Strong decays of charmed baryons

Chong Chen, Xiao-Lin Chen, Xiang Liu,[*] Wei-Zhen Deng, and Shi-Lin Zhu[†]

Department of physics, Peking University, Beijing, 100871, China
(Received 5 April 2007; published 16 May 2007)

There has been important experimental progress in the sector of heavy baryons in the past several years. We study the strong decays of the S-wave, P-wave, D-wave, and radially excited charmed baryons using the $^3P_0$ model. After comparing the calculated decay pattern and total width with the available data, we discuss the possible internal structure and quantum numbers of those charmed baryons observed recently.

I. INTRODUCTION

BABAR and Belle Collaborations observed several excited charmed baryons: $\Lambda_c(2880\,2940)^+$, $\Xi_c(2980\,3077)^{+,0}$, and $\Omega_c(2768)^0$ last year [1–5], which inspired several investigations of these states in literature [6–9]. We collect the experimental information of these recently observed hadrons in Table I. Their quantum numbers have not been determined except $\Lambda_c(2880)^+$. In order to understand their structures using the present experimental information, we study the strong decay pattern of the excited charmed baryons systematically in this work. In the past decades, there has been some research work on heavy baryons [8,11,12].

The quantum numbers and decay widths of S-wave and some P-wave charmed baryons are known [13]. We first systematically analyze their strong decays in the framework of the $^3P_0$ strong decay model. Accordingly one can extract the parameters and estimate the accuracy of the $^3P_0$ model when it is applied in the charmed baryon system. Then we go one step further and extend the same formalism to study the decay patterns of these new charmed baryons $\Lambda_c(2880\,2940)^+$, $\Xi(2980\,3077)^{+,0}$ under different assignments of their quantum numbers. After comparing the theoretical results with the available experimental data, we can learn their favorable quantum numbers and assignments in the quark model.

Very recently CDF Collaboration reported four particles [14], which are consistent with $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ predicted in the quark model [15]. Their masses are $M_{\Sigma_b^+} = 5808^{+2.0}_{-2.3} \pm 1.7$ MeV, $M_{\Sigma_b^-} = 5816^{+1.0}_{-1.0} \pm 1.7$ MeV, $M_{\Sigma_b^{*+}} = 5829^{+1.6}_{-1.8} \pm 1.7$, $M_{\Sigma_b^{*-}} = 5837^{+2.1}_{-1.9} \pm 1.7$ MeV. The mass splitting between Σ_b and Σ_b^* was discussed in Refs. [16,17] while the strong decays of $\Sigma_b^{\pm(*)}$ were studied in Ref. [18]. As a by-product, we also calculate the strong decays of $\Sigma_b^{(*)\pm}$ and other S-wave bottom baryons in this work.

This paper is organized as follows. We give a short theoretical review of S-wave, P-wave, and D-wave charmed baryons and introduce our notations for them in Sec. II. Then we give a brief review of the $^3P_0$ model in Sec. III. We present the strong decay amplitudes of charmed baryons in Sec. IV. Section V is the numerical results. The last section is our discussion and conclusion. Some lengthy formulae are collected in the appendix.

II. THE NOTATIONS AND CONVENTIONS OF CHARMED BARYON

We first introduce our notations for the excited charmed baryons. Inside a charmed baryon there are one charm quark and two light quarks (u, d, or s). It belongs to either the symmetric 6_F or antisymmetric $\bar{3}_F$ flavor representation (see Fig. 1). For the S-wave charmed baryons, the total color-flavor-spin wave function and color wave function must be symmetric and antisymmetric, respectively. Hence the spin of the two light quarks is $S = 1$ for 6_F or $S = 0$ for $\bar{3}_F$. The angular momentum and parity of the S-wave charmed baryons are $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$ for 6_F and $J^P = \frac{1}{2}^+$ for $\bar{3}_F$. The names of the S-wave charmed baryons are listed in Fig. 1, where we use the star to denote $\frac{3}{2}^+$ baryons and the prime to denote the $J^P = \frac{1}{2}^+$ baryons in the 6_F representation.

In Fig. 2 we introduce our notations and conventions for the P-wave charmed baryons. l_ρ is the orbital angular momentum between the two light quarks while l_λ denotes the orbital angular momentum between the charm quark and the two light quark system. We use the prime to label the Ξ_{cJ_l} baryons in the 6_F representation and the tilde to discriminate the baryons with $l_\rho = 1$ from that with $l_\lambda = 1$.

The notation for the D-wave charmed baryons is more complicated (see Fig. 3). Besides the prime, l_ρ and l_λ defined above, we use the hat and check to denote the charmed baryons with $l_\rho = 2$ and $l_\rho = 1$, respectively. For the baryons with $l_\rho = 1$ and $l_\lambda = 1$, we use the superscript L to denote the different total angular momentum in $\check{\Lambda}_{cJ_l}^L$, $\check{\Sigma}_{cJ_l}^L$, and $\check{\Xi}_{cJ_l}^L$.

---

*Electronic address: xiangliu@pku.edu.cn
†Electronic address: zhusl@phy.pku.edu.cn

TABLE I. A summary of recently observed charmed baryons by *BABAR* and Belle Collaborations.

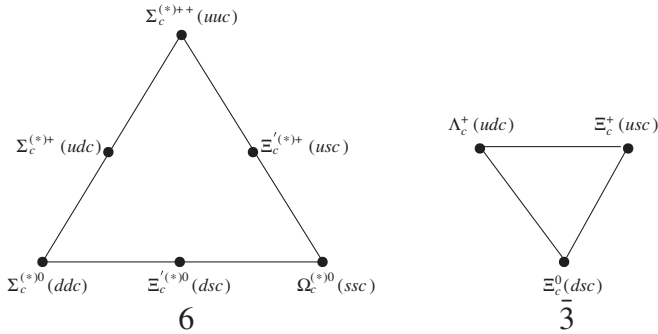| State	Mass and width (MeV)	Decay channels in experiments	Other information
$\Lambda_c(2880)^+$	$2881.5 \pm 0.3, < 8$ [10]	$\Lambda_c \pi^+ \pi^-$	J^P favors $\frac{5}{2}^+$ [2],
	$2881.9 \pm 0.1 \pm 0.5, 5.8 \pm 1.5 \pm 1.1$ [1]	$D^0 p$	$\frac{\Gamma(\Sigma_c^\star(2520)\pi^\pm)}{\Gamma(\Sigma_c(2455)\pi^\pm)} = 0.225 \pm 0.062 \pm 0.025$ [2]
	$2881.2 \pm 0.2^{+0.4}_{-0.3}, 5.5^{+0.7}_{-0.5} \pm 0.4$ [2]	$\Sigma_c^{\star 0, ++}(2520)\pi^{+,-}$	
$\Lambda_c(2940)^+$	$2939. \pm 1.3 \pm 1.0, 17.5 \pm 5.2 \pm 5.9$ [1]	$D^0 p$	—
	$2937.9 \pm 1.0^{+1.8}_{-0.4}, 10 \pm 4 \pm 5$ [2]	$\Sigma_c(2455)^{0,++}\pi^{+,-}$	
$\Xi_c(2980)^+$	$2967.1 \pm 1.9 \pm 1.0, 23.6 \pm 2.8 \pm 1.3$ [3]	$\Lambda_c^+ K^- \pi^+$	—
	$2978.5 \pm 2.1 \pm 2.0, 43.5 \pm 7.5 \pm 7.0$ [4]	$\Lambda_c^+ K^- \pi^+$	
$\Xi_c(2980)^0$	$2977.1 \pm 8.8 \pm 3.5, 43.5$ [4]	$\Lambda_c^+ K_S^0 \pi^-$	—
$\Xi_c(3077)^+$	$3076.4 \pm 0.7 \pm 0.3, 6.2 \pm 1.6 \pm 0.5$ [3]	$\Lambda_c^+ K^- \pi^+$	—
	$3076.7 \pm 0.9 \pm 0.5, 6.2 \pm 1.2 \pm 0.8$ [4]	$\Lambda_c^+ K^- \pi^+$	
$\Xi_c(3077)^0$	$3082.8 \pm 1.8 \pm 1.5, 5.2 \pm 3.1 \pm 1.8$ [4]	$\Lambda_c^+ K_S^0 \pi^-$	—
$\Omega_c(2768)^0$	2768.3 ± 3.0 [5]	$\Omega_c^0 \gamma$	$J^P = \frac{3}{2}^+$

FIG. 1. The SU(3) flavor multiplets of charmed baryons.

FIG. 2. The notations for P-wave charmed baryons. $f_S(6_F)$ and $f_A(\bar{3}_F)$ denote the SU(3) flavor representation. $S_{q_1 q_2}$ is the total spin of the two light quarks. L denotes the total orbital angular momentum of the charmed baryon system.

III. THE $^3P_0$ MODEL

The $^3P_0$ model was first proposed by Micu [19] and further developed by Yaouanc *et al.* later [20–22]. Now this model is widely used to study the strong decays of hadrons [23–30].

According to this model, a pair of quarks with $J^{PC} = 0^{++}$ is created from the vacuum when a hadron decays, which is shown in Fig. 4 for the baryon decay process $A \to B + C$. The new $q\bar{q}$ pair created from the vacuum together with the qqq within the initial baryon regroup into the outgoing meson and baryon via the quark rearrangement process. In the nonrelativistic limit, the transition operator is written as

$$T = -3\gamma \sum_m \langle 1m; 1-m|00\rangle \int d^3\mathbf{k}_4 d^3\mathbf{k}_5 \delta^3(\mathbf{k}_4 + \mathbf{k}_5)$$
$$\times \mathcal{Y}_1^m\left(\frac{\mathbf{k}_4 - \mathbf{k}_5}{2}\right)\chi_{1,-m}^{45}\varphi_0^{45}\omega_0^{45}b_{4i}^\dagger(\mathbf{k}_4)d_{5j}^\dagger(\mathbf{k}_5), \quad (1)$$

where i and j are the color indices of the created quark and antiquark. $\varphi_0^{45} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\omega_0^{45} = \delta_{ij}$ for

the flavor and color singlet, respectively. $\chi_{1,-m}^{45}$ is for the spin triplet state. $\mathcal{Y}_1^m(\mathbf{k}) \equiv |\mathbf{k}|Y_1^m(\theta_k, \phi_k)$ is a solid harmonic polynomial corresponding to the p-wave quark pair. γ is a dimensionless constant related to the strength of the quark pair creation from the vacuum, which was extracted by fitting to data. The hadron and meson state are defined as, respectively, according to the definition of the mock state [31]

(a) $l_\rho = 0$, $l_\lambda = 2$

$f_S(6)$: $L = 2 \otimes S_{q_1 q_2} = 1$

- $J_l = 1$: $\Sigma_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$ $\Xi'_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$
- $J_l = 2$: $\Sigma_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$ $\Xi'_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$
- $J_l = 3$: $\Sigma_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$ $\Xi'_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$

$f_A(\bar{3})$: $L = 2 \otimes S_{q_1 q_2} = 0 \implies J_l = 2$: $\Lambda_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$ $\Xi_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$

(b) $l_\rho = 2$, $l_\lambda = 0$

$f_S(6)$: $L = 2 \otimes S_{q_1 q_2} = 1$

- $J_l = 1$: $\hat{\Sigma}_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$ $\hat{\Xi}'_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$
- $J_l = 2$: $\hat{\Sigma}_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$ $\hat{\Xi}'_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$
- $J_l = 3$: $\hat{\Sigma}_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$ $\hat{\Xi}'_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$

$f_A(\bar{3})$: $L = 2 \otimes S_{q_1 q_2} = 0 \implies J_l = 2$: $\hat{\Lambda}_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$ $\hat{\Xi}_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$

(c) $l_\rho = 1$, $l_\lambda = 1$

$f_A(\bar{3})$

$L = 1 \otimes S_{q_1 q_2} = 1$
- $J_l = 0$: $\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$ $\check{\Xi}^1_{c0}(\frac{1}{2}^+)$
- $J_l = 1$: $\check{\Lambda}^1_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$ $\check{\Xi}^1_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$
- $J_l = 2$: $\check{\Lambda}^1_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$ $\check{\Xi}^1_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$

$L = 0 \otimes S_{q_1 q_2} = 1 \implies J_l = 1$: $\check{\Lambda}^0_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$ $\check{\Xi}^0_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$

$L = 2 \otimes S_{q_1 q_2} = 1$
- $J_l = 1$: $\check{\Lambda}^2_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$ $\check{\Xi}^2_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$
- $J_l = 2$: $\check{\Lambda}^2_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$ $\check{\Xi}^2_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$
- $J_l = 3$: $\check{\Lambda}^2_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$ $\check{\Xi}^2_{c3}(\frac{5}{2}^+, \frac{7}{2}^+)$

$f_S(6)$
- $L = 0 \otimes S_{q_1 q_2} = 0 \implies J_l = 0$: $\check{\Sigma}^0_{c0}(\frac{1}{2}^+)$ $\check{\Xi}'^0_{c0}(\frac{1}{2}^+)$
- $L = 1 \otimes S_{q_1 q_2} = 0 \implies J_l = 1$: $\check{\Sigma}^1_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$ $\check{\Xi}'^1_{c1}(\frac{1}{2}^+, \frac{3}{2}^+)$
- $L = 2 \otimes S_{q_1 q_2} = 0 \implies J_l = 2$: $\check{\Sigma}^2_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$ $\check{\Xi}'^2_{c2}(\frac{3}{2}^+, \frac{5}{2}^+)$
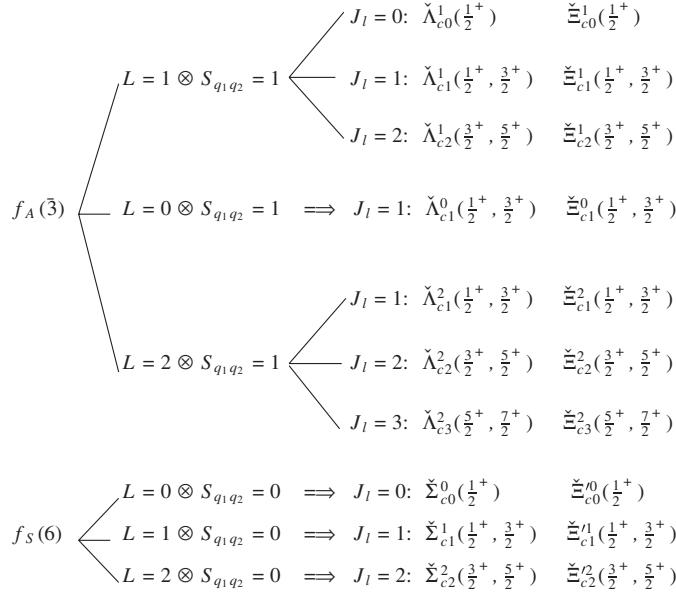
FIG. 3. The notations for the D-wave charmed baryons.

$$|A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\mathbf{P}_A)\rangle = \sqrt{2E_A} \sum_{M_{L_A},M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A}\rangle \int d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{P}_A)$$

$$\times \psi_{n_A L_A M_{L_A}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \chi^{123}_{S_A M_{S_A}} \varphi^{123}_A \omega^{123}_A | q_1(\mathbf{k}_1) q_2(\mathbf{k}_2) q_3(\mathbf{k}_3)\rangle, \tag{2}$$

$$|B(n_B^{2S_B+1}L_{BJ_BM_{J_B}})(\mathbf{P}_B)\rangle = \sqrt{2E_B} \sum_{M_{L_B},M_{S_B}} \langle L_B M_{L_B} S_B M_{S_B} | J_B M_{J_B}\rangle \int d^3\mathbf{k}_a d^3\mathbf{k}_b \delta^3(\mathbf{k}_a + \mathbf{k}_b - \mathbf{P}_B)\psi_{n_B L_B M_{L_B}}(\mathbf{k}_a, \mathbf{k}_b)$$

$$\times \chi^{ab}_{S_B M_{S_B}} \varphi^{ab}_B \omega^{ab}_B | q_a(\mathbf{k}_a) \bar{q}_b(\mathbf{k}_b)\rangle \tag{3}$$
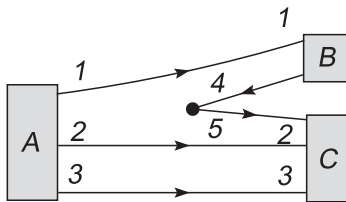
and satisfy the normalization condition

FIG. 4. The decay process of $A \to B + C$ in the $^3P_0$ model.

$$\langle A(\mathbf{P}_A)|A(\mathbf{P}'_A)\rangle = 2E_A\delta^3(\mathbf{P}_A - \mathbf{P}'_A), \qquad \langle B(\mathbf{P}_B)|B(\mathbf{P}'_B)\rangle = 2E_B\delta^3(\mathbf{P}_B - \mathbf{P}'_B). \tag{4}$$

The subscripts 1, 2, 3 denote the quarks of parent hadron A. a and b refer to the quark and antiquark within the meson B, respectively. $\mathbf{k}_i(i = 1, 2, 3)$ are the momentum of quarks in hadron A. $\mathbf{k}_a$ and $\mathbf{k}_b$ are the momentum of the quark and antiquark in meson B. $\mathbf{P}_{A(B)}$ represents the momentum of state $A(B)$. $S_{A(B)}$ and $J_{A(B)}$ denote the total spin and the total angular momentum of state $A(B)$.

The S-matrix is defined as

$$\langle f|S|i\rangle = I - i2\pi\delta(E_f - E_i)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}. \tag{5}$$

The helicity amplitude of the process $A \to B + C$ in the center of mass frame of meson A is

$$\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(A \to BC) = \sqrt{8E_AE_BE_C}\gamma \sum_{\substack{M_{L_A}, M_{S_A}, \\ M_{L_B}, M_{S_B}, \\ M_{L_C}, M_{S_C}, m}} \langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle\langle L_BM_{L_B}S_BM_{S_B}|J_BM_{J_B}\rangle$$

$$\times \langle L_CM_{L_C}S_CM_{S_C}|J_CM_{J_C}\rangle\langle 1m; 1 - m|00\rangle\langle \chi^{235}_{S_CM_{S_C}}\chi^{14}_{S_BM_{S_B}}|\chi^{123}_{S_AM_{S_A}}\chi^{45}_{1-m}\rangle\langle \varphi^{235}_C\varphi^{14}_B|\varphi^{123}_A\varphi^{45}_0\rangle$$

$$\times I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{p}), \tag{6}$$

where the spatial integral $I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{p})$ is defined as

$$I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{p}) = \int d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3 d^3\mathbf{k}_4 d^3\mathbf{k}_5 \delta^3(\mathbf{k}_4 + \mathbf{k}_5)\delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{P}_A)\delta^3(\mathbf{k}_1 + \mathbf{k}_4 - \mathbf{P}_B)\delta^3(\mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_5 - \mathbf{P}_C)$$

$$\times \psi^*_{n_BL_BM_{L_B}}(\mathbf{k}_1, \mathbf{k}_4)\psi^*_{n_CL_CM_{L_C}}(\mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_5)\psi_{n_AL_AM_{L_A}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\mathcal{Y}^m_1\left(\frac{\mathbf{k}_4 - \mathbf{k}_5}{2}\right). \tag{7}$$

$\langle \chi^{235}_{S_CM_{S_C}}\chi^{14}_{S_BM_{S_B}}|\chi^{123}_{S_AM_{S_A}}\chi^{45}_{1-m}\rangle$ and $\langle \varphi^{235}_C\varphi^{14}_B|\varphi^{123}_A\varphi^{45}_0\rangle$ denote the spin and flavor matrix element, respectively.

The decay width of the process $A \to B + C$ is

$$\Gamma = \pi^2\frac{|\mathbf{p}|}{M_A^2}\frac{s}{2J_A + 1}\sum_{M_{J_A},M_{J_B},M_{J_C}}|\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}|^2,$$

where $|\mathbf{p}|$ is the momentum of the daughter baryon in the parent's center of mass frame. $s = 1/(1 + \delta_{BC})$ is a statistical factor which is needed if B and C are identical particles.

IV. THE STRONG DECAYS OF CHARMED BARYON

According to the $^3P_0$ model, the decay occurs through the recombination of the five quarks from the initial charmed baryon and the created quark pair. So there are

three ways of regrouping:

$$\mathcal{A}(q_1, q_2, c_3) + \mathcal{P}(q_4, \bar{q}_5) \to \mathcal{B}(q_2, q_4, c_3) + \mathcal{C}(q_1, \bar{q}_5), \tag{8}$$

$$\mathcal{A}(q_1, q_2, c_3) + \mathcal{P}(q_4, \bar{q}_5) \to \mathcal{B}(q_1, q_4, c_3) + \mathcal{C}(q_2, \bar{q}_5), \tag{9}$$

$$\mathcal{A}(q_1, q_2, c_3) + \mathcal{P}(q_4, \bar{q}_5) \to \mathcal{B}(q_1, q_2, q_4) + \mathcal{C}(c_3, \bar{q}_5), \tag{10}$$

where q_i and c_3 denote the light quark and charm quark, respectively.

When the excited charmed baryon decays into a charmed baryon plus a light meson as shown in Eqs. (8) and (9), the total decay amplitude reads

$$M^{M_{J_A}M_{J_B}M_{J_C}} = -2\gamma\sqrt{8E_AE_BE_C}\sum_{M_{\rho_A}}\sum_{M_{L_A}}\sum_{M_{\rho_B}}\sum_{M_{L_B}}\sum_{m_1,m_3,m_4,m}\langle J_{12}M_{12}; s_3m_3|J_AM_{J_A}\rangle\langle l_{\rho A}m_{\rho A}; l_{\lambda A}m_{\lambda A}|L_AM_{L_A}\rangle$$

$$\times \langle L_AM_{L_A}; S_{12}m_{12}|J_{12}M_{12}\rangle\langle s_1m_1; s_2m_2|S_{12}m_{12}\rangle\langle J_{14}M_{14}; s_3m_3|J_BM_{J_B}\rangle\langle l_{\rho B}m_{\rho B}; l_{\lambda B}m_{\lambda B}|L_BM_{L_B}\rangle$$

$$\times \langle L_BM_{L_B}; S_{14}m_{14}|J_{14}M_{14}\rangle\langle s_1m_1; s_4m_4|S_{14}m_{14}\rangle\langle 1m; 1 - m|00\rangle\langle s_4m_4; s_5m_5|1 - m\rangle$$

$$\times \langle L_CM_{L_C}; S_CM_C|J_CM_{J_C}\rangle\langle s_2m_2; s_5m_5|S_CM_C\rangle \times \langle \phi^{1,4,3}_B\phi^{2,5}_C|\phi^{4,5}_0\phi^{1,2,3}_A\rangle \times I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{p}), \tag{11}$$

where the prefactor 2 in front of γ arises from the fact that the amplitude from Eq. (8) is the same as that from Eq. (9).

The overlap integral in the momentum space is

$$I_{M_{L_B},M_{L_C}}^{M_{L_A},m}(\mathbf{p}) = \delta^3(\mathbf{P}_B - \mathbf{P}_C)$$

$$\times \int d^3\mathbf{p}_1 d^3\mathbf{p}_2 \psi_B^*(l_{\rho B}, m_{\rho B}, l_{\lambda B}, m_{\lambda B})$$

$$\times \psi_C^*(L_C M_{L_C}) \mathcal{Y}_1^m\left(\frac{\mathbf{p}_4 - \mathbf{p}_5}{2}\right)$$

$$\times \psi_A(l_{\rho A}, m_{\rho A}, l_{\lambda A}, m_{\lambda A}). \quad (12)$$

Since all hadrons in the final states are S-wave in this work, Eq. (12) can be further expressed as

$$I_{M_{L_B},M_{L_C}}^{M_{L_A},m}(\mathbf{p}) = \delta^3(\mathbf{P}_B - \mathbf{P}_C)\Pi(l_{\rho A}, m_{\rho A}, l_{\lambda A}, m_{\lambda A}, m), \quad (13)$$

where we have used the harmonic oscillator wave functions for both the meson and baryon. The expressions of $\Pi(l_{\rho A}, m_{\rho A}, l_{\lambda A}, m_{\lambda A}, m)$ for the decays of the S-wave, P-wave, and D-wave charmed baryons are collected in the appendix. We also move the lengthy expressions of momentum space integration of the S-wave, P-wave, and D-wave charmed baryons to the appendix.

V. NUMERICAL RESULTS

The decay widths of charmed baryons from the $^3P_0$ model involve several parameters: the strength of quark pair creation from vacuum γ, the R value in the harmonic oscillator wave function of the meson, and the $\alpha_{\rho,\lambda}$ in the baryon wave functions. We follow the convention of Ref. [32] and take $\gamma = 13.4$, which is considered as a universal parameter in the $^3P_0$ model. The R value of π and K mesons is 2.1 GeV^{-1} [32] while it is $R = 2.3$ GeV^{-1} for the D meson [33]. $\alpha_\rho = \alpha_\lambda = 0.5$ GeV for the proton and Λ [30]. For the S-wave charmed bary-

ons, the parameters α_ρ and α_λ in the harmonic oscillator wave functions can be fixed to reproduce the mass splitting through the contact term in the potential model [34]. Their values are $\alpha_\rho = 0.6$ GeV and $\alpha_\lambda = 0.6$ GeV. For the P-wave and D-wave charmed baryons, α_ρ and α_λ are

TABLE III. The strong decay widths of S-wave bottom baryons Σ_b, Σ_b^*, Ξ_b', and Ξ_b^*. Here all results are in units of MeV.

	J^P	Channel	Width	Experimental results [14]
Σ_b^+	$\frac{1}{2}^+$	$\Lambda_b^0 \pi^+$	3.5	~8
Σ_b^-	$\frac{1}{2}^+$	$\Lambda_b^0 \pi^-$	4.7	
Σ_b^{*+}	$\frac{3}{2}^+$	$\Lambda_b^0 \pi^+$	7.5	~15
Σ_b^{*-}	$\frac{3}{2}^+$	$\Lambda_b^0 \pi^-$	9.2	
Ξ_b'	$\frac{1}{2}^+$	$\Xi_b \pi$	0.10	—
Ξ_b^*	$\frac{3}{2}^+$	$\Xi_b \pi$	0.85	—

TABLE IV. The decay widths of P-wave charmed baryons $\Lambda_c^+(2593, 2625)$ and $\Xi_c^{+,0}(2790, 2815)$ with the fixed structure and quantum number assignments. Here all results are in units of MeV.

	Assignment	Channel	Γ	Γ_{exp} [13]
$\Lambda_c^+(2593)$	$\Lambda_{c1}(\frac{1}{2}^-)$	$\Sigma_c^{++}\pi^-$	3.4	$3.6^{+2.0}_{-1.3}$
		$\Sigma_c^+\pi^0$	6.4	
		$\Sigma_c^0\pi^+$	3.4	
$\Lambda_c^+(2625)$	$\Lambda_{c1}(\frac{3}{2}^-)$	$\Sigma_c^{++}\pi^-$	1.9×10^{-3}	<0.10
		$\Sigma_c^+\pi^0$	2.6×10^{-3}	<1.9
		$\Sigma_c^0\pi^+$	1.9×10^{-3}	<0.10
$\Xi_c^+(2790)$	$\Xi_{c1}(\frac{1}{2}^-)$	$\Xi_c'^+\pi^0$	5.0	<15
		$\Xi_c'^0\pi^+$	4.9	
$\Xi_c^0(2790)$	$\Xi_{c1}(\frac{1}{2}^-)$	$\Xi_c'^+\pi^-$	5.2	<12
		$\Xi_c'^0\pi^0$	5.1	
$\Xi_c^+(2815)$	$\Xi_{c1}(\frac{3}{2}^-)$	$\Xi_c^{\star+}\pi^0$	2.7	<3.5
		$\Xi_c^{\star0}\pi^+$	2.6	
$\Xi_c^0(2815)$	$\Xi_{c1}(\frac{3}{2}^-)$	$\Xi_c^{\star+}\pi^-$	2.7	<6.5
		$\Xi_c^{\star0}\pi^0$	2.8	

TABLE II. The strong decay widths of S-wave charmed baryons $\Sigma_c^{++,+,0}(2455)$, $\Sigma_c^{*++,+,0}(2520)$, and $\Xi_c^{*+,0}(2645)$. Here all results are in units of MeV.

	J^P	Channel	Width	Total width (Exp) [13]
$\Sigma_c^{++}(2455)$	$\frac{1}{2}^+$	$\Lambda_c^+\pi^+$	1.24	2.23 ± 0.30
$\Sigma_c^+(2455)$	$\frac{1}{2}^+$	$\Lambda_c^+\pi^0$	1.40	<4.6
$\Sigma_c^0(2455)$	$\frac{1}{2}^+$	$\Lambda_c^+\pi^-$	1.24	2.2 ± 0.40
$\Sigma_c^{*++}(2520)$	$\frac{3}{2}^+$	$\Lambda_c^+\pi^+$	11.9	14.9 ± 1.9
$\Sigma_c^{*+}(2520)$	$\frac{3}{2}^+$	$\Lambda_c^+\pi^0$	12.1	<17
$\Sigma_c^{*0}(2520)$	$\frac{3}{2}^+$	$\Lambda_c^+\pi^-$	11.9	16.1 ± 2.1
$\Xi_c^{*+}(2645)$	$\frac{3}{2}^+$	$\Xi_c^+\pi^0$	0.64	<3.1
$\Xi_c^{*+}(2645)$	$\frac{3}{2}^+$	$\Xi_c^0\pi^+$	0.49	
$\Xi_c^{*0}(2645)$	$\frac{3}{2}^+$	$\Xi_c^+\pi^-$	0.54	<5.5
$\Xi_c^{*0}(2645)$	$\frac{3}{2}^+$	$\Xi_c^0\pi^0$	0.54	

TABLE V. The decay widths of $\Sigma_c^{++}(2800)$ in different P-wave charmed baryons assignments. $\mathcal{R} = \Sigma_c^{+,++}\pi^{+,0}/\Sigma_c^{\star+,++}\pi^{+,0}$. The total width of $\Sigma_c^{++}(2800)$ is 75^{+22}_{-17} MeV [13]. Here all results are in units of MeV.

Assignment	$\Lambda_c^+\pi^+$	$\Sigma_c^{+,++}\pi^{+,0}$	$\Sigma_c^{\star+,++}\pi^{+,0}$	$\mathcal{R}$
$\Sigma_{c0}(\frac{1}{2}^-)$	307	0.0	0.0	—
$\Sigma_{c1}(\frac{1}{2}^-)$	0.0	296	0.4	740
$\Sigma_{c1}(\frac{3}{2}^-)$	0.0	0.7	220	3×10^{-3}
$\Sigma_{c2}(\frac{3}{2}^-)$	8.1	1.3	0.3	4.3
$\Sigma_{c2}(\frac{5}{2}^-)$	8.1	0.6	0.5	1.2
$\tilde{\Sigma}_{c1}(\frac{1}{2}^-)$	0.0	75	69	1.1
$\tilde{\Sigma}_{c1}(\frac{3}{2}^-)$	0.0	75	69	1.1

expected to lie in the range $0.5 \sim 0.7$ GeV. In the following, our numerical results are obtained with the typical values $\alpha_\rho = \alpha_\lambda = 0.6$ GeV.

The strong decay widths of the S-wave charmed baryons $\Sigma_c^{++,+,0}(2455)$, $\Sigma_c^{*++,+,0}(2520)$, and $\Xi_c^{*+,0}(2645)$ are listed in Table II. Accordingly the decay widths of the S-wave bottomed baryons are presented in Table III.

Because Ξ_b, Ξ_b', and Ξ_b^* have not been observed so far, their masses are taken from the theoretical estimate in Ref. [35], which are $m_{\Xi_b} = 5805.7$ MeV, $m_{\Xi_b'} = 5950$ MeV, and $m_{\Xi_b^*} = 5966.1$ MeV.

The quantum number and internal structure of the following P-wave charmed baryons $\Lambda_c^+(2593)$, $\Lambda_c^+(2625)$, $\Xi_c^{+,0}(2790)$, and $\Xi_c^{+,0}(2815)$ are relatively known experi-

TABLE VI. The decay widths of $\Lambda_c^+(2880)$ with different D-wave assignments. All results are in units of MeV.

Assignment	$\Sigma_c^{0,+,++}\pi^{+,0,-}$	$\Sigma_c^{\star 0,+,++}\pi^{+,0,-}$	$\frac{\Gamma(\Sigma_c^\star \pi^\pm)}{\Gamma(\Sigma_c \pi^\pm)}$	$D^0 p$	Remark
$\Lambda_{c2}(\frac{3}{2}^+)$	7.8	0.9	0.11	0.0	×
$\Lambda_{c2}(\frac{5}{2}^+)$	0.06	5.34	89	0.0	×
$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	78.3	59.1	75	0.0	×
$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	78.3	59.1	0.75	0.0	×
$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	0.9	2.3	2.6	2.3	×
$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	0.22	6.0	27	2.3	×
$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	132	144	1.1	0.0	×
$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	66.3	18.0	0.27	150	×
$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	16.5	45.0	2.7	150	×
$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	82.8	9.0	0.10	0.0	×
$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	0.0	54.1	—	0.0	×
$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	25.7	8.1	0.32	64	×
$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	6.5	20.4	3.1	64	×
$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	57.9	14.2	0.24	0.0	×
$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	9.4	47.1	5.0	0.0	×
$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	10.8	5.5	0.51	12	
$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	6.1	7.4	1.2	12	×

TABLE VII. The decay widths of $\Lambda_c^+(2940)$ with different D-wave assignments. Here all results are in units of MeV.

Assignment	$\Sigma_c^{0,+,++}\pi^{+,0,-}$	$\Sigma_c^{\star 0,+,++}\pi^{+,0,-}$	$\frac{\Gamma(\Sigma_c^\star \pi^\pm)}{\Gamma(\Sigma_c \pi^\pm)}$	$D^0 p$	Remark
$\Lambda_{c2}(\frac{3}{2}^+)$	11.7	9.1	0.77	0.0	×
$\Lambda_{c2}(\frac{5}{2}^+)$	0.2	9.1	46	0.0	×
$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	170	150	0.88	0.0	×
$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	170	150	0.88	0.0	×
$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	2.2	0.5	0.23	11	
$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	0.6	1.4	2.3	11	
$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	212	259	1.2	0.0	×
$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	106	32.4	0.31	340	×
$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	26.5	81.0	3.1	340	×
$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	142	16.2	0.11	0.0	×
$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	0.0	97.0	—	0.0	×
$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	34.5	12.6	0.37	95	×
$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	8.6	31.7	3.7	95	×
$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	77.7	27.7	0.36	0.0	×
$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	19.5	75.6	3.9	0.0	×
$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	22.2	12.9	0.58	49	×
$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	12.4	17.5	1.4	49	×

mentally [13]. Their strong decay modes and widths from the $^3P_0$ model are collected in Table IV. The quantum number of $\Sigma_c^{++}(2800)$ is still unknown [13]. Thus under different P-wave assignments of $\Sigma_c^{++}(2800)$, we present the strong decay widths of its possible decay modes in Table V. In the heavy quark limit, the process $\Sigma_c^{++}(2800) \to \Lambda_c^+ \pi^+$ is forbidden if $\Sigma_c^{++}(2800)$ is assigned as $\Sigma_{c1}(\frac{1}{2}^-)$, $\Sigma_{c1}(\frac{3}{2}^-)$, $\tilde{\Sigma}_{c1}(\frac{1}{2}^-)$, and $\tilde{\Sigma}_{c1}(\frac{3}{2}^-)$, which is observed in our calculation as can be seen from Table V.

$\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ are observed in the invariant mass spectrum of $D^0 p$ [1]. The first radial excitation of Λ_c does not decay into $D^0 p$ from the $^3P_0$ model. Hence the

possibility of $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ being a radial excitation is excluded. We calculate their strong decays assuming they are D-wave charmed baryons. The results are shown in Tables VI and VII.

With positive parity, $\Xi(2980)^{+,0}$ and $\Xi(3077)^{+,0}$ can be either the first radially excited charmed baryons or the D-wave charmed baryons. With different assumptions of their quantum numbers we present their strong decay widths in Tables VIII and IX and Fig. 5.

The numerical results depend on the parameters α_ρ and α_λ in the harmonic oscillator wave functions of the charmed baryons. We illustrate such a dependence in

TABLE VIII. The decay widths of $\Xi_c^+(2980)$ with different D-wave assignments. Here all results are in units of MeV.

Assignment	$\Xi_c^0 \pi^+$	$\Xi_c'^0 \pi^+$	$\Xi_c^{\star 0} \pi^+$	$\Sigma_c^{++} k^-$	$\Lambda_c^+ \bar{k}^0$	Remark
$\Xi_{c2}(\frac{3}{2}^+)$	0.0	1.1	0.11	0.37	0.0	$\times$
$\Xi_{c2}(\frac{5}{2}^+)$	0.0	0.12×10^{-2}	0.67	0.11×10^{-3}	0.0	$\times$
$\Xi_{c1}'(\frac{1}{2}^+)$	4.4	0.72	0.18	0.25	5.3	
$\Xi_{c1}'(\frac{3}{2}^+)$	4.4	0.18	0.46	0.062	5.3	
$\Xi_{c2}'(\frac{3}{2}^+)$	0.0	0.16	0.17	0.56	0.0	$\times$
$\Xi_{c2}'(\frac{5}{2}^+)$	0.0	0.47×10^{-2}	1.0	0.71×10^{-4}	0.0	$\times$
$\Xi_{c3}'(\frac{5}{2}^+)$	0.054	0.53×10^{-2}	0.14×10^{-2}	0.82×10^{-4}	0.053	$\times$
$\Xi_{c3}'(\frac{7}{2}^+)$	0.054	0.30×10^{-2}	0.19×10^{-2}	0.46×10^{-4}	0.053	$\times$
$\hat{\Xi}_{c2}(\frac{3}{2}^+)$	0.0	9.5	6.1	0.61	0.0	
$\hat{\Xi}_{c2}(\frac{5}{2}^+)$	0.0	9.5	6.1	0.61	0.0	
$\hat{\Xi}_{c1}'(\frac{1}{2}^+)$	74	6.3	1.0	0.40	78	$\times$
$\hat{\Xi}_{c1}'(\frac{3}{2}^+)$	74	1.6	2.5	0.10	78	$\times$
$\hat{\Xi}_{c2}'(\frac{3}{2}^+)$	0.0	14	4.5	0.91	0.0	
$\hat{\Xi}_{c2}'(\frac{5}{2}^+)$	0.0	6.3	7.1	0.40	0.0	
$\hat{\Xi}_{c3}'(\frac{5}{2}^+)$	48	7.2	2.9	0.46	50	$\times$
$\hat{\Xi}_{c3}'(\frac{7}{2}^+)$	48	4.1	3.9	0.26	50	$\times$
$\check{\Xi}_{c0}'^0(\frac{1}{2}^+)$	0.0	0.30	1.4	1.3	0.0	$\times$
$\check{\Xi}_{c1}^0(\frac{1}{2}^+)$	1.0	0.40	0.46	1.7	0.46	$\times$
$\check{\Xi}_{c1}^0(\frac{3}{2}^+)$	1.0	0.10	1.2	0.43	0.46	$\times$
$\check{\Xi}_{c1}'^1(\frac{1}{2}^+)$	0.0	18	4.4	5.5	0.0	
$\check{\Xi}_{c1}'^1(\frac{3}{2}^+)$	0.0	4.5	11	1.4	0.0	
$\check{\Xi}_{c0}^1(\frac{1}{2}^+)$	0.0	18	18	5.5	0.0	
$\check{\Xi}_{c1}^1(\frac{1}{2}^+)$	6.2	9.1	2.2	2.8	7.2	$\times$
$\check{\Xi}_{c1}^1(\frac{3}{2}^+)$	6.2	2.3	5.5	0.69	72	$\times$
$\check{\Xi}_{c2}^1(\frac{3}{2}^+)$	0.0	11	1.1	0.34	0.0	$\times$
$\check{\Xi}_{c2}^1(\frac{5}{2}^+)$	0.0	0.0	6.6	0.0	0.0	$\times$
$\check{\Xi}_{c2}'^2(\frac{3}{2}^+)$	0.0	5.6	1.8	2.4	0.0	
$\check{\Xi}_{c2}'^2(\frac{5}{2}^+)$	0.0	1.7	4.32	0.24	0.0	
$\check{\Xi}_{c1}^2(\frac{1}{2}^+)$	19	3.7	1.1	1.6	23	
$\check{\Xi}_{c1}^2(\frac{3}{2}^+)$	19	0.93	2.6	0.40	23	
$\check{\Xi}_{c2}^2(\frac{3}{2}^+)$	0.0	8.4	1.7	0.36	0.0	
$\check{\Xi}_{c2}^2(\frac{5}{2}^+)$	0.0	1.2	6.0	0.16	0.0	
$\check{\Xi}_{c3}^2(\frac{5}{2}^+)$	8.1	1.3	60	0.19	8.7	$\times$
$\check{\Xi}_{c3}^2(\frac{7}{2}^+)$	8.1	0.75	0.81	0.10	8.7	

TABLE IX.   The decay widths of $\Xi_c^+(3077)$ with different D-wave assignments. Here all results are in units of MeV.

Assignment	$\Xi_c^0\pi^+$	$\Xi_c'^0\pi^+$	$\Xi_c^{\star 0}\pi^+$	$\Sigma_c^{++}k^-$	$\Sigma_c^{++}k^-$	$\Lambda_c^+\bar{k}^0$	$D^+\Lambda$	Remark
$\Xi_{c2}(\frac{3}{2}^+)$	0.0	2.1	0.30	0.73	0.054	0.0	0.0	
$\Xi_{c2}(\frac{5}{2}^+)$	0.0	0.037	1.7	0.42×10^{-2}	0.32	0.0	0.0	
$\Xi_{c1}'(\frac{1}{2}^+)$	7.0	1.4	0.46	0.49	0.089	4.4	3.2	
$\Xi_{c1}'(\frac{3}{2}^+)$	7.0	0.36	1.1	0.12	0.22	4.4	3.2	
$\Xi_{c2}'(\frac{3}{2}^+)$	0.0	3.2	0.43	1.1	0.081	0.0	0.0	
$\Xi_{c2}'(\frac{5}{2}^+)$	0.0	0.025×10^{-2}	2.5	0.28×10^{-2}	0.48	0.0	0.0	$\times$
$\Xi_{c3}'(\frac{5}{2}^+)$	0.19	0.029	0.012	0.32×10^{-2}	0.32×10^{-3}	0.12	0.026	$\times$
$\Xi_{c3}'(\frac{7}{2}^+)$	0.19	0.016×10^{-3}	0.016	0.18×10^{-2}	0.44×10^{-3}	0.12	0.026	$\times$
$\hat{\Xi}_{c2}(\frac{3}{2}^+)$	0.0	34	29	6.0	2.0	0.0	0.0	$\times$
$\hat{\Xi}_{c2}(\frac{5}{2}^+)$	0.0	34	29	6.0	2.0	0.0	0.0	$\times$
$\hat{\Xi}_{c1}'(\frac{1}{2}^+)$	201	23	4.8	4.0	0.33	130	38	$\times$
$\hat{\Xi}_{c1}'(\frac{3}{2}^+)$	201	5.7	12	1.0	0.83	130	38	$\times$
$\hat{\Xi}_{c2}'(\frac{3}{2}^+)$	0.0	51	22	8.9	1.5	0.0	0.0	$\times$
$\hat{\Xi}_{c2}'(\frac{5}{2}^+)$	0.0	23	34	4.0	2.3	0.0	0.0	$\times$
$\hat{\Xi}_{c3}'(\frac{5}{2}^+)$	129	26	14	4.5	0.94	84	25	$\times$
$\hat{\Xi}_{c3}'(\frac{7}{2}^+)$	129	15	19	2.6	0.13	84	25	$\times$
$\check{\Xi}_{c0}'^0(\frac{1}{2}^+)$	0.0	0.69	0.13	0.29	1.2	0.0	0.0	
$\check{\Xi}_{c1}^0(\frac{1}{2}^+)$	15	0.92	0.044	0.39	0.38	11	0.64×10^{-3}	$\times$
$\check{\Xi}_{c1}^0(\frac{3}{2}^+)$	15	0.23	0.11	0.096	0.96	11	0.64×10^{-3}	$\times$
$\check{\Xi}_{c1}'^1(\frac{1}{2}^+)$	0.0	39	12	12	0.21	0.0	0.0	$\times$
$\check{\Xi}_{c1}'^1(\frac{3}{2}^+)$	0.0	9.9	30	3.0	5.2	0.0	0.0	$\times$
$\check{\Xi}_{c0}^1(\frac{1}{2}^+)$	0.0	39	47	12	8.3	0.0	0.0	$\times$
$\check{\Xi}_{c1}^1(\frac{1}{2}^+)$	110	20	5.9	6.1	1.0	69	42	$\times$
$\check{\Xi}_{c1}^1(\frac{3}{2}^+)$	110	5.0	15	1.5	2.6	69	42	$\times$
$\check{\Xi}_{c2}^1(\frac{3}{2}^+)$	0.0	25	3.0	7.6	0.52	0.0	0.0	$\times$
$\check{\Xi}_{c2}^1(\frac{5}{2}^+)$	0.0	0.0	18	0.0	3.1	0.0	0.0	$\times$
$\check{\Xi}_{c2}'^2(\frac{3}{2}^+)$	0.0	9.2	6.0	3.9	0.75	0.0	0.0	
$\check{\Xi}_{c2}'^2(\frac{5}{2}^+)$	0.0	5.8	10	1.1	2.1	0.0	0.0	
$\check{\Xi}_{c1}^2(\frac{1}{2}^+)$	22	6.1	2.3	2.6	0.54	14	15	$\times$
$\check{\Xi}_{c1}^2(\frac{3}{2}^+)$	22	1.5	5.6	0.64	1.3	14	15	$\times$
$\check{\Xi}_{c2}^2(\frac{3}{2}^+)$	0.0	14	5.2	5.8	0.77	0.0	0.0	$\times$
$\check{\Xi}_{c2}^2(\frac{5}{2}^+)$	0.0	3.9	14	0.74	3.0	0.0	0.0	$\times$
$\check{\Xi}_{c3}^2(\frac{5}{2}^+)$	21	4.4	2.5	0.85	0.23	14	4.3	$\times$
$\check{\Xi}_{c3}^2(\frac{7}{2}^+)$	21	2.5	3.4	0.48	0.31	14	4.3	$\times$

Figs. 6–8 using several typical decay channels: $\Sigma_c^{++}(2455)\to\Lambda_c^+\pi^+$, $\Lambda_c^+(2593)\to\Sigma_c^{++}(2455)\pi^-$, and $\Lambda_c^+(2880)\to\Sigma_c^{*++}(2520)\pi^-$, where $\Sigma_c^{++}(2455)$, $\Lambda_c^+(2593)$, and $\Lambda_c^+(2880)$ are S-wave, P-wave, and D-wave baryons, respectively.

VI. DISCUSSION AND CONCLUSION

At the present it is still too difficult to calculate the strong decay widths of hadrons from the first principles of QCD. For this purpose, some phenomenological strong decay models were proposed such as the $^3P_0$ model, flux tube model, QCD sum rule, lattice QCD, etc., among which only the first two approaches can be applied to the strong decays of excited hadrons. To a large extent, the predictions from the $^3P_0$ and flux tube models roughly agree with each other.

The $^3P_0$ model possesses inherent uncertainties [20,28,32]. In certain cases, the result from the $^3P_0$ model may be a factor of $2\sim3$ off the experimental width. The uncertainty source of the $^3P_0$ model arises from the strength of the quark pair creation from the vacuum γ, the approximation of nonrelativity, and assuming the sim-
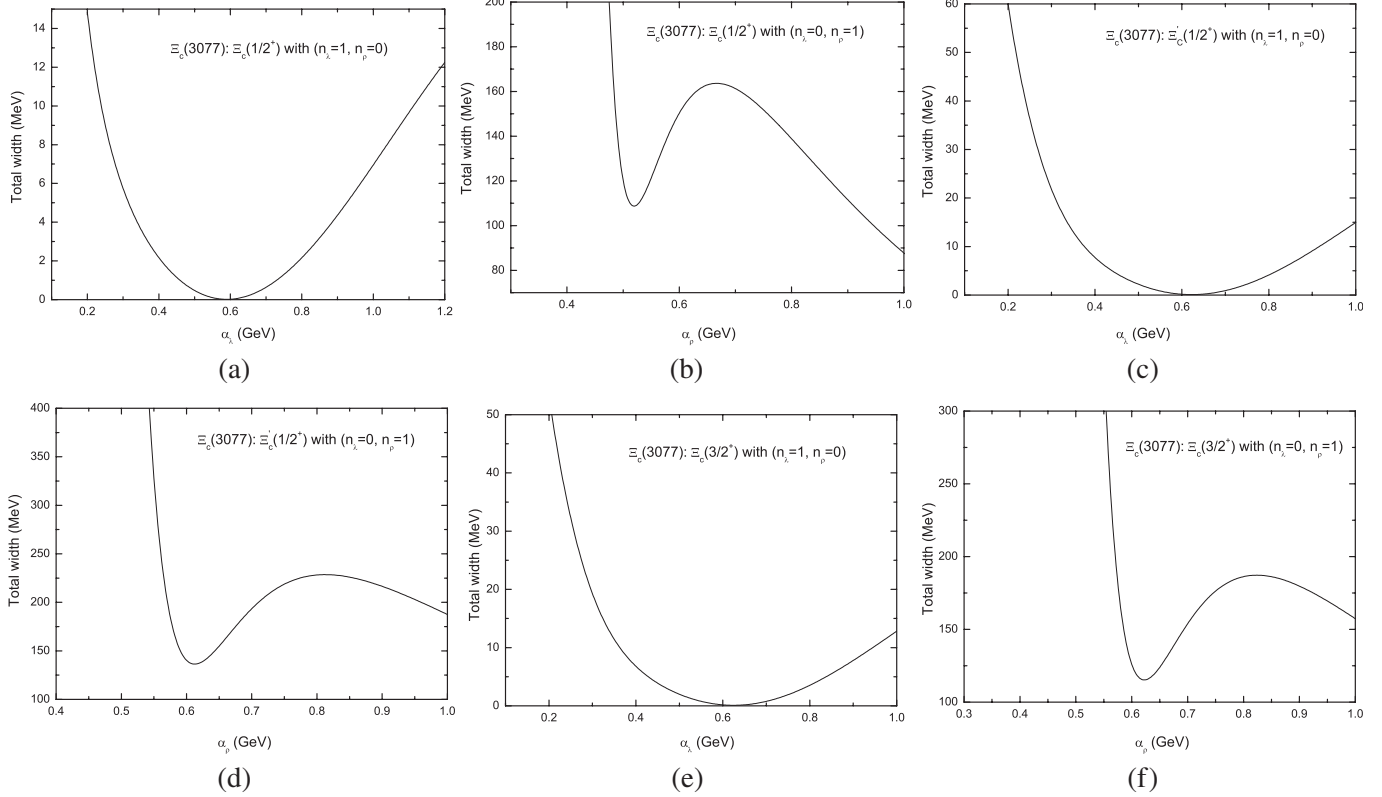
FIG. 5. The dependence of the total decay width of $\Xi_c(3077)$ on the parameter α_λ or α_ρ if $\Xi_c(3077)$ is a radial excitation. In these figures, we fix $\alpha_\rho = 0.6$ GeV for the case with $n_\lambda = 1$, and $\alpha_\lambda = 0.6$ GeV for $n_\rho = 1$. The situation of $\Xi_c(2980)$ as a radial excitation is very similar.
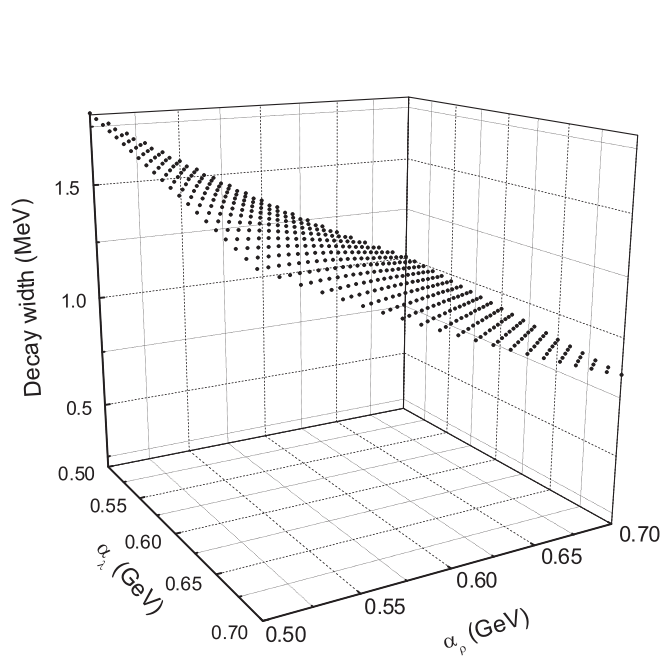

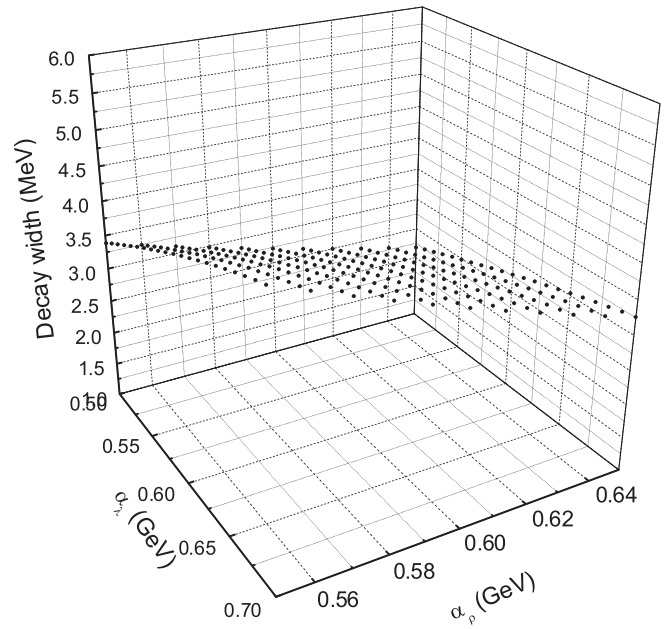FIG. 6. The variation of the decay width of $\Sigma_c^{++}(2455) \rightarrow \Lambda_c^+ \pi^+$ with α_ρ and α_λ.

FIG. 7. The variation of the decay width of $\Lambda_c^+(2593) \rightarrow \Sigma_c^{++}(2455)\pi^-$ with α_ρ and α_λ. Here $\Lambda_c^+(2593)$ is assigned as $\Lambda_{c1}(\frac{1}{2}^-)$.
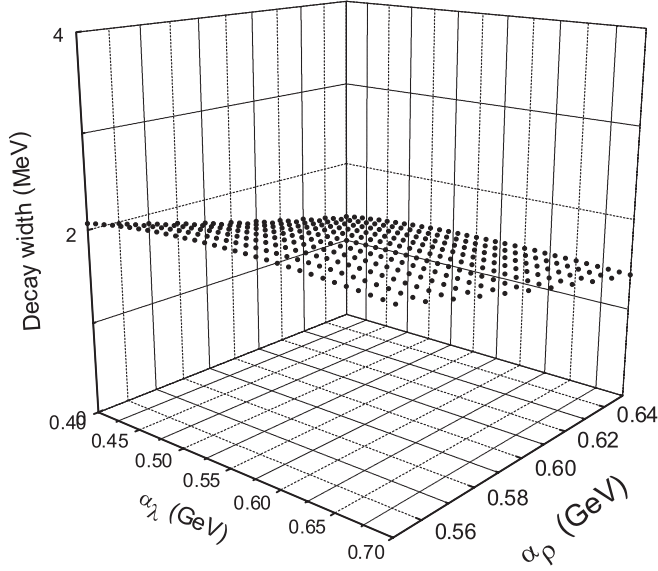
FIG. 8. The variation of the decay width of $\Lambda_c^+(2880) \rightarrow \Sigma_c^{*++}(2520)\pi^-$ with α_ρ and α_λ. Here $\Lambda_c^+(2880)$ is assigned as $\Lambda_{c2}(\frac{5}{2}^+)$.

ple harmonic oscillator radial wave functions for the hadrons. Even with the above uncertainty, the $^3P_0$ model is still the most systematic, effective, and widely used framework to study the hadron strong decays.

In this work, we have calculated the strong decay widths of charmed baryons using the $^3P_0$ model. Our numerical results do not strongly depend on the parameters α_ρ and α_λ as shown in Figs. 6–8. Thus the following qualitative features and conclusions remain essentially unchanged with reasonable variations of α_ρ and α_λ.

Our results for the S-wave charmed baryons $\Sigma_c^{++,+,0}(2455)$, $\Sigma_c^{*++,+,0}(2520)$, and $\Xi_c^{*+,0}(2645)$ are roughly consistent with experimental data within the inherent uncertainty of the $^3P_0$ model. As a by-product, we have also calculated the strong decays of $\Sigma_b^\pm$ and $\Sigma_b^{*\pm}$ observed by CDF Collaboration recently. The numerical results are consistent with the experimental values too.

The decay width of the P-wave baryon $\Lambda_c^+(2593)$ is 3 times larger than the experimental value. With the large experimental uncertainty and the inherent theoretical uncertainty of the $^3P_0$ model, such a deviation is still acceptable. The decay widths of $\Lambda_c^+(2625)$ and $\Xi_c^{+,0}(2790, 2815)$ are compatible with the experimental upper bound. By comparing our results with the experimental total width, we tend to exclude the $\Sigma_{c0}(\frac{1}{2}^-)$ assignment for $\Sigma_c^{++}(2800)$. Since the $\Sigma_c^{++}(2800)$ is observed in the $\Lambda_c^+\pi^+$ channel [36], there are only two assignments left for $\Sigma_c^{++}(2800)$, i.e. $\Sigma_{c2}(\frac{3}{2}^-)$ or $\Sigma_{c2}(\frac{5}{2}^-)$. More experimental information such as the ratio $\frac{\Gamma[\Sigma_c^{++}(2800)\rightarrow\Sigma_c^{+,++}\pi^{+,0}]}{\Gamma[\Sigma_c^{++}(2800)\rightarrow\Sigma_c^{*+,++}\pi^{+,0}]}$ will be helpful in the determination of the quantum number of $\Sigma_c^{++}(2800)$.

We have also calculated the strong decay widths of newly observed $\Lambda_c(2880, 2940)^+$, $\Xi(2980, 3077)^{+,0}$ as-

suming they are candidates of D-wave charmed baryons. We find that the only possible assignment of $\Lambda_c(2880)^+$ is $\check{\Lambda}_{c3}^2(\frac{5}{2})+$ after considering both its total decay width and the ratio $\Gamma(\Sigma_c^\star\pi^\pm)/\Gamma(\Sigma_c\pi^\pm)$, which agrees very well with the indication from the Belle experiment that $\Lambda_c(2880)^+$ favors $J^P = \frac{5}{2}^+$ by the analysis of the angular distribution [2].

Unfortunately the experiment information about the $\Lambda_c(2940)^+$, $\Xi(2980, 3077)^{+,0}$ is scarce at the present. From their calculated decay widths, we can only exclude some assignments which are marked with crosses in Tables VII, VIII, and IX. The decay width ratios of $\Lambda_c(2940)^+$, $\Xi(2980, 3077)^{+,0}$ from the $^3P_0$ model will be useful in the identification of their quantum numbers in the future since the inherent uncertainty cancels largely.

We have also discussed the strong decays of $\Xi(2980, 3077)^{+,0}$ assuming they are radial excitations. Unfortunately the numerical results in Fig. 5 depend quite strongly on the node of the spatial wave function which is related to the parameters of the harmonic oscillator wave functions as shown in Fig. 5. We are unable to make strong conclusions here.

ACKNOWLEDGMENTS

APPENDIX

1. The harmonic oscillator wave functions used in our calculation

For the S-wave charmed baryon,

$$\psi(0, 0, 0, 0) = 3^{3/4}\left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}$$
$$\times \exp\left[-\frac{\mathbf{p}_\rho^2}{2\alpha_\rho^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right]. \quad \text{(A1)}$$

For the P-wave charmed baryon,

$$\psi(1, m, 0, 0) = -i3^{3/4}\left(\frac{8}{3\sqrt{\pi}}\right)^{1/2}(1/\alpha_\rho^2)^{5/4}\mathcal{Y}_1^m(\mathbf{p}_\rho)$$
$$\times\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\exp\left[-\frac{\mathbf{p}_\rho^2}{2\alpha_\rho^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right], \quad \text{(A2)}$$

$$\psi(0, 0, 1, m) = -i3^{3/4}\left(\frac{8}{3\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\lambda^2}\right)^{5/4}\mathcal{Y}_1^m(\mathbf{p}_\lambda)\left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}$$
$$\times\exp\left[-\frac{\mathbf{p}_\rho^2}{2\alpha_\rho^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right]. \quad \text{(A3)}$$

For the D-wave charmed baryon,

$$\psi(2, m, 0, 0) = 3^{3/4}\left(\frac{16}{15\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\rho^2}\right)^{7/4}\mathcal{Y}_2^m(\mathbf{p}_\rho)\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}$$

$$\times \exp\left[-\frac{\mathbf{p}_\rho^2}{2\alpha_\rho^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right], \tag{A4}$$

$$\psi(0, 0, 2, m) = 3^{3/4}\left(\frac{16}{15\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\lambda^2}\right)^{7/4}\mathcal{Y}_2^m(\mathbf{p}_\lambda)\left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}$$

$$\times \exp\left[-\frac{\mathbf{p}_\rho^2}{2\alpha_\rho^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right], \tag{A5}$$

$$\psi(1, m, 1, m') = -3^{3/4}\left(\frac{8}{3\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\rho^2}\right)^{5/4}\mathcal{Y}_1^m(\mathbf{p}_\rho)$$

$$\times \left(\frac{8}{3\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\lambda^2}\right)^{5/4}\mathcal{Y}_1^{m'}(\mathbf{p}_\lambda)$$

$$\times \exp\left[-\frac{\mathbf{p}_\rho^2}{2\alpha_\rho^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right]. \tag{A6}$$

Here $\mathcal{Y}_l^m(\mathbf{p})$ is the solid harmonic polynomial.

The ground state wave function of the meson is

$$\psi(0, 0) = \left(\frac{R^2}{\pi}\right)^{3/4}\exp\left[-\frac{R^2(\mathbf{p}_2 - \mathbf{p}_5)^2}{8}\right]. \tag{A7}$$

The wave function of the first radially excited charmed baryon $\psi(n_\rho, n_\lambda)$ reads as

$$\psi(1, 0) = 3^{3/4}\sqrt{\frac{2}{3}}\left(\frac{1}{\pi^2\alpha_\rho\alpha_\lambda}\right)^{3/2}\left[\frac{3}{2} - \frac{\mathbf{p}_\rho^2}{\alpha_\rho^2}\right]$$

$$\times \exp\left[-\frac{\mathbf{p}_\rho^2}{2\alpha_\rho^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right],$$

$$\psi(0, 1) = 3^{3/4}\sqrt{\frac{2}{3}}\left(\frac{1}{\pi^2\alpha_\rho\alpha_\lambda}\right)^{3/2}\left[\frac{3}{2} - \frac{\mathbf{p}_\lambda^2}{\alpha_\lambda^2}\right]$$

$$\times \exp\left[-\frac{\mathbf{p}_\rho^2}{2\alpha_\lambda^2} - \frac{\mathbf{p}_\lambda^2}{2\alpha_\lambda^2}\right],$$

where n_ρ and n_λ denote the radial quantum number between the two light quarks and between the heavy quark and the two light quarks, respectively. Here $\mathbf{p}_\rho = \sqrt{\frac{1}{2}}(\mathbf{p}_1 - \mathbf{p}_2)$ and $\mathbf{p}_\lambda = \sqrt{\frac{1}{6}}(\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3)$ for the above expressions. All the above harmonic oscillator wave functions can be normalized as $\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 |\psi|^2 = 1$.

2. The momentum space integration

The momentum space integration $\Pi(l_{\rho A}, m_{\rho A}, l_{\lambda A}, m_{\lambda A}, m)$ includes:

For the S-wave charmed baryon decay,

$$\Pi(0, 0, 0, 0, 0) = \beta|\mathbf{p}|\Delta_{0,0}. \tag{A8}$$

For the P-wave charmed baryon decay,

$$\Pi(0, 0, 1, 0, 0) = \frac{1}{2f_1}[f_2\beta|\mathbf{p}|^2 - \zeta]\Delta_{0,1},$$

$$\Pi(0, 0, 1, 1, -1) = \Pi(0, 0, 1, -1, 1) = \frac{\zeta}{2f_1}\Delta_{0,1},$$

$$\Pi(1, 0, 0, 0, 0) = \left[\beta\varpi|\mathbf{p}|^2 + \frac{1}{2\sqrt{2}\lambda_1}^+ \frac{\lambda_2\zeta}{4\lambda_1 f_1}\right]\Delta_{1,0},$$

$$\Pi(1, 1, 0, 0, -1) = \Pi(1, -1, 0, 0, 1) = \beta\varpi|\mathbf{p}|^2\Delta_{1,0}.$$

For the D-wave charmed baryon decay,

$$\Pi(0, 0, 2, 0, 0) = -\frac{f_2}{f_1^2}\left[\frac{f_2}{2}\beta|\mathbf{p}|^3 + \zeta|\mathbf{p}|\right] \times \Delta_{0,2}, \qquad \Pi(0, 0, 2, 1, -1) = \Pi(0, 0, 2, -1, 1) = \frac{\sqrt{3}\zeta f_2}{2f_1^2}|\mathbf{p}|\Delta_{0,2},$$

$$\Pi(2, 0, 0, 0, 0) = -2\left[\left(\beta\varpi^2|\mathbf{p}|^3 + \frac{1}{\sqrt{2}\lambda_1}\varpi|\mathbf{p}| + \frac{\lambda_2}{2\lambda_1 f_1}\zeta\varpi\right)\right]\Delta_{2,0},$$

$$\Pi(2, 1, 0, 0, -1) = \Pi(2, -1, 0, 0, 1) = \left[\left(\frac{1}{\sqrt{2}\lambda_1}^+ \frac{\lambda_2}{2\lambda_1 f_1}\zeta\right)\right]\Delta_{2,0},$$

$$\Pi(1, 0, 1, 0, 0) = \left[\frac{f_2}{2f_1}\beta\varpi|\mathbf{p}|^3 + \frac{1}{2f_1}\left(\frac{\lambda_2\zeta}{2\lambda_1}\beta + \zeta\varpi + \frac{\lambda_2 f_2}{4\lambda_1 f_1}\right)|\mathbf{p}| + \frac{f_2}{4\sqrt{2}\lambda_1 f_1}|\mathbf{p}|\right]\Delta_{1,1},$$

$$\Pi(1, 1, 1, -1, 0) = \Pi(1, -1, 1, 1, 0) = \left[\frac{\lambda_2}{4\lambda_1 f_1}\beta|\mathbf{p}|\right]\Delta_{1,1}, \qquad \Pi(1, 0, 1, 1, -1) = \Pi(1, 0, 1, -1, 1) = \left[\frac{1}{2f_1}\varpi\zeta|\mathbf{p}|\right]\Delta_{1,1},$$

$$\Pi(1, 1, 1, 0, -1) = \Pi(1, -1, 1, 0, 1) = \left[\left(\frac{1}{2\sqrt{2}\lambda_1}^+ \frac{\lambda_2}{4\lambda_1 f_1}\zeta\right) \times \frac{f_2}{2f_1}|\mathbf{p}|\right]\Delta_{1,1}.$$

For the strong decay of the radial excitation, the momentum space integrals denoted as $\Pi(n_\rho, n_\lambda)$ are:

$$\Pi(0, 1) = \sqrt{\frac{2}{3}}\left[-\frac{\beta f_2^2}{4\alpha_\lambda^2 f_1^2}k^3 + \frac{3\beta}{2}k - \frac{3\beta}{2f_1\alpha_\lambda^2}k + \frac{f_2\zeta}{2f_1^2\alpha_\lambda^2}k\right]\Delta_{0,0},$$

$$\Pi(1, 0) = \sqrt{\frac{2}{3}}\left[\frac{1}{\alpha_\rho^2}\left(\beta\varpi^2 k^3 - \frac{3\beta\alpha_\rho^2}{2}k + \frac{3\beta}{2\lambda_1}k - \frac{\sqrt{2}\varpi}{2\lambda_1}k - \frac{\lambda_2\varpi\zeta}{3\lambda_1}k + \frac{\lambda_2^2}{4\lambda_1^2}\right)\right]\Delta_{0,0},$$

where

$$\lambda_1 = \frac{1}{\alpha_\rho^2} + \frac{1}{4}R^2, \qquad \lambda_2 = -\frac{1}{2\sqrt{3}}R^2, \qquad \lambda_3 = \frac{1}{\alpha_\lambda^2} + \frac{1}{12}R^2, \qquad \lambda_4 = \frac{1}{\sqrt{2}\alpha_\rho^2} + \frac{1}{2\sqrt{2}}R^2,$$

$$\lambda_5 = -\left(\frac{1}{\sqrt{6}\alpha_\lambda^2} + \frac{1}{2\sqrt{6}}R^2\right), \qquad \lambda_6 = \frac{1}{4\alpha_\rho^2} + \frac{1}{12\alpha_\lambda^2} + \frac{1}{8}R^2, \qquad f_1 = \lambda_3 - \frac{\lambda_2^2}{4\lambda_1}, \qquad f_2 = \lambda_5 - \frac{2\lambda_2\lambda_4}{4\lambda_1},$$

$$f_3 = \lambda_6 - \frac{\lambda_4^2}{4\lambda_1}, \qquad \zeta = \frac{\lambda_2}{2\sqrt{2}\lambda_1} + \frac{1}{\sqrt{6}}, \qquad \varpi = \frac{\lambda_2 f_2}{4\lambda_1 f_1} - \frac{\lambda_4}{2\lambda_1}, \qquad \beta = \left(1 + \frac{\sqrt{3}\lambda_2 f_2 - 2\sqrt{3}\lambda_4 f_1 + 2\lambda_1 f_2}{4\sqrt{6}\lambda_1 f_1}\right),$$

and

$$\Delta_{0,0} = \left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\left(\frac{R^2}{\pi}\right)^{3/4}\left(\frac{\pi^2}{\lambda_1 f_1}\right)^{3/2}\exp\left[-\left(f_3 - \frac{f_2^2}{4f_1}\right)|\mathbf{p}|^2\right]\left[-\sqrt{\frac{3}{4\pi}}\left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\right],$$

$$\Delta_{0,1} = \left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\left(\frac{R^2}{\pi}\right)^{3/4}\left(\frac{\pi^2}{\lambda_1 f_1}\right)^{3/2}\exp\left[-\left(f_3 - \frac{f_2^2}{4f_1}\right)|\mathbf{p}|^2\right]\left[\frac{3i}{4\pi}\left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{8}{3\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\lambda^2}\right)^{5/4}\right],$$

$$\Delta_{1,0} = \left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\left(\frac{R^2}{\pi}\right)^{3/4}\left(\frac{\pi^2}{\lambda_1 f_1}\right)^{3/2}\exp\left[-\left(f_3 - \frac{f_2^2}{4f_1}\right)|\mathbf{p}|^2\right]\left[\frac{3i}{4\pi}\left(\frac{8}{3\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\rho^2}\right)^{5/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\right],$$

$$\Delta_{0,2} = \left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\left(\frac{R^2}{\pi}\right)^{3/4}\left(\frac{\pi^2}{\lambda_1 f_1}\right)^{3/2}\exp\left[-\left(f_3 - \frac{f_2^2}{4f_1}\right)|\mathbf{p}|^2\right]\left[\frac{\sqrt{15}}{8\pi}\left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{16}{15\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\lambda^2}\right)^{7/4}\right],$$

$$\Delta_{2,0} = \left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\left(\frac{R^2}{\pi}\right)^{3/4}\left(\frac{\pi^2}{\lambda_1 f_1}\right)^{3/2}\exp\left[-\left(f_3 - \frac{f_2^2}{4f_1}\right)|\mathbf{p}|^2\right]\left[\frac{\sqrt{15}}{8\pi}\left(\frac{16}{15\sqrt{\pi}}\right)^{1/2}\left(\frac{1}{\alpha_\rho^2}\right)^{7/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\right],$$

$$\Delta_{1,1} = \left(\frac{1}{\pi\alpha_\rho^2}\right)^{3/4}\left(\frac{1}{\pi\alpha_\lambda^2}\right)^{3/4}\left(\frac{R^2}{\pi}\right)^{3/4}\left(\frac{\pi^2}{\lambda_1 f_1}\right)^{3/2}\exp\left[-\left(f_3 - \frac{f_2^2}{4f_1}\right)|\mathbf{p}|^2\right]\left[-\left(\frac{3}{4\pi}\right)^{3/2}\frac{8}{3\sqrt{\pi}}\left(\frac{1}{\alpha_\lambda^2\alpha_\rho^2}\right)^{5/4}\right].$$

In the above expressions, $|\mathbf{p}|$ reads as

$$|\mathbf{p}| = \frac{\sqrt{(m_A^2 - (m_B + m_C)^2)(m_A^2 - (m_B - m_C)^2)}}{2m_A}.$$

[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **98**, 012001 (2007).
[2] K. Abe *et al.* (Belle Collaboration), arXiv:hep-ex/0608043.
[3] B. Aubert *et al.* (*BABAR* Collaboration), arXiv:hep-ex/0607042.
[4] R. Chistov *et al.* (Belle Collaboration), Phys. Rev. Lett. **97**, 162001 (2006).
[5] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **97**, 232001 (2006).
[6] J. L. Rosner, arXiv:hep-ph/0612332; arXiv:hep-ph/0609195; AIP Conf. Proc. **870**, 63 (2006).

[7] X. G. He, Xue-Qian Li, Xiang Liu, and X. Q. Zeng, arXiv:hep-ph/0606015.

[8] H. Y. Cheng and C. K. Chua, Phys. Rev. D **75**, 014006 (2007).

[9] H. Garcilazo, J. Vijande, and A. Valcarce, J. Phys. G **34**, 961 (2007).

[10] M. Artuso *et al.* (CLEO Collaboration), Phys. Rev. Lett. **86**, 4479 (2001).

[11] S. Tawfiq, P. J. O'Donnell, and J. G. Körner, Phys. Rev. D **58**, 054010 (1998); M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and A. G. Rusetsky, Phys. Rev. D **60**, 094002 (1999); M. Q. Huang, Y. B. Dai, and C. S. Huang, Phys. Rev. D **52**, 3986 (1995); **55**, 7317(E) (1997); S. L. Zhu, Phys. Rev. D **61**, 114019 (2000).

[12] D. Pirjol and T. M. Yan, Phys. Rev. D **56**, 5483 (1997).

[13] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).

[14] I. V. Gorelov, arXiv:hep-ex/0701056.

[15] E. Jenkins, Phys. Rev. D **54**, 4515 (1996); **55**, R10 (1997); M. Karlinear and H. J. Lipkin, arXiv:hep-ph/0307243; Phys. Lett. B **575**, 249 (2003).

[16] J. L. Rosner, Phys. Rev. D **75**, 013009 (2007).

[17] M. Karliner and H. J. Lipkin, arXiv:hep-ph/0611306.

[18] C. W. Hwang, arXiv:hep-ph/0611221.

[19] L. Micu, Nucl. Phys. **B10**, 521 (1969).

[20] A. Le Yaouanc, L. Oliver, O. Pène, and J. Raynal, Phys. Rev. D **8**, 2223 (1973); **9**, 1415 (1974); **11**, 1272 (1975); Phys. Lett. B **72**, 57 (1977); **71**, 397 (1977).

[21] A. Le Yaouanc, L. Oliver, O. Pène, and J. Raynal, Phys. Lett. B **72**, 57 (1977).

[22] A. Le Yaouanc, L. Oliver, O. Pène, and J. Raynal, *Hadron Transitions in the Quark Model* (Gordon and Breach, New York, 1987).

[23] H. G. Blundell and S. Godfrey, Phys. Rev. D **53**, 3700 (1996).

[24] P. R. Page, Nucl. Phys. **B446**, 189 (1995); S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).

[25] S. Capstick and W. Roberts, Phys. Rev. D **49**, 4570 (1994).

[26] E. S. Ackleh, T. Barnes, and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996).

[27] H. Q. Zhou, R. G. Ping, and B. S. Zou, Phys. Lett. B **611**, 123 (2005).

[28] X. H. Guo, H. W. Ke, X. Q. Li, X. Liu, and S. M. Zhao, arXiv:hep-ph/0510146.

[29] J. Lu, W. Z. Deng, X. L. Chen, and S. L. Zhu, Phys. Rev. D **73**, 054012, (2006); B. Zhang, X. Liu, and S. L. Zhu, arXiv:hep-ph/0609013.

[30] S. Capstick and W. Roberts, Phys. Rev. D **47**, 1994 (1993).

[31] C. Hayne and N. Isgur, Phys. Rev. D **25**, 1944 (1982).

[32] H. G. Blundell and S. Godfrey, Phys. Rev. D **53**, 3700 (1996).

[33] F. E. Close and E. S. Swanson, Phys. Rev. D **72**, 094004 (2005).

[34] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).

[35] E. Jenkins, Phys. Rev. D **54**, 4515 (1996).

[36] R. Mizuk (Belle Collaboration), Phys. Rev. Lett. **94**, 122002 (2005).