Non-Gaussianity from tachyonic preheating in hybrid inflation

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In a previous work we showed that large non-Gaussianities and nonscale-invariant distortions in the cosmic microwave background power spectrum can be generated in hybrid inflation models, due to the contributions of the tachyon (waterfall) field to the second order curvature perturbation. Here we clarify, correct, and extend those results. We show that large non-Gaussianity occurs only when the tachyon remains light throughout inflation, whereas $n = 4$ contamination to the spectrum is the dominant effect when the tachyon is heavy. We find constraints on the parameters of warped-throat brane-antibrane inflation from non-Gaussianity. For F-term and D-term inflation models from supergravity, we obtain nontrivial constraints from the spectral distortion effect. We also establish that our analysis applies to complex tachyon fields.

I. INTRODUCTION

In the simplest models of inflation, the primordial density perturbations have a negligible degree of non-Gaussianity. The parameter f_{NL} which characterizes non-Gaussianity is of the order of $|n - 1| \ll 1$ (where n is the spectral index) in conventional inflation models [1–4], whereas the current experimental limit is $|f_{NL}| \lesssim 100$ [5]; one can additionally characterize the non-Gaussianity using the trispectrum, which is also small in conventional models [6]. Nevertheless, there have been intense theoretical efforts to find models which predict observably large levels [7] (see [8] for a review). It has been difficult to find examples which give large f_{NL}. In single field inflation models a small inflaton sound speed is necessary to achieve large non-Gaussianity [9], as in the models of [10,11], unless the inflaton potential has a sharp feature [12].

The simplest multifield models do not seem to give large non-Gaussianity [13], though it is not clear if this is true also of more complicated models. Thus it is quite significant that one of the most prevalent classes of models, hybrid inflation [14], is able to yield large non-Gaussianity for certain ranges of parameters [15]. The effect is due to the growth of the waterfall field—tachyonic preheating—which contributes to the curvature perturbation, and hence the temperature anisotropy, only starting at second order in cosmological perturbation theory. We show that, depending on the values of certain model parameters, two interesting effects are possible: $n = 4$ distortion of the spectrum or large non-Gaussianity. The same effect was observed in [16], though tachyonic preheating

after hybrid inflation was not considered in that paper.[1] Non-Gaussianity from preheating has also been studied in [17–22]. The calculations are complicated, and required numerical integrations over time and wave numbers; hence the results are not immediately intuitive. One of our goals in the present paper is to give a better understanding of this novel effect, and to present some new results concerning the application of these results to popular models of hybrid inflation including brane inflation [23] and P-term inflation [24], which is a synthesis of supergravity inflationary models interpolating between F-term and D-term inflation.

We begin by reviewing the results of [15] in Sec. II. In Sec. III we apply these results by establishing constraints on the parameters of hybrid inflation, coming either from the production of large non-Gaussianity, or from nonscale-invariant contributions to the spectrum (as opposed to bispectrum). These results extend and correct our previous limits [15]. We then adapt them to the cases of brane-antibrane inflation in Sec. IV and P-term inflation in Sec. V. We further extend our analysis to the more realistic case of a complex tachyon field in Sec. VI, showing that the extra components of the tachyon add in a simple way and amplify the real-field results by factors of order unity. Conclusions are given in Sec. VII. Appendix A gives details about the matching between early- and late-time WKB solutions of the tachyon fluctuation mode functions,

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[1]Reference [16] studied the behavior of metric perturbations during preheating in the model $V = \lambda \phi^4/4 + g^2 \phi^2 \chi^2/2 + \lambda' \chi^4/4$ and found that an $n = 4$ contamination of the spectrum is generated when the χ field is heavy throughout inflation while large non-Gaussianity is possible when the χ field is light. In this paper we consider a different model, tachyonic preheating, finding results which are qualitatively similar.

while Appendix B gives details about the source term of the curvature perturbation for complex tachyons.

II. REVIEW OF PREVIOUS RESULTS

A. Hybrid inflation

The hybrid inflation model which we study is defined by the potential

$$V(\varphi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{m_\varphi^2}{2}\varphi^2 + \frac{g^2}{2}\varphi^2\sigma^2, \quad (1)$$

where φ is the inflaton and σ is the tachyonic field. Its mass depends on φ as $m_\sigma^2 = -\lambda v^2 + g^2\varphi^2$, which changes sign when φ reaches the critical value $\varphi_c = (\sqrt{\lambda}/g)v$. At this time, fluctuations in the tachyon field start to grow exponentially. This phase of exponential growth is called tachyonic preheating [25–28]; see also [29] for a discussion of the general theory of preheating and [30] for a different type of tachyonic preheating.

During the early stages of preheating, before the fluctuations have become nonperturbatively large and before the backreaction has set in, the expansion of the universe will still be approximately de Sitter. Once the tachyon fluctuations become sufficiently large their backreaction modifies the expansion of the universe and brings inflation to an end. This happens at a time $N_\star \equiv Ht_\star$ when the fluctuations in σ grow to a certain value,

$$\langle \delta\sigma^2(N_*) \rangle = \int \frac{d^3k}{(2\pi)^3}|\xi_k|^2 \Big|_{N=N_\star} = \frac{v^2}{4}, \quad (2)$$

where ξ_k is the mode function for the fluctuations (discussed below). This happens at some time after the onset of the instability. For a wide range of parameters (including the values originally considered in [14]), one has $N_\star \ll 1$ so that the symmetry breaking completes on a time scale short compared to the Hubble time. This is the usual *waterfall condition* regime of hybrid inflation. In the present work we will consider both the possibilities that $N_\star \ll 1$ and also $N_\star \gtrsim 1$.

We find it convenient to measure time in terms of number of e-foldings, taking $N = 0$ to coincide with the onset of the instability, when $m_\sigma^2 = 0$, and N_* to be the end of inflation, defined by (2). Horizon crossing occurs at some $N_i < 0$, so the number of e-foldings since horizon crossing is $N_e = N_* - N_i$. We determine N_e using the standard relation

$$N_e = 62 - \ln\left(\frac{10^{16} \text{ GeV}}{V^{1/4}}\right) - \frac{1}{3}\ln\left(\frac{V^{1/4}}{\rho_{\text{r.h.}}}\right) \quad (3)$$

with the energy density at reheating ($\rho_{\text{r.h.}} \sim T_{\text{r.h.}}^4$) assumed to be limited by the gravitino bound $T_{\text{r.h.}} \lesssim 10^{10}$ GeV, though we have checked that this assumption has little effect on our results. Given N_* and N_e, N_i is determined by $N_i = N_* - N_e$.

B. Second order curvature perturbation

We work up to second order in perturbation theory, employing the longitudinal gauge throughout. The expanded metric and Einstein equations can be found in [15]. The matter content of the theory is expanded in perturbation theory as

$$\varphi(\tau, \vec{x}) = \varphi_0(\tau) + \delta^{(1)}\varphi(\tau, \vec{x}) + \tfrac{1}{2}\delta^{(2)}\varphi(\tau, \vec{x})$$

$$\sigma(\tau, \vec{x}) = \delta^{(1)}\sigma(\tau, \vec{x}) + \tfrac{1}{2}\delta^{(2)}\sigma(\tau, \vec{x}).$$

As discussed in [15], we are justified in dropping the homogeneous background of the tachyon field $\langle\sigma(\tau, \vec{x})\rangle \equiv \sigma_0(\tau) = 0$. Conformal time, τ, is related to cosmic time as $dt = a d\tau$. We denote derivatives with respect to conformal time as $f' = \partial_\tau f$ and with respect to cosmic time as $\dot{f} = \partial_t f$.

Similarly the gauge invariant curvature perturbation, ζ, is expanded in perturbation theory as

$$\zeta = \zeta^{(1)} + \tfrac{1}{2}\zeta^{(2)}.$$

Because $\sigma_0 = 0$, the first order contribution $\zeta^{(1)}$ is identical to the standard result from single field models. We split the second order curvature perturbation into a component which is due to the inflaton field and a component which is due to the tachyon field as

$$\zeta^{(2)} = \zeta_\varphi^{(2)} + \zeta_\sigma^{(2)}.$$

The second order inflaton curvature perturbation, $\zeta_\varphi^{(2)}$, coincides with the $\zeta^{(2)}$ in single field models. This contribution has been previously computed and is known to be small and conserved on large scales [1–4].

The quantity of interest is $\zeta_\sigma^{(2)}$, the tachyon curvature perturbation. Beyond linear order in perturbation theory there are nonadiabatic pressures in the model which will source the time evolution of $\zeta^{(2)}$ on large scales. The contribution $\zeta_\sigma^{(2)}$ is the term which is amplified during the preheating phase and which will come to dominate $\zeta^{(2)}$ at late times. We therefore focus on $\zeta_\sigma^{(2)}$, since any significant non-Gaussianity will arise due to this term.

One of the principal results of [15] was the computation of the tachyonic contribution to the second order tachyon curvature perturbation in terms of the first order tachyon fluctuations $\delta^{(1)}\sigma$:

$$\zeta_\sigma^{(2)} \cong \frac{\kappa^2}{\epsilon}\int_{\tau_i}^\tau d\tau'\left[\frac{(\delta^{(1)}\sigma')^2}{\mathcal{H}(\tau')} - \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}((\delta^{(1)}\sigma')^2\right.$$

$$\left. - a^2 m_\sigma^2(\delta^{(1)}\sigma)^2\right], \quad (4)$$

where $\kappa^2 = M_p^{-2} = 8\pi G_N$, ϵ is the slow-roll parameter, $\epsilon = \frac{1}{2}M_p^2(V'(\varphi)/V)^2$, τ is the conformal time, $\tau = -[Ha(1-\epsilon)]^{-1}$, $\mathcal{H}$ is the conformal time Hubble parameter, $\mathcal{H} = 1[\tau(1-\epsilon)]^{-1}$, and all factors in the integrand are evaluated at τ' unless otherwise indicated. In deriving (4), we have performed partial integrations in

which surface terms at the initial time were dropped; hence (4) is only valid for perturbations which are dominated by the tachyonic growth at late times. For such perturbations, there is little sensitivity to the value taken for τ_i. An analogous result was derived for hybrid inflation (not considering preheating) in [31]. Metric perturbations during preheating have also been discussed in [32–35].

Since $\zeta_\sigma^{(2)}$ depends only on the first order tachyon fluctuation $\delta^{(1)}\sigma$, and not on $\delta^{(2)}\sigma^2$ we can drop the superscript (1) and denote $\delta^{(1)}\sigma$ by $\delta\sigma$ in subsequent text.

The expression (4) satisfies an important consistency check, namely, that it is a local expression. Nonlocal operators Δ^{-n}, powers of the inverse Laplacian, arise at intermediate steps in the calculation, using the generalized longitudinal gauge, which separates metric perturbations into scalar, vector, and tensor components. In the process of decoupling these to solve for the curvature perturbation, one must apply Δ^{-1}. The nonlocal terms should cancel out of physical quantities, similarly to electrodynamics in Coulomb gauge. The second order curvature perturbation is related to the observable cosmic microwave background (CMB) temperature fluctuations in a nontrivial way, so this by itself does not prove that $\zeta^{(2)}$ must be local. However, [36] has recently shown that under the conditions present in our model, $\zeta^{(2)}$ should indeed be local.

Using (4), it is possible to compute tachyonic contributions to the spectrum and bispectrum of the curvature perturbation,

$$\langle \zeta_{k_1}^{(2)} \zeta_{k_2}^{(2)} \rangle \equiv \delta(\vec{k}_1 + \vec{k}_2) S(k_i) \quad (5)$$

$$\langle \zeta_{k_1}^{(2)} \zeta_{k_2}^{(2)} \zeta_{k_3}^{(2)} \rangle \equiv \frac{\delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)}{(2\pi)^{3/2}} B(\vec{k}_1, \vec{k}_2, \vec{k}_3). \quad (6)$$

C. Tachyon mode functions

To compute the correlations in (5) and (6), we express the tachyon fluctuation in terms of creation and annihilation operators,

$$\delta\sigma(\vec{x}, N) = \int \frac{d^3k}{(2\pi)^{3/2}} a_k \xi_k(N) e^{i\vec{k}\cdot\vec{x}} + \text{H.c.}, \quad (7)$$

where the mode functions obey the linearized tachyon equation of motion. To make this equation more tractable, we have approximated the tachyon mass dependence on time (N) as being linear, $m_\sigma^2 \cong -cH^2N$, so that

$$\frac{d^2}{dN^2}\delta\sigma_k + 3\frac{d}{dN}\delta\sigma_k + [\hat{k}^2 e^{-2N} - cN]\delta\sigma_k = 0, \quad (8)$$

where $\hat{k} = k/H$. We found that this technical assumption is nearly always satisfied for model parameter values consistent with the near scale invariance of the CMB fluctuations;

furthermore it is always satisfied for parameters which lead to large non-Gaussianity. The term cN is proportional to the first term in the Taylor series for $e^{2\eta N} - 1$, where $\eta \cong 4M_p^2 m_\phi^2/(\lambda v^4)$ is the usual slow-roll parameter for φ. We thus demand that $2\eta|N| \ll 1$ throughout inflation. Notice that inflation is ended by the tachyonic instability, not by the failure of the slow-roll conditions. The coefficient c is given by $c = 2\eta\lambda v^2/H^2$, and $H^2 = V/(3M_p^2)$, with $V \cong \frac{1}{4}\lambda v^4$. Using the COBE normalization $V/(M_p^4\epsilon) = 6 \times 10^{-7}$ to eliminate the inflaton mass m_φ, we find that $c = 2.2 \times 10^4 g M_p/v$.

Although Eq. (8) has exact solutions in terms of Airy functions when $k = 0$, for general k no closed-form solutions exist. We therefore constructed solutions, using the WKB approximation, or alternatively the adiabatic approximation. The WKB approximation is valid in the limit of $N \to \pm\infty$; these solutions are matched to each other at N_k, a k-dependent intermediate value of N. They have the form

$$\xi_k \cong \begin{cases} (2H\hat{k}^3)^{-1/2}(1 + i\hat{k}e^{-N}), & N < N_k \\ b_k e^{-(3/2)N + (9/4c)z^{3/2}}(1 + |z|)^{-1/4}, & N > N_k \end{cases} \quad (9)$$

with

$$b_k = \frac{1 - i\sqrt{c|N_k|}}{\sqrt{2H}(c|N_k|)^{3/4}} \frac{(1 + |z_k|)^{1/4}}{\exp(\frac{9}{4c}z_k^{3/2})}, \quad (10)$$

where $z \equiv (1 + \frac{4}{9}cN)$, $z_k \equiv (1 + \frac{4}{9}cN_k)$, and the dividing time between the small- and large-N behavior for a given mode is implicitly defined by $N_k = \ln(\hat{k}/\sqrt{c}) - \ln\sqrt{|N_k|}$. The matching time N_k is discussed in some detail in Appendix A. The alternative method, using the adiabatic approximation, will be reviewed in Sec. III. See also [37] for a discussion of the solutions of the mode function equation.

The solutions of (8) have been discussed in great detail in [15]. However, a few comments are in order about the solutions (9). At early times $N < N_k$ the gradient term in the Klein-Gordon equation dominates over the mass term, $k^2/a^2 > |m_\sigma^2|$, and the resulting mode functions look just like the solutions for a massless field in de Sitter space. These ultraviolet modes are redshifted by the expansion of the universe into the instability band where the mass term dominates the dynamics $|m_\sigma^2| > k^2/a^2$. (Because of the time dependence of m_σ the modes may reenter the massless regime for a brief period of time; we have verified that this does not alter any of our results.) In this infrared regime (where the mass term dominates) the Airy function solutions are appropriate. We have checked the solutions (9) against numerical solutions of (8), and found good agreement.

D. Integrated results

Using the solution (9) in (4), and going to the limit of vanishing wave numbers, $\zeta_\sigma^{(2)}$ takes the form

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[2] Indeed, as was shown in [15], $\delta^{(2)}\sigma$ decouples from the gauge invariant quantity up to second order in perturbation theory.

$$\zeta_{k=0}^{(2)}(N_*) = \frac{\kappa^2}{\epsilon} \int \frac{d^3p}{(2\pi)^{3/2}} (a_p b_p + a_p^\dagger b_{-p})^2 \int_{\max(N_p,N_i)}^{N_*} f(c, N, N_*) dN, \tag{11}$$

where $f(c, N, N_*)$ is given by

$$f(c, N, N_*) = e^{-3N+(9/2c)z^{3/2}}(1+|z|)^{-1/2}\left[\frac{9}{4}(1 - e^{3(N-N_*)})\left|\sqrt{z} - 1 - \frac{2c\,\mathrm{sign}(z)}{27(1+|z|)}\right|^2 - cNe^{3(N-N_*)}\right]. \tag{12}$$

It should be noted that the dominant time dependence is determined by the combination $-3N + \frac{9}{2c}z^{3/2}$ in the exponent. The e^{-3N} decay factor is typical of massive modes, which redshift as $\delta\sigma_k \sim a^{-3/2}$. The $e^{(9/2c)z^{3/2}}$ growth factor is a result of the tachyonic instability. As we noted in the discussion following Eq. (4), Eq. (11) is valid only when the late-time behavior dominates. Therefore an important consistency condition for all of our analysis is that

$$\frac{9}{2c}z_*^{3/2} \equiv \frac{9}{2c}\left(1 + \frac{4}{9}cN_*\right)^{3/2} > 3|N_i|. \tag{13}$$

If this condition is violated then the preheating is not playing any role in the dynamics and we can safely assume that no significant non-Gaussianity is produced.

Using (12), the correlation functions which give the spectrum and bispectrum from (5) and (6) can then be computed as

$$S = 2\frac{\kappa^4}{\epsilon^2} \int \frac{d^3p}{(2\pi)^3} |b_p|^4 \left[\int_{\max(N_p,N_i)}^{N_*} dN f(c, N, N_*)\right]^2 \tag{14}$$

and

$$B = 8\frac{\kappa^6}{\epsilon^3} \int \frac{d^3p}{(2\pi)^3} |b_p|^6 \left[\int_{\max(N_p,N_i)}^{N_*} dN f(c, N, N_*)\right]^3. \tag{15}$$

The tachyonic contribution to the spectrum cannot exceed the experimentally inferred inflaton power spectrum, $\sqrt{P_\varphi(k)} \cong 2\pi \times 10^{-5}k^{-3/2}$, leading to the bound $S < P_\varphi$. Moreover, the bispectrum is related to the nonlinearity parameter f_{NL} by $B = -\frac{6}{5}f_{\mathrm{NL}}(P_\varphi(k_1)P_\varphi(k_2) + \mathrm{perms})$, which at equal momenta k_i leads to

$$f_{\mathrm{NL}} = -\frac{5}{18}\frac{B}{P_\phi^2}. \tag{16}$$

The current experimental constraint is $|f_{\mathrm{NL}}| \lesssim 100$. By analogy, we also define a parameter f_L for the spectrum as

$$f_L = \frac{S}{P_\phi} \tag{17}$$

and demand that $|f_L| < 1$.[3]

III. CONSTRAINTS ON HYBRID INFLATION PARAMETER SPACE

By numerically evaluating the integrals (14) and (15) and applying the experimental limits on the inflaton power spectrum and bispectrum, we were able to find constraints on the hybrid inflation parameter space. We update these bounds in the present section.

A. The issue of scale invariance

In [15], we noted that it is possible for the tachyonic contributions to the spectrum or bispectrum either to be nearly scale invariant ($S \sim 1/k^3$, $B \sim 1/k^6$), or else to badly violate scale invariance ($S, B \sim k^0$). In the latter case, the spectral index for the tachyon contribution to the two-point function is $n = 4$. However, we did not clearly differentiate between these two regimes in the limits presented in [15], an omission which we rectify here.

The two regimes, scale invariant and nonscale invariant, can be understood in reference to condition (13) which must be satisfied in order for tachyonic preheating to play any significant role. There are two ways to satisfy (13). One is to take $cN_\star \gg 1$, which usually requires $c > 1$. This is the regime in which the tachyon mass is not small compared to H during most of inflation, and so it corresponds to nonscale-invariant fluctuations of $\delta\sigma$. The tachyon fluctuations are Hubble damped as $\delta\sigma \sim a(t)^{-3/2}$ prior to inflation, but this suppression can be overcome on large scales if the amplification during the preheating phase is sufficiently large, which typically requires very small values of the self-coupling $\lambda \ll 1$. This nonscale-invariant regime corresponds to a region of the parameter space where the waterfall condition of hybrid inflation is satisfied.

The second way to satisfy (13) is to take $c|N| < 1$ for all $N \in [N_i, N_\star]$. This gives a scale-invariant spectrum for the tachyon and also for $\zeta_\sigma^{(2)}$, which is most easily seen by

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[3]One might consider being more conservative and imposing, say, $|f_L| < 0.01$, rather than $|f_L| < 1$, as we have done. Because the effect turns on exponentially fast, our exclusion plots are actually quite insensitive to the value assumed for f_L and f_{NL}. For example, the exclusion plots for $|f_{\mathrm{NL}}| < 1$ are visually hard to distinguish from those for $|f_{\mathrm{NL}}| < 100$.
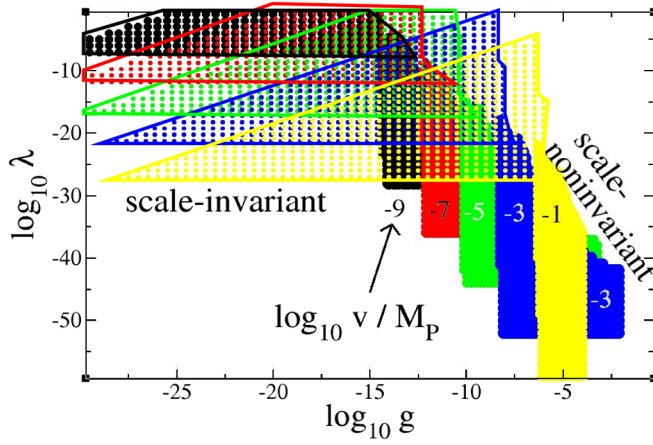
FIG. 1 (color online). Excluded regions of the hybrid inflation parameter space, for $\log_{10} v/M_p = -1, -3, -5, -7$, and -9.

writing the tachyon mass squared as $|m_\sigma^2|/H^2 = c|N|$. It is clear that if $c|N| < 1$ for all $N \in [N_i, N_\star]$ then the tachyon field will have been light compared to the Hubble scale for all ~60 e-foldings of inflation which ensures a nearly scale-invariant spectrum for the tachyon. Also, in this case the instability will typically take several e-foldings to complete so that $N_\star > 1$. This scale-invariant regime corresponds to a region of the parameter space where the usual waterfall condition of hybrid inflation is violated.

B. Nonscale-invariant case

In the nonscale-invariant case f_L and f_{NL} depend on k. Because the tachyon spectrum is blue in this case the strongest constraint comes from evaluating f_L, f_{NL} at the largest values of k which are measured by the CMB. In deriving our constraints we conservatively take this to be $k = e^6 H e^{-N_e}$ where N_e is the total number of e-foldings of inflation (3). The resulting constraints in the plane of $\log_{10} g$ and $\log_{10} \lambda$ are shown in the right-hand region of Fig. 1, for several values of $\log_{10} v/M_p$. We find that the most stringent constraints come from f_L rather than f_{NL}, so we expect to see distortions of the spectrum rather than non-Gaussianity at the left-hand boundaries of the excluded regions. (The other boundaries are unexcluded for the different reasons described in the next paragraph.) In comparing these excluded regions to Fig. 12 of [15], one sees that they are smaller than in our previous work, in the upper right-hand corner. This is due to correcting an error in [15], in which we failed to apply the condition (13) restricting the validity of our analysis.

In computing f_L, f_{NL} over a wide range of $g, \lambda, v/M_p$ we also checked that the additional assumptions were respected: the tachyon mass-squared m_σ^2 varies linearly with the number of e-foldings, which was shown in [15] to require $gv/M_p < 10^{-5}$; the false vacuum energy density $\lambda v^4/4$ dominates during inflation, leading to the bound $g > 460\lambda(v/M_p)^3$; the reheat temperature exceeds

100 GeV, so that baryogenesis can occur at least during the electroweak phase transition, leading to the lower bounds on λ.

C. Scale-invariant case; the adiabatic approximation

On the left-hand side of Fig. 1, we display new constraints for which the spectrum and bispectrum are scale invariant. In contrast to the right-hand side, f_{NL} provides the dominant constraint here, so that non-Gaussianity is playing the important role. To obtain these results, we employed a different approximation for the tachyon mode functions, namely, the adiabatic approximation, described in Appendix F of [15]. Because the tachyon has a small mass during the entirety of inflation (subsequent to horizon crossing), its mass is changing slowly, and we can use the standard mode functions for light fields, but with a time-dependent mass:

$$\delta\sigma(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{H}{\sqrt{2k^3}} (-k\tau)^{\eta_\sigma(\tau)} e^{ikx} a_k + \text{H.c.} \quad (18)$$

Here $\eta_\sigma = M_p^2 V_{,\sigma\sigma}/V$ is the slow-roll parameter for the tachyon, given by

$$\eta_\sigma(\tau) = 8\eta \frac{M_p^2}{v^2} \ln|H\tau|, \quad (19)$$

where $\eta = M_p^2 V_{,\varphi\varphi}/V$. We have also verified that the solution (18) can be reproduced by (9) in the appropriate limit.

Since the mode functions have a simple form in the adiabatic approximation, it is possible to go farther analytically in this case. Notably, we could find an implicit equation for N_* after evaluating the integral (2):

$$N_* \cong \frac{v/M_p}{15\,000 N_e g} \ln\left[1 + 2 \times 10^6 N_* g \left(\frac{M_p}{v}\right)^3 \right]. \quad (20)$$

This expression for N_* is much easier to evaluate than the one which arises in the WKB approximation since the latter leads to a numerical integral $f(N; \lambda, g, v)$ which must be inverted to find the N_* which satisfies $f(N_*; \lambda, g, v) = v^2/4$.

Moreover, the time (N) integral in (4) can be evaluated explicitly using the saddle point method, since it is dominated by the exponential growth near $N = N_*$. Namely, an integral of the form $\int dN e^g$ is approximated by $e^{g_*}/\sqrt{|g_*'|}$, where g_* is the maximum value (at $N = N_*$) and g_*' is the derivative evaluated at the same point. Defining η_f to be $|\eta_\sigma|$ at $N = N_*$, this results in the expression

$$B(\vec{k}_i) = 4H^6 \eta_f d_*^3 \int \frac{d^3p}{(2\pi)^3} |p|^{-3-2\eta_f} |p - k_3|^{-3-2\eta_f}$$
$$\times (|p + k_2|^{-3-2\eta_f} + |p + k_1|^{-3-2\eta_f}) \quad (21)$$

for the bispectrum,[4] where $d_* = H^2\kappa^2\eta_f e^{2\eta_f N_*}/(2\epsilon)$. In the limit of small $c \sim \eta_f$, this is manifestly nearly scale invariant, $B \sim 1/k_i^{6(1+\eta_f)}$, by power counting the divergent behavior of the integral in the infrared. The divergence must be cut off in the usual way, by ignoring modes with p smaller than the horizon. Numerically evaluating the remaining p integral for $k_i \sim k$, and using $k\tau_* \sim e^{N_i}$ for modes near the horizon,[5] we find

$$B(k) \cong 45k_i^{-6(1+\eta_f)}\left(\frac{\kappa^2 H^2 \eta_f e^{2\eta_f N_e}}{2\pi\epsilon}\right)^3. \qquad (22)$$

Further using the COBE normalization to write $\kappa^2 H^2/\epsilon = 2 \times 10^{-7}$, we find that the nonlinearity parameter is

$$f_{\text{NL}} = -2.6 \times 10^{-5}(\eta_f e^{2\eta_f N_e})^3. \qquad (23)$$

Moreover, the COBE normalization implies $\eta_f = 7360N_* g M_p/v$. Demanding that $|f_{\text{NL}}| < 100$ gives the new excluded regions on the left-hand side where ξ_k is the mode function for the fluctuations (discussed below). This happens at some time after the onset of the instability. For a wide range of parameters (including the values originally considered in [14]) one has $N_\star \ll 1$ so that the symmetry breaking completes on a time scale short compared to the Hubble time. This is the usual waterfall condition regime of hybrid inflation. In the present work we will consider both the possibilities that $N_\star \ll 1$ and also $N_\star \gtrsim 1$ of Fig. 1. Unlike the nonscale-invariant regions, these have non-Gaussianity being the dominant effect, rather than the tachyon contribution to the spectrum.

We have claimed that in the scale-invariant regime the dominant constraint is coming from f_{NL} and not from f_L, contrarily to the nonscale-invariant regime. We now justify this claim. Repeating the steps above for the tachyon spectrum, S, one obtains

$$|f_L| \sim 10^{-6}(\eta_f e^{2\eta_f N_e})^2. \qquad (24)$$

Thus, in the scale-invariant regime, the linearity and nonlinearity parameters are related as

$$|f_{\text{NL}}| \sim 10^6 |f_L|^{3/2} \qquad (25)$$

so that $|f_{\text{NL}}| > |f_L|$ except when $|f_L|$ is extremely small. This demonstrates that it is indeed possible to obtain significant non-Gaussianity in this region of the parameter space. We have verified that the result (25) can also be derived using the mode function solutions (9).

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[4]In [15], the conformal time when the instability starts is (perhaps confusingly named) $\tau_* = -1/H$, due to our choice of $N = 0$ for the beginning of the instability.

[5]The horizon-crossing condition is $k\tau_i = 1$, and $\tau_*/\tau_i = e^{N_i}/e^0$.

IV. IMPLICATIONS FOR BRANE-ANTIBRANE INFLATION

We now apply our results to a popular model of inflation from string theory, brane-antibrane inflation, correcting and extending our preliminary results in this direction in [15]. This can be done by mapping the low-energy effective action for the brane-antibrane system onto the hybrid inflation model. We focus on the popular Kachu-Kallosh-Linde-Maldacena-McAllister (KKLMMT) scenario [23,38,39] which reconciles brane inflation with modulus stabilization using warped geometries with background fluxes for type IIB string theory vacua [40]. In this model, the antibrane is at the bottom of a Klebanov-Strassler (KS) throat [41], with warp factor $a_i \ll 1$, and the brane moves down the throat.[6] Within the KS throat the geometry is well approximated by

$$ds^2 = a(y)^2 g_{\mu\nu} dx^\mu dx^\nu + dy^2 + y^2 d\Omega_5^2,$$

where y is the distance along the throat, $a(y) \cong e^{ky}$ is the warp factor, and $d\Omega_5^2$ is the metric on the base space of the corresponding conifold singularity of the underlying Calabi-Yau space. In the subsequent analysis we ignore the base space and treat the geometry as AdS$_5$.

In the following we compute only the non-Gaussianity which is due to the preheating dynamics and ignore the possible effects of the inflaton sound speed [10,11,43]; hence our results may be thought of as a lower bound on the non-Gaussianity from brane inflation.

In string theory the open string tachyon T between a D3-brane and antibrane,[7] separated by a distance y, is described by the action [45]

$$S_{\text{tac}} = -\int d^4x\sqrt{-g}\mathcal{L} \qquad (26)$$

$$\mathcal{L} = V(T, y)\sqrt{1 + (a_i M_s)^{-2}g^{\mu\nu}\partial_\mu T^*\partial_\nu T}.$$

Here the small-$|T|$ expansion of the potential is

$$V(T, y) = 2a_i^4 \tau_3\left[1 + \frac{1}{2}\left(\left(\frac{M_s y}{2\pi}\right)^2 - \frac{1}{2}\right)\right.$$
$$\left. \times |T|^2 + \mathcal{O}(|T|^4) + \cdots\right], \qquad (27)$$

where M_s is the string mass scale, $\tau_3 = g_s^{-1} M_s^4/(2\pi)^3$ is the D3-brane tension, and g_s is the string coupling. Notice that in the warped-throat scenario the instability does not set in until the branes are separated by the unwarped string length,[8] $(a_i M_s)^{-1}$. An interesting difference between the

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[6]See [42] for other discussions on non-Gaussianity in string theory models of inflation.

[7]We restrict ourselves to inflation models driven by D3-branes since inflation driven by higher dimensional branes have problems with overclosure of the universe by defect formation [44].

[8]There is some confusion in the literature on this point, with some papers having stated that the instability is determined by the warped string length, but this is not the case [11]. We thank Louis Leblond for pointing this out to us.

string tachyon and that of ordinary hybrid inflation is that (at $y = 0$) the tachyon potential $V(|T|)$ in the string case does not have a local minimum; rather

$$V(T, 0) \sim \tau_3 e^{-|T|^2/4}. \tag{28}$$

The potential is minimized as $T \rightarrow \infty$. Therefore T does not have a vacuum expectation value (VEV). Nevertheless, the unstable brane-antibrane system decays into closed strings soon after the instability begins, and the large-T part of the potential is not meaningful for determining the actual evolution of the tachyon. In hybrid inflation, it is also true that the end of inflation occurs somewhat before the fluctuations of the tachyon become as large as the VEV. We will see that, even in the absence of a T^4 coupling, we can still define the equivalent of λ and v for the brane-antibrane system, by equating $\frac{1}{4}\lambda v^4$ to the false vacuum energy, and λv^2 to the tachyon mass scale. This amounts to replacing the condition for the end of inflation (2) by

$$\int \frac{d^3 k}{(2\pi)^3} |\xi_k|^2 |_{N=N_\star} = \frac{\text{false vacuum energy}}{|\text{tachyon mass}|^2}. \tag{29}$$

Despite the fact that the tachyon potential is only minimized at $|T| \rightarrow \infty$ the condition (29) is quite reasonable. Detailed numerical simulations of the symmetry breaking in the theory (26) were performed in [44]. Comparing to the analysis of [44], one finds that $N_\star$ as defined in (29) roughly corresponds to the time at which singularities in the spatial gradients of the tachyon field form [46]. The appearance of singularities within a finite time corresponds to the formation of lower dimensional branes [47] and hence by $N = N_\star$ the inflaton field ceases to exist as a physical degree of freedom. This means that, as in our previous analysis, for $N > N_\star$ there no longer exists any nonadiabatic pressure (since only one field, the tachyon, is dynamical) and the large scale curvature perturbation becomes conserved to all orders in perturbation theory [48].

The effective values of the couplings can be found by rewriting the action in terms of the canonically normalized fields $\sigma = a_i \sqrt{\tau_3} T/M_s$ and $\varphi = \sqrt{\tau_3} y$ [see Eqs. 3.6, 3.10, or C.1 in [23]], and then matching to the hybrid inflation potential (1). This gives the correspondence

$$v = \sqrt{\frac{2}{\pi^3}} \frac{a_i M_s}{\sqrt{g_s}} \tag{30}$$

$$\lambda = \frac{\pi^3}{4} g_s \tag{31}$$

$$g = \sqrt{2\pi g_s} a_i. \tag{32}$$

For the analysis of the preceding sections to be valid, the inflaton potential must be well described by $V_0 + \frac{1}{2} m_\varphi^2 \varphi^2$ during the relevant e-foldings of inflation. The full potential can be written as

$$V_{\text{inf}} = \frac{m_\varphi^2}{2} \varphi^2 + V_0 \left(1 - \frac{\nu}{4\pi^2} \frac{V_0}{\varphi^4}\right), \tag{33}$$

where $V_0 = 2a_i^4 \tau_3$ and ν is a geometrical factor which is given $\nu = 27/16$ for the KS throat. It is typical to parametrize the inflaton mass in terms of the dimensionless quantity β as $m_\varphi^2 = \beta H_0^2$, where $H_0^2 = V_0/(3M_p^2)$. Using the COBE normalization, we find that

$$\beta = 10^{7/2} a_i^3 \left(\frac{M_s}{M_p}\right)^3. \tag{34}$$

Demanding that the mass term in (33) dominate over the Coulomb term even when the branes are separated by the local string length yields a lower bound on β:

$$\beta > 324 \pi^4 g_s^2 a_i^{10} \left(\frac{M_p}{M_s}\right)^2.$$

The parameter β is also bounded from above by the requirement that $|n - 1| \lesssim 10^{-1}$ which corresponds to $g_s^2 a_i^{10} (M_p/M_s)^2 \ll 5 \times 10^{-6}$.

Our results apply only in the case that $\beta > 0$; moreover, the case where $\beta < 0$ does not make sense from the string theory point of view, since $\varphi = 0$ denotes the bottom of the throat, and the brane must roll toward that point, not away from it [49].

Our preliminary results about this in [15] suffered from the neglect of the condition (13); moreover, we unduly restricted the full string parameter space by assuming that the scale of inflation was determined by the COBE normalization; that is not the case. As in hybrid inflation, we still have three parameters even after normalizing the spectrum, which we can take to be g_s, a_i and the ratio of the warped string scale to the Planck scale, $a_i M_s/M_p$. Taking into account the additional restrictions on β, we find that the scale-noninvariant exclusions (right-hand side of Fig. 1) do not survive at all in the KKLMMT model; however all the scale invariant ones do. Therefore this model has the potential for producing large non-
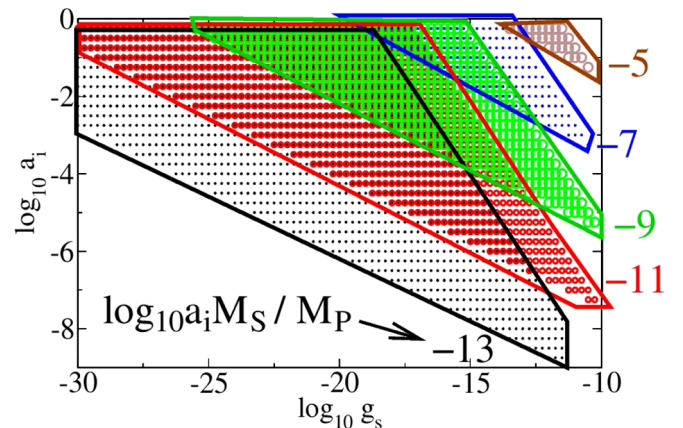

FIG. 2 (color online). Excluded regions of the KKLMMT brane-antibrane inflation parameter space, in the plane of $\log_{10}a_i$ versus $\log_{10}g_s$ for $\log_{10}a_i M_s/M_p = -13, -11, \ldots, -5$ from non-Gaussianity.

Gaussianity, and is even constrained by producing too high levels of non-Gaussianity.

The constraints in the string parameter space are shown in Fig. 2. The excluded regions shown here correspond to very small values of $g_s \lesssim 10^{-10}$. In the simplest way of connecting type IIB string theory to low-energy phenomenology, the gauge couplings of the standard model are related to g_s by running down from the string scale, which would render such small values of g_s incompatible with observations. However, type IIB strings are dual to themselves under SL(2,Z) transformations which take $g_s \rightarrow 1/g_s$. In the dual picture, the string coupling is very large, and the gauge dynamics at the string scale would be confining. It is conceivable that the standard model arises as a remnant of a strongly coupled gauge theory at the high scale, similar to technicolor models. In this case, the small values of g_s which give rise to large non-Gaussianity could still be compatible with particle physics constraints.

V. IMPLICATIONS FOR P-TERM INFLATION

In realistic models of hybrid inflation from supergravity, the potential is generated by F- or D-terms. P-term inflation is a class of models which combines the two kinds of terms and can interpolate between them [24]. The potential for P-term inflation, along the inflationary trajectory, is

$$V_{\text{inf}} = \frac{g^2 \xi^2}{2} \left(1 + \frac{g^2}{8\pi^2} \ln\frac{\varphi^2}{\varphi_c^2} + \frac{f}{8} \frac{\varphi^4}{M_p^4} + \cdots \right), \quad (35)$$

where $\cdots$ denotes terms of order φ^6/M_p^6 and higher. Here $\varphi_c = \xi = \sqrt{|\xi_+|^2 + \xi_3^2}$ is defined in terms of two Fayet-Iliopoulos parameters ξ_+ and ξ_3, and in (35) f must lie in the interval $0 \leq f \leq 1$, since it is defined as $f = |\xi_+|^2/\xi^2$. The limits $f = 0$ and 1 correspond to D-term [50] and F-term [51] models, respectively. We will consider each of these limits separately. We do not consider the complications which arise when these models are coupled to moduli fields [52].

As in the models previously considered, the false vacuum energy dominates during inflation and the Hubble scale is given by

$$H \cong \frac{g\xi}{\sqrt{6}M_p}. \quad (36)$$

The inflaton couples to two scalar fields $\Phi_{\pm}$, of which one linear combination σ is tachyonic. Its mass squared is given by

$$m_\sigma^2 = g^2(\varphi^2 - \xi). \quad (37)$$

By comparing (36) and (37) to the hybrid inflation potential (1) we can determine the hybrid inflation model parameters as

$$\lambda = \frac{g^2}{2} \quad (38)$$

$$v = \sqrt{2\xi}. \quad (39)$$

The coupling g retains its original meaning in P-term inflation.

A. D-term inflation

D-term inflation corresponds to taking $f = 0$ in (35). During a slow-roll phase the inflaton field evolves as

$$\varphi_0(t)^2 = \varphi_c^2 - \frac{g^3 \xi}{2\sqrt{6}\pi^2}(t - t_c)$$

which implies that m_σ^2 varies linearly with the number of e-foldings. Scales relevant for the CMB left the horizon when $\varphi = \varphi_N$, where

$$\varphi_N^2 = \varphi_c^2 + \frac{g^2 N}{2\pi^2} M_p^2 = \xi + \frac{g^2 N}{2\pi^2} M_p^2.$$

Two distinct regimes are possible depending on the value of the coupling g. It is often assumed that g is relatively large so that $g^2 N/(2\pi^2) \gg \xi/M_p^2$ [53] which gives the correct amplitude of density perturbations with $\xi \cong 10^{-5} M_p^2$ and requires $g \gtrsim 2 \times 10^{-3}$ for consistency. In this regime $\varphi_N \gg \varphi_c$ so that slow-roll at the onset of the instability is not guaranteed and our previous analysis of the tachyon mode functions does not apply. However, in this regime it is also difficult to satisfy the constraints coming from the cosmic string tension, to avoid overproduction of cosmic strings, $\xi \lesssim 4 \times 10^{-7}$ [54]. (Reference [55] has pointed out that the constraints on the cosmic string tension can be weakened by incorporating the effect which strings have on the observed spectral index.)

We are therefore driven to consider D-term inflation in the regime of small coupling g^2 so that $g^2 N/(2\pi^2) \ll \xi/M_p^2$ and $\varphi_N \cong \varphi_c$. In this case we are guaranteed that the universe will still be in a slow-roll phase at the onset of the instability and our previous analysis holds without modification. This corresponds to very small couplings $g \ll 2 \times 10^{-3}$; however, there is no obstruction to taking such a small coupling since g^2 is not necessarily related to the gauge coupling constant in a GUT [24]. In this regime the COBE normalization fixes $\xi \cong 7 \times 10^{-4} g^{2/3} M_p^2$ so that g is the only independent model parameter. The cosmic string constraint $\xi \lesssim 4 \times 10^{-7} M_p^2$ then restricts the coupling g to be smaller than $g \lesssim 1.3 \times 10^{-5}$.

Applying our previous analysis of hybrid inflation to D-term inflation,[9] including the additional constraints mentioned above, we find that there is a range of couplings,

$$-10.0 < \log_{10} g \lesssim -8.7 \quad (40)$$

which is ruled out because of the spectral distortion con-

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[9]See [56] for further discussion of preheating in D-term inflation.

straint, in the scale-noninvariant region of Fig. 1. On the other hand, there is no constraint coming from non-Gaussianity for this model.

B. F-term inflation

F-term inflation [51,57] corresponds to taking $f = 1$ in (35). In this case the dynamics are somewhat more complicated than the D-term model. Again there are two possible regimes: a large coupling regime where $\varphi_N \gg \varphi_c$ and our previous analysis does not apply and also a small coupling regime where $\varphi_N \cong \varphi_c$ and our previous analysis does apply. The large coupling regime corresponds to $g \gtrsim 2 \times 10^{-3}$ and again the cosmic string tension constraints are difficult to satisfy (see, however, [58]). We are therefore driven to consider only the small coupling regime, $g \lesssim 2 \times 10^{-3}$. For couplings $3 \times 10^{-7} \ll g \lesssim 2 \times 10^{-3}$ it can be shown that the quadratic term φ^4/M_p^4 in the potential (35) can be neglected and the dynamics is identical to D-term inflation, which we have already considered. Thus we consider only the F-term model for $g \ll 3 \times 10^{-7}$ since this is the only region of parameter space for which the model differs significantly from the D-term model.[10]

For $g \lesssim 3 \times 10^{-7}$, so the f-term is dominating the potential, the slow-roll parameter $\epsilon = \xi^3/(8M_p^6)$, and the COBE normalization fixes $\xi \cong 6.7 \times 10^6 g^2 M_p^2$ and the cosmic string tension is within observational bounds for $g \lesssim 2 \times 10^{-7}$. Again applying the general hybrid inflation constraints, we find the excluded region

$$- 13.0 \lesssim \log_{10} g \lesssim -9.5 \tag{41}$$

which, as in the case of D-term inflation, comes from the k^3 spectral distortion effect rather than non-Gaussianity.

For more general P-term models with $0 < f < 1$, we expect the excluded regions to interpolate between (40) and (41). In deriving our constraints, we have been driven to the small coupling regime by the requirement that the cosmic sting tension be within observational bounds. Our analysis does not give any significant constraint on the string theoretic D3/D7 model [59] since in this case the cosmic strings are not stable and there is no motivation to consider the small values of the coupling g in (40) and (41). Indeed, such small couplings are difficult to motivate from string theory [60].

VI. THE CASE OF A COMPLEX TACHYON

In the preceding sections we have applied the results of [15], which were derived under the assumption that σ is a real field, to models in which the tachyon is actually complex. In doing so we have assumed that the general-

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[10]We have neglected the intermediate regime $0.06 \lesssim g \lesssim 0.15$ which will not yield significant non-Gaussianity or spectral distortion.

ization of the analysis of [15] to the case of a complex tachyon does not significantly modify the exclusion plot, Fig. 1. Here we verify this claim.

A. Cosmological perturbation theory for an $O(M)$ multiplet

Before restricting to the case of a complex tachyon, we consider the somewhat more general case of an $O(M)$ symmetric multiplet of tachyon fields σ_A with $A = 1, \ldots, M$. The matter sector is expanded in perturbation theory as

$$\varphi(\tau, \vec{x}) = \varphi^{(0)}(\tau) + \delta^{(1)}\varphi(\tau, \vec{x}) + \tfrac{1}{2}\delta^{(2)}\varphi(\tau, \vec{x}) \tag{42}$$

$$\sigma_A(\tau, \vec{x}) = \delta^{(1)}\sigma_A(\tau, \vec{x}) + \tfrac{1}{2}\delta^{(2)}\sigma_A(\tau, \vec{x}). \tag{43}$$

As in [15] the time-dependent vacuum expectation value (VEV) of the tachyon fields are set to zero $\langle \sigma_A \rangle \equiv \sigma^{(0)} = 0$ which is a consequence of the $O(M)$ symmetry of the theory. Notice, however, that the tachyon field *does* develop an effective VEV for the radial component:

$$\langle |\sigma| \rangle \equiv \langle \sqrt{\sigma_A \sigma^A} \rangle \neq 0.$$

We also assume that

$$\frac{\partial V}{\partial \sigma_A} = \frac{\partial^2 V}{\partial \sigma_A \partial \varphi} = 0$$

but V is, for the time being, otherwise arbitrary. Here and elsewhere the potential and its derivatives are understood to be evaluated on background values of the fields so that $V = V(\varphi^{(0)}, \sigma^{(0)})$, for example.

We consider only the $\delta^{(2)}G_0^0 = \kappa^2 \delta^{(2)}T_0^0$, $\partial_i \delta^{(2)}G_0^i = \kappa^2 \partial_i \delta^{(2)}T_0^i$, and $\delta_j^i \delta^{(2)}G_i^j = \kappa^2 \delta_j^i \delta^{(2)}T_i^j$ equations since the second order vector and tensor fluctuations decouple from this system. In the case that $\sigma_A^{(0)} = 0$, the second order tachyon fluctuations $\delta^{(2)}\sigma_A$ decouple from the inflaton and gravitational fluctuations up to second order and hence we do not need to solve for $\delta^{(2)}\sigma_A$. Note also that the Klein-Gordon equation for the inflaton fluctuations is not necessary to close the system. In this section we sometimes insert the slow-roll parameters ϵ and η explicitly though we do not yet assume that they are small. We also introduce the shorthand notation $m_\varphi^2 \equiv \partial^2 V/\partial \varphi^2$.

The second order (0, 0) equation is

$$3\mathcal{H}\psi'^{(2)} + (3 - \epsilon)\mathcal{H}^2\phi^{(2)} - \partial^k\partial_k\psi^{(2)}$$
$$= -\frac{\kappa^2}{2}\left[\varphi_0'\delta^{(2)}\varphi' + a^2\frac{\partial V}{\partial \varphi}\delta^{(2)}\varphi\right] + \Upsilon_1, \tag{44}$$

where Υ_1 is constructed entirely from first order quantities. Dividing Υ_1 into inflaton and tachyon contributions we have

$$\Upsilon_1 = \Upsilon_1^\varphi + \Upsilon_1^\sigma,$$

where

$$Y_1^\varphi = 4(3 - \epsilon)\mathcal{H}^2(\phi^{(1)})^2 + 2\kappa^2\varphi_0'\phi^{(1)}\delta^{(1)}\varphi'$$
$$- \frac{\kappa^2}{2}(\delta^{(1)}\varphi')^2 - \frac{\kappa^2}{2}a^2m_\varphi^2(\delta^{(1)}\varphi)^2$$
$$- \frac{\kappa^2}{2}(\vec{\nabla}\delta^{(1)}\varphi)^2 + 8\phi^{(1)}\partial^k\partial_k\phi^{(1)} + 3(\phi'^{(1)})^2$$
$$+ 3(\vec{\nabla}\phi^{(1)})^2 \tag{45}$$

and

$$Y_1^\sigma = -\frac{\kappa^2}{2}\left[\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A + \partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A \right.$$
$$\left. + a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_A\delta^{(1)}\sigma_B\right]. \tag{46}$$

The divergence of the second order $(0, i)$ equation is

$$\partial^k\partial_k[\psi'^{(2)} + \mathcal{H}\phi^{(2)}] = \frac{\kappa^2}{2}\varphi_0'\partial^k\partial_k\delta^{(2)}\varphi + Y_2, \tag{47}$$

where $Y_2 = Y_2^\varphi + Y_2^\sigma$ is constructed entirely from first order quantities. The inflaton part is

$$Y_2^\varphi = 2\kappa^2\varphi_0'\partial_i(\phi^{(1)}\partial^i\delta^{(1)}\varphi) + \kappa^2\partial_i(\delta^{(1)}\varphi'\partial^i\delta^{(1)}\varphi)$$
$$- 8\partial_i(\phi^{(1)}\partial^i\phi'^{(1)}) - 2\partial_i(\phi'^{(1)}\partial^i\phi^{(1)}) \tag{48}$$

and the tachyon part is

$$Y_2^\sigma = \kappa^2\partial_i(\delta^{(1)}\sigma_A'\partial^i\delta^{(1)}\sigma^A). \tag{49}$$

The trace of the second order (i, j) equation is

$$3\psi''^{(2)} + \partial^k\partial_k[\phi^{(2)} - \psi^{(2)}] + 6\mathcal{H}\psi'^{(2)} + 3\mathcal{H}\phi'^{(2)}$$
$$+ 3(3 - \epsilon)\mathcal{H}^2\phi^{(2)}$$
$$= \frac{3\kappa^2}{2}\left[\varphi_0'\delta^{(2)}\varphi' - a^2\frac{\partial V}{\partial\varphi}\delta^{(2)}\varphi\right] + Y_3, \tag{50}$$

where $Y_3 = Y_3^\varphi + Y_3^\sigma$ is constructed entirely from first order quantities. The inflaton part is

$$Y_3^\varphi = 12(3 - \epsilon)\mathcal{H}^2(\phi^{(1)})^2 - 6\kappa^2\varphi_0'\phi^{(1)}\delta^{(1)}\varphi'$$
$$+ \frac{3\kappa^2}{2}(\delta^{(1)}\varphi')^2 - \frac{3\kappa^2}{2}a^2m_\varphi^2(\delta^{(1)}\varphi)^2$$
$$- \frac{\kappa^2}{2}(\vec{\nabla}\delta^{(1)}\varphi)^2 + 3(\phi'^{(1)})^2 + 8\phi^{(1)}\partial^k\partial_k\phi^{(1)}$$
$$+ 24\mathcal{H}\phi^{(1)}\phi'^{(1)} + 7(\vec{\nabla}\phi^{(1)})^2 \tag{51}$$

and the tachyon part is

$$Y_3^\sigma = \kappa^2\left[\frac{3}{2}\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A - \frac{1}{2}\partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A \right.$$
$$\left. - \frac{3}{2}a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_A\delta^{(1)}\sigma_B\right]. \tag{52}$$

The derivation of the master equation which was presented in Appendix B of [15] follows here unmodified

except for the new definitions of Y_1^σ, Y_2^σ, and Y_3^σ. The master equation is

$$\phi''^{(2)} + 2(\eta - \epsilon)\mathcal{H}\phi'^{(2)} + [2(\eta - 2\epsilon)\mathcal{H}^2 - \partial^k\partial_k]\phi^{(2)} = J, \tag{53}$$

where the source is

$$J = Y_1 - Y_3 + 4\Delta^{-1}Y_2' + 2(1 - \epsilon + \eta)\mathcal{H}\Delta^{-1}Y_2$$
$$+ \Delta^{-1}\gamma'' - (1 + 2\epsilon - 2\eta)\mathcal{H}\Delta^{-1}\gamma', \tag{54}$$

and the quantity γ is defined as

$$\gamma = Y_3 - 3\Delta^{-1}Y_2' - 6\mathcal{H}\Delta^{-1}Y_2. \tag{55}$$

We can split the source into tachyon and inflaton contributions $J = J^\varphi + J^\sigma$ in the obvious manner, by taking the tachyon and inflaton parts of Y_1, Y_2, Y_3, γ.

In Appendix B we prove the identity [see Eq. (B2)]

$$\gamma_\sigma = -\frac{\kappa^2}{2}(\partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A)$$
$$- 3\kappa^2\Delta^{-1}\partial_i(\partial^k\partial_k\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A)$$

which is analogous to the result for a real tachyon field, derived in [15].

We now proceed to derive the tachyon curvature perturbation. The derivation of $\zeta_\sigma^{(2)}$ presented in [15] follows unmodified except, of course, for the change in the definitions of Y_1^σ, Y_2^σ, Y_3^σ, and γ_σ. From this point onwards we assume that ϵ, $|\eta| \ll 1$. The leading contribution to the tachyon curvature perturbation is

$$\zeta_\sigma^{(2)} \cong \frac{1}{\epsilon}\int_{\tau_i}^\tau d\tau'\left[-\frac{Y_1^\sigma}{\mathcal{H}(\tau')} + \frac{1}{3}\frac{Y_3^\sigma}{\mathcal{H}(\tau')} - \frac{2}{3}\frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}Y_3^\sigma\right].$$

Now, using Eqs. (46) and (52) we can write this in terms of the tachyon fluctuation $\delta^{(1)}\sigma$ as

$$\zeta_\sigma^{(2)} \cong \frac{\kappa^2}{\epsilon}\int_{-1/a_iH}^\tau d\tau'\left[\frac{\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A}{\mathcal{H}(\tau')} - \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}\right.$$
$$\left. \times\left(\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A - a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_A\delta^{(1)}\sigma_B\right)\right]. \tag{56}$$

The corrections to (56) are either total gradients or are subleading in the slow-roll expansion. In deriving (56) we have restricted ourselves to the preheating phase during which the fluctuations $\delta^{(1)}\sigma_A$ grow exponentially.

Using (56) the second order tachyon curvature perturbation can be computed once the fluctuations $\delta^{(1)}\sigma_A$ are determined. The first order tachyon fluctuations are described by the perturbed Klein-Gordon equation

$$\delta^{(1)}\sigma_A'' + 2\mathcal{H}\delta^{(1)}\sigma_A - \partial^k\partial_k\delta^{(1)}\sigma_A + a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_B = 0. \tag{57}$$

B. Complex tachyon mode functions

At this point we restrict our attention to the case with $M = 2$ and the potential

$$V = \frac{\lambda}{4} (\sigma_A \sigma^A - v^2)^2 + \frac{g^2}{2} \varphi^2 \sigma_A \sigma^A + \frac{m_\varphi^2}{2} \varphi^2. \quad (58)$$

For $\sigma_A^{(0)} = 0$ the mass matrix is diagonal

$$\frac{\partial^2 V}{\partial \sigma_A \partial \sigma_B} = (-\lambda v^2 + g^2 \varphi_0^2) \delta_{AB} \equiv m_\sigma^2 \delta_{AB}$$

so that the tachyon fluctuations with $A = 1$ and $A = 2$ evolve independently [see Eq. (57)].

As previously the quantum mechanical solutions $\delta^{(1)} \sigma_A$ are written in terms of annihilation and creation operators a_k^A, $a_k^{A\dagger}$ in the usual way

$$\delta^{(1)} \sigma_A(x) = \int \frac{d^3 k}{(2\pi)^{3/2}} a_k^A \xi_k(t) e^{ikx} + \text{H.c.} \quad (59)$$

Both components $A = 1$ and $A = 2$ have the same time dependence owing to the fact that the mass matrix is diagonal. The ξ_k in (59) are thus identical to the solutions of (8), which we have already studied.

C. The end of symmetry breaking

For a multicomponent tachyon, the condition defining $N_\star$ must be modified as $\langle \delta^{(1)} \sigma_A \delta^{(1)} \sigma^A \rangle (N = N_\star) = v^2/4$ which, for the case $M = 2$, changes (2) to

$$\int \frac{d^3 k}{(2\pi)^3} |\xi_k|^2|_{N=N_\star} = \frac{v^2}{8}.$$

D. Tachyon curvature perturbation

For the potential (58), the tachyon curvature perturbation $\zeta_\sigma^{(2)}$ decomposes into a sum of term

$$\zeta_\sigma^{(2)} = \sum_{A=1,2} \zeta_A^{(2)},$$

where $\zeta_A^{(2)}$ is the contribution to $\zeta_\sigma^{(2)}$ coming from σ_A. Consider, as an example, the spectrum of the tachyon curvature perturbation

$$\langle \zeta_{\sigma,k_1}^{(2)} \zeta_{\sigma,k_2}^{(2)} \rangle = \langle \zeta_{1,k_1}^{(2)} \zeta_{1,k_2}^{(2)} \rangle + \langle \zeta_{2,k_1}^{(2)} \zeta_{2,k_2}^{(2)} \rangle + \langle \zeta_{1,k_1}^{(2)} \zeta_{2,k_2}^{(2)} \rangle$$
$$+ \langle \zeta_{1,k_2}^{(2)} \zeta_{2,k_1}^{(2)} \rangle.$$

Because the annihilation/creation operators a_k^1 and a_k^2 are independent the cross terms on the last line do not contribute to the connected part of the correlation function. This means that

$$\langle \zeta_{\sigma,k_1}^{(2)} \zeta_{\sigma,k_2}^{(2)} \rangle = 2 \langle \zeta_{1,k_1}^{(2)} \zeta_{1,k_2}^{(2)} \rangle.$$

The quantity $\langle \zeta_{1,k_1}^{(2)} \zeta_{1,k_2}^{(2)} \rangle = \langle \zeta_{2,k_1}^{(2)} \zeta_{2,k_2}^{(2)} \rangle$ will be identical to $\langle \zeta_{\sigma,k_1}^{(2)} \zeta_{\sigma,k_2}^{(2)} \rangle$ which we have already computed. We see, then, that the effect of having a complex tachyon field (as opposed to a real field) is to multiply f_L and f_{NL} by a factor of 2 and also to slightly reduce $N_\star$. The net change in f_L, f_{NL} is order unity and the new exclusion plots are difficult to visually distinguish from Fig. 1. This justifies our previous claims that our constraints do not change significantly when one considers a complex tachyon field.

VII. CONCLUSIONS

In this paper we have studied the evolution of the second order curvature perturbation during tachyonic preheating at the end of hybrid inflation. We have found that, depending on the values of certain model parameters, two interesting effects are possible:

(i) Preheating generates a scale-invariant contribution to the curvature perturbation. In this case significant non-Gaussianity can be generated during preheating and the model is even constrained by producing too high a level of non-Gaussianity.

(ii) Preheating generates a nonscale-invariant contribution to the curvature perturbation with spectral index $n = 4$. In this case the strongest constraint comes from the distortion of the power spectrum and no significant non-Gaussianity can be produced.

In both cases, one typically requires fairly small values of the dimensionless couplings g, λ in order to obtain a strong effect. Note that a small coupling g does not require fine-tuning in the technical sense, since g^2 is only multiplicatively renormalized: $\beta(g^2) \sim \mathcal{O}(g^2 \lambda, g^4)/(16\pi^2)$. That is, if g is small at tree level then loop corrections do not change its effective value significantly.

We have applied our constraints on hybrid inflation to several popular models: brane inflation, D-term inflation, and F-term inflation. In the case of brane inflation we have found that significant non-Gaussianity from preheating is possible for sufficiently small values of the warp factor. For both D- and F-term inflation, we have shown that no non-Gaussianity is produced during preheating; however, we still put interesting constraints on the model due to the distortion of the spectrum by nonscale-invariant fluctuations.

We have also generalized the results of [15] to the case of a complex tachyon field, confirming our previous claims that this modification does not significantly alter our exclusion plots.

We should note that the model of hybrid inflation considered here always gives a small blue tilt to the spectral index, $n > 1$, which is disfavored by recent data [5]. One avenue for future study [61] is to generalize our results to the case of inverted hybrid inflation [62] which always gives $n < 1$.

APPENDIX A: THE MATCHING TIME N_k

The matching time N_k which determines the boundary between large- and small-scale behavior of the mode functions (9) is determined by the transcendental equation

$$|N_k|e^{2N_k} = \frac{\hat{k}^2}{c}. \tag{A1}$$

The solutions may be written exactly in terms of the branches of the Lambert-W functions. In the region $\hat{k} < \sqrt{c/(2e)}$, the solution is triple valued (see Fig. 1 of [15]) and may be written as

$$N_k = \begin{cases} \frac{1}{2}W_{-1}(-\frac{2\hat{k}^2}{c}) & \text{for the branch with } N_k < -1; \\ \frac{1}{2}W_0(-\frac{2\hat{k}^2}{c}) & \text{for the branch with } -1 < N_k < 0; \\ \frac{1}{2}W_0(+\frac{2\hat{k}^2}{c}) & \text{for the branch with } N_k > 0. \end{cases} \tag{A2}$$

In the region $\hat{k} > \sqrt{c/(2e)}$ the solution is single valued and can be written as

$$N_k = \frac{1}{2}W_0\left(+\frac{2\hat{k}^2}{c}\right). \tag{A3}$$

One may derive some asymptotic expressions for N_k in various regions of interest. When $|N_k| \gg 1$ we have

$$N_k \cong \ln\left(\frac{\hat{k}}{\sqrt{c}}\right) \tag{A4}$$

which describes N_k at $\hat{k} \gg \sqrt{c/(2e)}$ and also the lower branch of N_k at $\hat{k} \ll \sqrt{c/(2e)}$. For $\hat{k} \lesssim \sqrt{c/(2e)}$ there are two more branches of the solution with approximate behavior

$$N_k \cong \pm\frac{\hat{k}^2}{c}. \tag{A5}$$

In our analysis we have used the approximation that N_k is a single-valued function, described by

$$N_k^{\text{s.v.}} = \frac{1}{2}\Theta(\sqrt{c/(2e)} - \hat{k})W_{-1}\left(-\frac{2\hat{k}^2}{c}\right)$$
$$+ \frac{1}{2}\Theta(\hat{k} - \sqrt{c/(2e)})W_0\left(+\frac{2\hat{k}^2}{c}\right),$$

where $\Theta(x)$ is the Heaviside step function. We have verified both numerically [15] and analytically that the single-valued approximation does not significantly alter our results.

APPENDIX B: AN IDENTITY CONCERNING γ_σ

In this Appendix we derive an identity concerning the tachyon source term γ_σ (55):

$$\gamma_\sigma = Y_3^\sigma - 3\Delta^{-1}\partial_\tau Y_2^\sigma - 6\mathcal{H}\Delta^{-1}Y_2^\sigma.$$

Using Eqs. (49)–(52) we can write this

$$\gamma_\sigma = \kappa^2\Delta^{-1}\Big[\frac{3}{2}\partial^k\partial_k(\delta^{(1)}\sigma_A'\delta^{(1)}\sigma'^A)$$
$$- \frac{1}{2}\partial^k\partial_k(\partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A)$$
$$- \frac{3}{2}a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\partial^k\partial_k(\delta^{(1)}\sigma_A\delta^{(1)}\sigma_B)$$
$$- 3\partial_\tau\partial_i(\delta^{(1)}\sigma_A'\partial^i\delta^{(1)}\sigma^A)$$
$$- 6\mathcal{H}\partial_i(\delta^{(1)}\sigma_A'\partial^i\delta^{(1)}\sigma^A)\Big]$$

and, after some algebra, we have

$$\gamma_\sigma = \kappa^2\Delta^{-1}\Big[-\frac{1}{2}\partial^k\partial_k(\partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A)$$
$$- 3\partial_i\Big(\delta^{(1)}\sigma_A'' + 2\mathcal{H}\delta^{(1)}\sigma_A' + a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_B\Big)$$
$$\times \partial^i\delta^{(1)}\sigma^A - 3\Big(\delta^{(1)}\sigma_A'' + 2\mathcal{H}\delta^{(1)}\sigma_A'$$
$$+ a^2\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B}\delta^{(1)}\sigma_B\Big)\partial^k\partial_k\delta^{(1)}\sigma^A\Big]. \tag{B1}$$

In deriving this equation we have used the fact that

$$\frac{\partial^2 V}{\partial\sigma_A\partial\sigma_B} = \frac{\partial^2 V}{\partial\sigma_B\partial\sigma_A}$$

which follows from the $O(M)$ symmetry of the theory. The last two lines of (B1) can be simplified using the equation of motion for the tachyon fluctuation (57) which gives

$$\gamma_\sigma = -\frac{\kappa^2}{2}(\partial_i\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A)$$
$$- 3\kappa^2\Delta^{-1}\partial_i(\partial^k\partial_k\delta^{(1)}\sigma_A\partial^i\delta^{(1)}\sigma^A). \tag{B2}$$

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