Spectral broadening of radiation from relativistic collapsing objects

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We discuss light curves and the spectral broadening of the radiation emitted during a finite interval of time from the surface of a spherically symmetric collapsing object. We study a simplified model of monochromatic radiations. We discuss how one can obtain information about the physical parameters of the collapsing body, such as its mass and radius, from the light curves and spectral broadenings.

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Light propagating in the vicinity of astrophysical compact objects, like neutron stars and black holes, is affected by the gravitational field. It has been demonstrated that the general relativistic effects might be important for understanding the features of the radiation coming from the neutron-star-like objects [1–3]. The gravitational redshift and bending of light rays emitted by a compact object affect the form and spectrum of the observed signals. For the emission of light from the vicinity of a black hole these effects are more profound than for neutron stars. For example, the line broadening of X-rays observed by the ASCA satellite can be explained by the strong gravitational effect on the light emitted from the accretion disk located near to the central black hole [4,5].

Astrophysical black holes are believed to be formed as a result of the gravitational collapse of massive stars [6]. Then it is natural to expect that the radiation emitted during the gravitational collapse is also affected by the strong gravity. For a spherical collapse and continuous emission of light, this effect was studied in detail [7–9]. Recently the light curves for collapsing objects were studied in a slightly different setup [10] assuming that the radiation has a profile of a sharp pulse in time. Such radiation may occur during the collapse of a star when its matter density becomes much higher than the nuclear density. Under these conditions hadronic phase transitions are expected [11] which may result in sharp-in-time emission of massless particles (photons and neutrinos) [12].

In this work, we consider the radiation emitted by a collapsing star during a finite time interval and calculate light curves and the spectrum of this radiation as seen by a distant observer. As in the previous work [10], we adopt a simplified model of a freely-falling spherical surface and assume that the radiation is originally monochromatic. But instead of instant radiation, we focus on the radiation emitted during a finite interval of time. The main goal of this study is to analyze how one can extract information about the characteristics of a collapsing object (its mass and radius) from the observed spectra and light curves.

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In the adopted simplified approach, we consider a spherical collapse and assume that the points of the collapsing surface follow radial geodesics in the Schwarzschild geometry [7,10,13,14]. Thus the photons emitted from the surface propagate to the observer at infinity in the background of the Schwarzschild metric

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2. \tag{1}$$

Here $f = f(r) = 1 - 2M/r$, M is the mass of the collapsing object, and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is a line element on a unit sphere. We use the natural units, $c = G = \hbar = 1$ throughout this paper.

Denote by τ the proper time as measured by an observer comoving with the collapsing surface. We suppose that the collapse starts at $\tau = 0$ with the initial surface radius R_0. We denote by $t^{(e)}(\tau)$ the Schwarzschild time t corresponding to τ and take $t^{(e)}(\tau = 0) = 0$. In the coordinates (t, r, θ, ϕ) the four-velocity of the collapsing surface is $v^\mu = (dt^{(e)}/d\tau, dR/d\tau, 0, 0)$. We define v_I as the invariant radial velocity which measures the proper length change as measured by the proper time of the observer at rest at a given radius. One has $v_I = f^{-1}(R)\frac{dR}{dt}$, $|v_I| \leq 1$. For a freely-falling surface with the initial radius R_0 the invariant velocity as a function of R is

$$v_I = -\sqrt{\frac{2M}{R}}\frac{\sqrt{1 - R/R_0}}{\sqrt{1 - 2M/R_0}}. \tag{2}$$

Because of the spherical symmetry the trajectories of particles and light are plane, so that we can always put $\phi = 0$. Denote by $p^\mu = (p^t, p^r, p^\theta, 0)$ the 4-momentum of a photon, then $E = -p_t$ (the energy at infinity) and $L = p_\theta$ (the angular momentum) are constants of motion. In the adopted units, where $\hbar = 1$, E coincides with the photon frequency as measured by a far distant observer, and $\mathbf{p}$ coincides with its wave vector. Instead of the angular momentum L we shall use the impact parameter defined by $l = L/E$. The radial momentum p^r is given by $p^r = \sigma E Z$, where $Z(l, r) = \sqrt{1 - l^2 f(r)/r^2}$. Here and later σ denotes a sign function which takes the values + and − for

a forward ($p_r > 0$) and backward ($p_r < 0$) motion of the photon, respectively.

For a photon emitted at the radius R and propagating to the infinity there exists an upper limit for the impact parameter $l_{\max} = R/\sqrt{f(R)}$ determined by the condition $Z(l_{\max}, R) = 0$. Let us consider the emission angle, β, of the radiation as measured by an observer comoving with the surface. The emission angle is zero for the emission in vertical outward direction and $\pi/2$ for the tangentially emitted radiation. Since the matter inside the collapsing surface is supposed to be opaque, it is natural to consider only the light emitted with the emission angle, $\beta \leq \pi/2$. It gives a lower bound on the impact parameter for the backward emission, $l_T \leq l \leq l_{\max}$, where $l_T = R/\sqrt{f(R_0)}$ is determined by the condition for tangentially emitted radiation, $Z(l_T, R) = -v_I$. Hence the possible ranges of the impact parameter are $0 \leq l \leq l_{\max}$ and $l_T \leq l \leq l_{\max}$ for a forward and backward emission, respectively. In this work we consider only $R > 3\sqrt{3}\sqrt{1 - 2M/R_0}M$. A discussion of the allowed ranges of the impact parameter for the smaller radius up to $R \sim 2M$, can be found in [7,8].

We choose the direction of the axis $\theta = 0$ such that a photon emitted with $\beta = 0$ at $\theta = 0$ propagates along the radial direction to a distant observer. For a photon emitted from a collapsing surface at the angle θ to reach the distant observer, θ coincides with the bending angle. For a null ray from the collapsing surface when its radius is R, the bending angle for a forward-emission, θ_+ is

$$\theta_+ = \Theta(l, R) \equiv l \int_R^\infty \frac{dr}{r^2 Z(l, r)}. \tag{3}$$

For a backward emission, a photon, before it reaches infinity, should first pass through a turning point $r_t (< R)$. The turning point is determined by $Z(l, r_t) = 0$. Then we get the bending angle for backward emission as $\theta_-(l, R) = 2\Theta(l, r_t) - \Theta(l, R)$. One can easily check that for a tangentially emitted backward photon the angle $\theta_-(l_T, R)$ is greater than $\pi/2$. This means that a distant observer can see a part of the "opposite side" of the spherical surface as a result of strong gravity effect.

Let $p_\mu^{(e)}$ and $p_\mu^{(o)}$ be the 4-momentum of a photon emitted by a collapsing surface and of a photon at infinity, respectively. Then $\nu^{(e)} = -p_\mu^{(e)} v^\mu$ is the energy (frequency) of the photon as measured by a comoving observer. For the observer at rest at with the 4-velocity, $v^{(o)\mu} = \delta_0^\mu$, the observed energy (frequency) is determined by $\nu^{(o)} = -p_\mu^{(o)} v^{(o)\mu}$. For a given ray with the impact parameter l, which is emitted from the freely-falling surface, Eq. (2), when its radius is R, the redshift factor Φ defined as the ratio of the emitted frequency to the observed frequency at infinity, $\Phi \equiv \nu^{(e)}/\nu^{(o)}$, is given by

$$\Phi_\sigma(l, R) = \frac{1 - \sigma v_I Z(l, R)}{\sqrt{f}\sqrt{1 - v_I^2}}. \tag{4}$$

Consider a light ray with the impact parameter l emitted from the collapsing surface at the moment of the proper time τ, when it has the radius $R(\tau), R_f(\tau_f) \leq R(\tau) \leq R_i(\tau_i)$, and let t be the time when it reaches the distant observer. The first ray reaching the distant observer is the ray with $l = 0$ emitted from a point "a" at τ_i (see Fig. 1). We use it as a reference ray. We characterize the arrival time of other rays by their time delay Δt with respect to the arrival time of the reference ray. In Fig. 1 we show the sets of points of emission $\{R, \theta\}$, which have the same Δt, by dotted lines. For the forward ray the time delay Δt is given by the following expression

$$\Delta t_+(l; \tau, \tau_i) = t^{(e)}(\tau) - t^{(e)}(\tau_i) + T(l, R(\tau)) + R_i - R(\tau) + 2M \ln\frac{R_i - 2M}{R(\tau) - 2M}, \tag{5}$$

$$T(l, R) \equiv \int_R^\infty \frac{dr}{f(r)}\left[\frac{1}{Z(l, r)} - 1\right]. \tag{6}$$

Similarly for the backward ray one has

$$\Delta t_-(l; \tau, \tau_i) = t^{(e)}(\tau) - t^{(e)}(\tau_i) + 2T(l, r_t) - T(l, R(\tau)) + R_i + R(\tau) - 2r_t + 2M \ln\frac{(R(\tau) - 2M)(R_i - 2M)}{(r_t - 2M)^2}. \tag{7}$$

The integrals for Θ and T, Eqs. (3) and (6) respectively, can be expressed in terms of the elliptic functions. However, for practical calculations it is very convenient to use the analytical approximations for these quantities in terms of simple elementary functions [10,15].

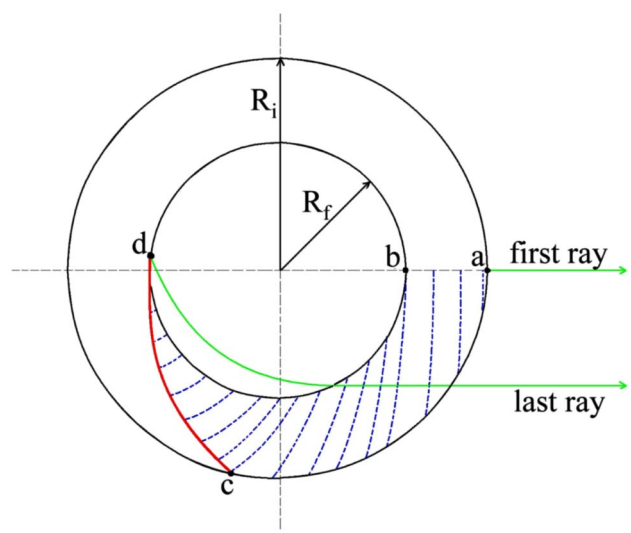

FIG. 1 (color online). The dotted lines are the emission points for the same arrival time parameter. "a" and "b" denote the points of the first lights with $l = 0$ from R_i and R_f respectively. The solid line from "c" to "d" are the points where the last tangential ray for each R, $R_i \geq R \geq R_f$, is emitted. "c" denotes where the light is emitted tangentially from R_i. "d" is the point from which the last ray is emitted from R_f.

For the monochromatic emission with frequency $\nu^{(e)} = \nu$, the flux as measured by a distant observer at r_0 can be written as an integral over the proper time, τ:

$$F_\nu^{(o)}(t) = \frac{2\pi}{r_0^2} \int d\tau l \left| \frac{dl}{d\tau} \right| \Phi^{-4} I_\nu^{(e)}(l, \tau), \tag{8}$$

where $I_\nu^{(e)}(l, \tau)$ is the intensity of the radiation (see e.g. [10]). In the present paper we focus on the finite-duration monochromatic emission, and take the intensity in the form $I_\nu^{(e)}(l, \tau) = f(\tau) \mathcal{I}^{(e)}(l)$. For simplicity we assume that the profile $f(\tau)$ is a step function, $f(\tau) = 1$ for $\tau_i \leq \tau \leq \tau_f$ and $f(\tau) = 0$ otherwise. We further simplify calculations assuming the isotropic emission: $\mathcal{I}^{(e)}(l) = \mathcal{I}^{(e)}$. Then we get

$$F_\nu^{(o)}(t) = \frac{2\pi \mathcal{I}^{(e)}}{r_0^2} \int d\tau f(\tau) l \left| \frac{dl}{d\tau} \right| \Phi^{-4}. \tag{9}$$

The integration over τ for a given t is equivalent to the integration along the dotted line in Fig. 1 for the same arrival time difference. The duration of the observed light, ΔT, is determined by the difference between the arrival time of the last ray from R_f emitted at the point "d" and the arrival time of the first ray from R_i emitted at "a" with $l = 0$:

$$\begin{aligned} \Delta T &\equiv \Delta t_-(l_T, \tau_f, \tau_i) \\ &= t^{(e)}(\tau_f) - t^{(e)}(\tau_i) + 2T(l_T, r_t) - T(l_T, R(\tau)) + R_i \\ &\quad + R(\tau_f) - 2r_t + 2M \ln \frac{(R(\tau_f) - 2M)(R_i - 2M)}{(r_t - 2M)^2}. \end{aligned} \tag{10}$$

In what follows it is convenient to use a dimensionless time parameter δ, which is a normalized arrival time difference defined as $\delta \equiv \Delta t_\pm / \Delta T$. The time parameter δ changes in the interval [0, 1]. For a given radius R, the time parameter for forwardly emitted light increases as l increases from $l = 0$ to $l_{\max}(R)$. Then the backward emissions takes place for $l < l_{\max}(R)$ and ends with $l_T(R)$ when $\delta = 1$.

For the radial rays ($l = 0$) emitted from "a" and "b" in Fig. 1 the time parameter δ are 0 and δ_f, respectively. Denote by δ_T the time parameter for the point "c" (the last ray from R_i, see Fig. 1). Then during the interval $[\delta_f, \delta_T]$ the distant observer receives light emitted in the radius domain $[R_i, R_f]$. In other time intervals, earlier, $0 \leq \delta \leq \delta_f$, or later, $\delta_T \leq \delta \leq 1$, only part of this radius domain contributes. For this reason it is natural to expect the maximum flux at some δ between δ_f and δ_T.

To illustrate characteristic features of the light curves, we plot in Fig. 2 the observable flux for the case when the collapse starts at the radius $R_0 = 9.0M$ and the emission takes place from $R_i = 6.0M$ to $R_f = 4.6M$. We use the analytic approximation developed in [10] for calculating the arrival time differences, $l|dl/d\tau|$ and bending angles. The arrival time difference between the first ray and the last ray is calculated to be $\Delta T = 42.0M$, while the corresponding proper time interval is $\Delta\tau = 3.6M$. The invariant velocities are $v_I = -0.38$ and -0.52 for R_i and R_f, respectively. The first ray from the surface R_f arrives at $\delta_f = 0.17$ and the last ray from R_i arrives at $\delta_T = 0.37$. One can see in Fig. 2 that the maximum of the flux is observed around $\delta \sim 0.2$ as discussed above.

Let us discuss now the time dependence of the observed redshift factor. For a moment, we consider the radiation of a sharp-in-time profile emitted at the moment of τ when the radius is R_e. Figure 3 shows the redshift factor as a function of δ for $R_e = R_i = 6.0M$, $R_e = 5.3M$, and $R_e = R_f = 4.6M$. One can see that for all three curves that have the same minimum value of the redshift factor, $\Phi_T \equiv \Phi(R, l_T) = 1.13$, at $\delta = 1$. As discussed in Ref. [10], the basic reason is that the redshift factor of the last ray ($l = l_T$) for a freely-falling surface does not depend on the radius of radiation but depends only on the initial radius R_0:

$$\Phi_T = \frac{1}{\sqrt{1 - 2M/R_0}}. \tag{11}$$

For the emission during the *finite* interval of time, the lights rays which arrive at the time δ are emitted at different radii, and hence have different redshifts. For each value of δ (except for $\delta = 0$ and $\delta = 1$) one has a finite range of possible redshifts and the redshift curves are broaden. The spectral broadening of the monochromatic radiation is a function of arrival time.

The shadowed region in Fig. 4 shows the broadening of redshift factor as a function of the time parameter δ. We again choose $R_0 = 9M$. The redshift factor of the first ray (emitted from "a" in Fig. 1) is $\Phi_0 \equiv \Phi(R = 6.0M, l = 0) = 1.82$. The maximum redshift occurs for the ray emitted from "b" in Fig. 1, which arrives to the distant observer at $\delta_f = 0.18$. The maximum redshift factor $\Phi_{\max} \equiv \Phi(R = 4.6, l = 0) = 2.36$ in Fig. 3 corresponds to the spectral broadening at δ_f in Fig. 4. From $\delta_T = 0.37$ to $\delta = 1$, the distant observer receives the last rays, which are emitted tangentially for given radii, with the same redshift

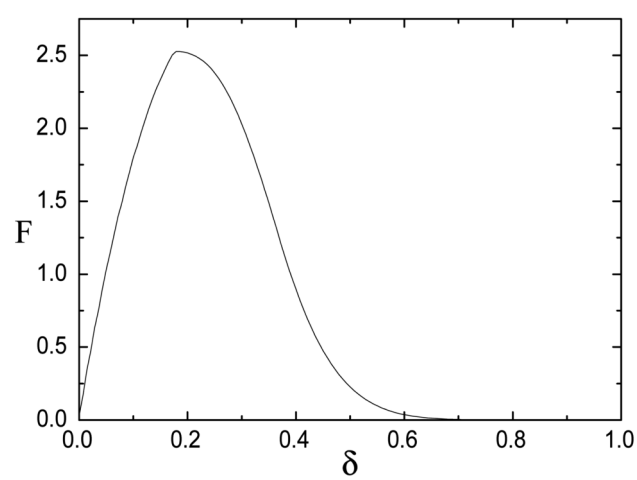

FIG. 2. The flux from $R_i = 6.0M$ through $R_f = 4.6M$ in arbitrary unit. The horizontal axis is the normalized arrival time δ.

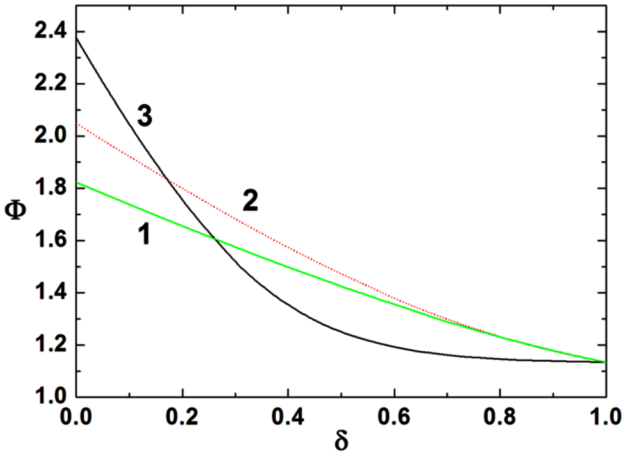

FIG. 3 (color online). The three lines 1, 2, and 3 are the redshift factors for the radiation emitted from $R = 6.0M$, $5.3M$, and $4.6M$ respectively. Arrival time differences are those from sharp-in-time pulses [10].

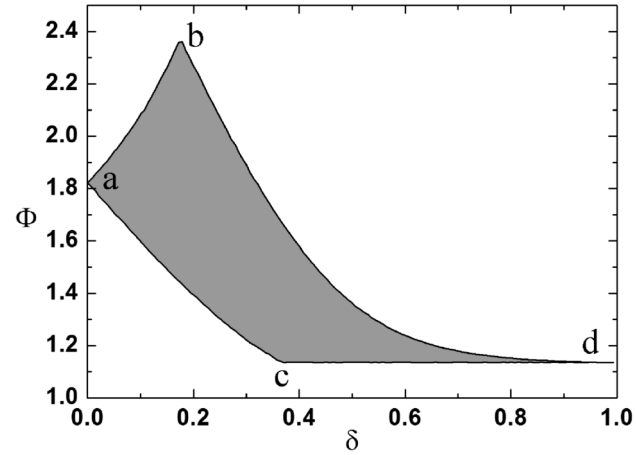

FIG. 4. The broadening of red shift factors. "a, b, c," and "d" are those in Fig. 1.

factor Φ_T. Hence the spectral broadening for $\delta_T \leq \delta \leq 1$ has a constant minimum value of Φ_T between c and d, as shown in Fig. 4.

In our model, the physical parameters characterizing the collapsing object are its mass M and three of dimensionless parameters R_0/M, R_i/M and R_f/M, which determine the initial radius and the initial and final radiation emitting radii, respectively. In case we do not know the frequency of the emitted radiation ν we cannot determine the redshift factor directly by observing the spectrum of the radiation. Nevertheless, if it is possible to determine the frequency $\nu_{\rm last}^{(o)}$ of the last ray with sufficient accuracy, then the relative (normalized) redshift factors $\tilde{\Phi} \equiv \Phi/\Phi_T = \nu^{(o)}/\nu_{\rm last}^{(o)}$ can be determined. Then by measuring δ_f and δ_T as well as $\tilde{\Phi}_0$ and $\tilde{\Phi}_{\rm max}$ one can determine R_0/M, R_i/M and R_f/M. Once we know R_0/M, Φ_T in Eq. (11) can be evaluated and we can determine the original frequency of emission as given by $\nu = \Phi_T \nu_{\rm last}^{(o)}$. The mass of the collapsing object can be inferred from the observed value of Δt to complete the determination of the physical characteristics of the collapsing object.

To summarize, we discuss in this work the light curves and the spectral broadening of the radiation emitted from a surface of a collapsing object. We demonstrate that the spectral broadening occurs when the radiation from the collapsing surface takes place during a finite duration of time. It is because of the variance of the frequency shifts of the light rays subjected to the gravitational redshift and the Doppler shift of the collapsing surface. In a simplified model of monochromatic radiation from a freely-falling spherical surface, we discuss the possible way of how to infer the physical parameters, the mass, the radii of the emission, and the frequency of the radiation from the light curves and spectral broadenings.

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