplished during the LHC era, there is an unprecedented level of enthusiasm to decipher the signal of high scale physics, where the SM is a low energy manifestation of the same. Whether the physics at a higher scale leaves its trace at LHC or not, it is certain that the enormous data will provide us a unique opportunity to study all the important aspects of physics under the framework of the SM with a greater accuracy.

In the SM, the CP violation is elegantly described by the Cabibbo-Kobayashi-Maskawa (CKM) mechanism. In this context, one of the main ingredients of the SM description of CP violation is the CKM unitarity triangle (UT) and the angles of the UT are termed as α (ϕ_2), β (ϕ_1), and γ (ϕ_3) [1]. Large CP violation, as was expected, has been established already in B-systems in the currently running B-factories at SLAC and KEK. The present status is that we have measured, with the huge data sets available, the angle β [actually, $\sin(2\beta)$] with a reasonable accuracy, and we expect to have a precision measurement of angle β in the years to come, with the help of the golden mode $B_d^0 \to J/\psi K_S$. Unfortunately, we do not have three golden modes to determine the three angles of the UT. So we have to be content with the best available modes like $B \to \pi\pi$ (and some related modes) for the determination of the angle α, but these modes are accompanied by a generic problem called penguin contamination, whose remedy has not been found yet by the theoretical community. Therefore, we are left with the angle $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, which was believed to be the most difficult one, among all the three angles, at the beginning. But, fortunately, in this case, nature has been very kind to provide us many options to determine the angle γ in various avenues.

There have been many attempts in the past to devise methods to determine the CKM angle γ as cleanly as possible. The golden method to determine γ is the Gronau-London-Wyler (GLW) method [2], which uses the interference of two amplitudes ($b \to c\bar{u}s$ and $b \to u\bar{c}s$) in $B \to DK$ modes. In this method γ can be determined by measuring the decay rates $B^- \to D^0K^-$, $B^- \to \bar{D}^0K^-$, and $B^- \to D_+^0K^-$ (where D_+^0 is the CP-even eigenstate of the neutral D meson system) and their corresponding CP conjugate modes. However, because the mode $B^- \to \bar{D}^0K^-$ is both color and CKM suppressed with respect to $B^- \to D^0K^-$ the corresponding amplitude triangles are expected to be highly squashed, and it is also very difficult to measure the rate of $B^- \to \bar{D}^0K^-$. To overcome the problems of the GLW method Atwood-Dunietz-Soni (ADS) [3] proposed an improved method where they have considered the decay chains $B^- \to K^-D^0[\to f]$ and $B^- \to K^-\bar{D}^0[\to f]$, where f is the doubly Cabibbo suppressed (Cabibbo favored) non-CP eigenstate of $D^0(\bar{D}^0)$. These methods are being explored in the currently running B-factory experiments and also will be taken up at the collider experiments along with another golden method called Aleksan-Dunietz-Kayser method [4], which uses the time dependent measurement of $B_s^0(\bar{B}_s^0) \to D_s^\mp K^\pm$ modes. Because of its importance and, of course, possible options available, there are many methods that exist in the literature. Some of the alternative methods to obtain γ are those using B and B_s decays [5–13], B_c decays [14], and also Λ_b decays [15].

In the meantime, another exciting method, Giri-Grossman-Soffer-Zupan (GGSZ) method (otherwise also known as the Dalitz method) [5] has been proposed [using $B \to D^0(\bar{D}^0)K \to K_S\pi\pi K$], which has many attractive features and has been explored already at both of the B-factories. It should be noted here that the GGSZ method uses the ingredients of the GLW and ADS methods where the $D^0(\bar{D}^0)$ decays to multiparticle final states. This method in turn helps us to constrain the angle γ directly from the experiments. But at present the error bars are quite large, which are expected to come down in the coming years. It may be worthwhile to emphasize here that one has to measure the angle with all possible clean methods available to arrive at a conclusion and thereby reducing the error in γ to a minimum.

In this continued effort, we now wish to explore yet another method with the decays $B_c^\pm \to D_s^\pm D^0 \to D_s^\pm(K^{*+}K^-)_{D^0}$ and $B_c^\pm \to D_s^\pm \bar{D}^0 \to D_s^\pm(K^{*+}K^-)_{\bar{D}^0}$. It has been shown earlier in [14] that the decay $B_c^\pm \to D^0(\bar{D}^0)D_s^\pm$

modes can be used to determine the CKM angle γ in a better way since the interfering amplitudes in the B_c case are roughly of equal sizes, whereas the corresponding ones in the GLW method (using B mesons) are not so. In our earlier work [14], we have shown that γ can be determined from the decay rates $B_c^\pm \to D^0 D_s^\pm$, $B_c^\pm \to \bar{D}^0 D_s^\pm$, and $B_c^\pm \to D_\pm^0 D_s^\pm$ (where $D_\pm^0$ are the CP eigenstates of neutral D meson system with CP eigenvalues ± 1, which can be identified by the CP-even and CP-odd decay products of neutral D meson). In this work we propose another method where we consider the $B_c^\pm \to D^0(\bar{D}^0) D_s^\pm$ decay modes, that are followed by $D^0(\bar{D}^0)$ decaying to $K^{*+}K^-$, which is a non-CP eigenstate.

The decay modes $B_c^- \to D_s^- D^0$ and $B_c^- \to D_s^- \bar{D}^0$ are described by the quark level transition $b \to c\bar{u}s$ and $b \to u\bar{c}s$, respectively, and the amplitudes for these processes are given as

$$\begin{aligned} \mathcal{A}(B_c^- \to D^0 D_s^-) &= \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (C + A), \\ \mathcal{A}(B_c^- \to \bar{D}^0 D_s^-) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* (\tilde{C} + \tilde{T}), \end{aligned} \tag{1}$$

where C and A denote the color suppressed tree and annihilation topologies for $b \to c$ transition, and $\tilde{C}$ and $\tilde{T}$ denote the color suppressed tree and color allowed tree contributions for $b \to u$ transition. It should be noted here that the amplitude with the smaller CKM element V_{ub} is color allowed while the larger element V_{cb} comes with color suppression factor (and along with the appropriate V_{cs} and V_{us} elements); the two amplitudes are of comparable sizes. Now let us denote these amplitudes as

$$A_B = \mathcal{A}(B_c^- \to D^0 D_s^-), \qquad \bar{A}_B = \mathcal{A}(B_c^- \to \bar{D}^0 D_s^-), \tag{2}$$

and their ratios as

$$\frac{\bar{A}_B}{A_B} = r_B e^{i(\delta_B - \gamma)}, \quad \text{with} \quad r_B = \left| \frac{\bar{A}_B}{A_B} \right| \quad \text{and} \quad \arg(\bar{A}_B/A_B) = \delta_B - \gamma, \tag{3}$$

where δ_B and $(-\gamma)$ are the relative strong and weak phases between the two amplitudes. The ratio of the corresponding CP conjugate processes are obtained by changing the sign of the weak phase γ. One can then obtain a rough estimate of r_B from dimensional analysis, i.e.,

$$r_B = \left| \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right| \cdot \frac{a_1^{\rm eff}}{a_2^{\rm eff}} \approx \mathcal{O}(1), \tag{4}$$

where $a_1^{\rm eff}$ and $a_2^{\rm eff}$ are the effective QCD coefficients describing the color allowed and color suppressed tree level transitions. For the sake of comparison, we would like to point out here that the corresponding ratio between the $B^- \to D^0(\bar{D}^0)K^-$ amplitudes are given as $|\mathcal{A}(B^- \to \bar{D}^0 K^-)/\mathcal{A}(B^- \to D^0 K^-)| = |(V_{ub}V_{cs}^*)/(V_{cb}V_{us}^*)| \cdot (a_2^{\rm eff}/a_1^{\rm eff}) \approx \mathcal{O}(0.1)$. The D^0 decay amplitudes are denoted as

$$A_D = \mathcal{A}(D^0 \to K^{*+}K^-), \qquad \bar{A}_D = \mathcal{A}(\bar{D}^0 \to K^{*+}K^-), \tag{5}$$

and their ratios as

$$\frac{\bar{A}_D}{A_D} = r_D e^{i\delta_D}, \quad \text{with} \quad r_D = \left| \frac{\bar{A}_D}{A_D} \right|. \tag{6}$$

It is interesting to note that the parameters r_D and δ_D have been measured recently by the CLEO Collaboration [16], with values $r_D = 0.52 \pm 0.05 \pm 0.04$ and $\delta_D = 332° \pm 8° \pm 11°$, rendering our study, at this point in time, more appealing.

With these definitions, the four amplitudes are given as

$$\begin{aligned} \mathcal{A}(B_c^- \to D_s^-(K^{*+}K^-)_D) &= |A_B A_D|[1 + r_B r_D e^{i(\delta_B + \delta_D - \gamma)}], \\ \mathcal{A}(B_c^- \to D_s^-(K^{*-}K^+)_D) &= |A_B A_D| e^{i\delta_D}[r_D \\ &\quad + r_B e^{i(\delta_B - \delta_D - \gamma)}], \\ \mathcal{A}(B_c^+ \to D_s^+(K^{*-}K^+)_D) &= |A_B A_D|[1 + r_B r_D e^{i(\delta_B + \delta_D + \gamma)}], \\ \mathcal{A}(B_c^+ \to D_s^+(K^{*+}K^-)_D) &= |A_B A_D| e^{i\delta_D}[r_D \\ &\quad + r_B e^{i(\delta_B - \delta_D + \gamma)}]. \end{aligned} \tag{7}$$

From these amplitudes one can obtain the four observables $(R_1, \cdots, R_4)$, with the definition

$$R_i = |\mathcal{A}_i(B_c^\mp \to D_s^\mp(K^{*\pm}K^\mp)_D)/A_B A_D|^2. \tag{8}$$

We can now write $R_1 = 1 + r_B^2 r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D - \gamma)$ and similarly R_2, R_3, and R_4.

Here we assume that the amplitudes $|A_B|$ and $|A_D|$ are known [so also r_B, which is $\mathcal{O}(1)$].

Thus, one can obtain an analytical expression for γ as

$$\sin^2\gamma = \frac{4([R_1 - R_3]^2 - [R_2 - R_4]^2)}{[[R_2 - (r_B^2 + r_D^2)][R_4 - (r_B^2 + r_D^2)] - [R_1 - (1 + r_B^2 r_D^2)][R_3 - (1 + r_B^2 r_D^2)]]}. \tag{9}$$

Now let us study the sensitivity of γ in some limiting cases in the method described above.

(a) If the relative strong phase between $\bar{A}_B$ and A_B is zero then Eq. (9) can no longer be used to extract the angle γ as both numerator and denominator vanish in this limit. However, still γ can be extracted, in this limit, from either of the observables R_1 and R_3 or R_2 and R_4. Now, considering the observables R_2 and R_4, for example, one can obtain an expression for γ as

$$\tan\gamma = \frac{\cot\delta_D(R_4 - R_2)}{R_2 + R_4 - 2(r_B^2 + r_D^2)}. \quad (10)$$

An analogous expression for γ also can be obtained from R_1 and R_3 with the replacement of $R_{2,4} \leftrightarrow R_{3,1}$ and $(r_B^2 + r_D^2) \leftrightarrow (1 + r_B^2 r_D^2)$. Let us now consider another limiting case.

(b) If $r_B = 1$ and $\delta_B = 0$, then the four observables $(R_1, \cdots, R_4)$ are no longer independent of each other and we have two degenerate sets with $(R_1 = R_4)$ and $(R_2 = R_3)$. One can then define two parameters

$$C_- \equiv \cos(\delta_D - \gamma) = \frac{1}{2r_B r_D}(R_4 - r_B^2 - r_D^2),$$
$$C_+ \equiv \cos(\delta_D + \gamma) = \frac{1}{2r_B r_D}(R_2 - r_B^2 - r_D^2), \quad (11)$$

where, we have deliberately retained the r_B term in the above expressions, so that one can still use this method for the $r_B \neq 1$ case. Thus one can now obtain the solution for γ, in terms of these observables, as

$$\sin^2\gamma = \frac{1}{2}[1 - C_+C_- \pm \sqrt{(1 - C_+^2)(1 - C_-^2)}], \quad (12)$$

one solution of which will give $\sin^2\gamma$ while the other being $\sin^2\delta_D$. Since δ_D has been measured already, $\sin^2\gamma$ could be extracted from these observables, once we know the values of R_2, R_4 (otherwise R_1 and R_3) and r_B (it may be noted that the value of r_D is already known now).

Our method consists of two parts, the first one being the $B_c^\pm \to D^0(\bar{D}^0)D_s^\pm$, which will be measured at the hadron colliders, such as LHC, whereas the second part consists of the measurement of $D^0(\bar{D}^0) \to K^{*+}K^-$, which can be measured also at the same collider experiments. Moreover, since we already have experiments and there are upcoming dedicated experiments to measure the parameters in the charm sector, like at CLEO-c and the BEPCII, which will provide us half of the parameters needed in our study, it is meaningful to combine the data from various experiments, mentioned above, to obtain γ with a better accuracy. In principle, one can study the $D^0 \to K^+\pi^0K^-$ (where K^{*+} decays to $K^+\pi^0$), but since CLEO and other charm experiments are doing precisely the same job we, therefore, leave it to these experiments to provide us the values of r_D and δ_D.

We would like to comment here that the possible effect of $D^0 - \bar{D}^0$ mixing for the determination of γ is not taken into account in our analysis since it has been well studied in the literature [5,17] and found that the effect is very small, unless we are doing a precision measurement of γ. To be quantitative the error could be around 1°, with the present data available, which for all practical purposes can be ignored at this moment.

Now, with r_D already known (so also δ_D), we are left with only two unknowns (δ_B and γ). Therefore, we have two unknowns and four observables. We can consider different non-CP eigenstates (like $\rho^+\pi^-$), which will increase the observables by four and unknowns by two (r'_D and δ'_D) for each additional eigenstate. One can also take $B_c^\pm \to D^0(\bar{D}^0)D_s^{*\pm}$ mode thereby further increasing the number of observables by four and unknowns by two (say r'_B and δ'_B, in fact it could be just δ'_B). Hence we hope to have enough observables and at best half the number of unknowns (actually, it will always be less than half since new unknown parameters, namely, r'_D and δ'_D also can be inferred from the D decay data), and we can obtain the value of γ without hadronic uncertainties. Also, it should be reminded here that by the time the actual measurement could be done, using this method, results from the other methods, mentioned earlier, might be available.

Now let us estimate the branching ratios for these modes. Using the generalized factorization approximation, the amplitudes are given as

$$\mathcal{A}(B_c^- \to D^0D_s^-) = \frac{G_F}{\sqrt{2}}V_{cb}V_{us}^*(a_2^{\rm eff}X + a_1^{\rm eff}Y),$$
$$\mathcal{A}(B_c^- \to \bar{D}^0D_s^-) = \frac{G_F}{\sqrt{2}}V_{ub}V_{cs}^*(a_1^{\rm eff}X_1 + a_2^{\rm eff}X), \quad (13)$$

where, $X = if_{D^0}(m_{B_c}^2 - m_{D_s}^2)F_0^{B_cD_s}(m_{D^0}^2)$, $X_1 = if_{D_s}(m_{B_c}^2 - m_{D^0}^2)F_0^{B_cD^0}(m_{D_s}^2)$, and $Y = if_{B_c}(m_{D_s}^2 - m_{D^0}^2)F_0^{D_sD^0}(m_{B_c}^2)$ are the factorized hadronic matrix elements. For numerical evaluation we use the values of the form factors at zero recoil from [18] as $F_0^{B_cD^0}(0) = 0.352$, $F_0^{B_cD_s}(0) = 0.37$, the decay constants (in MeV) as $f_{D^0} = 235$, $f_{D_s} = 294$, $f_{B_c} = 360$, the QCD coefficients $a_1^{\rm eff} = 1.01$, $a_2^{\rm eff} = 0.23$, particle masses, lifetime of B_c and CKM matrix elements from [19]. We thus obtain the branching ratios as

$$\mathrm{BR}(B_c^- \to D^0D_s^-) = 7.0 \times 10^{-6},$$
$$\mathrm{BR}(B_c^- \to \bar{D}^0D_s^-) = 4.5 \times 10^{-5}. \quad (14)$$

Let us now make a crude estimate of the number of reconstructed events that could be observable at LHC per year of running. At LHC, one expects about 10^{10} untriggered B_c's per year [20]. For the estimation we use the branching ratios as $\mathrm{BR}(B_c^- \to D^0D_s^-) = 7.0 \times 10^{-6}$ and $\mathrm{BR}(D^0 \to K^{*+}K^-) = 3.7 \times 10^{-3}$ [19] and assume that the D_s can be reconstructed efficiently by combining a number of hadronic decay modes. As the LHCb trigger system has a good performance for hadronic modes, we assume an overall efficiency of 30% and hence we expect to get nearly 80 events per year of running at LHC.

We have outlined here that $B_c^{\pm} \to (D^0)D_s^{\pm} \to (K^{*\pm}K^{\mp})D_s^{\pm}$ and $B_c^{\pm} \to (\bar{D}^0)D_s^{\pm} \to (K^{*\pm}K^{\mp})D_s^{\pm}$ can be used to determine the CKM angle γ at the LHC. Since the interfering amplitudes are of equal order (which is not the case with $B \to DK$ methods) and furthermore neither tagging nor time dependent studies are required to undertake this strategy and above all the final particles are charged ones (and of course with reduced background), this method may be very well suited for the determination of γ without hadronic uncertainties. But one has to pay the price for all the niceties of this method in the sense that the branching ratios are smaller by an order compared to the earlier modes. Nevertheless, we hope that this should not cause any hindrance for the clean determination of angle γ using this method, and even if we get lesser number of events the predictive power will not be diluted.

In conclusion, in this paper we have looked into the possibility of extracting the CKM angle γ using multibody B_c decays and in view of the fact that the LHC will be operating shortly, this method can be found to be very useful to obtain γ in yet another method to supplement the results from other methods. We believe that during the first few years of the LHC run we will have a meaningful value of angle γ with reduced errors and emphasize that the strategy presented here will be an added asset to our endeavor to measure the angle γ.

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