

Baryonic mass formula in large N_c QCD versus quark modelClaude Semay,^{1,*} Fabien Buisseret,^{1,†} Nicolas Matagne,^{2,‡} and Florica Stancu^{2,§}¹*Groupe de Physique Nucléaire Théorique, Université de Mons-Hainaut, Académie universitaire Wallonie-Bruxelles, Place du Parc 20, BE-7000 Mons, Belgium*²*University of Liège, Institute of Physics B5, Sart Tilman, B-4000 Liège 1, Belgium*

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We establish a connection between the quark model results and the $1/N_c$ expansion mass formula used in the description of baryon resonances. We show that a remarkable compatibility exists between the two methods.

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I. INTRODUCTION

So far the standard approach to baryon spectroscopy is the constituent quark model where the Hamiltonian contains a spin independent part formed of the kinetic plus the confinement energies and a spin dependent part given by a hyperfine interaction. The latter can be either due to one gluon exchange or to Goldstone boson exchange between quarks, or it can be an instanton induced interaction. The results are naturally model dependent.

It is therefore very important to develop model independent methods that can help in alternatively understanding baryon spectroscopy and that can support quark model assumptions. Large N_c QCD offers such a method. In 1974 't Hooft proposed to generalize QCD from $SU(3)$ to $SU(N_c)$ [1] where N_c is an arbitrary number of colors and suggested a perturbative expansion in the parameter $1/N_c$, applicable to all QCD regimes. Witten has generalized the approach to baryons [2] and this has led to a powerful $1/N_c$ expansion method to study static properties of baryons, as for example, the masses, the magnetic moments, the axial currents, etc. The method is systematic and predictive. It is based on the discovery that, in the limit $N_c \rightarrow \infty$, QCD possesses an exact contracted $SU(2N_f)$ symmetry [3,4] where N_f is the number of flavors. This symmetry is only approximate for finite N_c so that corrections have to be added in powers of $1/N_c$. The $1/N_c$ expansion method has extensively and successfully been applied to ground state baryons [5–8] (for recent developments see Ref. [9]). Its applicability to excited states is a subject of current investigations. In this case the symmetry under consideration is assumed to be $SU(2N_f) \times O(3)$ where $SU(2N_f)$ is related QCD, as introduced above. However $O(3)$ is not related to QCD but it brings an additional degree of freedom. It is of common practice to introduce it in order to construct orbitally excited states. The direct product $SU(2N_f) \times O(3)$ is also used in quark models to classify

three quark states, but there $SU(2N_f)$ is not an intrinsic symmetry. Thus the two approaches have formally the same symmetry in common which does not imply common dynamical assumptions. The only common feature is that the excited states are stable in a first approximation.

The purpose of the present study is to see whether or not there is a compatibility between the two methods. If such a compatibility exists, an important support to the constituent quark model can be provided by the model independent $1/N_c$ expansion method, and a better understanding of the physical content of large N_c mass formulas can be gained.

In the language of quark models, the baryon states can roughly be classified into excitation bands with $N = 0$ for the ground state band and $N = 1, 2, 3, \dots$ for excited states, where N represents units of excitation, like in a harmonic oscillator picture. The key tool of this comparative study is that one can analyze both the $1/N_c$ expansion results and the quark model basic ingredients in terms of N which makes the comparison between the two methods possible and very convenient.

The paper is organized as follows. The next section introduces the mass formula used in the $1/N_c$ expansion method. Section III gives a mass formula obtained from a Hamiltonian quark model where the kinetic energy is relativistic, the confinement is an Y -junction flux tubes and the hyperfine interaction is of an one-gluon exchange nature. Section IV is devoted to the comparison between terms of the mass formula which are common in the two approaches. The last section is devoted to conclusions.

II. BARYONS IN LARGE N_c QCD

For simplicity, we illustrate the method with the $N_f = 2$ case but the arguments are similar to any N_f . So, here we deal with $SU(4)$ which has 15 generators, the spin subgroup generators S_i ($i = 1, 2, 3$), the isospin subgroup generators T_a ($a = 1, 2, 3$) and G_{ia} which act both on spin and isospin degrees of freedom. The $SU(4)$ generators are components of an irreducible tensor operator which transforms according to the adjoint representation [211] of dimension **15** of $SU(4)$. The $SU(4)$ algebra is

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$$\begin{aligned}
[S_i, T_a] &= 0, & [S_i, G_{ja}] &= i\varepsilon_{ijk}G_{ka}, & [T_a, G_{ib}] &= i\varepsilon_{abc}G_{ic}, \\
[S_i, S_j] &= i\varepsilon_{ijk}S_k, & [T_a, T_b] &= i\varepsilon_{abc}T_c, \\
[G_{ia}, G_{jb}] &= \frac{i}{4}\delta_{ij}\varepsilon_{abc}T_c + \frac{i}{4}\delta_{ab}\varepsilon_{ijk}S_k.
\end{aligned} \tag{1}$$

Together with the generators ℓ_i of $\text{SO}(3)$, the $\text{SU}(4)$ generators form the building blocks of the mass operator. Then in the $1/N_c$ expansion the mass operator M has the general form

$$M = \sum_i c_i O_i, \tag{2}$$

where the coefficients c_i are reduced matrix elements that encode the QCD dynamics and are determined from a fit to the existing data, and the operators O_i are $\text{O}(3)$ scalars of the form

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \tag{3}$$

where $O_\ell^{(k)}$ is a k -rank tensor in $\text{O}(3)$ and $O_{SF}^{(k)}$ a k -rank tensor in $\text{SU}(2)$ -spin (homomorphic to $\text{SO}(3)$), but invariant in $\text{SU}(2)$ -flavor. Generally the operators $O_{SF}^{(k)}$ are combinations of the $\text{SU}(2N_f)$ generators and here, in particular, of $\text{SU}(4)$ generators. The lower index i in the left hand side represents a specific combination. Each n -body operator is multiplied by an explicit factor of $1/N_c^{n-1}$ resulting from the power counting rules [2]. For the ground state, one has $k = 0$. For excited states the $k = 2$ tensor is also important. The sum in the mass operator is finite. Operator reduction rules simplify the expansion. In addition, in practical applications, it is customary to include terms up to $1/N_c$ and drop higher order corrections of order $1/N_c^2$. As an example, in Eqs. (4), we exhibit the list of operators used in the calculation of the masses of the $[\mathbf{70}, 1^-]$ multiplet up to order $1/N_c$ included [10]. Note that although O_5 and O_6 carry a factor of $1/N_c^2$ their matrix elements are of order $1/N_c$ because they contain the coherent operator G^{ia} which brings an extra factor of N_c .

$$\begin{aligned}
O_1 &= N_c \mathbb{1}, & O_2 &= \frac{1}{N_c} \ell^i S^i, & O_3 &= \frac{1}{N_c} T^a T^a, \\
O_4 &= \frac{1}{N_c} S^i S^i, & O_5 &= \frac{15}{N_c^2} \ell^{(2)ij} G^{ia} G^{ja}, \\
O_6 &= \frac{3}{N_c^2} \ell^i T^a G^{ia}.
\end{aligned} \tag{4}$$

Here $O_1 = N_c \mathbb{1}$ is the trivial operator, proportional to N_c and the only one which survives when $N_c \rightarrow \infty$ [2], where the $\text{SU}(4)$ symmetry is exact. It is the only spin-isospin independent term in the mass formula. The $\text{SU}(4)$ quadratic operators $S^i S^i$, $T^a T^a$ and $G^{ia} G^{ia}$ should all enter the mass formula (the sum over repeated indices is implicit). But they are related to each other by the operator identity [7]

$$\{S^i, S^i\} + \{T^a, T^a\} + 4\{G^{ia}, G^{ia}\} = \frac{1}{2}N_c(3N_c + 4), \tag{5}$$

so one can express $G^{ia} G^{ia}$ in terms of $S^i S^i$ and $T^a T^a$. Note that the right hand side of Eq. (5) is the eigenvalue of the Casimir operator for the irreducible representation $[N_c - 1, 1]$ of $\text{SU}(4)$. The operators O_2 , O_5 and O_6 are relevant for orbitally excited states. Among them, the role of O_2 will be discussed below.

A. The ground state band

The mass formula for the ground state up to order $1/N_c$ is simple because one can replace $T^a T^a$ by $S^i S^i$, due to an identity which holds for symmetric $[N_c]$ states [7]. As there is no orbital excitation, the mass formula (2) takes the following simple form

$$M = c_1 N_c + c_4 \frac{1}{N_c} S^2 + \mathcal{O}\left(\frac{1}{N_c^3}\right), \tag{6}$$

which means that for $N = 0$ only the operators O_1 and O_4 contribute to the mass. Thus the fit gives quantitative information only for c_1 and c_4 . For $N_c = 3$, $M_N = 940$ MeV for $S = 1/2$, and $M_\Delta = 1232$ MeV for $S = 3/2$, one gets

$$c_1 = 289 \text{ MeV}, \quad c_4 = 292 \text{ MeV}. \tag{7}$$

B. Excited states

Among the excited states, those belonging to the $N = 1$ band, or equivalently to the $[\mathbf{70}, 1^-]$ multiplet, have been most extensively studied, either for $N_f = 2$ [11–20] or for $N_f = 3$ [21]. In the latter case, first order corrections in $\text{SU}(3)$ symmetry breaking were also included.

The $N = 2$ band contains the $[\mathbf{56}', 0^+]$, $[\mathbf{56}, 2^+]$, $[\mathbf{70}, \ell^+]$ ($\ell = 0, 2$) and $[\mathbf{20}, 1^+]$ multiplets. There are no physical resonances associated to $[\mathbf{20}, 1^+]$. The few studies related to the $N = 2$ band concern the $[\mathbf{56}', 0^+]$ for $N_f = 2$ [22], $[\mathbf{56}, 2^+]$ for $N_f = 3$ [23], and $[\mathbf{70}, \ell^+]$ for $N_f = 2$ [24], later extended to $N_f = 3$ [25]. The method has also been applied [26] to highly excited nonstrange and strange baryons belonging to $[\mathbf{56}, 4^+]$, the lowest multiplet of the $N = 4$ band [27].

The group theoretical similarity of excited symmetric states to the ground state makes the analysis of these states simple [23,26]. For mixed symmetric states, the situation is more complex. There is a standard procedure which reduces the study of mixed symmetric states to that of symmetric states. This is achieved by the decoupling of the baryon into an excited quark and a symmetric core of $N_c - 1$ quarks. This procedure has been applied to the $[\mathbf{70}, 1^-]$ multiplet [11–21] and to the $[\mathbf{70}, \ell^+]$ ($\ell = 0, 2$) multiplet [24,25]. In fact the decoupling is not necessary, provided one knows the matrix elements of the $\text{SU}(2N_f)$ generators between mixed symmetric states. The case of $\text{SU}(4)$ has been presented in Ref. [10].

In Sec. IV, we collect the values of c_1 , c_2 and c_4 obtained in the above studies in order to make a comparison between those values and their analogs resulting from the quark model described below.

III. QUARK MODEL FOR BARYONS

A. Confining interaction

In the framework of potential models, it is generally assumed that a baryon, viewed as a bound state of three quarks, can be described in a first approximation by the following spinless Salpeter Hamiltonian

$$H = \sum_{i=1}^3 \sqrt{\vec{p}_i^2 + m_i^2} + V_Y, \quad (8)$$

where m_i is the current mass of the quark i , and V_Y the confining interaction potential.

The nonperturbative part of the gluon exchanges, responsible for the confinement, can be successfully described in the flux tube model [28]. In this framework, each quark is assumed to generate a string, or a flux tube, characterized by its energy density (string tension). Recent developments in lattice QCD tend to confirm the Y -junction as the correct configuration for the flux tubes in baryons [29]. In this picture, a flux tube starts from each quark and the tubes meet at the Toricelli point of the triangle formed by the three quarks. This point, denoted by \vec{x}_T , is such that it minimizes the sum of the flux tube lengths, and its position is a complicated function of the quark coordinates \vec{x}_i . Moreover, the energy density of the tubes appears to be equal for mesons and baryons. The Y -junction potential reads

$$V_Y = a \sum_{i=1}^3 |\vec{x}_i - \vec{x}_T|. \quad (9)$$

In Ref. [30], it has been shown that this complicated potential is successfully approximated by the more easily computable expression

$$V = a \left[\alpha \sum_{i=1}^3 |\vec{x}_i - \vec{R}| + (1 - \alpha) \frac{1}{2} \sum_{i < j} |\vec{x}_i - \vec{x}_j| \right], \quad (10)$$

where \vec{R} is the position of the center of mass. If $\alpha = 1$, Eq. (10) is a simplified Y -junction, where the Toricelli point is replaced by the center of mass. If $\alpha = 0$, this interaction reduces to a Δ -type potential. Results of Ref. [30], obtained in the framework of a potential model, show that $\alpha = 1$ gives a better description than $\alpha = 0$, and that the Y -junction is approximated at best by α close to $1/2$.

B. Mass formula

Let us now introduce auxiliary fields, in order to get rid of the square roots appearing in the Hamiltonian (8). We

get

$$H(\mu_i, \nu_j, \lambda_{ij}) = \sum_{j=1}^3 \left[\frac{\vec{p}_j^2 + m_j^2}{2\mu_j} + \frac{\mu_j}{2} \right] + \alpha \sum_{j=1}^3 \left[\frac{a^2(\vec{x}_j - \vec{R})^2}{2\nu_j} + \frac{\nu_j}{2} \right] + \frac{(1 - \alpha)}{2} \sum_{j < k} \left[\frac{a^2(\vec{x}_j - \vec{x}_k)^2}{2\lambda_{jk}} + \frac{\lambda_{jk}}{2} \right]. \quad (11)$$

The auxiliary fields, denoted as μ_i , ν_j , and λ_{ij} are, strictly speaking, operators. Although being formally simpler, $H(\mu_i, \nu_j, \lambda_{ij})$ is equivalent to H up to the elimination of the auxiliary fields thanks to the constraints

$$\delta_{\mu_i} H(\mu_i, \nu_j, \lambda_{ij}) = 0 \Rightarrow \mu_{i,0} = \sqrt{\vec{p}_i^2 + m_i^2}, \quad (12a)$$

$$\delta_{\nu_j} H(\mu_i, \nu_j, \lambda_{ij}) = 0 \Rightarrow \nu_{j,0} = a|\vec{x}_j - \vec{R}|, \quad (12b)$$

$$\delta_{\lambda_{ij}} H(\mu_i, \nu_j, \lambda_{ij}) = 0 \Rightarrow \lambda_{ij,0} = a|\vec{x}_i - \vec{x}_j|. \quad (12c)$$

It is worth mentioning that $\langle \mu_{i,0} \rangle$ can be seen as a dynamical mass of a quark of current mass m_i , while $\langle \nu_{j,0} \rangle$ is, in this case, the static energy of the straight string linking the quark i to the Toricelli point [31]. Similarly, $\langle \lambda_{ij,0} \rangle$ can be interpreted as the static energy of a straight string joining the quarks i and j . Although the auxiliary fields are operators, the calculations are considerably simplified if one considers them as real numbers. They are then finally eliminated by a minimization of the masses with respect to them [32]. The extremal values of μ_i , ν_j , and λ_{ij} , considered as numbers, are logically close to the values of $\langle \mu_{i,0} \rangle$, $\langle \nu_{j,0} \rangle$, and $\langle \lambda_{ij,0} \rangle$ given by relations (12). This procedure leads to a spectrum which is an upper bound of the ‘‘true spectrum’’ (computed without auxiliary fields) [33]: it can be shown that, the more auxiliary fields are introduced, the higher are the masses compared to those without auxiliary fields [34]. Let us finally mention that, for $\alpha = 1$, the Hamiltonian (11) can be related to the rotating string model for a baryon (see, for example, Ref. [35]).

In Ref. [36], it has been shown that the eigenvalues of a Hamiltonian of the form (11) can be analytically found by making an appropriate change of variables, the quark coordinates $\vec{x}_i = \{\vec{x}_1, \vec{x}_2, \vec{x}_3\}$ being replaced by new coordinates $\vec{x}'_k = \{\vec{R}, \vec{\xi}, \vec{\eta}\}$. The center of mass is defined as

$$\vec{R} = \frac{\mu_1 \vec{x}_1 + \mu_2 \vec{x}_2 + \mu_3 \vec{x}_3}{\mu_t}, \quad (13)$$

with $\mu_t = \mu_1 + \mu_2 + \mu_3$ and $\{\vec{\xi}, \vec{\eta}\}$ being the two relative coordinates. From Ref. [36], it can be immediately found that the mass spectrum of bound states of three massless particles ($m_i = 0$ for the u and d quarks) is given by

$$M(\mu, \nu, \lambda) = \omega(2n + \ell + 3) + \frac{3}{2} \left(\mu + \alpha\nu + \frac{(1 - \alpha)}{2} \lambda \right), \quad (14)$$

with

$$\omega = a \sqrt{\frac{1}{\mu} \left[\frac{\alpha}{\nu} + \frac{3(1 - \alpha)}{2\lambda} \right]}, \quad (15)$$

$n = n_\xi + n_\eta$ and $\ell = \ell_\xi + \ell_\eta$. An obvious symmetry argument helps us to make the identification $\mu_i = \mu$, $\nu_i = \nu$, and $\lambda_{ij} = \lambda$. In this symmetric case, properties of the equilateral triangle together with the relations (12) allow to make the following ansatz

$$\lambda = \sqrt{3}\nu. \quad (16)$$

Defining

$$Q = \alpha + \frac{(1 - \alpha)}{2} \sqrt{3}, \quad \tilde{\nu} = Q\nu, \quad (17)$$

and

$$N = 2n + \ell, \quad (18)$$

we find

$$M(\mu, \tilde{\nu}) = aQ \sqrt{\frac{1}{\mu\tilde{\nu}}} (N + 3) + \frac{3}{2} (\mu + \tilde{\nu}). \quad (19)$$

Formula (19) is clearly symmetric in μ and $\tilde{\nu}$. That means that we can set $\mu = \tilde{\nu}$. This equality can be viewed as a sort of virial theorem. Then we have

$$M(\mu) = \frac{aQ}{\mu} (N + 3) + 3\mu. \quad (20)$$

One can easily find that the relation $\delta_\mu M(\mu) = 0$ implies

$$\mu_0 = \left[\frac{a}{3} Q(N + 3) \right]^{1/2}, \quad (21)$$

and $M(\mu_0) = 6\mu_0$, as observed in Ref. [37]. Writing explicitly the square mass, we see that the model of Ref. [37] also predicts Regge trajectories, which are in agreement with the experimental data for light baryons

$$M^2(\mu_0) = 12aQ(N + 3). \quad (22)$$

The Regge slope is here given by $12aQ$. However, from experiment we know that the Regge slope for light baryons and light mesons are approximately equal. For light mesons, the exact value in the relativistic flux tube model is $2\pi a$, a lower value than the one obtained from formula (22). This is due to the auxiliary fields method: the more auxiliary fields we introduce, the more the masses are overestimated [34]. What can be done to cure this problem is to rescale a : let us define σ such that $12aQ = 2\pi\sigma$. Then, formula (22) is able to reproduce the light baryon Regge slope for a physical value $\sigma \lesssim 0.2 \text{ GeV}^2$. Note that

the best value for α is $1/2$. Consequently, the best value for Q is $1/2 + \sqrt{3}/4 \approx 0.93$. It is worth mentioning that such a rescaling of the string tension has already given good results in the study of hybrid mesons [38].

C. One gluon exchange and quark self-energy

Although including only the confining energy is sufficient to understand the Regge trajectories of light baryons, it is well-known that the absolute value of the masses which are obtained are too high with respect to the experimental data. Other contributions are needed to decrease these masses and we shall estimate their effect perturbatively. The most widely used is a Coulomb interaction term of the form

$$\Delta M_{\text{oge}} = -\frac{2}{3} \alpha_s \sum_{i < j} \left\langle \frac{1}{|\vec{x}_i - \vec{x}_j|} \right\rangle, \quad (23)$$

arising from one gluon exchange processes, where α_s is the strong coupling constant, usually assumed to be around 0.4 for light hadrons [39]. Lattice QCD calculations also support this value [40]. Assuming that $\langle 1/A \rangle \approx 1/\langle A \rangle$, and using symmetry arguments, relations (12) lead to

$$\sum_{i < j} \left\langle \frac{1}{|\vec{x}_i - \vec{x}_j|} \right\rangle \approx \frac{3a}{\lambda_0} = \frac{\sqrt{3}aQ}{\mu_0}, \quad (24)$$

$$\Delta M_{\text{oge}} = -2\alpha_s \frac{aQ}{\sqrt{3}\mu_0}. \quad (25)$$

Another interesting contribution to the mass, which can be added perturbatively, is the quark self-energy. Recently, it was shown that the quark self-energy, which is created by the color magnetic moment of a quark propagating through the vacuum background field, adds a negative constant to the hadron masses [41]. Its negative sign is due to the paramagnetic nature of the particular mechanism at work in this case. The quark self-energy contribution for three massless quarks is given by [41]

$$\Delta M_{\text{qse}} = -\frac{3fa}{2\pi\mu_0}. \quad (26)$$

The factor f has been computed in lattice QCD studies. First quenched calculations gave $f = 4$ [42]. A more recent unquenched work [43] gives $f = 3$. Since its value is still a matter of research, we will only assume that $f \in [3, 4]$.

With the unperturbed baryon mass $M(\mu_0)$, given by Eq. (22), the total mass is given by the sum $M_0 = M(\mu_0) + \Delta M_{\text{oge}} + \Delta M_{\text{qse}}$. Then, in the first order of perturbation and for $\alpha = 1/2$, it is straightforward to obtain the following mass formula for baryons

$$M_0^2 = 2\pi\sigma(N + 3) - \frac{4}{\sqrt{3}} \pi\sigma\alpha_s - \frac{12}{(2 + \sqrt{3})} f\sigma, \quad (27)$$

where the scaling $12aQ = 2\pi\sigma$ has been used. The effects of the one gluon exchange term and of the quark self-energy are thus to shift the square mass spectrum by a global negative amount. Let us note that the symbol N defined by Eq. (18) and the quantity N used to classify baryon states and used to plot results from the $1/N_c$ expansion are the same. This common N will be used in the next section to perform a comparison between the results obtained in both approaches.

The mass formula (27) does not take into account spin relativistic contributions, as the spin-spin or spin-orbit forces. Within the auxiliary field formalism, all these corrections to the static potential are expanded in powers of $1/\mu^2$ where μ is the constituent quark mass [44]. All the spin corrections to the mass formula (27) must depend both on the matrix elements of the interaction and on the coefficient $1/\mu^2$. In the following, we shall consider that the dominant dynamical effect is due to the constituent mass, while the matrix elements remain roughly constant with N , as presented in the next section.

IV. COMPARISON OF THE TWO APPROACHES

In the $1/N_c$ expansion method, the first term $c_1 N_c$ in the mass formula of Eq. (2) contains the main spin-independent contribution to the baryon mass, which in a quark model language, represents the confinement and the kinetic energy. So, it is natural to identify this term with the mass given by the formula (27). Then, for $N_c = 3$, we assume the relation

$$c_1^2 = \frac{M_0^2}{9}, \quad (28)$$

which gives

$$c_1^2 = \frac{2\pi}{9} \sigma N + c_0 \quad (29)$$

$$= \frac{2\pi}{9} \sigma(N+3) - \frac{4}{9\sqrt{3}} \pi \sigma \alpha_s - \frac{4}{3(2+\sqrt{3})} f \sigma. \quad (30)$$

Figure 1 shows a comparison between the values of c_1^2 obtained in the $1/N_c$ expansion method and those derived from the Eq. (29) for various values of N . From this comparison one can see that the results of large N_c QCD are entirely compatible with the formula (29). From a fit, one has $\sigma = 0.163 \pm 0.004$ GeV², a rather low but still acceptable value according to usual potential models, and $c_0 = 0.085 \pm 0.007$ GeV². To reproduce c_0 , we can set $\alpha_s = 0.4$, $f = 3.5$: these are very standard values.

In most of the quark models however, the string tension is generally assumed to lie in the range $[0.17, 0.20]$ GeV². If the value of σ is chosen in this interval, the corresponding values for c_1^2 , given by Eq. (29), are located in the shaded area of Fig. 1. Although the agreement with large N_c data is not so good than in the optimal case, where $\sigma = 0.163$ GeV², it remains satisfactory if we choose $f =$

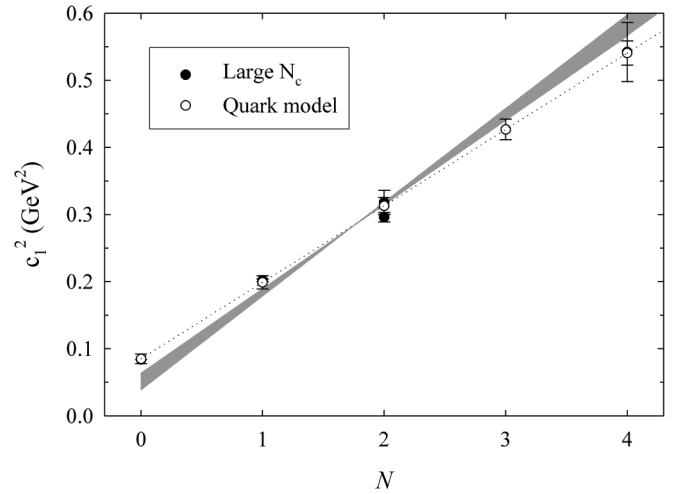


FIG. 1. Values of c_1^2 computed in the $1/N_c$ expansion (full circles) from a fit to experimental data (Eq. (7) for $N = 0$, Refs. [21,23] for $N = 1$, Ref. [24] for $N = 2$ and Ref. [26] for $N = 4$), compared with results from a fit (see text) of the formula (29) (empty circles and dotted line to guide the eyes). No data is available for $N = 3$ in large N_c studies. Values of c_1^2 as predicted by formula (29) for $\sigma \in [0.17, 0.20]$ GeV² are located in the shaded area.

3.98(4.42) for $\sigma = 0.17(0.20)$ GeV², together with $\alpha_s = 0.4$. These values are larger than what is expected. It could be argued that other mechanisms than the quark self-energy are present, their contribution decreasing the total mass M_0 . In mesons, for example, retardation effects due to the finite interaction speed were shown to be also proportional to μ^{-2} , like the quark self-energy [45]. It is possible that, when retardation effects are included, f can again be chosen in the interval $[3, 4]$ with a standard value of σ . But,

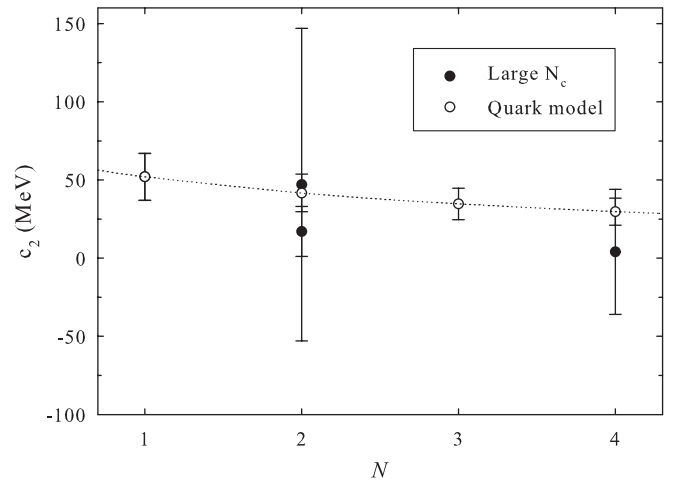


FIG. 2. Values of c_2 computed in the $1/N_c$ expansion (full circles) from a fit to experimental data (Refs. [21,23] for $N = 1$, Ref. [24] for $N = 2$ and Ref. [26] for $N = 4$), compared with results from formula (31) (empty circles and dotted line to guide the eyes). No data is available for $N = 3$ in large N_c studies.

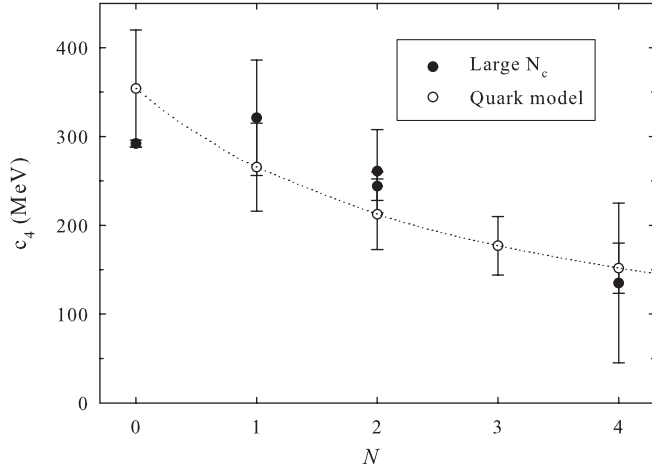


FIG. 3. Values of c_4 computed in the $1/N_c$ expansion (full circles) from a fit to experimental data (Eq. (7) for $N = 0$, Refs. [21,23] for $N = 1$, Ref. [24] for $N = 2$ and Ref. [26] for $N = 4$), compared with results from formula (31) (empty circles and dotted line to guide the eyes). No data is available for $N = 3$ in large N_c studies.

no model for retardation effects in baryon has been proposed yet.

Within the auxiliary field formalism, we can expect that c_2 and $c_4 \propto \mu_0^{-2}$, and thus

$$c_2 = \frac{c_2^0}{N+3}, \quad c_4 = \frac{c_4^0}{N+3}. \quad (31)$$

We see that this behavior is coherent with the large N_c results in Figs. 2 and 3. We chose $c_2^0 = 208 \pm 60$ MeV so that the point with $N = 1$, for which the uncertainty is minimal, is exactly reproduced. Let us note that the spin-

orbit term is vanishing for $N = 0$, so no large N_c result is available in this case. To compute the parameter c_4^0 a fit is performed on all the large N_c data. We obtain then $c_4^0 = 1062 \pm 198$ MeV. Note that $c_4^0 \gg c_2^0$. This shows that the spin-spin contribution is much larger than the spin-orbit contribution, which justifies the neglect of the spin-orbit one in quark model studies.

V. CONCLUSIONS

This study supports the quark model basic assumptions by the compatibility of its mass formula with the mass formula derived from the model independent $1/N_c$ expansion. These assumptions are: relativistic kinetic energy for light quarks, Y -junction confining interaction, negligible spin-orbit interaction, hyperfine interaction dominated by a spin-spin term. A recent analysis shows that a flux tube model and a feeble spin-orbit interaction give a successful account of hadron spectroscopy [46].

In addition this study suggests that a good description of the bulk content of the baryon mass can be obtained with a spin independent energy eigenvalue of the form $M_0 \propto \sqrt{N+3}$ where $N = 0, 1, 2, \dots$ is the number of excitation units, as in the harmonic oscillator. It also shows that the spin-orbit and spin-spin interactions vanish with the excitation energy. Moreover this comparative study gives a better insight into the large N_c mass operator where the coefficients c_i encode the QCD dynamics.

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