Very strong and slowly varying magnetic field as source of axions

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An investigation of the production of particles in a slowly varying but extremely intense magnetic field is extended to the case of axions. The motivation is, as for some previously considered cases, the possibility that such kind of magnetic field may exist around very compact astrophysical objects.

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I. STATEMENT OF THE PROBLEM

A magnetic field of huge strength can give rise to real particles even if its rate of variation is very small. This possibility could be of some interest from a purely theoretical point of view, but it gains more physical relevance if one accepts that such field configurations may be present around some very compact astrophysical objects [1–3]. In this case, the time variation is related to the evolution of the source, by collapse, rotations, or other processes, and it is therefore very slow, in comparison to the times typical of elementary-particle processes. We can call the former time the macroscopic time and the latter the microscopic time. The production of light particles in these processes has been analyzed in some detail in some previous papers [4,5], with the suggestion that it is one of the mechanisms at work in the phenomenon of *gamma-ray bursts;* the typical microscopic time is related to the electron mass since photons are produced through real or virtual intermediate states of $e^- - e^+$-pairs. The lightest particles that could be produced are massive neutrinos, but the magnetic-moment coupling induced by the standard electroweak interactions is extremely small.

There is, at least in the theoretical realm, another very light particle, that is, the axion [6]: owing to its dynamical characteristics it must be coupled also to the electromagnetic field [7]. Its electromagnetic coupling is being actively studied from an experimental side [8], and the possibility of detecting such particles as coming from nonterrestrial sources has already been foreseen [9]. It is immediately seen that the production of axions by a varying magnetic field must be realized through a mechanism different from the previously considered one. In fact, the axions are coupled [7] only to the pseudoscalar density $\mathbf{E}\cdot\mathbf{B}$, so the presence of an electric field is necessary as a starting point, but a nonstatic magnetic field always creates an electric field; even though the rate of variation is small, the very large magnetic strength makes the electric field not small.

In the present paper the coupling of the axion field with a given $\mathbf{E}\cdot\mathbf{B}$ density is written in standard second-quantized formalism, and then the effect of time variation of that density on the axion vacuum is determined and the consequent production is calculated. The result depends both on the spatial shape and on the time variation of the magnetic field. In accordance with the prevailing astrophysical hypotheses [1–3], the magnetic field is seen as a bundle of lines of force which may safely be considered straight in comparison with the microscopic scale. The time variation could affect both the shape and the strength of the fields; both are effective in the production process. The calculation procedure is not the standard adiabatic approximation [10] as was used in previous investigations [4], but the factor that one has to deal with a two-scale problem is still fully relevant.

II. GENERAL FORM OF THE PRODUCTION PROBABILITY

The starting point is a second-quantized axion field in the presence of given, classical, magnetic, and electric fields. The axion field $\phi(x)$ is coupled to the pseudoscalar density[1] $G(x)=\mathbf{E}(x)\cdot\mathbf{B}(x)$ and the coupling constant, of dimension of length, is here indicated by C. We assume that the interaction lasts from an initial time t_o until a final time t.

So we have for the axion field the expression:

$$\phi(x)=\phi_o(x)+\chi(x) = \phi_o(x)+C\int\Delta_R(x-y)G(y)d^4y. \quad (1)$$

Here $\Delta_R(x-y)$ is the standard retarded Green function, and the source is a c-number, so the same holds for χ. The field ϕ is free before t_o, where it has the standard expansion:

$$\phi_o(x)=\frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{\sqrt{2\omega_k}}[a_o(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}-i\omega t} + a_o^\dagger(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}+i\omega t}]. \quad (2)$$

When it acquires a contribution from χ, this term has the following actual expression:

[1] the same kind of coupling is possible also for the neutral pion, but in this case other channels of production are present [11].

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$$\chi(x) = \frac{\mathrm{i}}{(2\pi)^{3/2}} \int \frac{d^3k}{2\omega_k}\left[\mathrm{e}^{-\mathrm{i}\omega_k t}\int_{t_o}^{t} \mathrm{e}^{\mathrm{i}\omega_k \tau} g_k(\tau) d\tau \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} - \mathrm{c.c.}\right], \tag{3.1}$$

$$g_k(t) = \frac{C}{(2\pi)^{3/2}} \int G(\mathbf{r}, t)\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}. \tag{3.2}$$

The reality condition for G, that gives $g_k^* = g_{-k}$, has been used, together with the initial condition $\dot{\chi}(t_o) = 0$. The separation into positive and negative frequencies in $\chi(x)$ is unambiguous until the typical frequencies of G are small.

Having found the time evolution of the field, the total production of axions is calculated in Heisenberg description of motion, i.e., we take an initial state $a_o(\mathbf{k})|\circ\rangle = 0 \quad \forall\ \mathbf{k}$, the vacuum in the absence of interaction, and then we express the mean particle number as the time dependent term

$$\mathcal{N}(\mathbf{k}, t) = \langle\circ|a^\dagger(\mathbf{k}, t)a(\mathbf{k}, t)|\circ\rangle.$$

Since the only effect of the interaction is a c-number shift on the operators $a(\mathbf{k}, t) = a_o(\mathbf{k}) + b(\mathbf{k})$, the calculation is easy. In particular, when the interaction no longer acts we get

$$\mathcal{N}_f(\mathbf{k}) = |b_f(\mathbf{k})|^2. \tag{4}$$

The expression of $b_f(\mathbf{k})$ can be read from Eqs. (2), (3.1), and (3.2). It is:

$$b_f(\mathbf{k}) = \frac{\mathrm{i}}{\sqrt{2\omega_k}} \int_{t_o}^{t_f} \mathrm{e}^{\mathrm{i}\omega_k\tau} g_k(\tau) d\tau. \tag{5}$$

III. DETAILED CALCULATIONS

As anticipated in Sec. I, a more definite model of the magnetic field can be one of uniform direction (at a given time) with some transverse shape. Both the direction and the shape may vary in time; one possible restriction is the conservation of the total flux. More explicitly these conditions are realized by giving:

$$\begin{aligned}\mathbf{A} &= \frac{\Phi}{2\pi}\frac{\mathbf{n}\wedge\mathbf{r}}{r_\perp^2}[1 - w(\mu r_\perp)] \\ \mathbf{B} &= -\frac{\Phi}{\pi r_\perp}\mathbf{n}\partial_\perp w(\mu r_\perp).\end{aligned} \tag{6}$$

The unit vector $\mathbf{n}$ gives the instantaneous direction of the magnetic field, $r_\perp = \sqrt{r^2 - \mathbf{n}\cdot\mathbf{r}^2}$ and $\partial_\perp$ the corresponding derivative, and Φ is the total magnetic flux; the parameter μ defines the size of the field in the transverse directions, and w is taken to be cylindrically symmetric and, obviously, must go to zero at infinity. The requirement $w(0) = 1$ avoids singularities in $\mathbf{E}$.

Since $\mathbf{E} = -\dot{\mathbf{A}}$ and $\mathbf{A}\cdot\mathbf{B} = 0$, only the terms coming from the time variation of the direction of $\mathbf{A}$ contribute to $G = \mathbf{E}\cdot\mathbf{B}$. So we get:

$$G = \left(\frac{\Phi}{\pi}\right)^2 \frac{\mathbf{J}\cdot\mathbf{r}}{r_\perp^3}[(1 - w)\partial_\perp w]. \tag{7}$$

Here $\mathbf{J}$ gives the angular velocity of $\mathbf{n}$, i.e., $\mathbf{J} = \mathbf{n}\wedge\dot{\mathbf{n}}$. In this configuration the Fourier transform of the source is

$$g_k(t) = \frac{\mathrm{i}}{(2\pi)^{3/2}} C(2\Phi)^2\delta(\mathbf{n}\cdot\mathbf{k})\mathbf{J}\cdot\mathbf{k}S(k_\perp/\mu), \tag{8.1}$$

$$S(k_\perp/\mu) = (\mu/k_\perp)\int J_1(\rho k_\perp/\mu)[(1 - w(\rho))w'(\rho)]d\rho/\rho. \tag{8.2}$$

Here J_1 is the Bessel function of order 1 and w' indicates the derivative with respect to the argument. It is useful to remember that, owing to the presence of the factor $\delta(\mathbf{n}\cdot\mathbf{k})$ in the expression of S, we can substitute $k_\perp^2$ simply with k^2.

We now remember that the model of magnetic field we have at hand is such that it is uniform along one direction. However, this direction is continuously varying, so a most significant quantity is obtained by an angular integration

$$\frac{\partial\mathcal{A}}{\partial k} = k^2\int d\Omega_k\mathcal{N}_f(\mathbf{k}). \tag{9}$$

So we need a quantity like $\int d\Omega_k g_k(\tau)g_k^*(\tau')$ that contains a singularity due to the presence, in the domain of integration, of a δ-square term which arises for $\tau = \tau'$. This is clearly due to the unphysical assumption that, at every time, there is a direction in which the pseudoscalar density G is absolutely uniform. Since we are integrating over the direction at the end, the effect of the singularity is mild enough; it results in a logarithmic divergence. However, this fact must be explicitly dealt with, considering a finite extension of the fields. A more careful treatment is mathematically heavier, so it is presented in an Appendix, where also the role of Ψ, the total rotation angle of the magnetic field, is discussed in detail. The final result is given by

$$\frac{\partial\mathcal{A}}{\partial k} \approx C^2(2\Phi)^4\frac{k^2}{2\omega_k}\Psi[S(k/\mu)]^2\sqrt{2}\ln 4kL. \tag{10}$$

If we give a definite transverse shape to the fields, we get, evidently, a definite answer. In the situation where the fields change direction but not intensities, i.e., we keep μ constant, we may give, tentatively, the form $w(\rho) = \mathrm{e}^{-\rho^2}$. Then the expression for the function $S(k/\mu)$ is [12]:

$$S(k/\mu) = \frac{2\mu^2}{k^2}\left[\exp\left(-\frac{k^2}{4\mu^2}\right) - \exp\left(-\frac{k^2}{8\mu^2}\right)\right]. \tag{11}$$

The limit $k \to 0$ of this expression is finite, so the whole production goes to zero only owing to the phase-space factor k^2 in front of the expression in Eq. (10). This result, as it appears from the whole derivation, can be valid only for axion masses, and so for energies that are definitely larger than the typical frequencies of the astrophysical phenomena, no resonant dynamics is included.

IV. SOME CONCLUSIONS

The rate of production of axions by a slowly varying but very strong magnetic field has been calculated. The conditions are such that the astrophysical frequencies are very much lower than the proper frequencies of the axion field. In fact, even with an axion mass m_A of the order of 1 eV [12], which is considered allowed on cosmological grounds, the typical frequencies of the field shall exceed 10^{15} Hz, and so no resonance conditions appear realistic. If we take a limit $m_A \leq 25$ keV, as suggested by stellar dynamics [13], we are even further. The relevant parameter is the dimension of the spatial inhomogeneity, which in the present model is $1/\mu$. It is very reasonable to assume that $\mu \ll m_A$. In these situations the energies of the produced particles cannot exceed their masses by very much, so it is easy to give an expression for the total number of produced particles.

$$\mathcal{A} = \zeta \frac{C^2 \Phi^4 \mu^3}{m_A} \Psi \ln L\mu; \tag{12}$$

in particular, the total energy taken away turns out to be independent of the axion masses.

Some of the factors owe their origin to the general form of the interaction, Eq. (1) , and to dimensional requirements. It appears clear the role of the total rotation of the field (Ψ) in determining the overall production; so when the rotation is uniform, the rate is proportional to the angular velocity.

The numerical factor is more model dependent, in the chosen case it is $\zeta = 8\sqrt{\pi}[2\sqrt{3} - 2 - \sqrt{2}] = 0.704\ldots$. It must be said also that some of the parameters entering in Eq. (12) are little known, in particular, the inhomogeneity μ, which has a fundamental role in the quantitative result. For this reason it seems that the relevance of the particular form of production presented here can be estimate only when more detailed form of the possible sources is given.

The transformation from the incoming Heisenberg field to the outgoing field can be implemented by the simple unitary operator

$$\mathcal{U} = \exp\left[\int d^3k[a(\mathbf{k})b^*(\mathbf{k}) - a^\dagger(\mathbf{k})b(\mathbf{k})]\right]$$

as:

$$\mathcal{U} a(\mathbf{k}) \mathcal{U}^\dagger = a(\mathbf{k}) + b(\mathbf{k}).$$

The actual form of the evolution operator gives the further information that the axions are produced with a Poissonian distribution of multiplicity. Strictly speaking, this is true for a production in a totally defined state; in operations like the one leading to Eq. (9), this particular form can be blurred.

APPENDIX

We want to calculate $\int d\Omega_k g_k(\tau) g_k^*(\tau')$, with the functions g given by Eq. (8.1) and taking care of the finite extension of the magnetic field. This is implemented by substituting the δ-functions as:

$$\delta(\mathbf{n}\cdot\mathbf{k}) \rightarrow L\pi^{-1/2}\exp[-L^2(\mathbf{n}\cdot\mathbf{k})^2]. \tag{A1}$$

The calculation is performed in the particular case in which the rotation takes place in a constant plane, so $\mathbf{J} \parallel \mathbf{J}'$. The integration over the angles is performed in Cartesian coordinates. It is useful to introduce the unit vector of the three-momentum direction $\mathbf{k} = k\mathbf{v}$. Then $\int d\Omega_k = 2\delta(v^2 - 1)d^3v$ and, through standard although lengthy calculations, the representation is obtained:

$$\begin{aligned} \mathcal{I} &= \int d\Omega_k \delta(\mathbf{n}\cdot\mathbf{k})\mathbf{J}\cdot\mathbf{k}\,\delta(\mathbf{n}'\cdot\mathbf{k})\mathbf{J}'\cdot\mathbf{k} \\ &\rightarrow \frac{1}{\pi} JJ' \times \int d\lambda \exp[i\lambda(w^2-1)]w^2 dw[(\mathbf{n}\wedge\mathbf{n}')^2 \\ &\quad - 2i\lambda/(Lk)^2 - \lambda^2/(Lk)^4]^{-1/2}. \end{aligned} \tag{A2}$$

In the limit $L \rightarrow \infty$ it results that

$$\mathcal{I} = JJ'/|\mathbf{n}\wedge\mathbf{n}'|,$$

which can be obtained in a simpler way.

Now we must integrate over τ and τ', times an oscillating factor $e^{i\omega_k(\tau-\tau')}$. In the conditions that have been chosen, the motion takes place in a plane, so $\mathbf{n}$ is characterized by a unique angle ψ and $\mathbf{n}'$ by ψ'. Hence the integration over time amounts to an angular integration, in fact, $(\mathbf{n}\wedge\mathbf{n}')^2 = (\sin(\psi-\psi'))^2$ and, moreover, $J = \dot{\psi}$ and $J' = \dot{\psi}'$. So we must integrate $\mathcal{I}$ in $d\psi$, $d\psi'$ from zero to some final angle Ψ_f. Defining $\gamma = \psi - \psi'$, we see that the integrand shows, in the limit $L \rightarrow \infty$, a singularity for $\gamma = 0$. So we perform the integration from $-\frac{1}{2}\pi$ to $\frac{1}{2}\pi$ because the domain which does not include zero has no singular behavior. The oscillating factor is approximated with its value on the singular point $\tau = \tau'$, so that the exponential factor reduces to 1. Then the integral is a complete elliptic integral [14], which can be conveniently expressed in terms of the hypergeometric function. In fact, the integration in $d\gamma$ is:

$$X_\gamma = \int \frac{d\gamma}{\sqrt{\sin^2\gamma + Q}} \quad \text{with} \quad Q = -\frac{2i\lambda}{(Lk)^2} - \frac{\lambda^2}{(Lk)^4}. \tag{A3}$$

The result of the integration is

$$X_\gamma = \frac{1}{2}\pi\sqrt{\frac{1}{Q+1}}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; \frac{1}{Q+1}\right)$$

and in the limit $L \rightarrow \infty$, which gives $Q \rightarrow 0$, we get

$$X_\gamma \rightarrow \sqrt{2}[2\ln 2 + \ln Lk].$$

Since the dominant term in the limit is independent of the auxiliary parameter λ, the rest of the integration in Eq. (A2) is straightforward and it gives

$$I = \Psi\sqrt{2}\ln 4kL. \tag{A4}$$

A comment.—What is excluded is the possibility that the magnetic field should perform more than a complete rotation. This would destroy the correspondence $\psi = \psi' \leftrightarrow \tau = \tau'$.

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