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# Quark spin content of the proton, hyperon semileptonic decays, and the decay width of the $\Theta^+$ pentaquark

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Using the exisiting experimental data for hyperon semileptonic decays and the flavor-singlet axial-vector charge $g_A^{(0)}$ from polarized deep inelastic scattering of the proton, we derive the decay width of the $\Theta^+$ pentaquark baryon. We take into account the effects of flavor SU(3) symmetry breaking within the framework of the chiral quark-soliton model. All dynamical parameters of the model are fixed by using the five experimental hyperon semileptonic decay constants and flavor-singlet axial-vector charge. We obtain the numerical results of the decay width of the $\Theta^+$ pentaquark baryon as a function of the pion-nucleon sigma term $\Sigma_{\pi N}$ and investigate the dependence of the decay width of the $\Theta^+$ on the $g_A^{(0)}$, varying the $g_A^{(0)}$ within the range of the experimental uncertainty. We demonstrate that the combined values of all known semileptonic decays with the generally accepted value of $g_A^{(0)} \approx 0.3$ for the proton are compatible with a small decay width $\Gamma_{\Theta KN}$ of the $\Theta^+$ pentaquark, i.e. $\Gamma_{\Theta KN} \leq 1$ MeV.

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## I. INTRODUCTION

Since Diakonov, Petrov, and Polyakov [1] predicted in the chiral quark-soliton model ($\chi$QSM) the mass and narrow decay width of the pentaquark baryon $\Theta^+$ with strange quantum number $S = +1$ and leading quark Fock structure $uudd\bar{s}$, there has been an enormous amount of theoretical and experimental works (see, for example, recent reviews for the experimental results [2] and for the theoretical investigations [3–5]). Note that there is an earlier prediction by Praszałowicz of the mass in the soliton approach of the Skyrme model [6]. Many experiments have announced the existence of the $\Theta^+$ after the first independent observations by the LEPS [7] and DIANA [8] collaborations, while the $\Theta^+$ has not been seen in almost all high-energy experiments. Moreover, an exotic $\Xi_{\overline{10}}$ state was observed by the NA49 experiment at CERN [9], though its existence is still under debate.

A very recent CLAS experiment dedicated to search for the $\Theta^+$ has announced null results of finding the $\Theta^+$ in the reaction $\gamma p \to \bar{K}^0\Theta^+$ [10]. The subsequent experiment has also not found any evidence for the $\Theta^+$ in $\gamma d \to pK^-\Theta^+$ [11]. Though these experiments are the measurements with high statistics, it is too early to conclude the absence of the $\Theta^+$. Note e.g. that the DIANA Collaboration has continued to search for the $\Theta^+$ and found the formation of a narrow $pK^0$ peak with mass of $1537 \pm 2\ \mathrm{MeV}/c^2$ and width of $\Gamma = 0.36 \pm 0.11$ MeV in the $K^+n \to K^0p$ reaction [12]. Moreover, several new experiments searching for the $\Theta^+$ are in progress [13,14]. In this obscure status for the $\Theta^+$, more efforts are required for understanding the $\Theta^+$ theoretically as well as experimentally. In addition, a recent GRAAL experiment [15] announced the evidence of a new nucleonlike resonance with a seemingly narrow decay width ~10 MeV and a mass ~1675 MeV in the $\eta$ photoproduction from the neutron target. This new nucleonlike resonance, $N^*(1675)$, may be regarded as a nonstrange pentaquark because of its narrow decay width and dominant excitation on the neutron target which are known to be characteristic for typical pentaquark baryons [16], though one should not exclude a possibility that it might be one of the already known $\pi N$ resonances (possibly, $D_{15}$) [17]. This GRAAL data is consistent with the results for the transition magnetic moments in the chiral quark-soliton model ($\chi$QSM) [18] as well as the partial-wave analysis for the nonstrange pentaquark baryons [19]. Moreover, a recent theoretical calculation of the $\gamma N \to \eta N$ reaction [20] describes qualitatively well the GRAAL data, based on the values of the transition magnetic moments in Refs. [18,19], which implies that the $N^*$ seen in the GRAAL experiment could be favorably identified as one of the pentaquark baryons.

However, there is general opinion that, if the $\Theta^+$ exists, its width should be extremely small. Its value may even possibly lie below 1 MeV [21,22]. As far as theory is concerned, the decay width of the $\Theta^+$ has been investigated in many different approaches [22–29] and is mostly estimated to be very small. In the present work, we want to study the decay width of the $\Theta^+$ baryon within the framework of the chiral quark-soliton model ($\chi$QSM), including

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the effects of flavor SU(3) symmetry breaking and using the "model-independent approach" [30].[1] Recently, this approach has been applied to evaluate the magnetic moments of the baryon decuplet and antidecuplet, with parameters fixed by experimental magnetic moments of the baryon octet [31,32] and the baryon octet, decuplet, and antidecuplet mass splittings and the mass of the $\Theta^+$. The same method was employed to get various transition magnetic moments [18] and the results are in good agreement with the SELEX and GRAAL data. Thus, in the present work, we want to analyze in the same way the axial-vector coupling constant of the $\Theta^+$, based on the experimental data for hyperon semileptonic decay (HSD) constants $(g_1/f_1)^{B_1\to B_2}$ and the flavor-singlet axial-vector constant of the proton $g_A^{(0)}$. It is, in particular, interesting to use the $g_A^{(0)}$ as an input, since it carries information on the quark spin content of the proton. It is extracted from deep inelastic polarized electron-proton scattering and hence its information is independent of HSD. In fact, the $g_A^{(0)}$ is related to the pseudoscalar coupling $G_2$ in Ref. [1] by the Goldberger-Treiman relation but there it has been neglected, its effect being assumed to be rather small. In Ref. [1] the $\Gamma_{\Theta KN}$ was determined by the empirical value of the pion-nucleon coupling constant $g_{\pi NN}$ [33]. Moreover, the effects of flavor SU(3) symmetry breaking were neglected.

In the present work, we will perform a more general analysis of the $\Gamma_{\Theta KN}$, emphasizing its dependence on $g_A^{(0)}$ of the proton and the effects of SU(3) symmetry breaking. While the decay constants of HSDs are relatively well determined [34], the value of the $g_A^{(0)}$ is experimentally only known to be in the range of 0.15–0.35 [35]. Thus, we need to examine explicitly the dependence of the $\Gamma_{\Theta KN}$ on the $g_A^{(0)}$. We will later show that the $\Gamma_{\Theta KN}$ is rather sensitive to $g_A^{(0)}$ which is in contrast to the assumptions made in Ref. [1]. Moreover, we will see that the $\Gamma_{\Theta KN}$ is constrained by the value of the $\Sigma_{\pi N}$. In the end, we will see that the known data on HSDs and the experimental value of the $g_A^{(0)}$ are compatible with a small width of $\Gamma_{\Theta KN} \leq$ 1 MeV.

## II. FORMALISM

Using a formalism similar to the one of Ref. [18], the form factors of HSDs are defined by the following transition matrix elements of the vector and axial-vector currents:

$$\begin{aligned}\langle B_2|V_\mu^X|B_1\rangle &= \bar{u}_{B_2}(p_2)\Big[f_1(q^2)\gamma_\mu - \frac{if_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{M_1}q_\mu\Big]u_{B_1}(p_1),\\ \langle B_2|A_\mu^X|B_1\rangle &= \bar{u}_{B_2}(p_2)\Big[g_1(q^2)\gamma_\mu - \frac{ig_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{M_1}q_\mu\Big]\gamma_5 u_{B_1}(p_1),\end{aligned} \tag{1}$$

where the vector and axial-vector currents are defined as

$$V_\mu^X = \bar{\psi}(x)\gamma_\mu\lambda_X\psi(x), \qquad A_\mu^X = \bar{\psi}(x)\gamma_\mu\gamma_5\lambda_X\psi(x), \tag{2}$$

with $X = \frac{1}{2}(1 \pm i2)$ for strangeness conserving $\Delta S = 0$ currents and $X = \frac{1}{2}(4 \pm i5)$ for $|\Delta S| = 1$. The $q^2 = -Q^2$ stands for the square of the momentum transfer $q = p_2 - p_1$. The form factors $g_i$ and $f_i$ are real quantities due to $CP$ invariance, depending only on the square of the momentum transfer. We can safely neglect $g_3$ for the reason that its contribution to the decay rate is proportional to the ratio $\frac{m_l^2}{M_1^2} \ll 1$, where $m_l$ represents the mass of the lepton ($e$ or $\mu$) in the final state and $M_1$ that of the baryon in the initial state. Taking into account the $1/N_c$ rotational and $m_s$ corrections, we can write the resulting axial-vector constants $g_1^{(B_1\to B_2)}(0)$ as follows:

$$\begin{aligned}g_1^{(B_1\to B_2)}(0) = {}& a_1\langle B_2|D_{X3}^{(8)}|B_1\rangle + a_2 d_{pq3}\langle B_2|D_{Xp}^{(8)}\hat{J}_q|B_1\rangle\\ &+ \frac{a_3}{\sqrt{3}}\langle B_2|D_{X8}^{(8)}\hat{J}_3|B_1\rangle\\ &+ m_s\Big[\frac{a_4}{\sqrt{3}}d_{pq3}\langle B_2|D_{Xp}^{(8)}D_{8q}^{(8)}|B_1\rangle\\ &+ a_5\langle B_2|(D_{X3}^{(8)}D_{88}^{(8)} + D_{X8}^{(8)}D_{83}^{(8)})|B_1\rangle\\ &+ a_6\langle B_2|(D_{X3}^{(8)}D_{88}^{(8)} - D_{X8}^{(8)}D_{83}^{(8)})|B_1\rangle\Big],\end{aligned} \tag{3}$$

where $a_i$ denote parameters encoding the specific dynamics of the chiral soliton model. $\hat{J}_q$ ($\hat{J}_3$) stand for the $q$th (third) components of the collective spin operator of the baryons, respectively. The $D_{ab}^{(\mathcal{R})}$ denote the SU(3) Wigner matrices in representation $\mathcal{R}$.

The collective Hamiltonian describing baryons in the SU(3) $\chi$QSM takes the following form [36]:

$$\hat{H} = \mathcal{M}_{\rm sol} + \frac{J(J+1)}{2I_1} + \frac{C_2(\mathrm{SU}(3)) - J(J+1) - \frac{N_c^2}{12}}{2I_2} + \hat{H}', \tag{4}$$

with the symmetry breaking piece given by

$$\hat{H}' = \alpha D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}}D_{8i}^{(8)}\hat{J}_i, \tag{5}$$

where parameters $\alpha$, $\beta$, and $\gamma$ are proportional to the strange current quark mass $m_s$.

[1]The approach is *model-independent* insofar that it does not perform self-consistent calculations leading to some solitonic profile but uses only the semiclassical rotational picture of the $\chi$QSM and determines the dynamical coefficients by fitting them to experimental data. In fact the pioneering paper on $\Theta^+$ [1] used this method

Taking into account the recent experimental observation of the mass of $\Theta^+$, the parameters in Eq. (5) can be conveniently parameterized in terms of the pion-nucleon $\Sigma_{\pi N}$ term [assuming $m_s/(m_u + m_d) = 12.9$] as [28]

$$\alpha = 336.4 - 12.9\Sigma_{\pi N}, \qquad \beta = -336.4 + 4.3\Sigma_{\pi N}, \qquad \gamma = -475.94 + 8.6\Sigma_{\pi N} \tag{6}$$

(in units of MeV). Moreover, the inertia parameters which describe the splittings of SU(3) baryon mass representations take the following values (in MeV):

$$\frac{1}{I_1} = 152.4, \qquad \frac{1}{I_2} = 608.7 - 2.9\Sigma_{\pi N}. \tag{7}$$

Equations (6) and (7) follow from the fit to the masses of octet and decuplet baryons and of $\Theta^+$ as well. If one imposes the additional constraint that $M_{\Xi_{\overline{10}}} = 1860$ MeV, then $\Sigma_{\pi N} = 73$ MeV [28] (see also [37]) in agreement with recent experimental estimates [38,39]. However, since the measurement of $M_{\Xi_{\overline{10}}}$ is still under debate, we will not fix $\Sigma_{\pi N}$ but vary it within a certain range, i.e. $\Sigma_{\pi N} = 45$–$75$ MeV.

Because the Hamiltonian of Eq. (5) mixes different SU(3) representations, the collective wave functions are given as linear combinations [40]:

$$\begin{aligned} |B_8\rangle &= |8_{1/2}, B\rangle + c^B_{\overline{10}}|\overline{10}_{1/2}, B\rangle + c^B_{27}|27_{1/2}, B\rangle, \\ |B_{\overline{10}}\rangle &= |\overline{10}_{1/2}, B\rangle + d^B_8|8_{1/2}, B\rangle + d^B_{27}|27_{1/2}, B\rangle \\ &\quad + d^B_{\overline{35}}|\overline{35}_{1/2}, B\rangle, \end{aligned} \tag{8}$$

where $|B_{\mathcal{R}}\rangle$ denotes the state which reduces to the SU(3) representation $\mathcal{R}$ in the formal limit $m_s \to 0$. The spin index $J_3$ has been suppressed. The $m_s$-dependent (through the linear $m_s$ dependence of $\alpha$, $\beta$, and $\gamma$) coefficients in Eq. (8) can be found in Ref. [31].

TABLE I. Experimental inputs for determining the dynamical parameters $a_i$.

| Decay modes | Experiments | Refs. |
|---|---|---|
| $g_1/f_1(n \to p)$ | $1.2695 \pm 0.0029$ | [34] |
| $g_1/f_1(\Lambda \to p)$ | $0.718 \pm 0.015$ | [34] |
| $g_1/f_1(\Sigma^- \to n)$ | $-0.34 \pm 0.017$ | [34] |
| $g_1/f_1(\Xi^- \to \Lambda)$ | $0.25 \pm 0.05$ | [34] |
| $g_1/f_1(\Xi^0 \to \Sigma^+)$ | $1.32^{+0.21}_{-0.17} \pm 0.05$ | [34,41] |
| $g_A^{(0)}$ | 0.2–0.4 | [42] |

In order to determine the dynamical parameters $a_i$ in Eq. (3), we want to use experimental information on HSD, as we did for the magnetic moments of the SU(3) baryons [31]. Since there exist five experimental data for the transition axial-vector coupling constants $g_1/f_1(B_1 \to B_2)$ as listed in Table I, we need at least one more data. Since the singlet axial-vector constant of the proton $g_A^{(0)}$ provides an independent information from deep inelastic scattering of the proton, we can use it for input as well. However, it has still a large uncertainty, so that we will examine judicially the dependence of the present analysis on $g_A^{(0)}$. With these six input parameters at hand, the dynamical paramters $a_i$ can be determined by solving the following matrix equation:

$$\mathbf{M}[\Sigma_{\pi N}] \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} 1.2695 \\ 0.718 \\ -0.34 \\ 0.25 \\ 1.32 \\ 0.2\text{–}0.4 \end{bmatrix}, \tag{9}$$

where

$$\mathbf{M}[\Sigma_{\pi N}] = \begin{bmatrix} -\frac{7}{15} - \frac{4c_{27}}{45} - \frac{2c_{\overline{10}}}{3} & \frac{7}{30} - \frac{8c_{27}}{45} - \frac{2c_{\overline{10}}}{3} & \frac{1}{30} + \frac{2c_{27}}{15} - \frac{c_{\overline{10}}}{3} & -\frac{11}{135} & -\frac{2}{9} & -\frac{2}{15} \\ -\frac{4}{15} + \frac{c_{27}}{15} + \frac{c_{\overline{10}}}{3} & \frac{2}{15} + \frac{2c_{27}}{15} + \frac{c_{\overline{10}}}{3} & \frac{1}{15} - \frac{c_{27}}{10} + \frac{c_{\overline{10}}}{6} & -\frac{2}{45} & 0 & -\frac{1}{15} \\ \frac{2}{15} - \frac{2c_{27}}{45} & -\frac{1}{15} - \frac{4c_{27}}{45} & \frac{2}{15} + \frac{c_{27}}{15} & \frac{1}{135} & -\frac{2}{45} & \frac{1}{15} \\ -\frac{1}{15} - \frac{c_{27}}{15} & \frac{1}{30} - \frac{2c_{27}}{15} & \frac{1}{10} + \frac{c_{27}}{10} & \frac{1}{90} & \frac{1}{15} & -\frac{1}{15} \\ -\frac{7}{15} + \frac{2c_{27}}{45} + \frac{c_{\overline{10}}}{3} & \frac{7}{30} + \frac{4c_{27}}{45} + \frac{c_{\overline{10}}}{3} & \frac{1}{30} - \frac{c_{27}}{15} + \frac{c_{\overline{10}}}{6} & -\frac{11}{270} & \frac{1}{9} & \frac{1}{15} \\ 0 & 0 & 1 & 0 & -\frac{1}{5} & \frac{1}{5} \end{bmatrix}. \tag{10}$$

Inverting Eq. (9), we finally obtain the values of dynamical parameters $a_i$ as functions of $\Sigma_{\pi N}$ and $g_A^{(0)}$.

## III. RESULTS AND DISCUSSION

Table II lists as an example the results of the dynamical parameters $a_i$ for $g_A^{(0)} = 0.3$. The $a_i$ depend in general on $g_A^{(0)}$ and $\Sigma_{\pi N}$ nonlinearly, except for $a_6$ which is independent of $g_A^{(0)}$ as well as of $\Sigma_{\pi N}$. As a matter of fact, the $a_i$ are generally weakly dependent on $\Sigma_{\pi N}$ and the $a_2$, $a_4$, $a_5$ are rather sensitive to $g_A^{(0)}$.

We now insert the values of $a_i$ listed in Table II and similar ones with different $g_A^{(0)}$ into Eq. (3) together with the matrix elements of the $D_{ab}^{(\mathcal{R})}$ functions so that we can determine the transition axial-vector constant for the decay $\Theta^+ \to K^+ n$.

Figure 1 draws for various values of $g_A^{(0)}$ the dependence of the $g_A^{*(\Theta\to n)}$ on the $\Sigma_{\pi N}$. The larger $g_A^{(0)}$ we use, the

TABLE II. The dynamical parameters $a_i$ determined with $g_A^{(0)} = 0.3$. The $\Sigma_{\pi N}$ is varied from 45 to 75 MeV.

| $\Sigma_{\pi N}$ [MeV] | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ |
|---|---|---|---|---|---|---|
| 45 | −2.4811 | 0.8933 | 0.3190 | 1.3580 | 0.1399 | 0.0450 |
| 50 | −2.4113 | 1.0303 | 0.3196 | 1.3464 | 0.1432 | 0.0450 |
| 55 | −2.3608 | 1.1255 | 0.3210 | 1.3226 | 0.1499 | 0.0450 |
| 60 | −2.3221 | 1.1952 | 0.3228 | 1.2904 | 0.1591 | 0.0450 |
| 65 | −2.2909 | 1.2481 | 0.3250 | 1.2517 | 0.1700 | 0.0450 |
| 70 | −2.2650 | 1.2895 | 0.3275 | 1.2076 | 0.1825 | 0.0450 |
| 75 | −2.2426 | 1.3224 | 0.3303 | 1.1587 | 0.1964 | 0.0450 |

smaller $g_A^{*(\Theta\to n)}$ we obtain. Moreover, the $g_A^{*(\Theta\to n)}$ turns out to be negative when $g_A^{(0)}$ is larger than around 0.37. For a given $g_A^{(0)}$ the $g_A^{*(\Theta\to n)}$ decreases as the $\Sigma_{\pi N}$ increases. The $g_A^{*(\Theta\to n)}$ at $\Sigma_{\pi N} = 70$ MeV is 70% smaller than that at $\Sigma_{\pi N} = 45$ MeV.

Using the effective Lagrangian for the $\Theta^+ \to K^+ n$ decay:

$$\mathcal{L} = -\frac{g_A^{*(\Theta\to n)}}{2f_K}\bar{\Theta}\gamma_\mu\gamma_5(\partial^\mu K^+)n + \text{H.c.}, \tag{11}$$

where H.c. stands for the Hermitian conjugate. We obtain from the effective Lagrangian the invariant amplitude for the decay as follows:

$$i\mathcal{M} = i\frac{g_A^{*(\Theta\to n)}}{2f_K}\bar{u}(p_n)p_\mu^{K^+}\gamma_5\gamma^\mu u(p_\Theta), \tag{12}$$

where $\bar{\Theta}$, $K^+$, and $n$ denote the fields of the $\Theta^+$, of the positively charged kaon, and of the neutron, respectively. The $f_K = 112$ MeV represents the kaon decay constant. The $\bar{u}(p_n)$ and $u(p_\Theta)$ are the Dirac spinors for the neutron and $\Theta^+$ with the corresponding momenta, respectively, and the $p^{K^+}$ denotes the kaon momentum. The decay width of the $\Theta^+ \to KN$ is proportional to the square of the transition axial-vector constant:

$$\Gamma_{\Theta KN} = 2\Gamma_{\Theta K^+ n} = \frac{(g_A^{*(\Theta\to n)})^2|\vec{p}|}{16\pi f_K^2 M_\Theta^2}[(M_\Theta - M_N)^2 - m_K^2](M_\Theta + M_N)^2, \tag{13}$$

where $|\vec{p}| = \sqrt{(M_\Theta^2 - (M_N + m_K)^2)(M_\Theta^2 - (M_N - m_K)^2)}/2M_\Theta$ is the kaon momentum and $M_\Theta = 1540$ MeV, $M_N = 939$ MeV, and $m_K = 494$ MeV stand for the masses of the $\Theta^+$, the nucleon, and the kaon, respectively. The factor 2 in front of the decay width $2\Gamma_{\Theta K^+ n}$ takes care of the fact that $\Theta^+$ has two distinct decay channels, $K^+ n$ and $K^0 p$, which are equally populated due to isospin-symmetry. The $\Gamma_{\Theta KN}$ is sensitive to the value of the $g_A^{*(\Theta\to n)}$. Moreover, since the $\Gamma_{\Theta KN}$ is given as a function of the square of the $g_A^{*(\Theta\to n)}$, it is independent of the sign of the $g_A^{*(\Theta\to n)}$. Thus, the decay width $\Gamma_{\Theta KN}$ decreases until the sign of the $g_A^{*(\Theta\to n)}$ changes and then increases again.

In Table III, we list the total decay width of the $\Theta^+ \to KN$ as a function of $\Sigma_{\pi N}$ and $g_A^{(0)}$. Actually, the decay width decreases until $g_A^{(0)} = 0.4$, and then starts to increase, while it gets smaller almost monotonically as the larger value of the $\Sigma_{\pi N}$ is used. The region where proper combinations of $g_A^{(0)}$ and $\Sigma_{\pi N}$ yield a small width $\Gamma_{\Theta KN} \leq 1$ MeV of the $\Theta^+$ pentaquark can easily be identified.

In Fig. 2 we draw the results of the total decay width of the $\Theta^+ \to KN$ as a function of $\Sigma_{\pi N}$ and $g_A^{(0)}$. The smaller the $\Gamma_{\Theta KN}$ the more restricted are the values $\Sigma_{\pi N}$ and $g_A^{(0)}$. The shaded rectangle indicates the area where one has generally accepted experimental values of $g_A^{(0)}$ and $\Sigma_{\pi N}$, i.e. 0.3–0.4 and 65–75 MeV, respectively, and simultaneously a $\Gamma_{\Theta KN} \leq 1$ MeV. It is of great interest to see

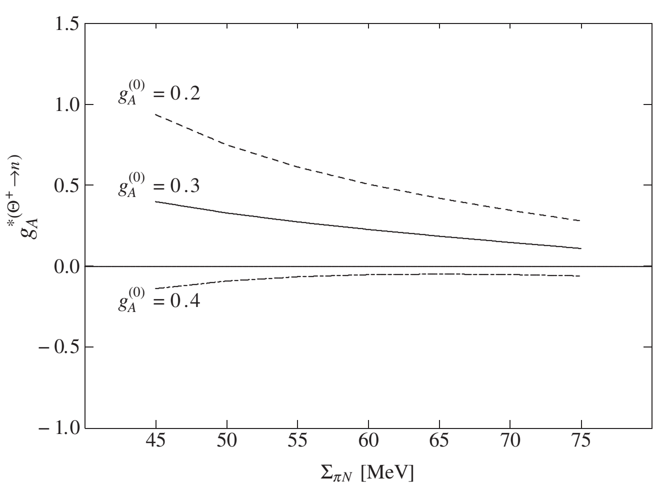


FIG. 1. The transition axial-vector coupling constant for $\Theta^+ \to K^+ n$ as a function of $\Sigma_{\pi N}$. The solid curve denotes that with $g_A^{(0)} = 0.3$, while the dashed and dotted-dashed ones represent that with $g_A^{(0)} = 0.2$, 0.4, respectively.

TABLE III. The decay width of $\Theta^+ \to KN$ determined with $g_A^{(0)}$ varied from 0.28 to 0.40. The $\Sigma_{\pi N}$ is varied from 45 to 75 MeV.

| $\Gamma_{\Theta KN}^{(\text{total})}$ | Input $g_A^{(0)}$ | | | |
|---|---|---|---|---|
| $\Sigma_{\pi N}$ [MeV] | 0.28 | 0.32 | 0.36 | 0.40 |
| 45 | 33.41 | 11.06 | 0.76 | 2.51 |
| 50 | 22.25 | 7.82 | 0.76 | 1.10 |
| 55 | 15.22 | 5.53 | 0.64 | 0.56 |
| 60 | 10.45 | 3.82 | 0.46 | 0.36 |
| 65 | 7.04 | 2.51 | 0.26 | 0.31 |
| 70 | 4.54 | 1.50 | 0.10 | 0.35 |
| 75 | 2.70 | 0.75 | 0.01 | 0.47 |

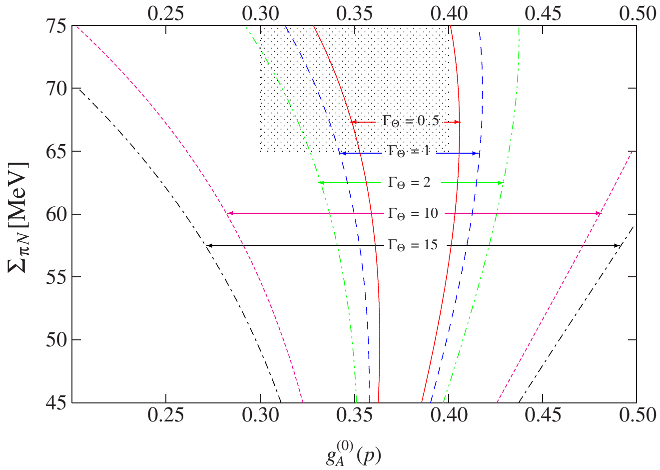


FIG. 2 (color online). The total decay width of $\Theta^+ \to KN$ in units of MeV as a function of $g_A^{(0)}$ and $\Sigma_{\pi N}$. The shaded square denotes the ranges of $g_A^{(0)}$: 0.3–0.4 and of $\Sigma_{\pi N}$: 65–75 MeV.

that the range of $g_A^{(0)}$ is compatible with a theoretical investigation [43], based on the $\chi$QSM, on the COMPASS and HERMES measurements of the deuteron spin-dependent structure function [44–46]. It is worthwhile to mention that the values of $g_A^{(0)}$ in the present analysis is almost the same as theoretical results within the $\chi$QSM [47,48]. The range of $\Sigma_{\pi N}$ given above is consistent with a recent analysis [37]. If one interprets the result of the DIANA Collaboration [12] as identification of the $\Theta^+$, namely, the formation of a narrow $pK^0$ peak with mass of $1537 \pm 2$ MeV$/c^2$ and width of $\Gamma = 0.36 \pm 0.11$ MeV in the $K^+ n \to K^0 p$ transition, then that result is inside the shaded area of Fig. 2.

## IV. SUMMARY AND CONCLUSION

In the present work, we analyzed within the framework of the chiral quark-soliton model the total decay width of the $\Theta^+ \to KN$, based on the experimental data of hyperon semileptonic decays and the flavor-singlet axial-vector constant $g_A^{(0)}$. The parameters in the collective Hamiltonian were fixed by the splittings of the SU(3) baryon mass representation [28]. The dynamical parameters in the collective axial-vector operators were fitted *model-independent approach* to the existing data of hyperon semileptonic decays and of $g_A^{(0)}$, where the value of the $g_A^{(0)}$ was varied within the range of 0.2–0.5. Since all these parameters depend on the value of the $\Sigma_{\pi N}$, we took its value to be 45–75 MeV.

We first computed the transition axial-vector coupling constant for the $\Theta^+ \to K^+ n$, $g_A^{*(\Theta\to n)}$. We showed that the $g_A^{*(\Theta\to n)}$ decreases as $g_A^{(0)}$ increases. Furthermore, the $g_A^{*(\Theta\to n)}$ depends on the $\pi N$ sigma term, $\Sigma_{\pi N}$: it is getting smaller as the $\Sigma_{\pi N}$ increases. Thus, the $g_A^{*(\Theta\to n)}$ turns out to be smaller with $\Sigma_{\pi N} = 70$ MeV by 70%, compared to that with $\Sigma_{\pi N} = 45$ MeV. It also was found that the $g_A^{*(\Theta\to n)}$ becomes negative around $g_A^{(0)} \simeq 0.37$.

The total width $\Gamma_{\Theta KN}$ of the $\Theta^+ \to KN$ decay was finally investigated. Since it is proportional to the square of the transition axial-vector constant $g_A^{*(\Theta\to n)}$, it is rather sensitive to the $g_A^{*(\Theta\to n)}$. The $\Gamma_{\Theta KN}$ is getting suppressed as the singlet axial-vector constant $g_A^{(0)}$ increases. However, since the $g_A^{*(\Theta\to n)}$ turns out to be negative around 0.37, the $\Gamma_{\Theta KN}$ starts to increase around 0.37. As a result, the total decay width $\Gamma_{\Theta KN}$ turns out to be smaller than 1 MeV for values of the $g_A^{(0)}$ and $\Sigma_{\pi N}$ larger than 0.31 and 65 MeV, respectively.

As a conclusion of the present analysis, which uses the model-independent approach to the chiral quark soliton, one can state: The known data of semileptonic decays combined with $0.3 \le g_A^{(0)} \le 0.4$ and $\Sigma_{\pi N} \ge 65$ MeV is compatible with the existence of a $\Theta^+$ pentaquark having a small width of the total decay $\Theta^+ \to KN$: $\Gamma_{\Theta KN} \le 1$ MeV. Since all dynamical parameters in the present approach are fixed by the existing experimental data, it is difficult to understand the origin of the strong correlation between the singlet axial-vector constant and the $\Theta^+$ decay width. The corresponding investigation is under way, in order to give a theoretical explanation of this strong correlation [49].

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