

Emanations of dark matter: Muon anomalous magnetic moment, radiative neutrino mass, and novel leptogenesis at the TeV scale

Thomas Hambye,¹ Kristjan Kannike,² Ernest Ma,³ and Martti Raidal²

¹*Instituto Física Teórica, Universidad Autónoma de Madrid, Cantoblanco, Spain*

²*National Institute of Chemical Physics and Biophysics, Ravala 10, Tallinn, Estonia*

³*Physics Department, University of California, Riverside, California 92521, USA*

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The evidence for dark matter signals a new class of particles at the TeV scale, which may manifest themselves indirectly through loop effects. In a simple model we show that these loop effects may be responsible for the enhanced muon anomalous magnetic moment, for the neutrino mass, as well as for leptogenesis in a novel way. This scenario can be verified at LHC and/or ILC experiments.

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There are at present two solid pieces of evidence for physics beyond the standard model (SM) of particle interactions. One is neutrino mass and the other is dark matter. As pointed out recently [1–5], the two may be intimately related. There is however another hint for physics beyond the SM, i.e. the muon anomalous magnetic moment [6]. It may also be related to neutrino mass, as pointed out some time ago [7,8]. Here we show how all three may be connected and allow in addition *novel* realistic leptogenesis [9] at the TeV scale. We assume that the particles responsible for the *strongly enhanced* muon anomalous magnetic moment are all members of a new class of particles which are *odd* under an exactly conserved discrete Z_2 symmetry, whereas all SM particles are even. The lightest particle of this class is absolutely stable and, assuming that it is also neutral, it becomes a good candidate for the cold dark matter of the Universe [10]. In particular we propose a specific minimal model where neutrinos also obtain small radiative Majorana masses from exactly the same particles.

Consider the particles listed in Table I. There are of course the SM lepton doublets L_α and singlets l_α^c , as well as the usual Higgs doublet Φ with $\langle\phi^0\rangle = v$. The neutral leptons N_i , N_i^c are new, as well as the scalar doublet (η^+, η^0) and singlet χ^- . They transform nontrivially under the global lepton $U(1)_L$ and the Z_2 symmetries as indicated. Although Table I contains several new particles, they are optimized to encompass the various otherwise disparate issues of dark matter, muon anomalous magnetic moment, neutrino mass, as well as leptogenesis at the TeV scale. Assuming the conservation of $U(1)_L$ and Z_2 , the relevant allowed terms in the Lagrangian involving these particles are given by

$$\begin{aligned} \mathcal{L} = & f_\alpha (\nu_\alpha \phi^- + l_\alpha \bar{\phi}^0) l_\alpha^c + h_{\alpha i} (\nu_\alpha \eta^0 - l_\alpha \eta^+) N_i^c \\ & + h'_{\alpha i} l_\alpha^c \chi^- N_i + M_i N_i N_i^c + \mu (\phi^+ \eta^0 - \phi^0 \eta^+) \chi^- \\ & + \frac{1}{2} \lambda_5 (\eta^+ \Phi)^2 + \text{H.c.}, \end{aligned} \quad (1)$$

where α refers to the (l_α, l_α^c) diagonal basis, i.e. e, μ, τ mass eigenstates, and i refers to the (N_i, N_i^c) diagonal basis.

Note that N_i^c is the Dirac mass partner of N_i . Note also that neutrinos are massless at tree level because $\langle\eta^0\rangle = 0$.

Consequently, there is a direct magnetic-dipole transition from μ to μ^c , given by the diagrams of Fig. 1, involving all the new particles of Table I in the loop. Note that the existence of a muon anomalous magnetic moment is consistent with the conservation of $U(1)_L$ as well as the new Z_2 .

The diagrams of Fig. 1 can be computed exactly by using the mass eigenstates

$$X^+ = \chi^+ \cos\theta - \eta^+ \sin\theta, \quad (2)$$

$$Y^+ = \chi^+ \sin\theta + \eta^+ \cos\theta, \quad (3)$$

where $\sin\theta \cos\theta (m_X^2 - m_Y^2) = \mu v$. Their contribution to the muon anomalous magnetic moment $a_\mu = (1/2)(g - 2)_\mu$ is then given by

$$\Delta a_\mu = \frac{-\sin\theta \cos\theta}{16\pi^2} \sum_i h_{\mu i} h'_{\mu i} \frac{m_\mu}{M_i} [F(x_i) - F(y_i)], \quad (4)$$

where $x_i = m_X^2/M_i^2$, $y_i = m_Y^2/M_i^2$, and

$$F(x) = \frac{1}{(1-x)^3} [1 - x^2 + 2x \ln x]. \quad (5)$$

Let $y_i \ll x_i \simeq 1$, $M_i \sim 1$ TeV, and $(-h_{\mu i} h'_{\mu i} \sin\theta \cos\theta / 24\pi^2) \sim 10^{-5}$, we then obtain $\Delta a_\mu \sim 10^{-9}$, which compares favorably with the experimental

TABLE I. Particle content of proposed model.

particles	$SU(2) \times U(1)$	$U(1)_L$	$(-1)^L$	Z_2
$L_\alpha = (\nu_\alpha, l_\alpha)$	$(2, -1/2)$	1	-	+
l_α^c	$(1, 1)$	-1	-	+
$\Phi = (\phi^+, \phi^0)$	$(2, 1/2)$	0	+	+
N_i	$(1, 0)$	1	-	-
N_i^c	$(1, 0)$	-1	-	-
$\eta = (\eta^+, \eta^0)$	$(2, 1/2)$	0	+	-
χ^-	$(1, -1)$	0	+	-

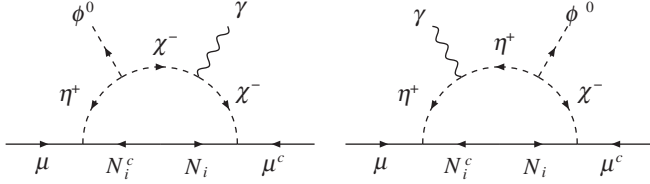


FIG. 1. Dominant contributions to muon anomalous magnetic moment.

value of [6]

$$\Delta a_\mu = [(22.4 \pm 10) \text{ to } (26.1 \pm 9.4)] \times 10^{-10}. \quad (6)$$

Because of the chirality flip in the internal fermion line, Δa_μ in our model is strongly enhanced by the factor $\mathcal{O}(v/m_\mu)$ compared to the usual result where the chirality flip occurs in the external muon line. In supersymmetry [11],

$$\begin{aligned} \Delta a_\mu^{\text{SUSY}} &= \frac{\tan\beta}{192\pi^2} \frac{m_\mu^2}{M_{\text{SUSY}}^2} (5g_2^2 + g_1^2) \\ &= 14 \tan\beta \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 10^{-10}, \end{aligned} \quad (7)$$

thus requiring a large $\tan\beta$ or a small M_{SUSY} or both. If M_{SUSY} turns out to be of order 1 TeV, it may be difficult to reconcile Eq. (7) with Eq. (6); whereas in our case, the experimental result, i.e. Eq. (6), can be obtained naturally with relatively small Yukawa couplings $h, h' \sim \mathcal{O}(10^{-1}-10^{-2})$.

As for dark matter, the usual R parity is identified as the new Z_2 of this model. The lightest particle which is odd under Z_2 can be $\text{Re}\eta^0$ or $\text{Im}\eta^0$ with mass $m_0 \sim 60$ to 80 GeV and mass splitting of a few GeV as discussed in Ref. [12]. Additional coupling to the 1 TeV singlet fermion in our more comprehensive model does not change this result. This fixes the λ_5 coupling to be $\mathcal{O}(10^{-2})$.

Consider now the *soft* explicit breaking of $U(1)_L$ down to its discrete subgroup $(-1)^L$ by the *small* Majorana mass terms

$$\frac{1}{2} m_{ij} N_i^c N_j^c + \frac{1}{2} m'_{ij} N_i N_j + \text{H.c.} \quad (8)$$

Neutrinos are still massless at tree level, but they may now acquire radiative masses in one loop as shown in Fig. 2.

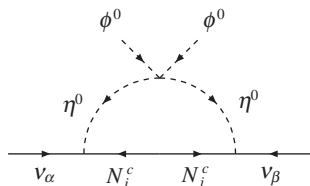


FIG. 2. Radiative Majorana neutrino mass.

As a result,

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_{i,j} h_{\alpha i} h_{\beta j} \tilde{m}_{ij}, \quad (9)$$

where $\tilde{m}_{ij} = |2\lambda_5 v^2 m_{ij} I_{ij}|$, and

$$\begin{aligned} I_{ij} &= \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{(k^2 - m_0^2)^2} \frac{1}{(k^2 - M_i^2)} \frac{1}{(k^2 - M_j^2)} \\ &= \frac{1}{16\pi^2 i (M_i^2 - M_j^2)} \left[\frac{M_i^2}{m_0^2 - M_i^2} + \frac{M_i^4 \ln(M_i^2/m_0^2)}{(m_0^2 - M_i^2)^2} \right. \\ &\quad \left. - (i \leftrightarrow j) \right]. \end{aligned} \quad (10)$$

Note that this expression for neutrino mass is not of the canonical seesaw form [13]. Parametrically it is suppressed by two powers of the heavy scale rather than one, and numerically it involves two extra suppression factors: the $\lambda_5/16\pi^2$ loop factor and the small m/M factor. The latter is due to the fact that the Majorana neutrino masses of this model are associated with the low breaking scale of global lepton number, i.e. m_{ij} which are small because they are explicit symmetry breaking terms. The situation is similar to that of the supersymmetric models of Ref. [14], but with renormalizable interactions. These successive suppressions allow naturally small neutrino masses to be generated at the TeV scale with $\mathcal{O}(10^{-2}-1)$ Yukawa couplings, leading to large production cross sections at colliders and to a large Δa_μ contribution, i.e. Eq. (4). Numerically, let $M_{i,j} \sim 1$ TeV, $m_{ij} \sim 0.1$ GeV, $h_{\alpha i} \sim \mathcal{O}(10^{-2})$, $\lambda_5 \sim \mathcal{O}(10^{-2})$, and $m_0 \sim v \sim 10^2$ GeV, then the entries of \mathcal{M}_ν are typically of order 0.1 eV. As a specific example, consider the possibility that (N_1, N_1^c) have zero Yukawa couplings and let

$$h_{\alpha i} \simeq h \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} \end{pmatrix}, \quad (11)$$

then in the basis $[\nu_e, (\nu_\mu + \nu_\tau)/\sqrt{2}, (-\nu_\mu + \nu_\tau)/\sqrt{2}]$, Eq. (9) becomes

$$\mathcal{M}_\nu \simeq h^2 \begin{pmatrix} \tilde{m}_{22} & \tilde{m}_{23} & 0 \\ \tilde{m}_{23} & \tilde{m}_{33} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (12)$$

which is a realistic representation of present neutrino-oscillation data, with $\theta_{23} = \pi/4$, $\theta_{13} = 0$, and an inverted ordering of neutrino masses, having $m_3 = 0$. Taking $h'_{\alpha i}$ to be, for example, of the form

$$h'_{\alpha i} \simeq h' \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} \end{pmatrix}, \quad (13)$$

the leading hh' contributions to $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, $\tau \rightarrow e\gamma$ vanish identically. At subleading level only nonvanishing $\tau \rightarrow \mu\gamma$ occurs from the hh and $h'h'$ contributions

suppressed by an extra chirality flip in the external fermion line. Those have been calculated in [7], and we estimate

$$f_{M,E} \sim \frac{1}{4} \frac{(hh + h'h')}{hh'} \frac{m_\tau^2}{m_\mu v} \Delta a_\mu, \quad (14)$$

where $f_{M,E}$ are the form factors giving rise to $\tau \rightarrow \mu \gamma$ and Δa_μ is given by Eq. (4). Numerically we predict

$$\text{Br}(\tau \rightarrow \mu \gamma) \sim 4.6 \times 10^{-11} \left(\frac{h'}{h} + \frac{h}{h'} \right)^2, \quad (15)$$

which is well below the experimental bound $\text{Br}(\tau \rightarrow \mu \gamma) < 6.8 \times 10^{-8}$.

To have successful leptogenesis at the TeV scale, we assume (N_1, N_1^c) to be the lightest among the 3 pairs of singlet neutral fermions, and $h_{\alpha 1}, h'_{\alpha 1}$ to be very small, i.e. of order 10^{-7} instead of zero. This allows the decay of (N_1, N_1^c) to satisfy the out-of-equilibrium condition, but will not affect Eq. (11), as far as \mathcal{M}_ν and Δa_μ are concerned. Because Δm_{atm}^2 and Δm_{sol}^2 are induced by the two heavier generations $(N_{2,3}, N_{2,3}^c)$, this scenario predicts the lightest neutrino to be almost massless.

Consider first the 2×2 mass matrix spanning N_1 and N_1^c and rotate it by $\pi/4$, i.e.

$$\begin{aligned} & \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} m'_{11} & M_1 \\ M_1 & m_{11} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \\ & = \begin{pmatrix} -M_1 + A & B \\ B & M_1 + A \end{pmatrix}, \end{aligned} \quad (16)$$

where $A = (m'_{11} + m_{11})/2$, $B = (m'_{11} - m_{11})/2$. Choose phases so that $M > 0$, $A > 0$ are real and $B = |B|e^{i\alpha}$. Let the above mass matrix be diagonalized by

$$\begin{pmatrix} ce^{i\beta} & -s \\ s & ce^{-i\beta} \end{pmatrix} \quad (17)$$

on the left and its transpose on the right, where $c = \cos\theta$ and $s = \sin\theta$. Then

$$\sin\beta = \frac{-M_1 \tan\alpha}{\sqrt{A^2 + M_1^2 \tan^2\alpha}}, \quad \cos\beta = \frac{A}{\sqrt{A^2 + M_1^2 \tan^2\alpha}}, \quad (18)$$

and

$$\tan 2\theta = |B| \cos\alpha \frac{\sqrt{A^2 + M_1^2 \tan^2\alpha}}{AM_1}. \quad (19)$$

If $\alpha = 0$, there is no CP violation. On the other hand, if $A^2 \ll M_1^2 \tan^2\alpha$, then

$$e^{i\beta} = \frac{A}{M_1 \tan\alpha} - i, \quad e^{2i\beta} = -1 - \frac{2iA}{M_1 \tan\alpha}. \quad (20)$$

The mass eigenvalues become $(M_1 - A \cos 2\theta - |B| \sin 2\theta \sin\alpha) e^{2i\delta}$, and $-(M_1 + A \cos 2\theta + |B| \sin 2\theta \sin\alpha) e^{-2i\delta}$, where $\delta = (c^2 A / M_1 \tan\alpha) + (s c |B| \cos\alpha / M_1)$. Absorbing their phases into the definitions of the mass eigenstates ψ_1, ψ'_1 , we then have

$$\psi_1 = \frac{e^{i\delta}}{\sqrt{2}} [(ce^{-i\beta} - s)N_1 - (ce^{-i\beta} + s)N_1^c], \quad (21)$$

$$\psi'_1 = \frac{ie^{-i\delta}}{\sqrt{2}} [(ce^{i\beta} + s)N_1 + (ce^{i\beta} - s)N_1^c]. \quad (22)$$

As a result, the self-energy contributions to the decay of ψ_1 through N_1 and N_1^c with ψ'_1 as an intermediate state generate a lepton asymmetry

$$\begin{aligned} \epsilon_1 &= \frac{1}{64\pi} \frac{\text{Im}[e^{-4i\delta}(ce^{i\beta} - s)^2(ce^{i\beta} + s)^2]}{\Delta M_1 / M_1} \\ &\quad \times \left(\frac{4(\sum_\alpha |h_{\alpha 1}|^2)^2 - (\sum_\alpha |h'_{\alpha 1}|^2)^2}{2\sum_\alpha |h_{\alpha 1}|^2 + \sum_\alpha |h'_{\alpha 1}|^2} \right) \\ &= -\frac{1}{64\pi} \frac{|B|^2 \sin\alpha \cos\alpha}{A^2 + |B|^2 \sin^2\alpha} \\ &\quad \times \left(\frac{4(\sum_\alpha |h_{\alpha 1}|^2)^2 - (\sum_\alpha |h'_{\alpha 1}|^2)^2}{2\sum_\alpha |h_{\alpha 1}|^2 + \sum_\alpha |h'_{\alpha 1}|^2} \right). \end{aligned} \quad (23)$$

This is a novel mechanism because there is no CP violation in the Yukawa couplings. Instead it comes from the Majorana phase of the (N_1, N_1^c) mass matrix. Unfortunately, the out-of-equilibrium condition for ψ_1 decay requires both $|h_1|$ and $|h'_1|$ to be of order 10^{-7} ; hence the above contribution to the lepton asymmetry is negligible and cannot explain the observed baryon asymmetry of the Universe.

We now consider the contributions of (N_2, N_2^c) to the lepton asymmetry in the decays of ψ_1 and ψ'_1 . We note first that if $m'_{11} = m_{11} = 0$ ($m'_{22} = m_{22} = 0$), so that N_1, N_1^c (N_2, N_2^c) combine to form an exactly Dirac fermion, then there is no contribution to the asymmetry because lepton number is not broken in that system. For simplicity let $m'_{11} = m_{11}$ so that the two mass eigenvalues are $-M + m_{11}$ and $M + m_{11}$ corresponding to the two mass eigenstates $i(N_1 - N_1^c)/\sqrt{2}$ and $(N_1 + N_1^c)/\sqrt{2}$ respectively. The (N_2, N_2^c) system is described by Eqs. (16)–(22) replacing “1” by “2”. The total lepton asymmetry in ψ_1 and ψ'_1 decays receives contributions from the h interactions by themselves, the h' interactions by themselves, as well as their interference. The general expression is long and, for the sake of simplicity, we present only the self-energy h' contribution. [Note that neutrino mass does not depend on h' .]

$$\epsilon_1^{h'} = \frac{\Delta M_1 \Delta M_2}{16\pi(M_2^2 - M_1^2)^3} \frac{\text{Im}[\sum_{\alpha} (h'_{\alpha 1} h_{\alpha 2}^{h'})^2 (\exp(-2i\theta_2)(M_1^4 + 6M_1^2 M_2^2 + M_2^4) - (\sin 2\theta_2 / \tan \alpha_2)(M_1^4 - M_2^4))]}{2\sum_{\alpha} |h_{\alpha 1}|^2 + \sum_{\alpha} |h'_{\alpha 1}|^2}. \quad (24)$$

Here $\Delta M_1 = 2m_{11}$, $\Delta M_2 = 2A_2 / \cos 2\theta_2$, and we have expanded assuming $\Delta M_1 \ll M_1$, $\Delta M_2 \ll M_2$. Note that CP violation is again not necessary in the Yukawa couplings. Whereas this asymmetry is suppressed by $\Delta M_1 \Delta M_2$, it is unsuppressed by $h'_{\alpha 2}$ and enhanced by $(M_2^2 - M_1^2)^3$. Notice that, in our example model given by Eqs. (11) and (13), h'_{e2} is not constrained by any observable, and can lead to large CP asymmetry via Eq. (24). The value $\epsilon_1 \sim 10^{-6}$ needed to explain the observed baryon asymmetry of the Universe may be obtained consistently with the neutrino mass parameters and Δa_{μ} in three ways.

- (1) If $M_1 \approx M_2$, there is a resonance factor which comes at the cubic power, not in the first power as in the canonical leptogenesis. This enhancement factor can easily compensate the $\Delta M_{1,2}/M_{1,2}$ and Yukawa coupling suppressions. It works essentially as ordinary resonant leptogenesis [15] with larger asymmetries due to more freedom from the h' couplings.
- (2) If $M_2 \gg M_1$, successful leptogenesis can be achieved in a nonresonant way provided m_{11} or m'_{11} is not much smaller than M_1 . Phenomenologically m_{11} and m'_{11} are not constrained by the neutrino mass measurements and can be large. A set of parameters satisfying all constraints is for example: $M_1 = 2$ TeV, $M_2 = 5$ TeV, $\Delta M_1/M_1 \sim \mathcal{O}(1)$, $\Delta M_2/M_2 \sim \mathcal{O}(10^{-4})$, $h_2 \sim \mathcal{O}(10^{-2})$, $h'_2 \sim \mathcal{O}(10^{-1})$, $h_1 \sim h'_1 < 10^{-7}$, $\lambda_5 \sim \mathcal{O}(10^{-2})$. The most dangerous effects here are the $\Delta L = 2$ washout processes mediated by off-shell N_2 system. We have estimated them by solving the corresponding Boltzmann equations and we are confident that the set of parameters above is consistent with the observed baryon asymmetry of the Universe. The discussion of the washout effects is similar to the one of Ref. [16], where nonresonant leptogenesis at TeV scale from h' type coupling with just 3 right-handed neutrinos (i.e. no α phases) has been considered (see also Ref. [17]).
- (3) Finally we may replace (N_1, N_1^c) in Table I with a single neutral fermion S with zero lepton number, leaving the heavier generations unchanged. The soft breaking of lepton number allows S to mix slightly

with $(N_{2,3}, N_{2,3}^c)$ and the decay of S with very much suppressed Yukawa couplings is another viable leptogenesis scenario [18].

Our model can be directly verified at LHC and/or ILC experiments by discovering $N_i, N_i^c, (\eta^+, \eta^0)$ and χ^+ which all have $\mathcal{O}(10^{-1})$ couplings except N_1 and N_1^c . While the discovery at LHC requires some of those particles to have a mass of order few hundred GeV, at ILC one can reach TeV scale masses.

In conclusion, we have shown that dark matter, Δa_{μ} , neutrino mass, and leptogenesis may all be attributed to particles which are *odd* under an exact Z_2 . We have presented an explicit model in which Δa_{μ} is strongly enhanced, thus explaining the experimental result without unnaturally large Yukawa couplings. The observed neutrino masses are radiatively induced at the TeV scale, with extra suppression factors compared to the canonical see-saw: a loop factor, *small soft* lepton number breaking terms, and an extra power of the heavy scale. There are no constraints on neutrino mass or Δa_{μ} coming from $\mu \rightarrow e\gamma$. Novel and successful leptogenesis at the TeV scale occurs from (N_1, N_1^c) decays due to the interference with $(N_{2,3}, N_{2,3}^c)$. It does not require CP violation from Yukawa couplings and can be induced by new phases coming from the (N_i, N_i^c) mass matrices. It predicts however a (nearly) massless neutrino. An example set of model parameters consistent with all experimental data is presented above Eq. (11) and after Eq. (24). Dark matter in the form of η^0 as well as all the other new particles (except N_1, N_1^c because of their small couplings) in Table I can be directly produced and discovered at the LHC or at other future accelerators, such as ILC. Although there are certainly other possibilities to explain the considered phenomenology with help of particles odd under the discrete symmetry associated with dark matter, our model can be considered as an explicit and minimal example of a more general class of TeV scale models.

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