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TeV scale mirage mediation and natural little supersymmetric hierarchy

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TeV scale mirage mediation has been proposed as a supersymmetry-breaking scheme reducing the fine-tuning for electroweak symmetry breaking in the minimal supersymmetric extension of the standard model. We discuss a moduli stabilization setup for TeV scale mirage mediation which allows an extradimensional interpretation for the origin of supersymmetry breaking and naturally gives a weak-scale size of the Higgs B parameter. The setup utilizes the holomorphic gauge kinetic functions depending on both the heavy dilaton and the light volume modulus whose axion partners are assumed to be periodic fields. We also examine the low-energy phenomenology of TeV scale mirage mediation, particularly the constraints from electroweak symmetry breaking and flavor changing neutral current processes.

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I. INTRODUCTION

Low-energy supersymmetry (SUSY) is one of the primary candidates for physics beyond the standard model (SM) above the weak scale [1]. One strong motivation for supersymmetric extension of the SM is to solve the hierarchy problem between the weak scale and grand unified theory (GUT)/Planck scale. In particular, the minimal supersymmetric standard model (MSSM) is quite interesting from the viewpoint of its minimality as well as the realization of gauge coupling unification at $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV.

In the supersymmetric standard model, the lightest Higgs boson h^0 is predicted to have a light mass. Including the one-loop correction while ignoring the effect of stop mixing [2], m_{h^0} in the MSSM is given by

$$m_{h^0}^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3y_t^2 m_{\tilde{t}}^2}{4\pi^2} \ln(m_{\tilde{t}}^2/m_t^2), \tag{1}$$

where M_Z is the Z-boson mass, $\tan\beta = \langle H_u^0\rangle/\langle H_d^0\rangle \gtrsim 3$, y_t is the top quark Yukawa coupling, and $m_{\tilde{t}}$ is the stop mass. Thus the current experimental bound for the SM-like Higgs $m_{h^0} > 114$ GeV can be satisfied within the MSSM, but it implies a rather heavy stop mass, e.g. $m_{\tilde{t}} \gtrsim 600$ GeV. In supersymmetric models, $m_{\tilde{t}}$ is tightly linked to the up-type Higgs soft mass m_{H_u} through the renormalization group (RG) evolution induced by the large value of y_t:

$$\delta m_{H_u}^2 \sim -\frac{3y_t^2 m_{\tilde{t}}^2}{4\pi^2} \ln(\Lambda/m_{\tilde{t}}), \tag{2}$$

where Λ is the (effective) messenger scale of SUSY breaking which is expected to be close to the GUT/Planck scale in generic high scale mediation models. Unless cancelled by other effects, this RG evolution implies that $|m_{H_u}^2| \sim m_{\tilde{t}}^2$ at the weak scale. On the other hand, the electroweak symmetry breaking (EWSB) conditions in the MSSM give rise to

$$\frac{M_Z^2}{2} \simeq -\mu^2(M_Z) - m_{H_u}^2(M_Z) + \frac{m_{H_d}^2(M_Z)}{\tan^2\beta}, \tag{3}$$

where μ is the Higgsino mass and m_{H_d} is the down-type Higgs soft mass. This EWSB condition requires a fine-tuning of parameters with an accuracy of $\mathcal{O}(1)\%$ if m_{H_u} is heavier than 600 GeV as suggested by the lower bound of m_{h^0} and the RG evolution of $m_{H_u}^2$. This is the so-called little SUSY hierarchy problem [3].

During the last years, several types of scenarios solving the little SUSY hierarchy problem have been proposed [4–18]. Many of them extend the MSSM to increase m_{h^0} while keeping the superparticle masses as light as possible. An alternative possibility is to have a particular pattern of SUSY-breaking soft terms within the MSSM [19,20], satisfying the EWSB condition (3) without fine-tuning. A particularly interesting proposal along this direction is the TeV scale mirage mediation of SUSY breaking [21,22] which gives a little hierarchy between m_{H_u} and $m_{\tilde{t}}$ in a natural manner[1]:

$$|m_{H_u}^2(M_Z)| \sim \frac{m_{\tilde{t}}^2(M_Z)}{8\pi^2}. \tag{4}$$

In mirage mediation [25], anomaly mediated SUSY breaking [26] and modulus-mediated SUSY breaking [27] are dynamically arranged to cancel the RG evolution of soft parameters [19]. Such a pattern of SUSY breaking is a natural outcome of KKLT-type moduli stabilization [28] in which the modulus F component is suppressed com-

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[1]The schemes proposed in [23,24] also give a qualitatively similar pattern of soft terms.

pared to the gravitino mass $m_{3/2}$ by the factor $1/\ln(M_{\rm Pl}/m_{3/2})$ [25]. The typical size of superparticle masses in this scheme is given by

$$m_{\rm SUSY} \sim \frac{m_{3/2}}{8\pi^2}, \tag{5}$$

while the detailed pattern is determined by the anomaly to modulus mediation ratio. Under certain assumption on the discrete parameters of underlying theory, the effective RG evolution of soft parameters in mirage mediation is determined by a ''mirage messenger scale''

$$\Lambda \sim M_{\rm mir} \equiv \frac{M_{\rm GUT}}{(M_{\rm Pl}/m_{3/2})^{\alpha/2}}, \tag{6}$$

where $\alpha = \mathcal{O}(1)$ parameterizes the anomaly to modulus mediation ratio [19]. Having $\alpha = 2$ leads to $M_{\rm mir} \sim 1$ TeV minimizing the effective RG evolution of $m_{H_u}^2$; it thereby allows the little hierarchy (4) realized without fine-tuning. The TeV scale mirage mediation solving the little hierarchy problem can give two different mass patterns at the weak scale suggested by the EWSB condition (3):

$$\begin{aligned}
&\text{(I)}\ \ \mu \sim m_{H_{u,d}} \sim M_Z, \qquad m_{\tilde{t}} \sim \sqrt{8\pi^2}M_Z,\\
&\qquad B \sim M_Z/\tan\beta,\\
&\text{(II)}\ \ \mu \sim m_{H_u} \sim M_Z, \qquad m_{H_d} \sim m_{\tilde{t}} \sim \sqrt{8\pi^2}M_Z,\\
&\qquad B \sim 8\pi^2 M_Z/\tan\beta,
\end{aligned} \tag{7}$$

where we have used another EWSB condition $\mu B \simeq (m_{H_d}^2 + m_{H_u}^2 + 2\mu^2)/\tan\beta$ for the estimate of the Higgs mass parameter B. In Ref. [21], it has been shown that both mass patterns can be obtained in a certain class of a (string-motivated) effective supergravity (SUGRA) model with SUSY-breaking uplifting potential. The same model giving the mass pattern (I) also has been discussed in [22], followed by a phenomenological study including the degree of fine-tuning, dark matter detection and collider signals [29].

Recently, it was pointed out that the uplifting potential which has been assumed in [21,22] to get $\alpha = 2$ is difficult to have an extradimensional interpretation [30]. This would cast a doubt on the naturalness of the whole setup. Indeed, if the uplifting potential originates from a SUSY-breaking brane stabilized at the IR end of a warped throat as in the KKLT moduli stabilization scenario, the minimal setup discussed in [19,25] gives $\alpha = 1$, and thus an intermediate scale value of $M_{\rm mir}$. In this paper, we propose an alternative scheme giving $M_{\rm mir} \sim 1$ TeV even when the uplifting potential originates from a brane-localized source located at the IR end of a warped throat. This scheme utilizes the holomorphic gauge kinetic function and nonperturbative superpotential depending on both the dilaton superfield S and the volume modulus superfield T whose axion components are periodic fields. Following KKLT [28], we assume that S is stabilized by flux with a mass hierarchically heavier than the gravitino mass $m_{3/2}$, while T is stabilized by a nonperturbative superpotential with $m_T \sim m_{3/2}\ln(M_{\rm Pl}/m_{3/2})$. In fact, such a scheme was studied recently in [31]; however, the possibility of $M_{\rm mir} \sim 1$ TeV has not been explored.

In mirage mediation, the Higgs mass parameter B can be another source of fine-tuning since the conventional SUGRA mechanism to generate μ typically gives $B \sim m_{3/2} \sim 8\pi^2 m_{\rm SUSY}$. As we will see, the dilaton-modulus mixing in gauge kinetic function and nonperturbative superpotential provides a nonperturbative mechanism to generate $B \sim m_{\rm SUSY}$ in mirage or anomaly mediation scenario with $m_{3/2} \sim 8\pi^2 m_{\rm SUSY}$. Also, this mechanism for $B \sim m_{\rm SUSY}$ automatically gives a real B/M_a, thus avoids the SUSY CP problem.

The mass patterns (I) and (II) differ by the values of m_{H_d} and B, leading to a significant difference in the Higgs spectrum and associated phenomenology. A potential difficulty of the pattern (I) is that it requires a rather small $B \sim M_Z/\tan\beta$, which might be difficult to be obtained even under a mechanism to guarantee $B \sim m_{\rm SUSY}$. On the other hand, the pattern (II) does not suffer from such difficulty and predicts $\tan\beta \sim \sqrt{8\pi^2}$ under a mechanism to give $B \sim m_{\rm SUSY} \sim \sqrt{8\pi^2}M_Z$. Although a rather extensive study of the mass pattern (I) has been performed in [29], no detailed study of the mass pattern (II) has been made yet. In the last part of this paper, we analyze the electroweak symmetry breaking and various constraints from flavor changing neutral current (FCNC) processes in both mass patterns of TeV scale mirage mediation.

This paper is organized as follows. In Sec. II, we discuss the mirage mediation resulting from a moduli stabilization setup with dilaton-modulus mixing and also a nonperturbative mechanism to generate $B \sim m_{\rm SUSY}$ in mirage mediation scenario. We will present an explicit example which leads to the TeV scale mirage mediation solving the little SUSY hierarchy problem while giving a desired size of $B \sim m_{\rm SUSY}$. In Sec. III, we discuss the electroweak symmetry breaking and the constraints from FCNC processes for the SUSY mass patterns (I) and (II). We give our conclusions in Sec. IV.

II. MIRAGE MEDIATION FROM A GENERALIZED MODULI STABILIZATION WITH DILATON-MODULUS MIXING

In mirage mediation [25], soft terms receive comparable contributions from anomaly mediation [26] and modulus mediation [27]. For the canonically normalized soft terms

$$\mathcal{L}_{\rm soft} = -\tfrac{1}{2}M_a\lambda^a\lambda^a - \tfrac{1}{2}m_i^2|\phi^i|^2 - \tfrac{1}{6}A_{ijk}y_{ijk}\phi^i\phi^j\phi^k + \text{H.c.}, \tag{8}$$

where λ^a are gauginos, ϕ^i are sfermions, y_{ijk} are the canonically normalized Yukawa couplings, the soft parameters at energy scale just below the GUT scale $M_{\rm GUT}$ are given by [25]

$$\begin{aligned} M_a &= M_0 + \frac{b_a}{16\pi^2} g_{\rm GUT}^2 m_{3/2}, \\ A_{ijk} &= \tilde{A}_{ijk} - \frac{1}{16\pi^2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}, \\ m_i^2 &= \tilde{m}_i^2 - \frac{1}{32\pi^2}\frac{d\gamma_i}{d\ln Q} m_{3/2}^2 \\ &\quad + \frac{1}{4\pi^2}\left[\sum_{jk}\frac{1}{4}|y_{ijk}|^2 \tilde{A}_{ijk} - \sum_a g_a^2 C_2^a(\phi^i) M_0\right] m_{3/2}, \end{aligned} \tag{9}$$

where M_0, $\tilde{A}_{ijk}$, and $\tilde{m}_i$ are the pure modulus-mediated gaugino mass, trilinear A parameters, and sfermion masses which are generically of the order of $m_{3/2}/8\pi^2$, and Q denotes the renormalization scale. Here b_a and γ_i are the one-loop beta function coefficients and the anomalous dimension given by

$$\begin{aligned} b_a &= -3\,{\rm tr}(T_a^2({\rm Adj})) + \sum_i {\rm tr}(T_a^2(\phi^i)), \\ \gamma_i &= 2\sum_a g_a^2 C_2^a(\phi^i) - \frac{1}{2}\sum_{jk}|y_{ijk}|^2, \end{aligned} \tag{10}$$

where the quadratic Casimir $C_2^a(\phi^i) = (N^2-1)/2N$ for a fundamental representation ϕ^i of the gauge group $SU(N)$, $C_2^a(\phi^i) = q_i^2$ for the $U(1)$ charge q_i of ϕ^i, and $\omega_{ij} = \sum_{kl} y_{ikl} y_{jkl}^*$ is assumed to be diagonal. Thus in our convention, b_a and γ_{H_u} of the MSSM are given by

$$b_a = (\tfrac{33}{5}, 1, -3), \qquad \gamma_{H_u} = \tfrac{3}{2}g_2^2 + \tfrac{1}{2}g_Y^2 - 3y_t^2, \tag{11}$$

where g_2 and $g_Y = \sqrt{3/5}g_1$ denote the $SU(2)_W$ and $U(1)_Y$ gauge couplings. For our later discussion, it is convenient to define

$$\alpha \equiv \frac{m_{3/2}}{M_0 \ln(M_{\rm Pl}/m_{3/2})}, \qquad a_{ijk} \equiv \frac{\tilde{A}_{ijk}}{M_0}, \qquad c_i \equiv \frac{\tilde{m}_i^2}{M_0^2}, \tag{12}$$

where α represents the anomaly to modulus mediation ratio, while a_{ijk} and c_i parameterize the pattern of the pure modulus-mediated soft masses. As was noted in [25], soft terms resulting from KKLT-type moduli stabilization [28] receive comparable contributions from both the anomaly mediation and the modulus mediation; therefore, α, a_{ijk}, and c_i generically have the values of order unity.

Taking into account the 1-loop RG evolution, the above soft masses at $M_{\rm GUT}$ lead to quite a distinctive pattern of low-energy soft masses which can be described in terms of the mirage messenger scale [19]:

$$M_{\rm mir} = \frac{M_{\rm GUT}}{(M_{\rm Pl}/m_{3/2})^{\alpha/2}}. \tag{13}$$

The low-energy gaugino masses are given by

$$\begin{aligned} M_a(Q) &= M_0\left[1 - \frac{1}{8\pi^2} b_a g_a^2(Q)\ln\left(\frac{M_{\rm mir}}{Q}\right)\right] \\ &= \frac{g_a^2(Q)}{g_a^2(M_{\rm mir})} M_0, \end{aligned} \tag{14}$$

showing that the gaugino masses are unified at $M_{\rm mir}$, while the gauge couplings are unified at $M_{\rm GUT}$. The low-energy values of A_{ijk} and m_i^2 generically depend on the associated Yukawa couplings y_{ijk}. However, if y_{ijk} are small enough or if

$$a_{ijk} = c_i + c_j + c_k = 1, \tag{15}$$

their low-energy values are given by [19]

$$\begin{aligned} A_{ijk}(Q) &= M_0\left[a_{ijk} + \frac{1}{8\pi^2}(\gamma_i(Q) + \gamma_j(Q) + \gamma_k(Q)) \times \ln\left(\frac{M_{\rm mir}}{Q}\right)\right], \\ m_i^2(Q) &= M_0^2\left[c_i - \frac{1}{8\pi^2} Y_i\left(\sum_j c_j Y_j\right) g_Y^2(Q)\ln\left(\frac{M_{\rm GUT}}{Q}\right) \right. \\ &\quad \left. + \frac{1}{4\pi^2}\left\{\gamma_i(Q) - \frac{1}{2}\frac{d\gamma_i(Q)}{d\ln Q}\ln\left(\frac{M_{\rm mir}}{Q}\right)\right\}\ln\left(\frac{M_{\rm mir}}{Q}\right)\right], \end{aligned} \tag{16}$$

where Y_i is the $U(1)_Y$ charge of ϕ^i. Quite often, the modulus-mediated squark and slepton masses have a common value, i.e. $c_{\tilde{q}} = c_{\tilde{\ell}}$. Then, according to the above expression of low-energy sfermion mass, the 1st and 2nd generation squark and slepton masses are unified again at $M_{\rm mir}$.

A TeV scale mirage mediation can provide a natural solution to the little SUSY hierarchy problem [21,22]. If $\alpha = 2$ and also the conditions of (15) are satisfied for the top quark Yukawa coupling, $M_{\rm mir}$ is of the order of 1 TeV and the troublesome RG running of $m_{H_u}^2$ is nearly cancelled by the anomaly mediation effect. Explicitly, we find

$$\begin{aligned} m_{H_u}^2(M_Z) &= M_0^2\left[c_{H_u} - 0.026\sum_i c_i Y_i - \frac{3}{4\pi^2} y_t^2 \ln\left(\frac{M_{\rm mir}}{m_{\tilde{t}}}\right) + \mathcal{O}\left(\frac{1}{4\pi^2}\right)\right] \\ &= c_{H_u} M_0^2 + \mathcal{O}\left(\frac{M_0^2}{4\pi^2}\right), \end{aligned} \tag{17}$$

where $M_{\rm mir} \sim 1$ TeV. Related to the little SUSY hierarchy problem, an attractive feature of mirage mediation arising from KKLT-type moduli stabilization is that α, a_{ijk}, and c_i take *rational values* [up to small corrections of $\mathcal{O}(1/4\pi^2)$] under suitable assumption. Then by choosing the discrete parameters of the model in such a way to give

$$\alpha = 2, \qquad c_{H_u} = 0, \qquad a_{H_u t_L t_R} = c_{\tilde{t}_L} + c_{\tilde{t}_R} = 1, \tag{18}$$

one can naturally obtain the little hierarchy:

$$m_{H_u}^2(M_Z) \sim \frac{m_{\rm SUSY}^2}{8\pi^2} \sim M_Z^2, \quad (19)$$

for which the correct EWSB can be achieved without any severe fine-tuning of parameters. Here $m_{\rm SUSY} \sim M_0$ denotes generic superparticle masses including the stop and gaugino masses. The discrete parameter values of (18) predict

$$M_{\tilde{g}} \simeq M_{\tilde{W}} \simeq M_{\tilde{B}} \simeq A_t \simeq \sqrt{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2} = \mathcal{O}(\sqrt{8\pi^2}M_Z) \quad (20)$$

at low-energy scales around 1 TeV, where M_a ($a = \tilde{g}$, $\tilde{W}$, $\tilde{B}$) are the MSSM gaugino masses, and $m_{\tilde{t}_{L,R}}$ are the left-handed and right-handed stop masses.

Mirage mediation is a natural outcome of KKLT-type moduli stabilization [28] which can be described by 4D effective action of the form [25]:

$$\int d^4\theta[-3CC^* e^{-K/3} - C^2C^{*2}\mathcal{P}_{\rm lift}\theta^2\bar{\theta}^2] + \left(\int d^2\theta\left[\frac{1}{4}f_a W^{a\alpha}W^a_\alpha + C^3W\right] + {\rm H.c.}\right), \quad (21)$$

where $C = C_0 + F^C\theta^2$ is the chiral compensator superfield, K and W are the Kähler potential and superpotential, and $\mathcal{P}_{\rm lift}\theta^2\bar{\theta}^2$ is the uplifting spurion operator induced by a SUSY-breaking brane which is assumed to be sequestered from the visible gauge and matter superfields. After integrating out heavy moduli which are fixed by fluxes, K and W appear to depend only on the light (volume) modulus T and the visible matter superfields Φ^i:

$$K = K_0(T + T^*) + Z_i(T + T^*)\Phi^{i*}\Phi^i, \quad W = W_0(T) + \tfrac{1}{6}\lambda_{ijk}\Phi^i\Phi^j\Phi^k. \quad (22)$$

Here we assume that the model possesses an axionic shift symmetry:

$${\rm Im}\,(T) \rightarrow {\rm Im}(T) + \text{real constant}, \quad (23)$$

which is broken by the nonperturbative term in W_0. This ensures that the modulus Kähler potential K_0 and the matter Kähler metric Z_i depend only on the invariant combination $T + T^*$, the holomorphic Yukawa couplings λ_{ijk} are complex constants, and finally $\partial_T f_a$ are real constants. These features eliminate the dangerous CP violating phases in soft terms deduced from (21) [32].

As long as the uplifting brane is sequestered from the visible gauge and matter fields, its low-energy consequence can be described by a spurion operator [25,30,33] of the form

$$\theta^2\bar{\theta}^2\mathcal{P}_{\rm lift}(T + T^*), \quad (24)$$

independently of the detailed feature of SUSY breakdown. The condition of a nearly vanishing cosmological constant requires

$$\mathcal{P}_{\rm lift} = \mathcal{O}(m_{3/2}^2M_{\rm Pl}^2). \quad (25)$$

On the other hand, if it is induced by SUSY breaking at the IR end of a warped throat as in the scenario proposed by [28], which is the case of our major concern, $\mathcal{P}_{\rm lift}$ is red-shifted as

$$\mathcal{P}_{\rm lift} \sim e^{4A}M_{\rm Pl}^4, \quad (26)$$

where $e^{2A} \ll 1$ is the metric warp factor at the end of throat, implying that $e^{2A} \sim m_{3/2}/M_{\rm Pl}$ in such scenario. Although it is possible that uplifting is achieved by conventional F-term SUSY breaking which is not necessarily sequestered from the volume modulus T [34], here we focus on a sequestered uplifting scenario since the sequestering of a visible sector is crucial for TeV scale mirage mediation to solve the little SUSY hierarchy problem.

In the original KKLT compactification of type IIB string theory [28], the uplifting operator is provided by an anti-$D3$ brane stabilized at the IR end of a warped throat, while the Calabi-Yau volume modulus T can be identified as a field living at the UV end of the throat [35]. In such case, T is also sequestered from the uplifting brane, and thus $\mathcal{P}_{\rm lift}$ is (approximately) independent of T [25]. More detailed analysis of the modulus potential induced by anti-$D3$ [36] and also the study of SUSY breaking transmitted through the warped throat [37] imply that $\partial_T \ln\mathcal{P}_{\rm lift} = \mathcal{O}(e^{4A})$ in the limit that T lives mostly in the unwarped region. As a result, practically $\mathcal{P}_{\rm lift}$ can be regarded to be independent of T in senarii that it originates from SUSY-breaking brane at the IR end of the warped throat.

The sequestering of visible matter, i.e. the suppression of the dependence of $\mathcal{P}_{\rm lift}$ on the visible matter fields Φ^i:

$$\frac{\partial\mathcal{P}_{\rm lift}}{\partial(\Phi^{i*}\Phi^j)} \ll m_{\rm SUSY}^2 \sim \left(\frac{m_{3/2}}{8\pi^2}\right)^2, \quad (27)$$

is crucial for mirage mediation to be able to give $|m_{H_u}^2(M_Z)| \sim m_{\rm SUSY}^2/8\pi^2$ which would solve the little SUSY hierarchy problem. It was noticed in [38,39] that generically geometric separation alone does not lead to such sequestering. In particular, for many geometric background realized in string/M theory, sizable contact interaction (in $N = 1$ superspace) between Φ^i and a SUSY-breaking field is induced by the exchange of bulk fields [38], implying that a rather special type of geometric background is required to realize sequestering.

On the other hand, studies of sequestering in some class of 4D CFT [40] and 5D warped geometry [41], and also an operator analysis for SUSY breaking transmitted through a warped throat [42], suggest that sequestering might be realized if the visible sector is separated from the SUSY-breaking brane by a warped throat. Based on these observations, sequestering of visible matter fields was assumed in the initial analysis of soft terms in KKLT setup [25]. Recently, it was argued in [33] that sizable contact interaction might be induced even for the case of a warped throat by the exchange of the throat isometry vector superfield. More recently, this issue of sequestering in warped string compactification has been examined in more detail [37], confirming that the desired sequestering can be achieved easily when the visible brane and SUSY-breaking brane are separated from each other by a strongly warped throat. For instance, it has been noticed that transmission of SUSY breaking through a Klebanov-Strassler-type throat [43] leads to (in the unit with $M_{\rm Pl} = 1$)

$$\frac{\partial \mathcal{P}_{\rm lift}}{\partial(\Phi^{i*}\Phi^j)} \lesssim \mathcal{O}(e^{\sqrt{28}A}) = \mathcal{O}(e^{1.29A} m_{3/2}^2), \qquad (28)$$

where we have used $e^{2A} \sim m_{3/2}/M_{\rm Pl}$ for the metric warp factor. The soft scalar masses of Φ^i resulting from this violation of sequestering are given by

$$\delta m_{i\bar{j}}^2 \lesssim m_{3/2}^2 \left(\frac{m_{3/2}}{M_{\rm Pl}}\right)^{0.65} \sim 10^{-9} m_{3/2}^2, \qquad (29)$$

which are small enough to be ignored compared to the modulus and anomaly mediated scalar mass squares of $\mathcal{O}(m_{3/2}^2/(8\pi^2)^2)$.

The size of the violation of sequestering can differ for different types of throat. Generically, the warped sequestering scenario discussed in [37] gives $\delta m_{i\bar{j}}^2 \sim e^{\gamma A} m_{3/2}^2$ with $\gamma = \mathcal{O}(1)$ for the metric warp factor which can be as small as $e^{2A} \sim m_{3/2}/M_{\rm Pl}$, and thereby the soft scalar masses of visible matter and Higgs fields are dominated by the modulus and anomaly mediated contributions given by (9). In the following, we start with a setup including the case that $\mathcal{P}_{\rm lift}$ has a nontrivial T dependence as in Refs. [19,21,22], while keeping that $\mathcal{P}_{\rm lift}$ is independent of the visible matter fields Φ^i. Later, we will focus on the specific case that $\mathcal{P}_{\rm lift}$ is independent of both T and Φ^i.

In the Einstein frame, the modulus potential from (21) takes the form:

$$V_{\rm TOT} = e^{K_0}[(\partial_T \partial_{\bar{T}} K_0)^{-1} |D_T W_0|^2 - 3|W_0|^2] + V_{\rm lift}, \qquad (30)$$

where $D_T W_0 = \partial_T W_0 + W_0 \partial_T K_0$ and the uplifting potential is given by

$$V_{\rm lift} = e^{2K_0/3} \mathcal{P}_{\rm lift}. \qquad (31)$$

The superspace Lagrangian density (21) also determines the auxiliary components of C and T as

$$\begin{aligned} \frac{F^C}{C_0} &= \frac{1}{3}\partial_T K_0 F^T + m_{3/2}^*, \\ F^T &= -e^{K_0/2}(\partial_T \partial_{\bar{T}} K_0)^{-1}(D_T W_0)^*, \end{aligned} \qquad (32)$$

where $m_{3/2} = e^{K_0/2} W_0$. For the minimal KKLT setup with

$$f_a = T, \qquad W_0 = w_0 - Ae^{-aT}, \qquad (33)$$

where w_0 is a hierarchically small constant of $\mathcal{O}(m_{3/2})$ and $A = \mathcal{O}(1)$ in the unit with $M_{\rm Pl} = 1$, it is straightforward to compute the vacuum values of T and F^T by minimizing the corresponding modulus potential (30) under the fine-tuning condition $\langle V_{\rm TOT} \rangle = 0$.[2] At leading order in $\epsilon = 1/\ln(M_{\rm Pl}/m_{3/2})$, one finds [19,45]:

$$\begin{aligned} aT &= [1 + \mathcal{O}(\epsilon)] \ln(M_{\rm Pl}/m_{3/2}), \\ M_0 &= F^T \partial_T \ln({\rm Re}(f_a)) = \frac{F^T}{T+T^*} \\ &= \frac{m_{3/2}}{\ln(M_{\rm Pl}/m_{3/2})}\left(1 + \frac{3\partial_T \ln(\mathcal{P}_{\rm lift})}{2\partial_T K_0} + \mathcal{O}(\epsilon)\right), \\ \alpha &= \frac{m_{3/2}}{M_0 \ln(M_{\rm Pl}/m_{3/2})} = \left(1 + \frac{3\partial_T \ln(\mathcal{P}_{\rm lift})}{2\partial_T K_0} + \mathcal{O}(\epsilon)\right)^{-1}. \end{aligned} \qquad (34)$$

In order to get $\alpha = 2$ giving $M_{\rm mir} \sim 1$ TeV within this minimal setup, one needs $\partial_T \ln(\mathcal{P}_{\rm lift})/\partial_T K_0 = -1/3$ as was assumed in [19,21,22]. However, as was pointed out in [30], $\partial_T \ln(\mathcal{P}_{\rm lift})/\partial_T K_0 < 0$ means that the uplifting sector couples more strongly for a larger value of T, which makes it difficult to give an extradimensional interpretation for $\mathcal{P}_{\rm lift}$. Thus, in order to get $M_{\rm mir} \sim 1$ TeV in a more plausible case with $\partial_T \ln(\mathcal{P}_{\rm lift})/\partial_T K_0 \geq 0$, one needs to modify the minimal setup given by (33).

As was pointed out recently [31], generalizing the gauge kinetic functions as

$$f_a = k_a T + l_a S \qquad (35)$$

can give rise to a different value of the anomaly to modulus mediation ratio α for a given form of $\mathcal{P}_{\rm lift}$, where S is the dilaton superfield and T is the volume modulus superfield. Such dilaton-modulus mixing in f_a is *not* an unusual feature of string compactification. For instance, in heterotic string/M theory, for an appropriate normalization of S and T, one finds l_a are positive rational numbers, while k_a are flux-induced rational number [46]: $k_a = \frac{1}{8\pi^2} \times \int_{\rm CY} J \wedge [{\rm tr}(F \wedge F) - \frac{1}{2}{\rm tr}(R \wedge R)]$, where J, F, and R are the Kähler, gauge, and curvature 2-forms, respectively. A similar form of f_a is obtained also in D-brane models of type II string compactification. For instance, the gauge kinetic function on $D7$ branes wrapping a 4-cycle Σ_4 is given by (35) where k_a are integer-valued wrapping number and l_a are flux-induced rational number [47]: $l_a = \frac{1}{8\pi^2}\int_{\Sigma_4} F \wedge F$.

[2] In fact, the correct condition should be $\langle V_{\rm TOT} \rangle + \Delta V_{\rm TOT} = 0$, where $\Delta V_{\rm TOT}$ denotes the quantum correction to the classical vacuum energy density $\langle V_{\rm TOT} \rangle$ [44]. This can alter the prediction of sfermion masses by an order of $\Delta V_{\rm TOT}/M_{\rm Pl}^2$. $\Delta V_{\rm TOT}$ is dominated by the quadratically divergent one-loop corrections with the cutoff scale Λ, i.e. $\Delta V_{\rm TOT} \sim N\Lambda^2 m_{\rm SUSY}^2/8\pi^2$, where N is the number of light superfields in 4D effective theory. In KKLT-type moduli stabilization, the volume modulus is stabilized at a value for which Λ is comparable to $M_{\rm GUT}$, and then $\Delta V_{\rm TOT}/M_{\rm Pl}^2$ is small enough to be ignored.

The dilaton-modulus mixing in f_a suggests that some nonperturbative terms in the superpotential depend on both S and T as

$$W_0 = W_{\rm flux}(S, Z_\alpha) + W_{\rm np} = W_{\rm flux}(S, Z_\alpha) + \sum_I A_I(Z_\alpha) e^{-8\pi^2(k_I T + l_I S)}, \tag{36}$$

where Z_α are complex structure moduli stabilized by $W_{\rm flux}$ together with S, and A_I generically have vacuum values of order unity. Here $W_{\rm np}$ might be induced by hidden gaugino condensation or string-theoretic instantons. For a confining hidden $SU(N)$ gauge group with gauge kinetic function $f_h = k_h T + l_h S$, gaugino condensation gives $W_{\rm np} \sim e^{-8\pi^2(k_h T + l_h S)/N}$. Similarly, Euclidean action of some stringy instanton might be given by a linear combination of S and T, $S_{\rm ins} = 8\pi^2(k_{\rm in} T + l_{\rm in} S)$, thereby yielding $W_{\rm np} \sim e^{-8\pi^2(k_{\rm in} T + l_{\rm in} S)}$.

An important feature of the gauge kinetic function (35) and the nonperturbative terms in (36) which will be crucial for our subsequent discussion is that

$$\{k_A/k_B, l_A/l_B\} = \text{rational numbers}, \tag{37}$$

where $k_A = \{k_a, k_I\}$ and $l_A = \{l_a, l_I\}$. Note that these ratios are determined by the topological or group theoretical data of the underlying string compactification. This feature can be understood easily by noting that ${\rm Im}(S)$ and ${\rm Im}(T)$ are *periodic* axion fields, thus the coefficients k_A and l_A should be quantized. In the following, we discuss the mirage mediation resulting from the effective SUGRA with the holomorphic gauge kinetic function (35) and the moduli superpotential (36), and examine the possibility of $M_{\rm mir} \sim 1$ TeV, i.e. $\alpha = 2$, for a *sequestered* uplifting function $\partial_T \mathcal{P}_{\rm lift} = 0$.

Let us start with the usual KKLT assumption that S and Z_α are fixed by $W_{\rm flux}$ at $\langle S\rangle = S_0$ and $\langle Z_\alpha\rangle = Z_{0\alpha}$ with a mass hierarchically heavier than $m_{3/2}$ [28]. To be specific, we consider a model with the following form of the visible sector gauge kinetic function and the moduli superpotential:

$$\begin{aligned} f_v &= T + lS, \\ W_0 &= W_{\rm flux}(S, Z_\alpha) + W_{\rm np}(S, Z_\alpha, T) \\ &= W_{\rm flux} - A_1 e^{-8\pi^2(k_1 T + l_1 S)}, \end{aligned} \tag{38}$$

where $A_1 = \mathcal{O}(1)$. Note that we have chosen the normalization of T for which $k_A = (k_a, k_I)$ take rational values. After integrating out the heavy S and Z_α, the effective gauge kinetic function and modulus superpotential are given by

$$\begin{aligned} f_v^{\rm (eff)} &= T + lS_0, \\ W_0^{\rm (eff)} &= \langle W_{\rm flux}\rangle + W_{\rm np} = \langle W_{\rm flux}\rangle - A_1 e^{-8\pi^2(k_1 T + l_1 S_0)}. \end{aligned} \tag{39}$$

Using $U(1)_R$ transformation and also the axionic shift of T, we can always make $\langle W_{\rm flux}\rangle$ and $A_1 e^{-8\pi^2 l_1 S_0}$ real.

In the scheme under consideration, the requirement of a nearly vanishing cosmological constant leads to

$$\mathcal{P}_{\rm lift} \sim \frac{|W_0^{\rm (eff)}|^2}{M_{\rm Pl}^2} \sim m_{3/2}^2 M_{\rm Pl}^2. \tag{40}$$

On the other hand, the volume modulus T is stabilized by $W_0^{\rm (eff)} = \langle W_{\rm flux}\rangle + W_{\rm np}$ at a vacuum value yielding $\langle W_{\rm np}\rangle \sim \langle W_{\rm flux}\rangle / \ln(M_{\rm Pl}/m_{3/2})$ [25,28]. As a result, the flux-induced superpotential is required to have a vacuum value

$$\frac{|\langle W_{\rm flux}\rangle|^2}{M_{\rm Pl}^2} \sim \langle \mathcal{P}_{\rm lift}\rangle \tag{41}$$

in order for the scheme to admit the fine-tuning for a nearly vanishing cosmological constant. In the case that $\mathcal{P}_{\rm lift}$ is induced by SUSY breaking at the IR end of a warped throat as proposed in [28], one finds [48]

$$\frac{\langle \mathcal{P}_{\rm lift}\rangle}{M_{\rm Pl}^4} \sim e^{4A} \sim \exp\left[-\left(\frac{-4\int_{\tilde{\Sigma}_3} H_3}{3\int_{\Sigma_3} F_3}\right) 8\pi^2\, {\rm Re}(S_0)\right], \tag{42}$$

where $4\pi\,{\rm Re}(S_0) = 1/g_{\rm st}$ for the string coupling $g_{\rm st}$ whose self-dual value is normalized to be unity, and $\int_{\Sigma_3} F_3$ and $\int_{\tilde{\Sigma}_3} H_3$ denote the integer-valued RR and NS-NS fluxes over the 3-cycle Σ_3 collapsing along the throat and its dual 3-cycle $\tilde{\Sigma}_3$. To summarize, to achieve the nearly vanishing cosmological constant, the flux-induced superpotential is required to be tuned as (in the unit with $M_{\rm Pl} = 1$)

$$|\langle W_{\rm flux}\rangle| \sim \exp\left[-\left(\frac{-2\int_{\tilde{\Sigma}_3} H_3}{3\int_{\Sigma_3} F_3}\right) 8\pi^2\, {\rm Re}(S_0)\right], \tag{43}$$

thus can be parameterized as

$$\langle W_{\rm flux}\rangle \equiv A_0 e^{-8\pi^2 l_0 S_0}, \tag{44}$$

where $l_0 = -(2\int_{\tilde{\Sigma}_3} H_3)/(3\int_{\Sigma_3} F_3)$ is a *positive rational* number of order unity and $A_0 = \mathcal{O}(1)$. As we will see, this feature of the flux-induced superpotential makes the vacuum value of ${\rm Re}(T)/l_0\,{\rm Re}(S)$ to be a *rational* number [up to small corrections of $\mathcal{O}(1/\ln(M_{\rm Pl}/m_{3/2}))$], which eventually yields the mirage mediation parameters α, c_i, and a_{ijk} taking *rational* values. In the following, we will adopt this parametrization of $\langle W_{\rm flux}\rangle$ while keeping in mind that it does not originate from a nonperturbative dynamics, but from the fine-tuning of the cosmological constant.

Minimizing the modulus potential (30) for

$$\begin{aligned} W_0^{\rm (eff)} &= \langle W_{\rm flux}\rangle - A_1 e^{-8\pi^2(k_1 T + l_1 S_0)} \\ &= A_0 e^{-8\pi^2 l_0 S_0} - A_1 e^{-8\pi^2(k_1 T + l_1 S_0)} \end{aligned} \tag{45}$$

and a generic uplifting function $\mathcal{P}_{\rm lift}$, we find the vacuum values of T and F^T are given by

$$k_1 T \simeq (l_0 - l_1) S_0 + \frac{1}{8\pi^2} \ln\left(\frac{-8\pi^2 k_1}{\partial_T K_0} \frac{A_1}{A_0}\right),$$
$$\frac{F^T}{T+T^*} \simeq \frac{l_0}{l_0 - l_1} \frac{m_{3/2}}{\ln(M_{\rm Pl}/m_{3/2})}\left(1 + \frac{3\partial_T \ln(\mathcal{P}_{\rm lift})}{2\partial_T K_0}\right). \tag{46}$$

On the other hand, the phenomenologically favored $m_{3/2} \sim 10$ TeV and $g_{\rm GUT}^{-2} \simeq 2$ require

$$8\pi^2 l_0 \,{\rm Re}(S_0) \simeq \ln(M_{\rm Pl}/m_{3/2}) \sim 4\pi^2,$$
$${\rm Re}(T) + l\,{\rm Re}(S_0) \simeq 2, \tag{47}$$

implying

$$4 \lesssim \frac{l_0 - l_1 + k_1 l}{k_1 l_0} \lesssim 5 \tag{48}$$

when the involved uncertainties are taken into account. The modulus-mediated gaugino mass is given by

$$M_0 = F^T \partial_T \ln({\rm Re}(f_v)) = \frac{F^T}{T+T^*}\left(\frac{l_0 - l_1}{l_0 - l_1 + k_1 l}\right), \tag{49}$$

thus we find

$$\alpha = \frac{m_{3/2}}{M_0 \ln(M_{\rm Pl}/m_{3/2})}$$
$$= \frac{l_0 - l_1 + k_1 l}{l_0}\left(1 + \frac{3\partial_T \ln(\mathcal{P}_{\rm lift})}{2\partial_T K_0}\right)^{-1} \tag{50}$$

up to small corrections of the order of $1/\ln(M_{\rm Pl}/m_{3/2}) \sim 1/4\pi^2$. Note that the F component of heavy dilaton S is given by $F^S/S_0 \sim m_{3/2}^2/m_S$, and thus is completely negligible since the dilaton mass m_S is hierarchically heavier than $m_{3/2}$.

The modulus-mediated A parameters and sfermion masses are determined by the following term in the superspace action (21):

$$\int d^4\theta C C^* e^{-K_0/3} Z_i \Phi^{i*}\Phi^i, \tag{51}$$

where K_0 is the modulus Kähler potential and Z_i is the matter Kähler metric. One then finds [27]

$$\tilde{A}_{ijk} = a_{ijk} M_0 = F^T \partial_T \ln(e^{-K_0} Z_i Z_j Z_k),$$
$$\tilde{m}_i^2 = c_i M_0^2 = -|F^T|^2 \partial_T \partial_{\bar{T}} \ln(e^{-K_0/3} Z_i). \tag{52}$$

In the absence of dilaton-modulus mixing, $e^{-K_0/3} Z_i$ typically takes the form:

$$e^{-K_0/3} Z_i = (T + T^*)^{n_i}, \tag{53}$$

where n_i is a rational number. The gauge flux leading to the modification of f_a can modify the matter Kähler metric Z_i also. For simplicity, here we consider the case that the matter Kähler metric of the visible sector is not affected by the involved dilaton-modulus mixing, thereby $e^{-K_0/3} Z_i$ takes the above simple form. Then the resulting a_{ijk} and c_i are found to be

$$a_{ijk} = (n_i + n_j + n_k)\left(\frac{l_0 - l_1 + k_1 l}{l_0 - l_1}\right),$$
$$c_i = n_i\left(\frac{l_0 - l_1 + k_1 l}{l_0 - l_1}\right)^2. \tag{54}$$

In mirage mediation, the Higgs mass parameter B can be another source of fine-tuning since the conventional SUGRA mechanism to generate μ generically gives $B \sim 8\pi^2 m_{\rm SUSY}$ which is too large to give successful electroweak symmetry breaking. For instance, the Higgs bilinear terms in the Kähler and superpotential:

$$\Delta K = \tilde{\kappa}(T + T^*) H_u H_d + {\rm H.c.}, \qquad \Delta W = \tilde{\mu}(T) H_u H_d \tag{55}$$

give the canonically normalized Higgsino mass:

$$\mu = \mu_K + \mu_W$$
$$= \frac{1}{\sqrt{Z_{H_u} Z_{H_d}}}(m_{3/2} + F^{\bar{T}}\partial_{\bar{T}})\tilde{\kappa} + \frac{1}{\sqrt{Z_{H_u} Z_{H_d}}} e^{K_0/2}\tilde{\mu}, \tag{56}$$

and the canonically normalized holomorphic Higgs mass:

$$B\mu = -[m_{3/2}^* + F^T \partial_T \ln(\tilde{\mu}) + \mathcal{O}(F^T)]\mu_W$$
$$+ [m_{3/2}^* + \mathcal{O}(F^T)]\mu_K, \tag{57}$$

where Z_{H_u} and Z_{H_d} are the Kähler metrics of H_u and H_d, respectively. Since $m_{3/2} \sim 8\pi^2 m_{\rm SUSY}$ in mirage mediation, this shows that indeed B is generically of $\mathcal{O}(8\pi^2 m_{\rm SUSY})$.

The moduli stabilization setup discussed above provides a nonperturbative mechanism to generate $B \sim m_{\rm SUSY}$ without fine-tuning. To obtain the desired size of μ and B, let us assume that $\tilde{\kappa} = \tilde{\mu} = 0$ in perturbation theory due to a symmetry G under which $H_u H_d$ has a nontrivial transformation, however an exponentially small $\tilde{\mu} \sim e^{-8\pi^2(k_2 T + l_2 S_0)}$ is generated by a nonperturbative effect which breaks G:

$$\Delta W = A_2 e^{-8\pi^2(k_2 T + l_2 S_0)} H_u H_d. \tag{58}$$

Adding the above nonperturbative μ term to the modulus superpotential (39), the total nonperturbative superpotential of the model is given by

$$W_{\rm TOT} = A_0 e^{-8\pi^2 l_0 S_0} - A_1 e^{-8\pi^2(k_1 T + l_1 S_0)}$$
$$+ A_2 e^{-8\pi^2(k_2 T + l_2 S_0)} H_u H_d, \tag{59}$$

yielding

$$\mu = \frac{e^{K_0/2} A_2 e^{-8\pi^2(k_2 T + l_2 S_0)}}{\sqrt{Z_{H_u} Z_{H_d}}} \sim A_2 m_{3/2}^{N_\mu},$$
$$B = 8\pi^2 k_2 F^T - m_{3/2} + \mathcal{O}(F^T)$$
$$= \left[\frac{2k_2}{k_1}\left(1 + \frac{3\partial_T \ln(\mathcal{P}_{\rm lift})}{2\partial_T K_0}\right) - 1\right] m_{3/2} + \mathcal{O}(F^T), \tag{60}$$

where

$$N_\mu = \frac{k_2}{k_1}\frac{l_0 - l_1}{l_0} + \frac{l_2}{l_0}, \tag{61}$$

and we have used the vacuum expectation values (46) and (47) for the last expressions of μ and B. This result shows that $B \sim m_{\rm SUSY}$ with a proper size of μ can be obtained by choosing the involved rational coefficients as

$$\frac{k_2}{k_1} = \frac{1}{2}\left(1 + \frac{3\partial_T \ln(\mathcal{P}_{\rm lift})}{2\partial_T K_0}\right)^{-1}, \qquad \frac{k_2}{k_1}\frac{l_0 - l_1}{l_0} + \frac{l_2}{l_0} = 1, \tag{62}$$

and A_2 has a value of $\mathcal{O}(10^{-2})$ or of $\mathcal{O}(10^{-3})$ depending upon the necessary value of μ. Note that $\partial_T \ln(\mathcal{P}_{\rm lift})/\partial_T K_0$ is typically a rational number for the volume modulus T, and A_2 naturally can be small since the symmetry G is restored in the limit $A_2 = 0$.

It would be nice if $k_2 = k_1$ and $l_2 = l_1$, so that the nonperturbative μ term $e^{-8\pi^2(k_2 T + l_2 S_0)} H_u H_d$ has the same dynamical origin as the nonperturbative term $e^{-8\pi^2(k_1 T + l_1 S_0)}$ which stabilizes T. However, in view of the condition (62), it is possible only when $\partial_T \ln(\mathcal{P}_{\rm lift})/\partial_T K_0 = -1/3$ [21], for which it is hard to give an extradimensional interpretation to $\mathcal{P}_{\rm lift}$ [30]. In a more plausible case that $\partial_T \ln(\mathcal{P}_{\rm lift})/\partial_T K_0 \geq 0$, these two terms cannot have the same origin. However, still they can have naturally the same order of magnitude by choosing the discrete parameters to satisfy $\frac{k_2}{k_1}\frac{l_0 - l_1}{l_0} + \frac{l_2}{l_0} = 1$. Another interesting feature of this mechanism to generate μ is that the resulting B is automatically real in the field basis that $m_{3/2}$ and F^T are real, and thus avoids the SUSY CP problem, as a consequence of the axionic shift symmetry of T [32]. In the most interesting case that the uplifting brane is located at the IR end of a warped throat, and thus is sequestered from the volume modulus T, i.e. $\partial_T \mathcal{P}_{\rm lift} = 0$, the values of k_2 and l_2 which give $\mu \sim A_2 m_{3/2}$ and $B \sim m_{\rm SUSY}$ are

$$k_2 = \tfrac{1}{2}k_1, \qquad l_2 = \tfrac{1}{2}(l_0 + l_1). \tag{63}$$

The nonperturbative μ term (58) can be generated by a confining hidden $SU(N_c)$ gauge interaction with N_f flavors of hidden quarks $Q_h + Q_h^c$ and a singlet X. As a specific example, let us consider a hidden sector with $G = Z_3$ symmetry under which

$$X \to e^{i2\pi/3}X, \qquad H_u H_d \to e^{i2\pi/3} H_u H_d, \qquad Q_h Q_h^c \to e^{-i2\pi/3} Q_h Q_h^c. \tag{64}$$

Up to ignoring irrelevant higher dimensional operators, the hidden gauge kinetic function and superpotential invariant under Z_3 are given by

$$f_h = k_h T + l_h S_0,$$
$$W_h = \lambda_1 X^3 + \lambda_2 X Q_h Q_h^c + h_1 Q_h Q_h^c H_u H_d + h_2 X^2 H_u H_d. \tag{65}$$

Note that Z_3 forbids a bare $H_u H_d$ term in gauge kinetic functions, Kähler potential and superpotential. The Z_3 symmetry is anomalous under the hidden $SU(N_c)$ gauge interaction, and thus is broken by the nonperturbative Affleck-Dine-Seiberg superpotential [49]

$$W_{\rm ADS} = (N_c - N_f)\left(\frac{e^{-8\pi^2 f_h}}{\det(Q_h Q_h^c)}\right)^{1/(N_c - N_f)}. \tag{66}$$

Then, after integrating out the confining hidden sector while including the effect of $W_{\rm ADS}$, one finds the following effective superpotential:

$$W_h^{(\rm eff)} = A_3 e^{-12\pi^2(k_2 T + l_2 S_0)} + A_2 e^{-8\pi^2(k_2 T + l_2 S_0)} H_u H_d, \tag{67}$$

where

$$k_2 = \frac{2k_h}{3N_c - N_f}, \qquad l_2 = \frac{2l_h}{3N_c - N_f}. \tag{68}$$

Since $e^{-12\pi^2(k_2 T + l_2 S_0)} \sim m_{3/2}^{3/2}$ in the unit $M_{\rm Pl} = 1$ for the rational coefficients (62), the first term of $W_h^{(\rm eff)}$ can be ignored safely.

Adding the above $W_h^{(\rm eff)}$ to (39), we obtain the total superpotential:

$$W_{\rm TOT} = A_0 e^{-8\pi^2 l_0 S_0} - A_1 e^{-8\pi^2(k_1 T + l_1 S_0)} + A_2 e^{-8\pi^2(k_2 T + l_2 S_0)} H_u H_d. \tag{69}$$

Let us recall that the first term in $W_{\rm TOT}$ corresponds to the flux-induced superpotential parameterized as $\langle W_{\rm flux}\rangle \equiv A_0 e^{-8\pi^2 l_0 S_0}$, which reflects the fine-tuning required for a nearly vanishing cosmological constant for an exponentially red-shifted uplifting operator $\mathcal{P}_{\rm lift} \sim e^{-16\pi^2 l_0 \,{\rm Re}(S_0)}$. The second term $e^{-8\pi^2(k_1 T + l_1 S_0)}$ might be induced by $D3$ brane instanton or $D7$ brane gaugino condensation with $f_{D7} \propto k_1 T + l_1 S_0$. It should be stressed that although each of the three terms in $W_{\rm TOT}$ has a different origin, they naturally have the same order of magnitudes. Independently of the value of l_1, T is stabilized at a vacuum value making the first and second terms comparable to each other. As for the μ term, we could get $\mu \sim A_2 m_{3/2}$ and $B \sim m_{3/2}/8\pi^2$ by assuming that the rational coefficients k_2 and l_2 satisfy (62).

So far, we have discussed generic mirage mediation resulting from moduli stabilization with dilaton-modulus mixing. Let us finally examine if this generalized setup allows a TeV scale mirage mediation solving the little hierarchy problem for the case that the uplifting function is sequestered as $\partial_T \mathcal{P}_{\rm lift} = 0$. Here we just present a simple example giving the parameters in (18). The model is defined by

$$f_v = T,$$
$$W_{\rm TOT} = A_0 e^{-8\pi^2 S_0} - A_1 e^{-4\pi^2(T-2S_0)} + A_2 e^{-2\pi^2 T} H_u H_d, \tag{70}$$

where the nonperturbative μ term is induced by hidden $SU(N_c)$ gauge interaction with $f_h = T$, $N_c = 3$, and $N_f = 1$. The first term of $W_{\rm TOT}$ is assumed to be induced by flux which admits the fine-tuning for a nearly vanishing cosmological constant for the uplifting function given by $\mathcal{P}_{\rm lift} \sim e^{-16\pi^2 \, {\rm Re}(S_0)}$, where the exponential suppression of $\mathcal{P}_{\rm lift}$ is due to the exponentially small warp factor. The second term of $W_{\rm TOT}$ might be induced by string-theoretic instanton and/or additional hidden gauge interaction with gauge kinetic function $\propto T - 2S_0$. The uplifting function is assumed to be sequestered from the volume modulus T, which would be the case if it originates from a SUSY-breaking brane at the IR end of a warped throat, so that

$$\partial_T \mathcal{P}_{\rm lift} = 0. \tag{71}$$

The modulus Kähler potential and the Kähler metric of H_u, t_L, and t_R are chosen to be

$$K_0 = -3\ln(T + T^*), \qquad e^{-K_0/3} Z_{H_u} = \text{constant},$$
$$e^{-K_0/3} Z_{t_L} = e^{-K_0/3} Z_{t_R} = (T + T^*)^{1/2}. \tag{72}$$

It is straightforward to see that this model gives the necessary TeV scale mirage mediation parameters:

$$\alpha = 2, \qquad c_{H_u} = 0, \qquad a_{H_u t_L t_R} = c_{\tilde{t}_L} + c_{\tilde{t}_R} = 1, \tag{73}$$

as well as

$$\mu \sim A_2 m_{3/2}, \qquad B \sim \frac{m_{3/2}}{8\pi^2}, \qquad g_{\rm GUT}^{-2} \simeq 2. \tag{74}$$

This model can give either the mass patterns (I) or (II) of (7), depending upon the choice of $e^{-K_0/3} Z_{H_d}$ and the possibility of a further suppression of B.

III. SPARTICLE SPECTRUM AND CONSTRAINTS FROM ELECTROWEAK SYMMETRY BREAKING AND FCNC

In this section we discuss the low-energy sparticle spectrum and the constraints from electroweak symmetry breaking and FCNC processes in the TeV scale mirage mediation scenario. The pattern of low-energy sparticle masses can be obtained easily by choosing $M_{\rm mir} \sim 1$ TeV in the analytic solution (16). In Fig. 1, we show the running of gauge coupling constants and gaugino masses. Here we take $M_{\rm mir} = M_0 = 1$ TeV as a benchmark point. Note that the gaugino masses are unified at $M_{\rm mir}$, while the gauge coupling constants are unified at $M_{\rm GUT} \simeq 2.0 \times 10^{16}$ GeV. In Figs. 2 and 3, we show the running of trilinear couplings and scalar masses for the mass pattern (I) and (II), respectively. We choose $c_i = 1/2$ for all quark and lepton superfields, while $c_{H_u} = c_{H_d} = 0$ for the mass pattern (I) and $c_{H_u} = 0$, $c_{H_d} = 1$ for the mass pattern (II). As anticipated from (16), the trilinear couplings and scalar masses are unified at $M_{\rm mir}$ while the Higgs soft masses cross zero for the case of the mass pattern (I). After taking into account the ambiguity in $M_{\rm mir}/M_0 = \mathcal{O}(1)$ and higher order effects such as the threshold at $M_{\rm GUT}$ and two-loop running, the model of Fig. 2 gives rise to the little hierarchy $|m^2_{H_u,H_d}| \sim M_0^2/8\pi^2$ at $M_0 \sim 1$ TeV. For the model of Fig. 3, although the bottom Yukawa coupling and the $U(1)_Y$ D-term contribution provide additional contribution to $m^2_{H_u}(M_0)$, still a sufficient little hierarchy is realized for $m^2_{H_u}/M_0^2$, while $m^2_{H_d}/M_0^2 \sim 1$ in this case.

In the TeV scale mirage mediation scenario, the squark/slepton mass squares renormalized at high-energy scale, e.g. at a scale near $M_{\rm GUT}$, are negative as was noticed in

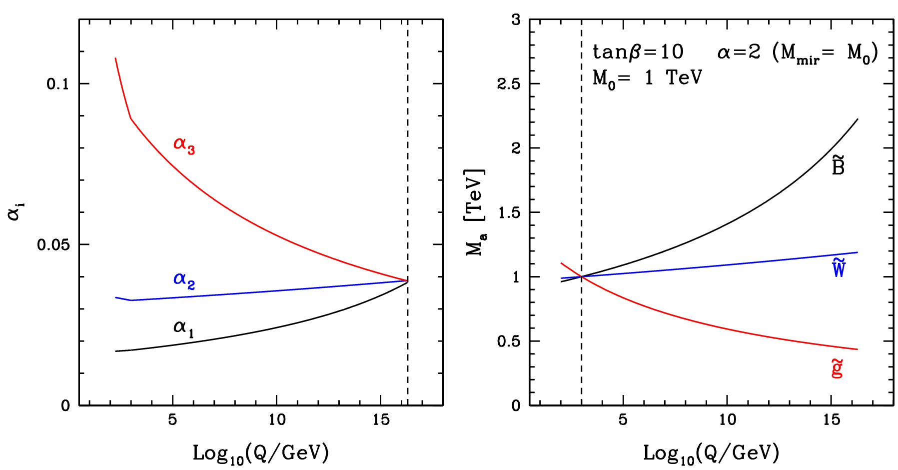

FIG. 1 (color online). Running of gauge couplings and gaugino masses in TeV scale mirage mediation.

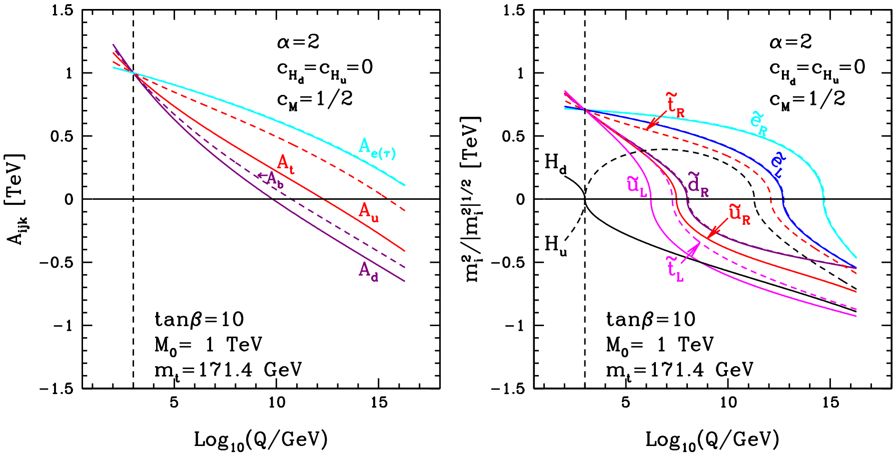

FIG. 2 (color online). Running of trilinear couplings and scalar masses leading to the mass pattern (I).

[50], while the values at low-energy scale below 10^6 GeV are positive. Such tachyonic *high-energy* squark/slepton mass squares might be considered as a problematic feature of the model. However, as long as the low-energy squark/slepton mass squares are positive, the model has a correct color/charge preserving (but electroweak symmetry breaking) vacuum which is a local minimum of the scalar potential over the squark/slepton values $|\phi| \lesssim 10^6$ GeV. On the other hand, tachyonic squark mass squares at the RG point $Q > 10^6$ GeV indicate that there might be a deeper color/charge breaking (CCB) minimum or an unbounded from below direction [51] at $|\phi| \gg 10^6$ GeV. In such a situation, we need a cosmological scenario in which our universe is settled down at the correct vacuum with $\phi = 0$.

In view of the fact that the squarks and sleptons get large positive mass squares in the high temperature limit, it is a rather plausible assumption [52] that squark/slepton fields are settled down at the color/charge preserving minimum after the inflation. However, as was pointed out in [53], the early universe might be trapped at the CCB minimum until it becomes the global minimum at low temperatures, depending upon the details of the model and also of the inflation scenario. This should be avoided in order for TeV scale mirage mediation to be viable. An examination of this issue is beyond this work as it requires an explicit scenario of early universe inflation. We thus simply assume that TeV scale mirage mediation can be combined with a successful early universe inflation leading to squark/slepton vacuum values settled down at the color/charge preserving local minimum.

Still we need to confirm that the color/charge preserving local minimum is stable enough against the decay into CCB vacuum. It has been noticed that the corresponding tunnelling rate is small enough, i.e. less than the Hubble expansion rate, as long as the RG points of vanishing squark/slepton mass squares are all higher than 10^4 GeV [52,54], which is satisfied safely by the TeV scale mirage mediation scenario solving the little SUSY hierarchy problem.

The analysis of electroweak symmetry breaking in TeV scale mirage mediation is more involved because

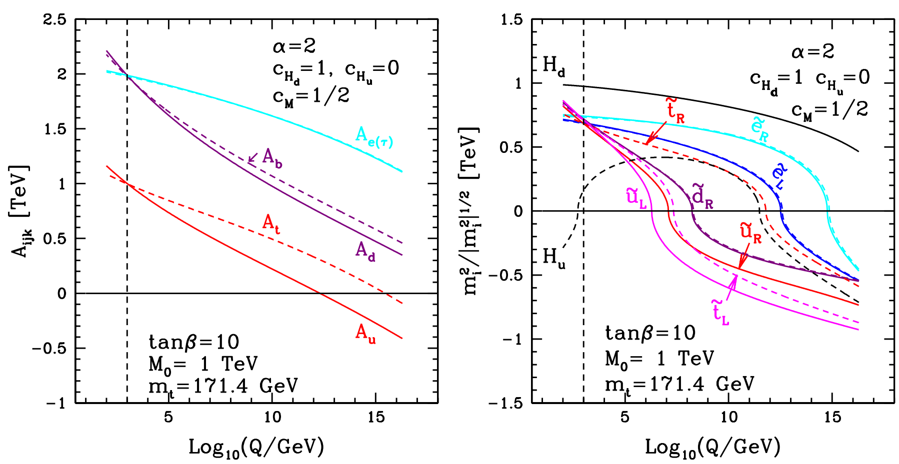

FIG. 3 (color online). Running of trilinear couplings and scalar masses leading to the mass pattern (II).

$$\frac{dm_{H_u}^2}{d\ln Q} \sim \frac{m_{\rm SUSY}^2}{8\pi^2} \sim m_{H_u}^2, \tag{75}$$

around the TeV scale, and thus the running Higgs parameter $m_{H_u}^2(Q)$ [$m_{H_d}^2(Q)$ also for the mass pattern (I)] is rather sensitive to the RG point Q. To express the conditions for electroweak symmetry breaking in terms of the RG-sensitive running parameters, one needs to include the Coleman-Weinberg one-loop potential [55,56] which cancels the Q dependence coming from the running parameters. This can be done efficiently [57] by replacing $m_{H_{d,u}}^2$ in the electroweak symmetry breaking conditions derived from the RG-improved tree level Higgs potential with

$$\bar{m}^2_{H_{d,u}} = m_{H_{d,u}}^2 - \frac{t_{d,u}}{\langle H_{d,u}^0\rangle}, \tag{76}$$

where the tadpoles $t_{d,u}$ are defined as

$$t_{d,u} = -\frac{1}{32\pi^2}\,{\rm Str}\left[\frac{\partial\mathcal{M}^2}{\partial\langle H_{d,u}^0\rangle}\mathcal{M}^2\left(\ln\left(\frac{\mathcal{M}^2}{Q^2}\right)-1\right)\right], \tag{77}$$

where Str stands for the supertrace and $\mathcal{M}$ represents the full mass matrix after $SU(2)_W\times U(1)_Y$ breaking.

Keeping this in mind, let us start with the RG-improved tree level scalar potential of the neutral Higgs bosons in the MSSM:

$$V = (m_{H_d}^2 + |\mu|^2)|H_d^0|^2 + (m_{H_u}^2 + |\mu|^2)|H_u^0|^2 - (B\mu H_d^0 H_u^0 + {\rm H.c.}) + \tfrac{1}{8}(g_2^2+g_Y^2)(|H_d^0|^2 - |H_u^0|^2)^2. \tag{78}$$

This Higgs potential leads to $\langle H_{d,u}^0\rangle \neq 0$ if the D-flat direction is stable,

$$m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2 - 2|B\mu| > 0, \tag{79}$$

and also the configuration $H_{d,u}^0 = 0$ is a saddle point,

$$(m_{H_d}^2 + |\mu|^2)(m_{H_u}^2 + |\mu|^2) - |B\mu|^2 < 0. \tag{80}$$

At the minimum of the potential, M_Z and $\tan\beta = \langle H_u^0\rangle/\langle H_d^0\rangle$ are determined as

$$\begin{aligned}\frac{M_Z^2}{2} &= \frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1} - |\mu|^2,\\ \frac{1+\tan^2\beta}{\tan\beta}|B\mu| &= m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2,\end{aligned} \tag{81}$$

which correspond to the electroweak symmetry breaking conditions in the MSSM.

The second of the above electroweak symmetry breaking conditions has a solution only when

$$\frac{m_{H_d}^2 + m_{H_u}^2}{|B|^2} \le \frac{1}{8}\left(\frac{1+\tan^2\beta}{\tan\beta}\right)^2. \tag{82}$$

We then find

$$|\mu| = \frac{1+\tan^2\beta}{4\tan\beta}|B| \times\left[1 \pm \sqrt{1 - 8\left(\frac{\tan\beta}{1+\tan^2\beta}\right)^2\frac{m_{H_d}^2 + m_{H_u}^2}{|B|^2}}\right], \tag{83}$$

where the minus sign is allowed only for $m_{H_d}^2 + m_{H_u}^2 \ge 0$. In the expansion in powers of $1/\tan\beta$, these two solutions can be approximated as

$$|\mu| = \begin{cases} \frac{\tan\beta}{2}|B|[1 + \mathcal{O}(\frac{1}{\tan^2\beta})] \\ \frac{1}{\tan\beta}\frac{m_{H_d}^2 + m_{H_u}^2}{|B|}[1 + \mathcal{O}(\frac{1}{\tan^2\beta})], \quad (m_{H_d}^2 + m_{H_u}^2 \ge 0). \end{cases} \tag{84}$$

Combining with the first condition of (81), we can find the required $|B|$ for given $m_{H_{d,u}}^2$ and $\tan\beta$. The mass pattern (I) favors the first solution because $m_{H_d}^2 + m_{H_u}^2$ tends to be negative due to the large negative anomalous dimension of H_u. On the other hand, the mass pattern (II) favors the second solution because the first solution requires a too small $|B|$ to allow the solution itself. This makes the two mass patterns behave in a qualitatively different manner. In particular, they require a quite different size of $|B|$:

$$\begin{aligned}&\text{Pattern (I): } |B| \simeq \frac{2|\mu|}{\tan\beta} \sim \frac{M_Z}{\tan\beta},\\ &\text{Pattern (II): } |B| \simeq \frac{1}{\tan\beta}\frac{m_{H_d}^2 + m_{H_u}^2}{|\mu|} \sim \frac{1}{\tan\beta}\frac{M_0^2}{M_Z}.\end{aligned} \tag{85}$$

In mirage mediation, even when one has a mechanism to eliminate the contribution of $\mathcal{O}(m_{3/2})$ to B as the one discussed in the previous section, it is hard to control $|B|$ to make it significantly smaller than $M_0 \sim m_{3/2}/4\pi^2$. Note that generically B can receive a contribution of $\mathcal{O}(m_{3/2}/8\pi^2)$ from a threshold effect at the UV cutoff scale. As a result, the mass pattern (I) might involve an additional fine-tuning to make $|B|$ as small as required. On the other hand, the mass pattern (II) fits well to the natural prediction $B \sim M_0$ which yields $\tan\beta \sim M_0/M_Z \sim \sqrt{8\pi^2}$. In the following, we ignore this potential fine-tuning for the mass pattern (I), and we compare its phenomenological aspects with those of the mass pattern (II).

Our theoretical framework for mirage mediation can predict the soft parameters at TeV with a precision of $\mathcal{O}(M_0/\sqrt{8\pi^2})$. As a result, it provides only an order of magnitude prediction for the soft parameters which have a size of $\mathcal{O}(M_0/\sqrt{8\pi^2})$ at TeV, i.e. m_{H_u}, m_{H_d}, B in the mass pattern (I) and m_{H_u} in the mass pattern (II). For these *small* parameters, we take a phenomenological approach treating them as free input parameters defined at the electroweak scale within the range of $\mathcal{O}(M_0/\sqrt{8\pi^2})$ as suggested by the mirage mediation scheme. To give a precise meaning to those input parameters, we define them at $Q = M_0/\sqrt{2}$ in the $\overline{DR}$ scheme [58].

The coupling constants and soft terms in the Higgs potential (78) are running parameters and the result of analysis depends on the RG point Q at which the potential is minimized. To deal with the Higgs parameters which have a size of $\mathcal{O}(M_0/\sqrt{8\pi^2})$, we need to reduce this renormalization scale dependence by including the Coleman-Weinberg one-loop effective potential [55,56]:

$$\Delta V_1 = \frac{1}{64\pi^2}\,\mathrm{Str}\left[\mathcal{M}^4\left(\ln\left(\frac{\mathcal{M}^2}{Q^2}\right) - \frac{3}{2}\right)\right]. \tag{86}$$

This one-loop correction can be effectively included in (81) [57] by replacing $m^2_{H_{d,u}}$ with $\bar{m}^2_{H_{d,u}}$ defined in (76). Taking $c_{\tilde{q},\tilde{u},\tilde{d}} = 1/2$ in (16), we obtain

$$\begin{aligned} \frac{t_d}{\langle H_d^0\rangle} &\approx -\delta m^2_{H_d} + \delta B\mu \cdot \mathrm{sgn}(B\mu)\tan\beta, \\ \frac{t_u}{\langle H_u^0\rangle} &\approx -\delta m^2_{H_u} + \delta B\mu \cdot \mathrm{sgn}(B\mu)\frac{1}{\tan\beta}, \end{aligned} \tag{87}$$

where

$$\begin{aligned} \delta m^2_{H_{d,u}} &= -\frac{M_0^2}{8\pi^2}\left[2\gamma_{H_{d,u}}\ln\left(\frac{M_0}{\sqrt{2}Q}\right) + (3g_2^2 + g_Y^2)\ln(\sqrt{2}) + \frac{1}{2}(3y^2_{b,t} - 3g_2^2 - g_Y^2)\right], \\ \delta B\mu &= \frac{\mu M_0}{8\pi^2}\left[(\gamma_{H_d} + \gamma_{H_u})\ln\left(\frac{M_0}{\sqrt{2}Q}\right) + (3g_2^2 + g_Y^2)\ln(\sqrt{2}) - \frac{1}{2}(3g_2^2 + g_Y^2)\right]. \end{aligned} \tag{88}$$

In the following numerical analysis, we use the electroweak symmetry breaking condition (81) supplemented by the replacement

$$m^2_{H_{d,u}} \rightarrow \bar{m}^2_{H_{d,u}} = m^2_{H_{d,u}} - \frac{t_{d,u}}{\langle H^0_{d,u}\rangle}, \tag{89}$$

which eliminates the sensitivity to the renormalization point Q as the Q dependence from $m^2_{H_{d,u}}$ and B is cancelled by the Q dependence of $t_{d,u}/\langle H^0_{d,u}\rangle$. In this regard, the following estimate turns out to be useful:

$$\bar{m}^2_{H_u} \approx m^2_{H_u}|_{Q=(M_0/\sqrt{2})} - 0.95\frac{M_0^2}{8\pi^2}. \tag{90}$$

In the model of the mass pattern (I), $m^2_{H_{d,u}}$ are free parameters of $\mathcal{O}(M_0^2/8\pi^2)$ at the weak scale. If $\tan\beta$ is not too small, the first condition of (81) is approximated as

$$\frac{M_Z^2}{2} \approx -\bar{m}^2_{H_u} - |\mu|^2 = -m^2_{H_u} + \frac{t_u}{\langle H_u^0\rangle} - |\mu|^2.$$

This leads to an upper bound of $m^2_{H_u}$,

$$m^2_{H_u} \lesssim -\frac{M_Z^2}{2} + \frac{t_u}{\langle H_u^0\rangle}, \tag{91}$$

which is saturated when $\mu = 0$. Combining this with the second condition of (81), we find

$$\begin{aligned} m_A^2 &= \frac{1+\tan^2\beta}{\tan\beta}|B\mu| \approx \bar{m}^2_{H_d} - \bar{m}^2_{H_u} - M_Z^2 \\ &\approx m^2_{H_d} - m^2_{H_u} - \frac{t_d}{\langle H_d^0\rangle} + \frac{t_u}{\langle H_u^0\rangle} - M_Z^2 \gtrsim 0, \end{aligned} \tag{92}$$

where m_A is the running pseudoscalar Higgs mass which is of $\mathcal{O}(M_Z)$ in this case. In Fig. 4, we show the parameter region leading to the correct electroweak symmetry breaking on the planes of $(m^2_{H_u}, M_0)$ and $(m^2_{H_u}, \tan\beta)$ for a benchmark scenario satisfying $m^2_{H_d}/m^2_{H_u} \approx \gamma_{H_d}/\gamma_{H_u}$.

In Fig. 5, we present similar plots for the mass pattern (II). In this case, B is of $\mathcal{O}(M_0)$, which would be naturally achieved by the nonperturbative mechanism discussed in the previous section, and also $m^2_{H_d} \approx M_0^2$ and $m^2_{H_u} \sim M_0^2/8\pi^2$ at TeV under the choice $c_{H_d} = 1$ and $c_{H_u} = 0$. Then the electroweak symmetry breaking conditions lead to

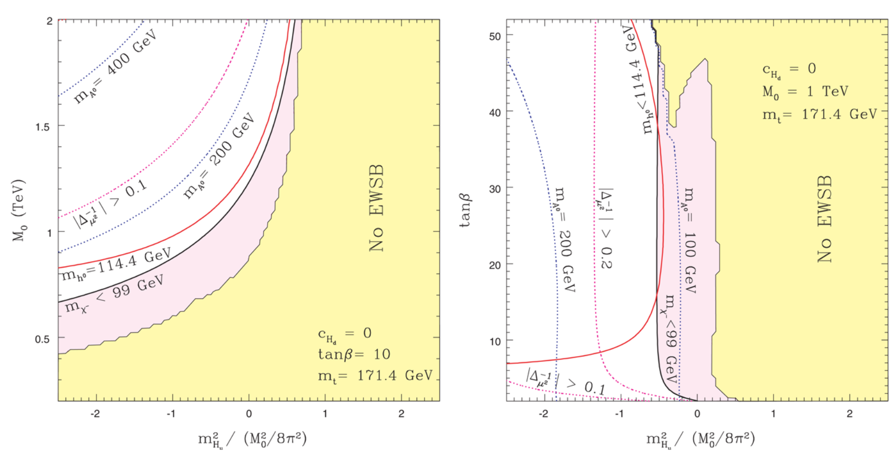

FIG. 4 (color online). Electroweak symmetry breaking, Higgs boson masses and the degree of fine-tuning in the mass pattern (I).

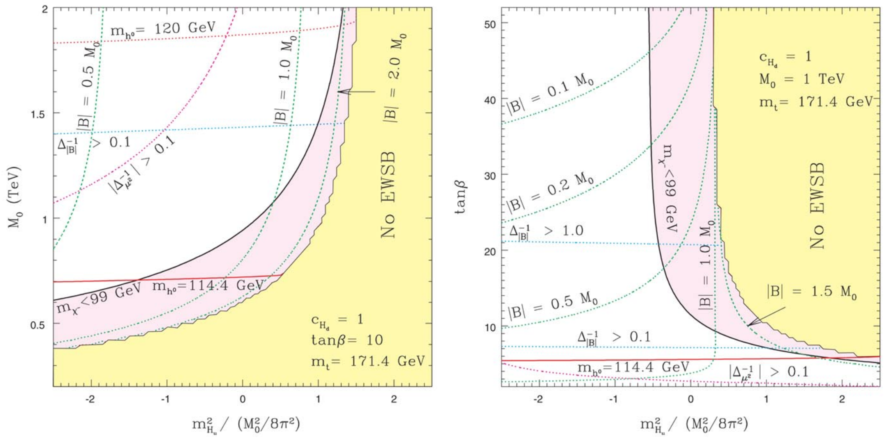

FIG. 5 (color online). Electroweak symmetry breaking, Higgs boson masses and the degree of fine-tuning in the mass pattern (II).

$$\frac{M_Z^2}{2} \approx \frac{\bar{m}_{H_d}^2}{\tan^2\beta} - \bar{m}_{H_u}^2 - |\mu|^2 \approx \left(1 - \frac{\bar{m}_{H_d}^2}{|B|^2}\right)\frac{\bar{m}_{H_d}^2}{\tan^2\beta} - \bar{m}_{H_u}^2. \tag{93}$$

Note that $m_A \simeq M_0$ in the case of the mass pattern (II).

Let us now estimate quantitatively the degree of fine-tuning for the electroweak symmetry breaking. Among the various possible measures of fine-tuning, we choose the sensitivity of M_Z^2 against a variation of the input parameter $\{a\} = \{\mu^2, B, m_{H_{d,u}}^2\}$ [3]:

$$\Delta_a \equiv \frac{\partial \ln M_Z^2}{\partial \ln a}. \tag{94}$$

We then find

$$\begin{aligned}
\Delta_{\mu^2} &= -\frac{2|\mu|^2}{M_Z^2} + \frac{2\tan^2\beta}{(\tan^2\beta-1)^2}\left(1 - \frac{4\tan\beta}{\tan^2\beta+1}\frac{|\mu|}{|B|}\right)\\
&\quad\times\left(1 + \frac{\tan^2\beta+1}{\tan\beta}\frac{|B\mu|}{M_Z^2}\right),\\
\Delta_{|B|} &= \frac{4\tan^2\beta}{(\tan^2\beta-1)^2}\left(1 + \frac{\tan^2\beta+1}{\tan\beta}\frac{|B\mu|}{M_Z^2}\right),\\
\Delta_{m_{H_d}^2} &= -\frac{2m_{H_d}^2}{M_Z^2}\frac{1}{(\tan^2\beta-1)^2}\\
&\quad\times\left(\tan^2\beta + 1 + \frac{2\tan^3\beta}{\tan^2\beta+1}\frac{M_Z^2}{|B\mu|}\right),\\
\Delta_{m_{H_u}^2} &= -\frac{2m_{H_u}^2}{M_Z^2}\frac{\tan^2\beta}{(\tan^2\beta-1)^2}\\
&\quad\times\left(\tan^2\beta + 1 + \frac{2\tan\beta}{\tan^2\beta+1}\frac{M_Z^2}{|B\mu|}\right),
\end{aligned} \tag{95}$$

where we have taken account of the μ and B dependence of $\tan\beta$. For the mass pattern (I), Δ_a are simplified as

$$\begin{aligned}
\Delta_{\mu^2} &= -\frac{2|\mu|^2}{M_Z^2} + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right),\\
\Delta_{|B|} &= \frac{4}{\tan^2\beta}\left(1 + \frac{2|\mu|^2}{M_Z^2}\right) + \mathcal{O}\left(\frac{1}{\tan^4\beta}\right),\\
\Delta_{m_{H_d}^2} &= -\frac{2m_{H_d}^2}{M_Z^2\tan^2\beta}\left(1 + \frac{M_Z^2}{|\mu|^2}\right) + \mathcal{O}\left(\frac{1}{\tan^4\beta}\right),\\
\Delta_{m_{H_u}^2} &= -\frac{2m_{H_u}^2}{M_Z^2} + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right).
\end{aligned} \tag{96}$$

The above results show that $\Delta_{|B|}$ and $\Delta_{m_{H_d}^2}$ are subdominant compared to $\Delta_{\mu^2} \sim \Delta_{m_{H_u}^2}$ if $|B|$ could be made to be small enough to give $\tan^2\beta \sim M_Z^2/|B|^2 \gg 1$. Note that $\Delta_{|B|}$ measures the sensitivity of M_Z^2 to $|B|$ under the assumption that $|B|$ is as small as $M_Z/\tan\beta$, not the degree of fine-tuning required to get such a small $|B|$. Δ_{μ^2} increases with $|\mu|$, but the degree of fine-tuning can be made to be better than 10%, i.e. $|\Delta_{\mu^2}^{-1}| > 0.1$, for $|\mu| \lesssim 200$ GeV. This is typically realized for a natural range of $m_{H_u}^2$ and M_0 as shown in the left panel of Fig. 4. We also plot in Fig. 4 the lightest Higgs mass using FEYNHIGGS1.2.2 [59]. The LEP bound on the physical Higgs boson mass, $m_{h^0} >$ 114.4 GeV, can be satisfied with a fine-tuning of μ^2 better than 10%.

For the mass pattern (II), the fine-tuning parameters are well approximated as

$$\begin{aligned}
\Delta_{\mu^2} &= \frac{2|\mu|^2}{M_Z^2}\left(\frac{|B|^2}{m_{H_d}^2} - 1\right) + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right),\\
\Delta_{|B|} &= \frac{4m_{H_d}^2}{M_Z^2\tan^2\beta} + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right),\\
\Delta_{m_{H_d}^2} &= -\frac{2m_{H_d}^2}{M_Z^2\tan^2\beta} + \mathcal{O}\left(\frac{1}{\tan^2\beta}\right),
\end{aligned} \tag{97}$$

where we have ignored the piece of $\mathcal{O}(m_{H_u}^2/m_{H_d}^2)$, and the

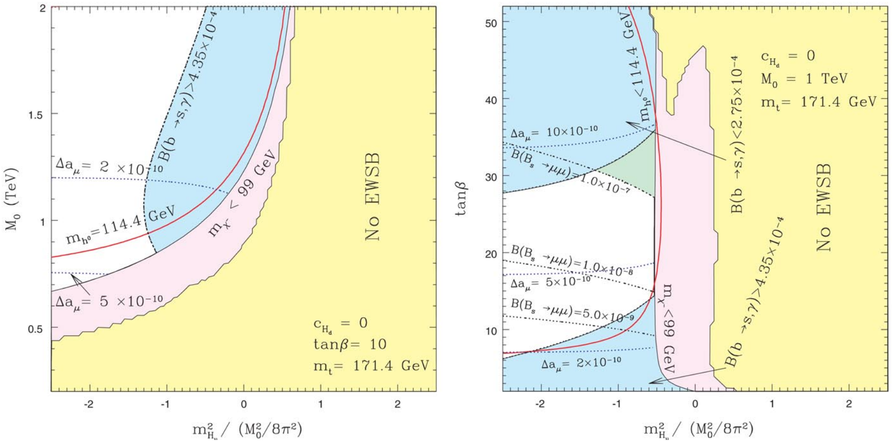

FIG. 6 (color online). Constraints from FCNC and the muon $g-2$ in the mass pattern (I).

expression for $\Delta_{m_{H_u}^2}$ is the same as the one for the mass pattern (I). In Fig. 5, we show the parameter region for which $|\Delta_{\mu^2}^{-1}| > 0.1$ and $|\Delta_{|B|}^{-1}| > 0.1$. Note that $|\mu|$ can be significantly bigger than M_Z while keeping $|\Delta_{\mu^2}| = \mathcal{O}(1)$ if $|B|^2 \simeq m_{H_d}^2$. This might open up an interesting possibility to raise $|\mu|$, thus raising the Higgsino mass, without causing a serious fine-tuning.

Let us finally discuss the constraints coming from various FCNC processes. In the mass pattern (I), all Higgs bosons and Higgsino have a light mass around a few hundred GeV. On the other hand, in the mass pattern (II), Higgs bosons other than the lightest one have a mass close to 1 TeV, while the Higgsino mass is around a few hundred GeV. In both cases, light particles can contribute to various FCNC processes through the flavor mixing in the F-term contribution of the squark masses induced by the Cabibbo-Kobayashi-Maskawa quark-mixing matrix. This consideration results in some constraints on the model, particularly for the large $\tan\beta$ region, and provides an opportunity to test the model with future experimental and theoretical improvements. In Figs. 6 and 7, we plot the constraints from $b \to s\gamma$ for the mass patterns (I) and (II), respectively. The current world average of the $b \to s\gamma$ branching ratio is given by [60]

$$B(b \to s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}, \tag{98}$$

where E_γ denotes the photon-energy cut. Theoretical prediction of the SM is estimated as [61][3]

$$B(b \to s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}} = 3.57 \pm 0.30 \times 10^{-4}. \tag{99}$$

In Figs. 6 and 7, we plot the 2-σ range by combining all the experimental and theoretical errors in quadrature:

$$2.75 \times 10^{-4} < B(b \to s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}} < 4.35 \times 10^{-4}. \tag{100}$$

In all plots, we choose a positive sign for μ for which the charged Higgs and chargino contributions to $b \to s\gamma$ tend to cancel each other. Negative μ gives a much stronger constraint due to the constructive interference.

In the mass pattern (I), both charged Higgs and chargino have a light mass, and their contributions to $b \to s\gamma$ compete to each other depending on the stop mass in the chargino contribution. The left panel of Fig. 6 shows that a large fraction of $(m_{H_u}^2, M_0)$ leading to electroweak symmetry breaking gives a $b \to s\gamma$ branching ratio within the allowed range (100). As the chargino contribution is enhanced by $\tan\beta$, the balance with the charged Higgs contribution is somewhat sensitive to the value of $\tan\beta$. In the right panel of Fig. 6 which is for the case of $M_0 = 1$ TeV, the upper left (lower right) region with large (small) $\tan\beta$ is disfavored by $b \to s\gamma$ due to the excessive chargino (charged Higgs) contribution which gives a too small (large) branching ratio.

In the mass pattern (II), only the chargino contribution to $b \to s\gamma$ is relevant. Then the small M_0 ($\lesssim 800$ GeV) and large $\tan\beta$ ($\gtrsim 15$) regions in the left and right panels of Fig. 7 are disfavored by giving a too small $b \to s\gamma$ branching ratio. In the right panel, the disfavored region quickly goes up and disappears if we increase M_0. Compared to the SM, the mass pattern (II) generically gives a smaller (larger) branching ratio for $\mu > 0$ ($\mu < 0$).

In the mass pattern (I), Higgs-mediated FCNC can give a sizable effect in large $\tan\beta$ regime [64] since all Higgs bosons have a relatively light mass. We calculated $B_s \to \mu\mu$ rate [65] and also the double penguin contribution to Δm_{B_s} in B_s-$\bar{B}_s$ mixing [66]. In the right panel of Fig. 6, we

[3]Theoretical uncertainty quoted here is inherited mostly from input parameters. It has been argued that the photon-energy cut introduces another uncertainty of similar size, which can be improved by perturbative calculation [62]. Recent NNLO calculation claims a central value 1.4σ lower than the experimental world average [63].

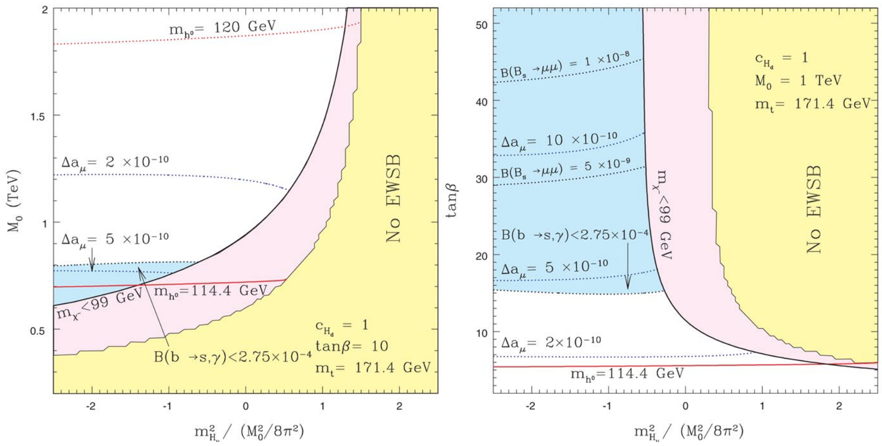

FIG. 7 (color online). Constraints from FCNC and the muon $g-2$ in the mass pattern (II).

plot contours for the branching ratio of $B_s \to \mu\mu$. The SM prediction is chirality suppressed, $B_{\rm SM}(B_s \to \mu\mu) = 3.46 \times 10^{-9}$; however, this is not the case for supersymmetric contribution. Current experimental bound, $B(B_s \to \mu\mu) < 1.0 \times 10^{-7}$, excludes a region above $\tan\beta \simeq 30$ for $M_0 = 1$ TeV. If the upper bound is improved to 1.0×10^{-8}, the excluded region comes down to $\tan\beta \simeq 20$. The branching ratio reaches to 5×10^{-9} around $\tan\beta \simeq 10$. However, we note that it is rather unlikely that $\tan\beta \gtrsim 10$ in the mass pattern (I) as it requires a very small $|B| \sim M_Z/\tan\beta$.

Recently a finite value of Δm_{B_s} has been measured at Tevatron with an unprecedented accuracy [67]:

$$\Delta m_{B_s} = 17.31^{+0.33}_{-0.18} \pm 0.07\ \text{ps}^{-1}. \tag{101}$$

A double Higgs penguin contribution to Δm_{B_s} can potentially cause a significant deviation from the SM prediction in the large $\tan\beta$ regime [68,69]. We examined an impact of this measurement on the mass pattern (I); however, we could not obtain a constraint stronger than the one from $B_s \to \mu\mu$. This is mainly due to a large ambiguity in hadronic parameters which determines the SM prediction. This uncertainty can be reduced if we consider the ratio $\Delta m_{B_s}/\Delta m_{B_d}$; however, in this case the dependence on the poorly known unitarity angle ϕ_3 (γ) introduces another source of ambiguity [70]. Considering the accuracy of the measurement, future progress in the lattice calculation of the involved hadronic parameters and also a precise determination of the unitarity angle might make Δm_{B_s} a strong probe for the mass pattern (I). For the mass pattern (II), these Higgs-mediated processes do not lead to any significant deviation from the SM predictions.

Anomalous magnetic moment of the muon provides a powerful tool to test new physics around the electroweak scale. Since the first report by BNL E821, the SM prediction has been carefully examined and refined including the semiempirical estimation of a hadronic vacuum polarization by dispersion relation and also the model dependent estimation of hadronic light-by-light contribution. See [71,72] for recent progress. Using the data set from e^+e^- collisions for the hadronic vacuum polarization, Passera [72] quotes the SM prediction as

$$a_\mu^{\rm SM} = (11\,659\,184.5 \pm 6.9) \times 10^{-10}, \tag{102}$$

while the latest experimental value is reported as [73]

$$a_\mu^{\rm exp} = (11\,659\,208.0 \pm 6.0) \times 10^{-10}. \tag{103}$$

This amounts to 2.6σ deviation from the SM[4]:

$$\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (23.5 \pm 9.1) \times 10^{-10}. \tag{104}$$

Analysis based on τ decays shows 0.7σ deviation; however, this result is still under debate due to the lack of full understanding of an isospin-breaking effect. Further theoretical and experimental effort will confirm or diminish the current disagreement based on e^+e^-.

In the MSSM, a charged Higgs contribution to the anomalous muon magnetic moment is suppressed by small Yukawa couplings, and then dominant contribution comes from chargino and neutralino loop diagrams. In the TeV scale mirage mediation scenario, the gaugino contribution to a_μ is small as the b-ino and W-ino masses are close to $M_0 \sim 1$ TeV. The Higgsino contribution is also small as it is suppressed by small Yukawa couplings. As a result, Δa_μ in TeV scale mirage mediation is significantly smaller than the value obtained in the conventional scenarios which have light gauginos and/or stau [75]. In Figs. 6 and 7, we plot the SUSY contribution to the muon $g-2$ for the mass patterns (I) and (II), respectively. Taking into account the constraints from FCNC processes and the lightest Higgs boson mass, TeV scale mirage mediation scenario predicts

[4] The latest analysis claims 3.4σ deviation with a 4.1×10^{-10} larger central value [74].

$$\begin{aligned} \Delta a_\mu &\lesssim 10\times 10^{-10} \qquad \text{(the mass pattern (I))}, \\ \Delta a_\mu &\lesssim 5\times 10^{-10} \qquad \text{(the mass pattern (II))}. \end{aligned} \tag{105}$$

If the discrepancy between the SM prediction based on e^+e^- scattering and the experimental measurement is confirmed with the current central value, it cannot be accommodated in the TeV scale mirage mediation setup discussed here. In this regard, an improvement of the theoretical and experimental errors on a_μ will have a considerable impact on TeV scale mirage mediation scenario.

IV. CONCLUSION

TeV scale mirage mediation has been proposed as a pattern of soft SUSY-breaking terms reducing the fine-tuning for the electroweak symmetry breaking in the MSSM [21,22], thereby solving the little SUSY hierarchy problem. The original proposal is based on a SUSY-breaking uplifting potential which is difficult to give an extradimensional interpretation [30]. In this paper, we note that the desired form of TeV scale mirage mediation can be achieved within a moduli stabilization scheme which has a brane-localized (sequestered) origin of the SUSY-breaking uplifting potential, if the holomorphic gauge kinetic functions and nonperturbative superpotential depend on both the dilaton superfield S and the volume modulus superfield T. We also propose a nonperturbative mechanism to generate the Higgs B parameter which has a desirable size $B \sim m_{\rm SUSY} \sim m_{3/2}/8\pi^2$ in mirage mediation scheme. An important feature of the scheme is that the axion components of S and T are periodic fields; therefore, the coefficients of S and T in gauge kinetic functions and nonperturbative superpotential can have discrete values only. As in the case of KKLT moduli stabilization, S is assumed to be stabilized by flux with m_S hierarchically heavier than $m_{3/2}$, while T is stabilized by nonperturbative effects yielding $m_T \sim m_{3/2}\ln(M_{\rm Pl}/m_{3/2})$. Then, under a proper choice of the involved discrete parameters, the TeV scale mirage mediation pattern of soft parameters solving the little SUSY hierarchy problem can be obtained.

The electroweak symmetry breaking conditions suggest that the TeV scale mirage mediation solving the little SUSY hierarchy problem can give two different mass patterns (I) and (II) at the weak scale, which differ by the values of m_{H_d} and B. In this paper, we analyzed the electroweak symmetry breaking as well as the constraints from various FCNC processes in both mass patterns. The results are summarized in Figs. 4–7, which show that a large fraction of the parameter space can give the correct electroweak symmetry breaking while satisfying the FCNC constraints with a reasonable degree of fine-tuning better than 10%. For the mass pattern (II), $|\mu|$ can be significantly bigger than M_Z while keeping the degree of fine-tuning better than 10%, if $|B|^2 \approx m_{H_d}^2$. This might open up a possibility to raise $|\mu|$, thus raising the Higgsino mass, without causing a fine-tuning problem.

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APPENDIX A

In this appendix, we summarize the notations and conventions used in this paper. The quantum effective action in $N = 1$ superspace can be written as

$$\begin{aligned} &\int d^4\theta\left[-3CC^* e^{-K/3} + \frac{1}{16}\left(G_a W^{a\alpha}\frac{D^2}{\partial^2}W^a_\alpha + \text{H.c.}\right)\right] + \left(\int d^2\theta C^3 W + \text{H.c.}\right) \\ &= \int d^4\theta\left[-3CC^* e^{-K_0/3} + CC^* e^{-K_0/3} Z_i \Phi^{i*} e^{2V^a T_a}\Phi^i + \frac{1}{16}\left(G_a W^{a\alpha}\frac{D^2}{\partial^2}W^a_\alpha + \text{H.c.}\right)\right] \\ &\quad + \left(\int d^2\theta C^3\left[W_0 + \frac{1}{6}\lambda_{ijk}\Phi^i\Phi^j\Phi^k\right] + \text{H.c.}\right) + \ldots, \end{aligned} \tag{A1}$$

where the gauge kinetic terms are written as a D-term operator to accommodate the radiative corrections to gauge couplings, and the ellipsis stands for the irrelevant higher dimensional operators. The Kähler potential K is expanded as

$$K = K_0(T_A, T_A^*) + Z_i(T_A, T_A^*)\Phi^{i*} e^{2V^a T_a}\Phi^i + \ldots, \tag{A2}$$

where V^a and Φ^i denote the visible gauge and matter superfields given by

$$\Phi^i = \phi^i + \sqrt{2}\theta\psi^i + \theta^2 F^i,$$
$$V^a = -\theta\sigma^\mu\bar{\theta}A^a_\mu - i\bar{\theta}^2\theta\lambda^a + i\theta^2\bar{\theta}\bar{\lambda}^a + \tfrac{1}{2}\theta^2\bar{\theta}^2 D^a, \tag{A3}$$

and $T_A = (C, T)$ are the SUSY-breaking messengers including the conformal compensator superfield $C = C_0 + \theta^2 F^C$ and the modulus superfield $T = T_0 + \sqrt{2}\theta\tilde{T} + \theta^2 F^T$. The radiative corrections due to renormalizable gauge and Yukawa interactions can be encoded in the matter Kähler metric Z_i and the gauge coupling superfield G_a which is given by

$$G_a = \mathrm{Re}(f_a) + \Delta G_a, \tag{A4}$$

where f_a is the holomorphic gauge kinetic function and ΔG_a includes the T_A-dependent radiative correction to gauge coupling. The superpotential is expanded as

$$W = W_0(T) + \tfrac{1}{6}\lambda_{ijk}(T)\Phi^i\Phi^j\Phi^k + \ldots, \tag{A5}$$

where $W_0(T)$ is the modulus superpotential stabilizing T. Here we do not specify the mechanism to generate the MSSM Higgs parameters μ and B, and treat them as free parameters.

For the canonically normalized component fields, the above superspace action gives the following form of the running gauge and Yukawa couplings, the supersymmetric gaugino-matter fermion coupling $\mathcal{L}_{\lambda\psi}$, and the soft SUSY-breaking terms:

$$\begin{aligned}
\frac{1}{g_a^2(Q)} &= \mathrm{Re}(G_a),\\
y_{ijk}(Q) &= \frac{\lambda_{ijk}}{\sqrt{e^{-K_0}Z_iZ_jZ_k}},\\
\mathcal{L}_{\lambda\psi} &= i\sqrt{2}(\phi^{i*}T_a\psi^i\lambda^a - \bar{\lambda}^a T_a\phi^i\bar{\psi}^i),\\
\mathcal{L}_{\mathrm{soft}} &= -m_i^2\phi^i\phi^{i*}\\
&\quad - \left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}A_{ijk}y_{ijk}\phi^i\phi^j\phi^k + \mathrm{h.c.}\right),
\end{aligned} \tag{A6}$$

where Q denotes the renormalization point and

$$\begin{aligned}
M_a(Q) &= F^A\partial_A\ln(\mathrm{Re}(G_a)),\\
A_{ijk}(Q) &= -F^A\partial_A\ln\left(\frac{\lambda_{ijk}}{e^{-K_0}Z_iZ_jZ_k}\right),\\
m_i^2(Q) &= -F^AF^{B*}\partial_A\partial_{\bar{B}}\ln(e^{-K_0/3}Z_i)
\end{aligned} \tag{A7}$$

for

$$\begin{aligned}
F^T &= -e^{K_0/2}(\partial_T\partial_{\bar{T}}K_0)^{-1}(D_TW_0)^*,\\
F^C &= m_{3/2}^* + \tfrac{1}{3}\partial_TK_0F^T \qquad (m_{3/2} = e^{K_0/2}W_0).
\end{aligned} \tag{A8}$$

In the approximation ignoring the off-diagonal components of $w_{ij} = \sum_{pq}y_{ipq}y^*_{jpq}$, the 1-loop RG evolution of soft parameters is determined by

$$\begin{aligned}
16\pi^2\frac{dM_a}{d\ln Q} &= 2\left[-3\,\mathrm{tr}(T_a^2(\mathrm{Adj})) + \sum_i\mathrm{tr}(T_a^2(\phi^i))\right]g_a^2M_a,\\
16\pi^2\frac{dA_{ijk}}{d\ln Q} &= \left[\sum_{pq}|y_{ipg}|^2A_{ipg} - 4\sum_a g_a^2C_2^a(\phi^i)M_a\right]\\
&\quad + [i\leftrightarrow j] + [i\leftrightarrow k],\\
16\pi^2\frac{dm_i^2}{d\ln Q} &= \sum_{jk}|y_{ijk}|^2(m_i^2 + m_j^2 + m_k^2 + |A_{ijk}|^2)\\
&\quad - 8\sum_a g_a^2C_2^a(\phi^i)|M_a|^2 + 2g_1^2q_i\sum_j q_jm_j^2,
\end{aligned} \tag{A9}$$

where the quadratic Casimir $C_2^a(\phi^i) = (N^2-1)/2N$ for a fundamental representation ϕ^i of the gauge group $SU(N)$, $C_2^a(\phi^i) = q_i^2$ for the $U(1)$ charge q_i of ϕ^i.

In mirage mediation, soft terms at M_{GUT} are determined by the modulus mediation of $\mathcal{O}(\frac{F^T}{T})$ and the anomaly mediation of $\mathcal{O}(\frac{F^C}{8\pi^2C_0})$ which are comparable to each other. In the presence of the axionic shift symmetry

$$U(1)_T\colon\ \mathrm{Im}(T) \to \mathrm{Im}(T) + \text{real constant}, \tag{A10}$$

which is broken by the nonperturbative term in the modulus superpotential

$$W_0 = w - Ae^{-aT}, \tag{A11}$$

one can always make that $m_{3/2}$ and F^T are simultaneously real. Also since $\frac{F^T}{T} \sim \frac{m_{3/2}}{4\pi^2}$, we have

$$\frac{F^C}{C_0} = m_{3/2}\left(1 + \mathcal{O}\left(\frac{1}{8\pi^2}\right)\right). \tag{A12}$$

Then, upon ignoring the parts of $\mathcal{O}(\frac{F^T}{8\pi^2T})$, the resulting soft parameters *at the scale just below* M_{GUT} are given by

$$\begin{aligned}
M_a(M_{\mathrm{GUT}}) &= M_0 + \frac{m_{3/2}}{16\pi^2}b_ag_a^2,\\
A_{ijk}(M_{\mathrm{GUT}}) &= \tilde{A}_{ijk} - \frac{m_{3/2}}{16\pi^2}(\gamma_i + \gamma_j + \gamma_k),\\
m_i^2(M_{\mathrm{GUT}}) &= \tilde{m}_i^2 - \frac{m_{3/2}}{16\pi^2}M_0\theta_i - \left(\frac{m_{3/2}}{16\pi^2}\right)^2\dot{\gamma}_i,
\end{aligned} \tag{A13}$$

where

$$\begin{aligned}
M_0 &= F^T\partial_T\ln\mathrm{Re}(f_a),\\
\tilde{A}_{ijk} &= -F^T\partial_T\ln\left(\frac{\lambda_{ijk}}{e^{-K_0}Z_iZ_jZ_k}\right) \equiv a_{ijk}M_0,\\
\tilde{m}_i^2 &= -|F^T|^2\partial_T\partial_{\bar{T}}\ln(e^{-K_0/3}Z_i) \equiv c_iM_0^2,
\end{aligned} \tag{A14}$$

and

$$\begin{aligned} b_a &= -3\,\mathrm{tr}(T_a^2(\mathrm{Adj})) + \sum_i \mathrm{tr}(T_a^2(\phi^i)), \\ \gamma_i &= 2\sum_a g_a^2 C_2^a(\phi^i) - \frac{1}{2}\sum_{jk} |y_{ijk}|^2, \\ \theta_i &= 4\sum_a g_a^2 C_2^a(\phi^i) - \sum_{jk} a_{ijk}|y_{ijk}|^2, \\ \dot{\gamma}_i &= 8\pi^2 \frac{d\gamma_i}{d\ln Q}, \end{aligned} \tag{A15}$$

where $\omega_{ij} = \sum_{kl} y_{ikl} y^*_{jkl}$ is assumed to be diagonal. Note that if λ_{ijk} are T-independent constant as required by the axionic shift symmetry $U(1)_T$, $\tilde{A}_{ijk} = F^T \partial_T \ln(e^{-K_0} Z_i Z_j Z_k)$.

Let us now summarize our conventions for the MSSM. The superpotential of canonically normalized matter superfields is given by

$$\begin{aligned} W = {} & y_D H_d \cdot Q D^c + y_L H_d \cdot L E^c - y_U H_u \cdot Q U^c \\ & - \mu H_d \cdot H_u, \end{aligned} \tag{A16}$$

where the $SU(2)_L$ product is $H \cdot Q = \epsilon_{ab} H^a Q^b$ with $\epsilon_{12} = -\epsilon_{21} = 1$, and color indices are suppressed. Then the chargino and neutralino mass matrices are given by

$$-\tfrac{1}{2}\tilde{\psi}^{-T} \mathcal{M}_C \tilde{\psi}^+ - \tfrac{1}{2}\tilde{\psi}^{0T} \mathcal{M}_N \tilde{\psi}^0 + \mathrm{H.c.}, \tag{A17}$$

where

$$\mathcal{M}_C = \begin{pmatrix} -M_2 & g_2\langle H_u^0\rangle \\ g_2\langle H_d^0\rangle & \mu \end{pmatrix}, \qquad \mathcal{M}_N = \begin{pmatrix} -M_1 & 0 & -\frac{1}{\sqrt{2}} g_Y\langle H_d^0\rangle & \frac{1}{\sqrt{2}} g_Y\langle H_u^0\rangle \\ 0 & -M_2 & \frac{1}{\sqrt{2}} g_2\langle H_d^0\rangle & -\frac{1}{\sqrt{2}} g_2\langle H_u^0\rangle \\ -\frac{1}{\sqrt{2}} g_Y\langle H_d^0\rangle & \frac{1}{\sqrt{2}} g_2\langle H_d^0\rangle & 0 & -\mu \\ \frac{1}{\sqrt{2}} g_Y\langle H_u^0\rangle & -\frac{1}{\sqrt{2}} g_2\langle H_u^0\rangle & -\mu & 0 \end{pmatrix}, \tag{A18}$$

in the field basis

$$\begin{aligned} \tilde{\psi}^{+T} &= -i(\tilde{W}^+, i\tilde{H}_u^+), \\ \tilde{\psi}^{-T} &= -i(\tilde{W}^-, i\tilde{H}_d^-), \\ \tilde{\psi}^{0T} &= -i(\tilde{B}, \tilde{W}^3, i\tilde{H}_d^0, i\tilde{H}_u^0), \end{aligned} \tag{A19}$$

for $\tilde{W}^\pm = (\tilde{W}^1 \mp i\tilde{W}^2)/\sqrt{2}$.

The one-loop beta function coefficients b_a and anomalous dimension γ_i in the MSSM are given by

$$\begin{aligned} b_3 &= -3, \\ b_2 &= 1, \\ b_1 &= \tfrac{33}{5}, \\ \gamma_{H_u} &= \tfrac{3}{2}g_2^2 + \tfrac{1}{2}g_Y^2 - 3y_t^2, \\ \gamma_{H_d} &= \tfrac{3}{2}g_2^2 + \tfrac{1}{2}g_Y^2 - 3y_b^2 - y_\tau^2, \\ \gamma_{Q_a} &= \tfrac{8}{3}g_3^2 + \tfrac{3}{2}g_2^2 + \tfrac{1}{18}g_Y^2 - (y_t^2 + y_b^2)\delta_{3a}, \\ \gamma_{U_a} &= \tfrac{8}{3}g_3^2 + \tfrac{8}{9}g_Y^2 - 2y_t^2\delta_{3a}, \\ \gamma_{D_a} &= \tfrac{8}{3}g_3^2 + \tfrac{2}{9}g_Y^2 - 2y_b^2\delta_{3a}, \\ \gamma_{L_a} &= \tfrac{3}{2}g_2^2 + \tfrac{1}{2}g_Y^2 - y_\tau^2\delta_{3a}, \\ \gamma_{E_a} &= 2g_Y^2 - 2y_\tau^2\delta_{3a} \end{aligned} \tag{A20}$$

where g_2 and $g_Y = \sqrt{3/5}g_1$ denote the $SU(2)_W$ and $U(1)_Y$ gauge couplings. The θ_i and $\dot{\gamma}_i$ which determine the soft scalar masses at M_{GUT} are given by

$$\begin{aligned} \theta_{H_u} &= 3g_2^2 + g_Y^2 - 6y_t^2 a_{H_u Q_3 U_3^c}, \\ \theta_{H_d} &= 3g_2^2 + g_Y^2 - 6y_b^2 a_{H_d Q_3 D_3^c} - 2y_\tau^2 a_{H_d L_3 E_3^c}, \\ \theta_{Q_a} &= \tfrac{16}{3}g_3^2 + 3g_2^2 + \tfrac{1}{9}g_Y^2 - 2(y_t^2 a_{H_u Q_3 U_3^c} + y_b^2 a_{H_d Q_3 D_3^c})\delta_{3a}, \\ \theta_{U_a} &= \tfrac{16}{3}g_3^2 + \tfrac{16}{9}g_Y^2 - 4y_t^2 a_{H_u Q_3 U_3^c}\delta_{3a}, \\ \theta_{D_a} &= \tfrac{16}{3}g_3^2 + \tfrac{4}{9}g_Y^2 - 4y_b^2 a_{H_d Q_3 D_3^c}\delta_{3a}, \\ \theta_{L_a} &= 3g_2^2 + g_Y^2 - 2y_\tau^2 a_{H_d L_3 E_3^c}\delta_{3a}, \\ \theta_{E_a} &= 4g_Y^2 - 4y_\tau^2 a_{H_d L_3 E_3^c}\delta_{3a}, \end{aligned} \tag{A21}$$

and

$$\begin{aligned} \dot{\gamma}_{H_u} &= \tfrac{3}{2}g_2^4 + \tfrac{11}{2}g_Y^4 - 3y_t^2 b_{y_t}, \\ \dot{\gamma}_{H_d} &= \tfrac{3}{2}g_2^4 + \tfrac{11}{2}g_Y^4 - 3y_b^2 b_{y_b} - y_\tau^2 b_{y_\tau}, \\ \dot{\gamma}_{Q_a} &= -8g_3^4 + \tfrac{3}{2}g_2^4 + \tfrac{11}{18}g_Y^4 - (y_t^2 b_{y_t} + y_b^2 b_{y_b})\delta_{3a}, \\ \dot{\gamma}_{U_a} &= -8g_3^4 + \tfrac{88}{9}g_Y^4 - 2y_t^2 b_{y_t}\delta_{3a}, \\ \dot{\gamma}_{D_a} &= -8g_3^4 + \tfrac{22}{9}g_Y^4 - 2y_b^2 b_{y_b}\delta_{3a}, \\ \dot{\gamma}_{L_a} &= \tfrac{3}{2}g_2^4 + \tfrac{11}{2}g_Y^4 - y_\tau^2 b_{y_\tau}\delta_{3a}, \\ \dot{\gamma}_{E_a} &= 22g_Y^4 - 2y_\tau^2 b_{y_\tau}\delta_{3a}, \end{aligned} \tag{A22}$$

where

$$\begin{aligned} b_{y_t} &= -\tfrac{16}{3}g_3^2 - 3g_2^2 - \tfrac{13}{9}g_Y^2 + 6y_t^2 + y_b^2, \\ b_{y_b} &= -\tfrac{16}{3}g_3^2 - 3g_2^2 - \tfrac{7}{9}g_Y^2 + y_t^2 + 6y_b^2 + y_\tau^2, \\ b_{y_\tau} &= -3g_2^2 - 3g_Y^2 + 3y_b^2 + 4y_\tau^2. \end{aligned} \tag{A23}$$

In our convention of the gaugino masses and A parameters, the 1-loop RG evolution of the stop trilinear coupling $A_t \equiv A_{H_u t_L t_R}$ in the MSSM is given by

$$\frac{dA_t}{d\ln Q} = \frac{1}{8\pi^2}\Big[6y_t^2 A_t + y_b^2 A_b - \Big(\frac{16}{3}g_3^2 M_3 + 3g_2^2 M_2 + \frac{13}{9}g_Y^2 M_1\Big)\Big]. \quad (A24)$$

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