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Revising limits on neutrino-Majoron couplings

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Any theory that has a global spontaneously broken symmetry will imply the existence of very light neutral bosons or massless bosons (sometimes called Majorons). For most of these models we have neutrino-Majoron couplings that appear as additional branching ratios in decays of mesons and leptons. Here we present an updated limit on the couplings between the electron, muon, and tau neutrinos and Majorons. For such we analyze the possible effects of Majoron emission in both meson and lepton decays. In the latter we also include an analysis of the muon decay spectrum. Our results are $|g_{e\alpha}|^2 < 5.5 \times 10^{-6}$, $|g_{\mu\alpha}|^2 < 4.5 \times 10^{-5}$ and $|g_{\tau\alpha}|^2 < 5.5 \times 10^{-2}$ at 90% C. L., where $\alpha = e, \mu, \tau$.

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I. INTRODUCTION

Recently neutrino physics has given us many surprises with strong evidences for flavor neutrino conversion to another type of neutrinos. Analysis of data from solar, atmospheric, and reactor neutrinos have shown us that no other mechanism can explain all the data, unless you have massive neutrinos [1,2]. These experiments are the first strong evidence for nonconservation of family lepton number and this may indicate that new symmetries and interactions are the source of this phenomena.

Experimental evidences of massive neutrinos imply that the minimal standard model (SM) is no longer correct. The simplest extension would be the inclusion of right-handed sterile neutrinos, what would allow Dirac mass terms for neutrinos. Despite its simplicity, this approach does not help us to understand the neutrino mass scale or predict the neutrino masses. Because of the large gap between neutrino mass scales and the other SM scales, several mechanisms have been suggested to generate neutrino masses, relating this mass scale to new physics. In many of these models the masses are of the Majorana type or a mix between Majorana and Dirac types, what implies nonconservation of lepton number.

As it is well known, lepton number is an accidental global symmetry $[U_L(1)]$ of the standard model. So, if the neutrino mass matrix includes Majorana terms, lepton number is broken either explicitly or spontaneously. If lepton number (L) is indeed a global symmetry,[1] its spontaneous breaking will generate a Goldstone boson, usually called Majoron [3,4]. In this case the breaking of L sets a new scale and requires a scalar which carries lepton number and acquires a non-null vacuum expectation value (vev). Several extensions of the SM allow spontaneous L breaking and predicts the existence of the Majoron. However, the simplest extensions (with a triplet scalar) are excluded due to the experimental results of LEP on the Z^0 invisible decay.

Another important class of models which predicts the existence of Majorons are supersymmetric extensions of the standard model with spontaneous R parity breaking. In these models, the introduction of antineutrino superfields (N^C) and new singlet superfields (Φ) (which contain neutral leptonic scalars), allows spontaneous breaking of lepton number [5–7]. In almost all of these models the Majoron will be the imaginary part of some linear combination of sneutrinos, the scalar component of the super Higgs fields (H_u and H_d) and the Φ superfields. Therefore we may safely assume as a model-independent coupling the following interaction term between J and ν[2]:

$$\mathcal{L} = \sum_{\alpha,\beta=e,\mu,\tau} i g_{\alpha\beta} \bar{\nu}_\alpha \gamma^5 \nu_\beta J, \tag{1}$$

where $g_{\alpha\beta}$ is a general complex coupling matrix in the flavor basis. Because in most models J is basically a singlet (avoiding the constraints imposed by the LEP results), the above couplings are usually the most relevant ones to phenomenological analysis (at least at low energies). In most models we must also include couplings between neutrinos and a new light scalar (that we call χ) with the same couplings as J:

$$\mathcal{L} = \sum_{\alpha,\beta=e,\mu,\tau} g_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta \chi. \tag{2}$$

Usually neutrino masses and mixings will depend on the vevs associated to the spontaneous breaking of L and the matrix g. In this context, knowledge of the couplings between neutrinos and Majorons may help us to understand the neutrino mass scale. However, this relation is very model dependent and may be very hard to realize in practice. Trying to make our results as model independent as possible, we will make no assumptions on $g_{\alpha\beta}$ and present our results with and without the existence of the massive scalar χ. Nevertheless, assuming Majorana neu-

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[1]In some grand unified theories (GUTs), lepton number is gauged and becomes a subgroup of a larger gauge symmetry.

[2]The same being valid for nonsupersymmetric models as well.

trinos (which is reasonable since lepton number is violated), bounds on $g_{\alpha\beta}$ may be transformed to the mass basis through the relation

$$G = U^T g U \tag{3}$$

where G_{ij} is the neutrino-Majoron coupling matrix in the basis where the neutrino mass matrix is diagonal ($M = \mathrm{diag}(m_1, m_2, m_3)$) and U rotates the mass eigenstates to the flavor eigenstates (see Sec. II D).

Majoron models can be interesting from the cosmological point of view because they can affect bounds to neutrino masses from large scale structure [8]. Neutrinos coming from astrophysical sources can also be significantly affected by fast decays, where the only mechanism not yet eliminated is due to neutrinos coupling to Majorons. This can affect the very high energy region, strongly changing the flavor ratios between different neutrino species [9] or the lower energy region, as supernova neutrinos [10,11].

Presently we know that the role of neutrino-Majoron couplings is marginal in solar and atmospheric neutrinos, therefore it is possible to put a limit on [12]

$$|G_{21}|^2 < |g_{\alpha\beta} U^*_{\alpha 2} U_{\beta 1}|^2 < 4 \times 10^{-6} \left(\frac{7 \times 10^{-5}\ \mathrm{eV}^2}{\Delta m^2_\odot} \right) \tag{4}$$

where G_{21} is the neutrino-Majoron coupling in the mass basis and $\Delta m^2_\odot$ is the solar mass difference squared ($\Delta m^2_{21} \equiv m^2_2 - m^2_1$). The observation of 1987A explosion ensures us that a large part of binding energy of supernova is released into neutrinos, what can be translated into the bounds [10]

$$\begin{aligned} |g_{ee}| &< (1 - 20) \times 10^{-5}, \\ 2 \times 10^{-11} < |g_{e\mu}||g_{\mu\mu}| &< 3 \times 10^{-10}. \end{aligned} \tag{5}$$

Such bounds were read off from Fig. 1 and Figs. 3,4 of Ref. [10] for g_{ee} and $|g_{e\mu}||g_{\mu\mu}|$, respectively. Also, limits from decay and scattering of Majorons inside supernova give the bounds [11]

$$|g_{\alpha\beta}| < 3 \times 10^{-7} \quad \text{or} \quad |g_{\alpha\beta}| > 2 \times 10^{-5}. \tag{6}$$

The first limit appears because if neutrino-Majoron coupling is strong enough the supernova energy is drained due Majoron emission and no explosion occurs; the second limit appears because if neutrino-Majoron coupling is too strong, the Majoron becomes trapped inside the supernova and no constraint is possible.

While neutrinoless double beta decays ($\beta\beta0\nu$) provide us the constraint

$$|g_{ee}| < 2 \times 10^{-4} \quad \text{and} \quad |g_{ee}| < 1.5 \tag{7}$$

where the first (second) bound corresponds to Majorons with lepton number equal to $L = 0$ ($L = 2$) at 90% C. L. [13]. Also, no evidence of Majoron production was seen in pion and kaon decays and therefore [14–16]

$$\sum_{l=e,\mu,\tau} |g_{el}|^2 < 3 \times 10^{-5}, \qquad \sum_{l=e,\mu,\tau} |g_{\mu l}|^2 < 2 \times 10^{-4}. \tag{8}$$

Besides the bounds mentioned above, there are bounds that depend on the rate of neutrino decay ($\nu \to \nu' J$). Such reaction depends on the neutrino lifetime, τ, that is a function of neutrino-Majoron couplings in the mass basis, which we denote by G. Without additional assumptions on neutrino hierarchy, we can not relate directly the neutrino-Majoron couplings and the neutrino lifetime. One example is Ref. [17] that, using cosmic microwave background data, puts a stronger constraint

$$G_{ij} \leq 0.6110^{-11} m^{-2}_{50} \quad \text{and} \quad G_{ii} \leq 10^{-7} \tag{9}$$

where $m_{50} = m/50$ meV and G is the neutrino-Majoron in the mass basis: G_{ii} and G_{ij} are, respectively, the diagonal and off-diagonal elements of G.

Future experiments can improve the present bounds on many orders of magnitude, we refer to Ref. [18,19] for details.

A summary of some of the previous bounds are shown in Table I, where we also show the respective relevant process used to constrain the neutrino-Majoron couplings. Almost

TABLE I. Some of the previous bounds on neutrino-Majoron couplings from different sources. In the last two columns are shown the process used to constraint the couplings and the respective references.

Category	Upper Bound	Process	Reference
solar neutrino constrain	$\lvert G_{21}\rvert^2 < 4 \times 10^{-6}\left(\frac{7 \times 10^{-5}\ \mathrm{eV}^2}{\Delta m^2_\odot}\right)$	$\nu_2 \to J + \nu_1$	[12]
supernova bounds	$\lvert g_{ee}\rvert < (1 - 20) \times \lvert 10^{-5}$, $2 \times 10^{-11} < \lvert g_{e\mu}\rvert\lvert g_{\mu\mu}\rvert < 3 \times 10^{-10}$, $\lvert g_{\alpha\beta}\rvert < 3 \times 10^{-7}$ or $\lvert g_{\alpha\beta}\rvert > 2 \times 10^{-5}$	$\nu \to J + \nu$, $\nu + \nu \to J$, $\nu \to J + \nu'$, $\nu + \nu \to J$	[10,11]
$\beta\beta0\nu$ decay	$\lvert g_{ee}\rvert < 2 \times 10^{-4}$	$(A, Z) \to (A, Z + 2) + 2e + J$	[13]
microwave background data	$G_{ij} \leq 0.6110^{-11} m^{-2}_{50}$ and $G_{ii} \leq 10^{-7}$	$\nu \to J + \nu'$	[17]
meson decay	$\sum_{l=e,\mu,\tau} \lvert g_{el}\rvert^2 < 3 \times 10^{-5}$, $\sum_{l=e,\mu,\tau} \lvert g_{\mu l}\rvert^2 < 2 \times 10^{-4}$	$\pi/K \to e + \nu + J$	[14–16]

all the bounds shown in Table I assume one particular model or class of models. Probably the most model-independent result is from [14–16], but in this case they assume not only neutrino-Majoron couplings but also neutrino-χ couplings to compute the upper bounds shown in Table I.

Here we will try to improve or make these limits more model-independent through an analysis of both meson and lepton decays. In Sec. II A we discuss the limits from pion, kaons, D, D_s and B decays, including decays of mesons into taus; in Secs. II B and II C we include bounds from lepton decays (from the total rate and from the spectral distortions). We conclude transforming our bounds to the mass basis in Sec. II D.

II. RESULTS

Here we try to improve the bounds on neutrino-Majoron couplings through the analysis of possible effects on mesons and leptons decays as well as on the spectrum of the muon decay. We also rewrite our results in the mass basis, which in many cases is more important for model analysis. All the bounds obtained here have 90% C.L. and were obtained through the chi-square method assuming Gaussian distributions and including both statistical and theoretical errors as follows

$$\chi^2 = \frac{(R_{\rm data} - R_{\rm theor})^2}{\sigma^2_{\rm data} + \sigma^2_{\rm theor}} \tag{10}$$

where $R_{\rm data}$, $R_{\rm theor}$, $\sigma_{\rm data}$, and $\sigma_{\rm theor}$ are, respectively, the experimental data of the rate R, the theoretical prediction for process R, assuming an incoherent sum of SM rate and Majoron contribution, the experimental error and the theoretical error.

A. Meson decay rates

The process $M \to l + \nu_l$ was extensively studied in the literature and has the following total decay rate [20]:

$$\Gamma_{SM} = \frac{G_F^2 |V_{qq'}|^2}{8\pi} f_m^2 m_l^2 m_M \left(1 - \frac{m_l^2}{m_M^2}\right)^2 f_{\rm rad}, \tag{11}$$

where the $f_{\rm rad}$ accounts for radiative corrections. In Eq. (11), m_M and m_l are the meson and lepton masses, G_F is the Fermi constant, f_m is the meson decay constant, and $V_{qq'}$ is the respective Cabibbo-Kobayashi-Maskawa (CKM) matrix element. Unless specified otherwise we are using the quantities as listed in the Particle Data Group compilation [20]. We also use the same source to compute the relevant radiative corrections for the mesons decay rates. An important feature of Eq. (11) is that, because it is a 2-body decay, $\Gamma_{\rm SM}$ is proportional to m_l^2, as it should be to conserve angular momentum.

In the last few years several of the meson decay constants were calculated through lattice QCD [21], which can be used to obtain stronger theoretical predictions. We used both the experimental [20] and theoretical values [22–25] of f_m on our calculations, but in most cases the results differ only by 10%. For this reason we will only show the results using the experimental values of f_m.

Because of the neutrino-Majoron couplings, the following process also contributes to mesons decay rates:

$$M \to l + \nu_{l'} + J, \tag{12}$$

where J stands for Majoron and $\nu_{l'}$ may be any neutrino flavor. A complex analytic expression for the total decay rate is given in [15]. Here we show a simpler result valid in the limit $m_l = m_\nu = 0$:

$$\Gamma_J = \frac{G_F^2 |V_{qq'}|^2}{768\pi^3} f_m^2 m_M^3 \sum_{m=e,\mu\tau} |g_{lm}|^2 \tag{13}$$

This result shows that when Majorons are included, the total decay rate is no longer proportional to the lepton mass (since now we have a 3-body decay). Therefore, the Majoron contribution (Γ_J) may easily overcome the SM predictions ($\Gamma_{\rm SM}$) if $g \sim 1$:

$$\frac{\Gamma_J}{\Gamma_{\rm SM}} \approx \frac{1}{48\pi^2} \frac{m_M^2}{m_l^2} \gg 1 \tag{14}$$

where we have assumed $m_l \ll m_M$. Assuming that the total decay rate is

$$\Gamma_{\rm total} = \Gamma_{\rm SM} + \Gamma_J, \tag{15}$$

the decay on J will be the dominant channel, unless g is small. Because only small deviations from the SM are allowed by experimental data, we must have $g \ll 1$. Following Eq. (10), we calculated upper bounds for $|g_{\alpha\beta}|^2$ at 90% C. L. The Table II shows the bounds obtained through this analysis.

As expected from the above remarks and the results on Table II, the most constrained matrix elements $g_{\alpha\beta}$ will be those concerning e, since the approximation $m_l \ll m_M$ is good in this case. We found that this bound can be improved using recent data [28] of the following ratio:

$$\frac{\Gamma(K^+ \to e^+ + \nu_e)}{\Gamma(K^+ \to \mu^+ + \nu_\mu)} = (2.416 \pm 0.043) \times 10^{-5} \tag{16}$$

TABLE II. Upper bounds on $\sum_{l=e,\mu,\tau} |g_{l\beta}|^2$ from meson decays with 90% C.L. The references for the experimental values used are shown in the last column. We only include the Majoron contribution, and not the new light scalar χ.

Mesons	$\lvert g_{e\alpha}\rvert^2$	$\lvert g_{\mu\alpha}\rvert^2$	$\lvert g_{\tau\alpha}\rvert^2$	Refs (exp. values) π
π	1.6×10^{-4}	2.1×10^{-1}		[20]
K	9.5×10^{-4}	9.3		[20]
D	1.6×10^{-1}	2.3	23	[20]
D_s		1	6.3	[26]
B	0.85	1.5	19	[27]

where the error is the quadrature of statistical and systematic errors. Because the Majoron contributions must be suppressed (as shown in Table II), we may approximate the above ratio:

$$\frac{\Gamma(K^+ \to e^+ + \nu_e)}{\Gamma(K^+ \to \mu^+ + \nu_\mu)} = \frac{\Gamma^e_{\rm SM} + \Gamma^e_J}{\Gamma^\mu_{\rm SM} + \Gamma^\mu_J} \approx \frac{\Gamma^e_{\rm SM} + \Gamma^e_J}{\Gamma^\mu_{\rm SM}}, \quad (17)$$

where $\Gamma^{e(\mu)}$ represents the decay rate with an e (μ) in the final state. In this way, using the previous statistical analysis, we can constrain the elements $g_{e\alpha}$ (at 90% C.L.):

$$\sum_{l=e,\mu,\tau} |g_{el}|^2 < 1.1 \times 10^{-5}. \quad (18)$$

When it comes to the μ matrix elements ($g_{\mu\alpha}$), the constraints from Table II may also be improved if we consider the decay channels of mesons in four leptons [20]:

$$BR(K^+ \to \mu^+ + \nu_\mu + \nu + \bar{\nu}) < 6 \times 10^{-6}. \quad (19)$$

Since the SM contribution to this decay is negligible, we may assume:

$$BR(K^+ \to \mu^+ + \nu_{l'} + J) < 6 \times 10^{-6} \quad (20)$$

resulting on (at 90% C.L.):

$$\sum_{l=e,\mu,\tau} |g_{\mu l}|^2 < 9 \times 10^{-5} \quad (21)$$

Finally, new experimental data for leptonic decay rates of heavy mesons such as the D^+, D_s^+ and B^+ mesons [20,26,27,29,30], allow us to impose limits to the τ matrix elements ($g_{\tau\alpha}$), as shown in Table II. The best bound being from the D_s^+ leptonic decay on $\tau^+ + \nu_\tau$ (at 90% C.L.)[26]:

$$\sum_{l=e,\mu,\tau} |g_{\tau l}|^2 < 6.3. \quad (22)$$

Because of large experimental uncertainty, this bound is quite weak, as can be seen above.

We stress that unlike [14–16], the results shown so far do not include possible decays on a light scalar χ and therefore are less model-dependent. If this new scalar is considered with a mass of 1 KeV (other choices for the χ mass do not change these results as long as it is well below the initial state masses), the previous results are basically improved by a factor of 2 (again, at 90% C.L.):

$$\sum_\alpha |g_{e\alpha}|^2 < 5.5 \times 10^{-6},$$
$$\sum_\alpha |g_{\mu\alpha}|^2 < 4.5 \times 10^{-5} \quad \text{and} \quad \sum_\alpha |g_{\tau\alpha}|^2 < 3.2 \quad (23)$$

B. Lepton decay rates

Because of its good experimental precision, lepton decays are good candidates for imposing bounds on neutrino-Majoron couplings. Moreover, in this case there are not uncertainties such as mesons decay constants and CKM elements. However, the leading term in $\Gamma(l_i \to l_j + \bar{\nu}_j + \nu_i)$ is no longer proportional to the final lepton mass (as it was in the case of mesons), because the SM decay is a 3-body decay already. For this reason $\Gamma_J < \Gamma_{\rm SM}$ even for $g \sim 1$. In fact the inclusion of Majorons in the final state decreases the decay rate by a factor of ≈ 10 ($\Gamma_J \approx \Gamma_{\rm SM}/10$, for $g = 1$), instead of increasing it as it was in the case of mesons. Therefore we expect much weaker bounds in this case. But, as we will show below, it is still possible to obtain good bounds for certain decays.[3] To calculate the 4-body decay rate [$\Gamma(l \to l' + \bar{\nu} + \nu + J)$] we used the programs FEYNARTS and FORMCALC [31,32].

As we did in the meson case, to constrain the $g_{\alpha\beta}$ matrix we assume that the total lepton decay rate receives contributions from Majoron emission:

$$\Gamma_{\rm total}(l_\alpha \to l_\beta + \bar{\nu}_\beta + \nu_\alpha) = \Gamma_J(l_\alpha \to l_\beta + \bar{\nu} + \nu + J) + \Gamma_{\rm SM}(l_\alpha \to l_\beta + \bar{\nu}_\beta + \nu_\alpha). \quad (24)$$

Because Majoron emission may change neutrino flavor (since $g_{\alpha\beta}$ may be nondiagonal), Γ_J may have any type of neutrinos in its final state. For this reason we omitted the subindex in Γ_J. Besides, both neutrinos (ν or $\bar{\nu}$) may emit Majorons, which implies:

$$\Gamma_J(l_\alpha \to l_\beta + \bar{\nu} + \nu + J) \propto \sum_\delta (|g_{\alpha\delta}|^2 + |g_{\beta\delta}|^2), \quad (25)$$

where $g_{\alpha\delta}$ and $g_{\beta\delta}$ are the couplings between Majoron and the α antineutrino and β neutrino, respectively. In Eq. (25), the interference terms $g_{\alpha\delta}g_{\beta\delta}$ are proportional to neutrino masses squared and were neglected. Because Table II shows that lighter leptons have stronger upper bounds, we will assume $g_{\alpha\delta} \gg g_{\beta\delta}$. Therefore we will consider that Majoron emission by $\bar{\nu}_\alpha$ is dominant:

$$\Gamma_J(l_\alpha \to l_\beta + \bar{\nu} + \nu + J) \propto \sum_\delta |g_{\alpha\delta}|^2. \quad (26)$$

Using the experimental values for the μ and τ decay rates [20] and the same kind of analysis used in the last section, the following bounds were obtained at 90% C.L.:

$$\sum_\alpha |g_{\mu\alpha}|^2 < 4 \times 10^{-4}, \qquad \sum_\alpha |g_{\tau\alpha}|^2 < 10 \times 10^{-2}, \quad (27)$$

where the first bound comes from μ decay and the second from τ decay, both at 90% C.L.. For the τ decay the same constraint is obtained if one considers decays in e's or μ's. If we include the contributions from χ emission (again with mass of 1 KeV and at 90% C.L.):

[3] We thank J. F. Beacom for the suggestion to use lepton decays to constrain neutrino-Majoron decays.

$$\sum_\alpha |g_{\mu\alpha}|^2 < 2.7 \times 10^{-4}, \qquad \sum_\alpha |g_{\tau\alpha}|^2 < 5.5 \times 10^{-2}. \tag{28}$$

C. Spectrum of lepton decay with Majorons

Another method that can be used to improve the limits obtained above is the analysis of the electron spectrum in the muon decay, which can be modified by the inclusion of Majorons. The normalized spectrum for the SM case and the Majoron case only are shown in Fig. 1.

Precision measurements of the electron spectrum (used to impose constraints on non V-A interactions) may constrain g if we consider the changes in the SM spectrum after including Majoron emission. The usual analysis parametrizes the electron spectrum using two parameters (ρ and η) [20]:

$$\frac{d\Gamma(x)}{dx} = \frac{G_F^2 m_\mu^5}{48\pi^3} x^2 \left[3(1-x) + \frac{2}{3}\rho(4x-3) + 3\eta \frac{m_e}{E_{\max}} \frac{1-x}{x} \right] \tag{29}$$

where $x = \dfrac{E}{E_{\max}}$ and $E_{\max} = \dfrac{m_\mu^2 + m_e^2}{2m_\mu}$. For the SM the predicted values are $\rho = 0.75$ and $\eta = 0$:

$$\frac{d\Gamma_{\rm SM}(x)}{dx} = \frac{G_F^2 m_\mu^5}{48\pi^3} \left[\frac{3}{2}x^2 - x^3 \right]. \tag{30}$$

The current experimental values are $\rho = 0.7509 \pm 0.001$ and $\eta = 0.001 \pm 0.024$ [20].

When the total spectrum (SM plus Majoron) is considered, we have found

$$\frac{d\Gamma_{\rm total}(x)}{dx} = \frac{G_F^2 m_\mu^5}{48\pi^3} \left[0.0066|g|^2 - 0.09|g|^2 x + \left(\frac{3}{2} + 0.35|g|^2\right)x^2 - (1 + 0.25|g|^2)x^3 \right] \tag{31}$$

where $|g|^2 = \sum_\alpha |g_{\mu\alpha}|^2$.

From the above expression and Fig. 1 we see that the most sensitive region is at the end of the spectrum (large x), which can be used to constrain g. Figure 1 also shows the allowed region by experimental data (region between solid gray lines) and the shape of the total spectrum (including the SM and Majoron contributions) with different values of $\sum_\alpha |g_{\mu\alpha}|^2$.

Because the spectrum is more sensitive to changes in the cubic term (or the ρ parameter), we consider the Majoron contributions to ρ:

$$\rho_{\rm total} = \tfrac{3}{8}(2 - 0.25|g|^2). \tag{32}$$

Using the chi-square method at 90% C.L. we obtain:

$$\sum_\alpha |g_{\mu\alpha}|^2 < 8 \times 10^{-3}. \tag{33}$$

As can be seen in Fig. 2, the Majoron main modifications to the spectra occurs in the neutrino spectrum, which has been measured by the Karmen experiment [33]. However, due to experimental uncertainties, the resulting bounds on g are too weak in this case.

Summarizing, the strongest bounds are given in the Table III, where we compared the previous limits and the newest constraints obtained here.

All bounds from Eqs. (23), (28), and (33) can be written as

$$\sum_{\alpha=e,\mu,\tau} |g_{l\alpha}|^2 < L_l^2 \tag{34}$$

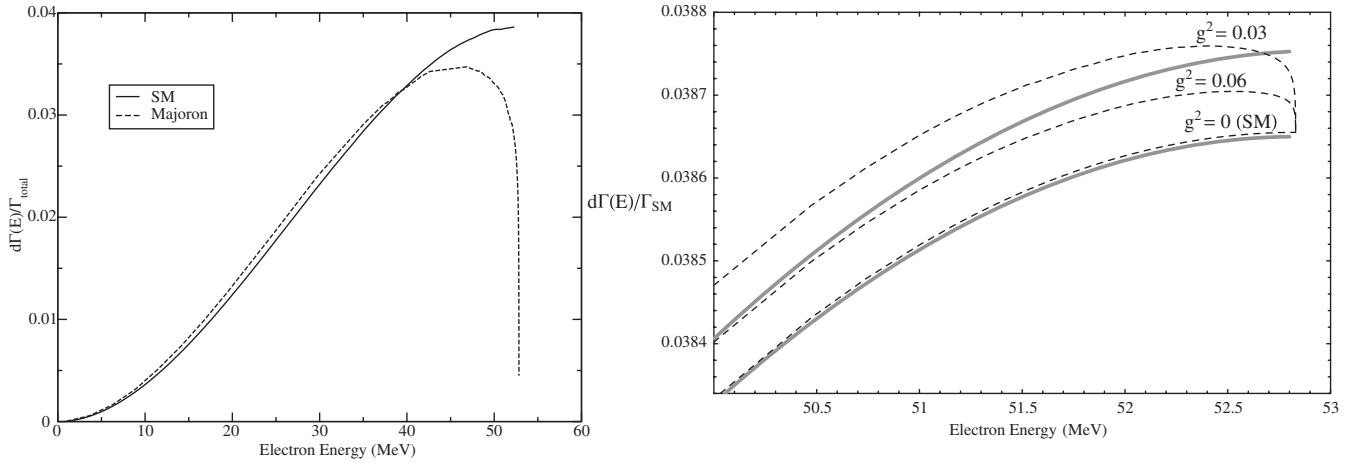

FIG. 1. At left, normalized electron spectra for muon decay in the SM (solid line) and with Majoron emission only (dashed line). At right, the experimental allowed region (between solid lines) for the electron spectrum and the total predicted spectrum (SM plus Majorons) for three values of $g^2 = \sum_\alpha |g_{\mu\alpha}|^2$.

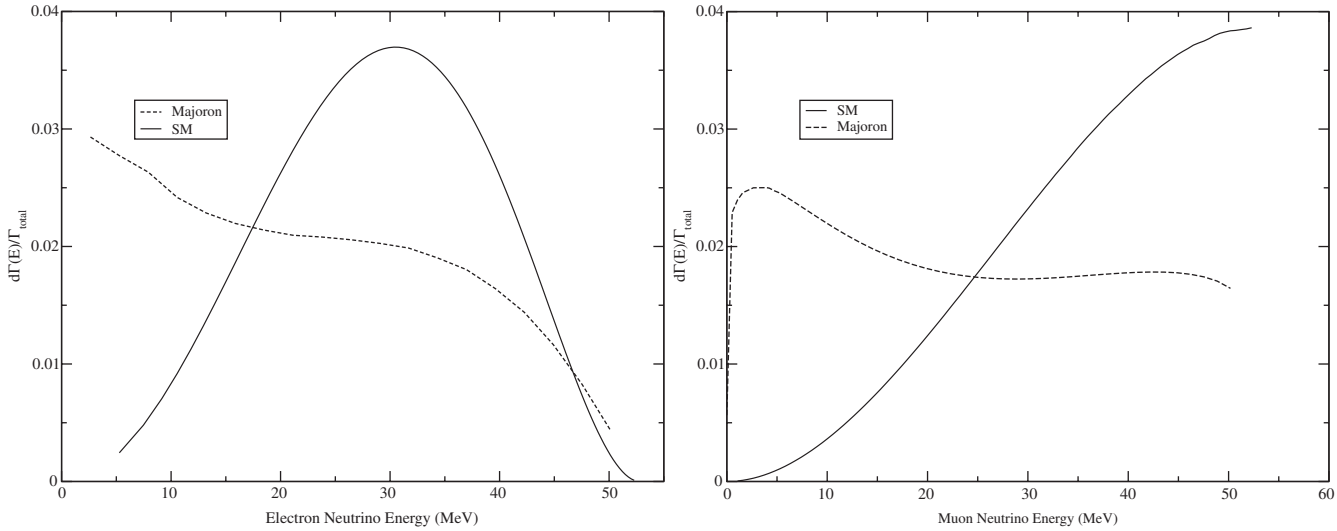

FIG. 2. At left (at right), normalized electron neutrino (muon neutrino) spectra for muon decay in the SM is the solid curve and with Majoron emission only is the dashed curve. In both cases we assume a diagonal $g_{\alpha\beta}$.

where L_l^2 is the strongest upper bound for $\sum_\alpha |g_{l\alpha}|^2$ (see Table III). From this constraints, we assume the conservative limit, where the upper bound applies not only for the sum, $\sum_\alpha |g_{l\alpha}|^2$, but also for the individual elements, as $|g_{l\alpha}|$:

$$|g_{l\alpha}| < L_l, \quad \forall\ \alpha = e, \mu, \tau. \tag{35}$$

D. Mass basis

All the results obtained so far are written in the flavor basis. However, in many cases, theoretical analyses are easier on the mass basis. We have two possible cases: Dirac or Majorana neutrinos. In this section we assume Majorana neutrinos to transform our bounds to the mass basis.

We can translate the previous results to the mass basis using the transformation matrix U [20]:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) \tag{36}$$

where $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$. The neutrino mass matrix is given by $M = \mathrm{diag}(m_1, m_2, m_3)$ and for a given mass m_1, we can written all other masses as a function of m_1 and the squared mass differences as follows

$$\begin{aligned} \Delta m_{12}^2 &\equiv m_2^2 - m_1^2 = \Delta m_\odot^2 \quad \text{and} \\ \Delta m_{23}^2 &\equiv m_3^2 - m_2^2 = \Delta m_{\mathrm{atm}}^2. \end{aligned} \tag{37}$$

Although the mass differences and angles have been measured experimentally [20], we have no information on the Majorana phases δ, α_1, and α_2.

TABLE III. Comparison between the strongest bounds (including the scalar χ) obtained here and the previous bounds from the same processes. All bounds are at 90% C.L. and the previous bounds are from [14–16].

Previous Bounds	Revised Bounds
$\sum_\alpha \lvert g_{e\alpha}\rvert^2 < 3 \times 10^{-5}$	$\sum_\alpha \lvert g_{e\alpha}\rvert^2 < 5.5 \times 10^{-6}$
$\sum_\alpha \lvert g_{\mu\alpha}\rvert^2 < 2.4 \times 10^{-4}$	$\sum_\alpha \lvert g_{\mu\alpha}\rvert^2 < 4.5 \times 10^{-5}$
none	$\sum_\alpha \lvert g_{\tau\alpha}\rvert^2 < 5.5 \times 10^{-2}$

To calculate the bounds in the mass basis, we will use the transformation rule for Majorana neutrinos

$$G = U^T g U \tag{38}$$

where g is the neutrino-Majoron coupling matrix in the flavor basis and G is the neutrino-Majoron coupling matrix in the mass basis.

Although it is not valid in general, many models [34–36] have the following property (at least in some limit)

$$G = \mathrm{diag}(g_1, g_2, g_3) \propto M = \mathrm{diag}(m_1, m_2, m_3). \tag{39}$$

Following [37], we calculate the allowed region for different values of δ, α_1, and α_2 and then choose the union of these regions as the final result, valid for any value of the phases, as shown in Fig. 3.

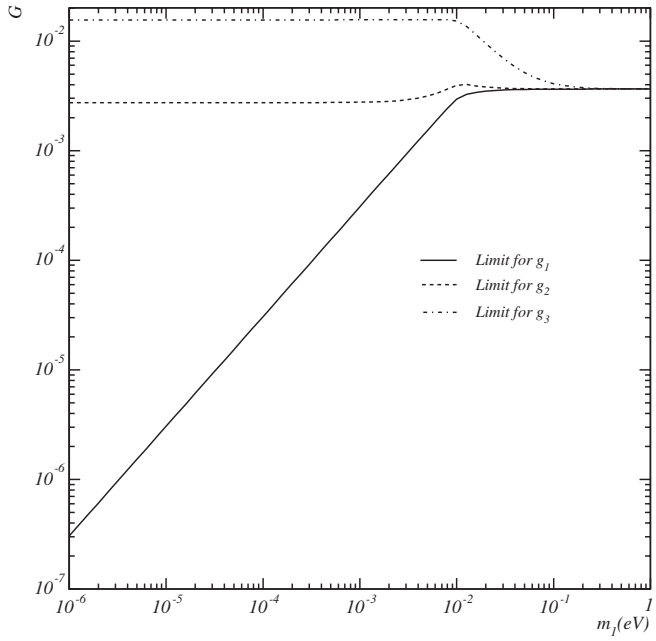

FIG. 3. Exclusion curves in the $m_1 - G$ plane. We show the curves for g_1, g_2, and g_3. These curves are for $\theta_{13} = 0$, nonzero values of θ_{13} are more restrictive.

III. CONCLUSIONS

Using three different techniques we were able to constrain the neutrino-Majoron couplings. The strongest constraints are shown in Table III. Considering only the limits from meson decays we improve by 1 order of magnitude the previous limits on $|g_{e\alpha}|^2$ and $|g_{\mu\alpha}|^2$ [14–16]. Although the best constraints were obtained from meson decay rates, we have shown that independent bounds can also be obtained from μ and τ decays. The latter one being the best to constrain the $g_{\tau\alpha}$ elements. We stress that the bounds on $g_{\tau\alpha}$ shown in Table III is probably the first model-independent constraint for this parameter.

The third alternative used was an analysis of the spectrum of muon decay. Despite its potential for constraining the $g_{\mu\alpha}$ elements, the experimental values are not precise enough to make such an analysis useful. Our best constraints are $|g_{e\alpha}|^2 < 5.5 \times 10^{-6}$, $|g_{\mu\alpha}|^2 < 4.5 \times 10^{-5}$, and $|g_{\tau\alpha}|^2 < 5.5 \times 10^{-2}$, $\alpha = e, \mu, \tau$ at 90% C. L..

Because the models cited here usually try to explain the neutrino mass scale, it may be convenient to analyze the limits on neutrino-Majoron couplings in the mass basis. With that in mind we transformed all our results from the flavor basis to the mass basis, using the current values for the angles of the neutrino mixing matrix. As shown in Fig. 3 the constraints on the mass basis are usually weaker than those on the flavor basis.

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