Baryons in holographic QCD

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We study baryons in holographic QCD with D4/D8/$\overline{\rm D8}$ multi-D-brane system. In holographic QCD, the baryon appears as a topologically nontrivial chiral soliton in a four-dimensional effective theory of mesons. We call this topological soliton brane-induced Skyrmion. Some review of D4/D8/$\overline{\rm D8}$ holographic QCD is presented from the viewpoints of recent hadron physics and QCD phenomenologies. A four-dimensional effective theory with pions and ρ mesons is uniquely derived from the non-Abelian Dirac-Born-Infeld (DBI) action of D8 brane with D4 supergravity background at the leading order of large N_c, without small amplitude expansion of meson fields to discuss chiral solitons. For the hedgehog configuration of pion and ρ-meson fields, we derive the energy functional and the Euler-Lagrange equation of brane-induced Skyrmion from the meson effective action induced by holographic QCD. Performing the numerical calculation, we obtain the soliton solution and figure out the pion profile $F(r)$ and the ρ-meson profile $\tilde{G}(r)$ of the brane-induced Skyrmion with its total energy, energy density distribution, and root-mean-square radius. These results are compared with the experimental quantities of baryons and also with the profiles of standard Skyrmion without ρ mesons. We analyze interaction terms of pions and ρ mesons in brane-induced Skyrmion, and find a significant ρ-meson component appearing in the core region of a baryon.

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I. INTRODUCTION

Based on the recent remarkable progress in the concept of gauge/gravity duality, the nonperturbative properties of QCD, especially the low-energy meson dynamics, are successfully described from the multi D-brane system consisting of D4/D8/$\overline{\rm D8}$ in the type IIA superstring theory [1]. This is called the Sakai-Sugimoto model, which is regarded as one of the reliable holographic models of QCD.

In the D4/D8/$\overline{\rm D8}$ holographic QCD, the compositions of the four-dimensional massless QCD, i.e., gluons and massless quarks, are represented in terms of the fluctuation modes of open strings on N_c-folded D4 branes and N_f-folded D8 and $\overline{\rm D8}$ branes. Since the mass of D-brane is proportional to the folding number of the D-brane as the Ramond-Ramond flux quantum, the existence of the D-brane with large folding number can be described by a curved space-time. Thus the supergravity description of the D-brane and also the construction of gauge theory from the D-brane eventually gives the concept of "duality" between the supergravity and gauge theory as the gauge/gravity duality mediated by the mutual D-brane. This duality was first realized as the discovery of AdS/CFT correspondence by Maldacena in 1997 [2] as the duality between the type IIB superstring theory on $\mathrm{AdS}_5 \times S^5$ and $N = 4$ SUSY Yang-Mills theory through the D3 brane. In the D4/D8/$\overline{\rm D8}$ holographic model, with the large-N_c condition as $N_f \ll N_c$, only D4 branes are represented by the classical supergravity background, and backreaction from the probe D8 and $\overline{\rm D8}$ branes to the total system is assumed to be small and neglected as a probe approximation. With the existence of the heavy D4 brane, there appears a horizon in ten-dimensional space-time as a classical D4 supergravity solution, and probe D8 and $\overline{\rm D8}$ branes are interpolated with each other in this curved space-time. This interpolation induces the spontaneous symmetry breaking of $\mathrm{U}(N_f)_{\rm D8} \times \mathrm{U}(N_f)_{\overline{\rm D8}}$ local symmetry into the single-valued $\mathrm{U}(N_f)_{\rm D8}$ symmetry, which is regarded as the holographic manifestation of chiral symmetry breaking. The existence of a D4 supergravity background is reflected on the metric of the non-Abelian Dirac-Born-Infeld (DBI) action of the D8 brane, and after integrating out the symmetric directions along the extra four-dimensional coordinate space, the effective action of the D8 brane is reduced into the five-dimensional Yang-Mills theory with flat four-dimensional space-time $(t, \mathbf{x})$ and another extra fifth dimension with curved measure. This five-dimensional Yang-Mills theory is regarded as a "unified" theory of mesons, including pions as massless pseudoscalar Nambu-Goldstone bosons with respect to chiral-symmetry breaking, and infinite tower of massive (axial) vector mesons. By diagonalizing this five-dimensional Yang-Mills action with regard to the parity eigenstates in the four-dimensional space-time, the meson spectrum is calculated, which is shown to achieve large coincidence with experimental data for mesons. Hidden local symmetry, which is phenomenologically introduced to discuss the dynamics of pions and vector mesons [3], naturally appears as a part of the local $\mathrm{U}(N_f)$ gauge symmetry of the D8 brane. Other phenomenological hypotheses for the low-energy meson dynamics like vector meson dominance [4]

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and Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relations among the couplings [5] are naturally reproduced in terms of the holographic QCD.

The $D4/D8/\overline{D8}$ holographic model describes the two-flavor three-color gauge theory as massless QCD from multi D-brane configurations. However the reliability of the classical supergravity description of D4 brane gives the constraint for the dual gauge theory side as a large 'tHooft coupling and also large N_c. In this sense the holographic model describes the nonperturbative large-N_c QCD. According to the investigation about the large-N_c gauge theory by 'tHooft in 1974 [6], large-N_c QCD is found to become equivalent with the weak coupling system of mesons and glueballs; the baryon does not directly appear as a dynamical degrees of freedom. In 1979, Witten showed that, in the large-N_c limit, the mass of a baryon increases and gets proportional to N_c, which explains the reason why the baryon does not directly appear in the large-N_c limit [7]. Witten also remarked that a baryon mass becomes proportional to the inverse of the coupling constant of meson fields in the large N_c limit, similar to a mass of a soliton wave in scalar field theories. This viewpoint gives the revival of the Skyrme model describing a baryon as a chiral soliton. From these considerations, a baryon is expected to appear as some topological object like chiral soliton in the large-N_c holographic QCD.

In this paper, we introduce the concept of a chiral soliton in the holographic QCD, i.e., we describe a baryon as a topological soliton in the four-dimensional meson effective action induced by the holographic QCD. We call this topological soliton a brane-induced Skyrmion. A baryon as a mesonic soliton was firstly investigated without (axial) vector mesons by Skyrme in 1961 [8], which is called the Skyrme model, based on the nonlinear sigma model as its meson effective action. By comparing the properties of the standard Skyrmion without (axial) vector mesons to the brane-induced Skyrmion, the roles of (axial) vector mesons for baryons are discussed in our context.

This paper is organized as follows. In Sec. II, we show the review of the $D4/D8/\overline{D8}$ holographic model as one of the reliable effective theories in holographic QCD. There we also add new insights to this holographic model from the viewpoints of recent hadron physics and QCD phenomenologies. In the end of this section, four-dimensional meson effective action is derived from the holographic QCD without small amplitude expansion to discuss the chiral soliton as a large amplitude mesonic soliton. We also show that the effects of higher mass excitation modes of (axial) vector mesons for chiral solitons are expected to be small by introducing the five-dimensional meson wave function picture. Therefore we take only pions and ρ mesons in the four-dimensional meson effective action for the argument of chiral solitons in later sections. In Sec. III, we investigate the brane-induced Skyrmion by using the four-dimensional meson effective action derived in the previous sections. In Sec. III A, we summarize the history of the standard Skyrmion with vector mesons from the beginning of the Skyrme model in 1961. The comparisons about the appearances of (axial) vector mesons in the traditional phenomenological treatment like the hidden local symmetry approach [3] and those in the brane-induced Skyrmion as the holographic QCD are discussed in detail. In Secs. III B and III C, configuration Ansatz as hedgehog type is taken for pion and ρ-meson fields in the effective action, and the energy function and Euler-Lagrange equation of brane-induced Skyrmion are derived. In Sec. III D, numerical results about the profiles of brane-induced Skyrmion are presented. We find that a stable soliton solution exists, which should be a correspondent of a baryon in the large-N_c holographic QCD. Numerical results about the meson wave functions, energy density, total energy, and root-mean-square radius of the brane-induced Skyrmion are summarized, where standard Skyrme profiles are also attached for comparison. These results are compared with the experimental data for baryons. The effects of ρ mesons for a baryon are also discussed from the holographic point of view. Section IV is devoted to summary and discussion. In the appendix, topological natures of Skyrmions are summarized.

II. MESON EFFECTIVE ACTION FROM HOLOGRAPHIC QCD

In this section we review the $D4/D8/\overline{D8}$ holographic model as one of the reliable effective theories in the holographic QCD [1], from the viewpoint of recent hadron physics and QCD phenomenologies. In Sec. II E, following this holographic model, we derive the effective action of pions and ρ mesons without small amplitude expansion of meson fields. This action is used later in Sec. III to discuss the chiral soliton, which is a nontrivial topological object in the low-energy meson field theory.

A. Holographic massless QCD with $D4/D8/\overline{D8}$

In this subsection we investigate the open-string spectrum of D4 and D8-$\overline{D8}$ branes in type IIA string theory, from which we obtain massless QCD in the weak coupling regime.

First, we review the construction of the four-dimensional pure Yang-Mills theory from D4 brane configuration following Witten's strategy [9]. In the superstring field theory, Dp branes appear as $(p+1)$-dimensional solitonlike objects in terms of the fundamental strings in the ten-dimensional space-time. In Witten's construction, N_c-folded D4 branes as the supersymmetric five-dimensional manifolds are introduced extending to $x_{\mu=0-3}$ and $x_4 \equiv \tau$ directions in the ten-dimensional space-time. From the fluctuation modes of 4-4 string (p-p' string means an open string with one end located on Dp brane and the other end on Dp' brane), there exist ten massless bosonic modes ($\mathcal{A}_{\mu=0-3}$, $\mathcal{A}_\tau$, $\Phi_{i=5-9}$) and massless fermionic modes as their superpartners. The scalar modes $\Phi_{i=5-9}$ are five Nambu-Goldstone modes with respect to the spontaneous breaking of the translational invariance of the D4 brane in ten-dimensional space-time.

In order to obtain the non-SUSY Yang-Mills theory which includes only massless gauge fields $\mathcal{A}_\mu$, the D4 brane is compactified to the S^1 circle along the τ direction with the periodicity of the Kaluza-Klein mass scale as $\tau \sim \tau + 2\pi M_{\rm KK}^{-1}$, and antiperiodic boundary condition in the τ direction is imposed for all the fermions on the D4 brane as

$$\psi(x_\mu, \tau + 2\pi M_{\rm KK}^{-1}) = -\psi(x_\mu, \tau). \tag{1}$$

Then, fermion fields can be Fourier-transformed with boundary condition (1) as

$$\psi(x_\mu, \tau) = \sum_{n=-\infty}^{+\infty} \psi_n(x_\mu) e^{i\tau M_{\rm KK}(n+(1/2))} \equiv \sum_{n=-\infty}^{+\infty} \psi_n(x_\mu, \tau). \tag{2}$$

These fermions are originally introduced as massless modes in five-dimensional space-time of the D4 brane. However the kinetic term of the fermion fields with the five-dimensional Klein-Gordon operator induces the large mass through the projection into the four-dimensional space-time as

$$\partial_M^2 \psi_n = (\partial_\mu^2 + \partial_\tau^2)\psi_n = \{\partial_\mu^2 - M_{\rm KK}^2(n + \tfrac{1}{2})^2\}\psi_n, \quad (M = 0\text{–}4,\ \mu = 0\text{–}3) \tag{3}$$

which means that, because of the boundary condition (1), all the fermions acquire large masses of Kaluza-Klein mass scale in four-dimensional space-time. Then supersymmetry on the D4 brane is completely broken, and scalar fields $\mathcal{A}_\tau$ and $\Phi_{i=5-9}$ are expected to acquire large mass of order $O(M_{\rm KK})$ through loop corrections. There also exists an infinite tower of massive Kaluza-Klein modes originated from the compactified S^1 direction of the D4 brane. These extra modes for the Yang-Mills theory are expected to be neglected by taking large $M_{\rm KK}$ scale. Then, only gauge fields $\mathcal{A}_{\mu=0-3}$ are left as massless modes and eventually give the pure Yang-Mills gauge theory.

Now, we discuss the construction of massless QCD in the weak coupling regime by taking the multi D-brane configurations. QCD is composed by two elements, i.e., massless gauge fields (gluons) and light flavors (quarks) as the matter fields. In order to include the flavor degrees of freedom, the idea of placing probe D6 brane in the D4 background is proposed by Kruczenski *et al.* [10]. In their construction, both D6 and D4 branes extend to the same flat four-dimensional space-time $x_{\mu=0-3}$ as their common part, and so D6 and D4 branes do not extend to all the directions of ten-dimensional space-time; there remain two dimensions in which both D6 and D4 branes can be seen as two localized points. This means that the 4-6 string has

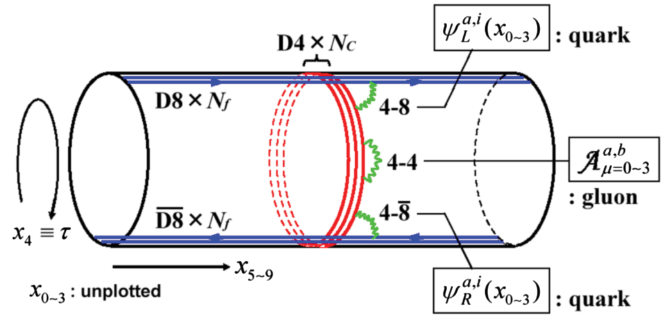

FIG. 1 (color online). Multi D-brane configurations of the D4/D8/$\overline{\rm D8}$ holographic QCD. N_c-folded D4 branes and N_f-folded D8-$\overline{\rm D8}$ branes. Flat four-dimensional space-time x_{0-3} are not plotted. Gluons and quarks appear as the fluctuation modes of 4-4, 4-8 and 4-$\bar{8}$ strings shown by the waving lines.

generally finite length, and, because of its string tension, it gives massive flavors as its fluctuation modes, which results in QCD with heavy flavors. In fact, the D4/D6 model does not correspond to massless QCD, and, in low-energy regime, it does not have the spectrum of light pseudoscalar mesons (pions) as the Nambu-Goldstone boson with respect to chiral-symmetry breaking in QCD.

With these considerations, Sakai and Sugimoto [1] place the N_f-folded D8-$\overline{\rm D8}$ branes with the N_c-folded D4 branes as shown in Fig. 1 and Table I. In this configuration, D8 and $\overline{\rm D8}$ branes are separately placed along the τ direction as shown in Fig. 1 to make tachyonic fluctuation modes of 8-$\bar{8}$ strings become massive and negligible [1].

Following Witten's construction mentioned above, D4 branes are compactified to the S^1 circle in the extra direction $\tau(\equiv x_4)$ with the Kaluza-Klein mass scale $M_{\rm KK}$, and the antiperiodic boundary condition (1) is imposed in the τ direction for all the fermions on D4 branes. Owing to the compactification, only the massless gauge fields $\mathcal{A}_\mu$ are left, and a non-SUSY four-dimensional U(N_c) gauge theory is obtained. Here, the gauge fields $\mathcal{A}_\mu$ as the fluctuation modes of the 4-4 string belong to the adjoint representation of gauge group U(N_c), and are identified as gluons in QCD.

Table I also shows that, comparing with the D4/D6 model [10], D4 and D8-$\overline{\rm D8}$ branes extend to all the directions of the ten-dimensional space-time. This means that 4-8 string can be arbitrarily shorter, especially localized around the physical-coordinate space $x_{\mu=0-3}$ shared by

TABLE I. Space-time extension of the D4 brane and D8-$\overline{\rm D8}$ branes to construct massless QCD. The circle denotes the extended direction of each D-brane. x_{0-3} correspond to the flat four-dimensional space-time.

	0	1	2	3	4	5	6	7	8	9
D4	○	○	○	○	○	—	—	—	—	—
D8-$\overline{\rm D8}$	○	○	○	○	—	○	○	○	○	○

both D4 and D8-$\overline{\text{D8}}$ branes (see Table I). Then 4-8 string gives N_f flavors of massless fermions as its fluctuation modes localized around $x_{\mu=0\text{–}3}$ in ten-dimensional space-time. These flavors are shown to belong to the fundamental representation of $\mathrm{U}(N_c)$ gauge group, and, in this holographic model, they are interpreted as quarks in QCD.

These flavors from 4-8 and 4-$\bar{8}$ strings are shown to have opposite chirality with each other as the chiral fermions [11]. Therefore $\mathrm{U}(N_f)_{\mathrm{D8}} \times \mathrm{U}(N_f)_{\overline{\mathrm{D8}}}$ local gauge symmetry of probe D8-$\overline{\text{D8}}$ branes can be interpreted as the correspondent of $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ chiral symmetry for flavor quarks in QCD. This consideration shows that the D4/D8/$\overline{\text{D8}}$ holographic model represents the four-dimensional massless QCD.

B. Supergravity description of background D4 brane

In the previous section, we showed that massless QCD can be constructed from the D4/D8/$\overline{\text{D8}}$ multibrane configurations by considering the symmetry of the induced gauge theory with the analysis of the open-string spectrum. On the other hand, according to the recent remarkable progress in the concept of gauge/gravity duality [12], the D-brane has a supergravity description, and, through this duality, the parameters among the gauge theory and the string theory (and also the supergravity solution) are found to be related with each other. In the actual calculations, classical supergravity solutions are tractable, and the reliability of this tree-level approximation with the classical supergravity solution gives some constraints for the parameters (g_{YM}, N_c) of the dual gauge theory. In this section, we show the relations between the gauge theory and the supergravity description of D-branes, and discuss the constrained properties of massless QCD which is taken to be dual with classical supergravity description of D-branes.

Here, we apply the probe approximation for the D4/D8/$\overline{\text{D8}}$ system; N_f-folded D8-$\overline{\text{D8}}$ flavor branes are introduced as the probes into the supergravity background solution of the N_c-folded D4 branes, and backreaction from the probe branes to the total system is neglected. This D4 background solution is shown to be holographic dual with the four-dimensional pure Yang-Mills theory at low energies [9]. This probe approximation is thought to be reliable for the large-N_c case as $N_f \ll N_c$, since the folding number of the D-brane, i.e., the Ramond-Ramond flux quantum, is proportional to the mass of the D-brane. The probe approximation is sometimes compared to the general coordinate system with a light probe object and the gravitational background induced by a heavier object.

This probe approximation is somehow similar to the quenched approximation, which is frequently used in lattice QCD calculations [13]. In quenched QCD, corresponding to $N_f = 0$ limit, the nonperturbative QCD vacuum is composed only with the gluonic part of QCD, and the (anti)quarks are treated as "probe" fermions traveling in the gluonic vacuum. In fact, the quark-loop effect is neglected in the quenched approximation like large N_c QCD, and the "backreaction" of the quark field to the nonperturbative gluonic vacuum is neglected. In spite of such reduction of quark dynamics, the quenched approximation with $N_c = 3$ is known to reproduce the low-lying hadron spectra only within 10% deviation in lattice QCD calculations relative to the experimental data, as well as the nonperturbative aspects of QCD such as the quark confinement properties in the interquark potentials and dynamical chiral-symmetry breaking [13]. These phenomenological success of the quenched approximation in QCD may be related with the viewpoint of the probe approximation in the holographic approach of QCD.

The supergravity solution of N_c-folded D4 branes compactified along the τ direction to S^1 circle can be written in the Euclidean metric as follows:

$$\begin{aligned} ds^2 &= \left(\frac{U}{R}\right)^{3/2}(g_{\mu\nu}dx_\mu dx_\nu + f(U)d\tau^2) \\ &\quad + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \\ e^\phi &= g_s\left(\frac{U}{R}\right)^{3/4}, \\ F_4 &= dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4, \\ f(U) &= 1 - \frac{U_{\mathrm{KK}}^3}{U^3}. \end{aligned} \tag{4}$$

ds is the infinitesimal invariant distance in ten-dimensional space-time: $ds^2 \equiv g_{MN}dx_M dx_N$ $(M, N = 0\text{–}9)$. $x_{\mu=0\text{–}3}$ and $x_4 \equiv \tau$ are the coordinates to which the D4 brane extends, and $x_{\mu=0\text{–}3}$ have the flat four-dimensional Euclidean metric $g_{\mu\nu}$. The D4 brane does not extend to the residual five dimensions $x_{\alpha=5\text{–}9}$, so that its classical supergravity solution has SO(5) rotational symmetry around $x_{\alpha=5\text{–}9}$, which is usefully taken as the polar coordinate (U, Ω_4); U corresponds to the radial coordinate and Ω_4 are four angle variables in $x_{\alpha=5\text{–}9}$ directions, parametrizing a unit sphere S^4. ϕ is a dilaton field from the fluctuation modes of closed strings. F_4 is the 4-form field strength and C_3 is the 3-form Ramond-Ramond field. ϵ_4 is a volume form on S^4 with total volume $V_4 = 8\pi^2/3$. The radial coordinate U is bounded from below as $U \geq U_{\mathrm{KK}}$, and, at $U = U_{\mathrm{KK}}$, the S^1 circle around τ direction completely shrinks with $f(U) \to 0$; U_{KK} corresponds to a horizon in ten-dimensional space-time with the D4 supergravity solution. The constant R can be written by the string coupling constant $g_s \equiv e^{\langle\phi\rangle}$ and string length l_s [1] as

$$R^3 = \pi g_s N_c l_s^3. \tag{5}$$

To avoid a conical singularity at $U = U_{\mathrm{KK}}$ and make the solution (4) everywhere regular, the period $\delta\tau$ of the compactified τ direction is taken [1] to be

$$\delta\tau \equiv \frac{4\pi}{3}\frac{R^{3/2}}{U_{\rm KK}^{1/2}}. \tag{6}$$

As discussed in the previous section about the open-string spectrum on the D4 brane, the four-dimensional pure Yang-Mills theory becomes effectively the same as the gauge theory induced on the compactified D4 brane below the Kaluza-Klein mass scale $M_{\rm KK}$, which is related to the period $\delta\tau$ as

$$M_{\rm KK} = \frac{2\pi}{\delta\tau} = \frac{3}{2}\frac{U_{\rm KK}^{1/2}}{R^{3/2}}. \tag{7}$$

The Yang-Mills coupling constant $g_{\rm YM}$ of the gauge theory can be also written by the parameters of the string theory by using the DBI action of the D4 brane as

$$\begin{aligned} S_{\rm D4}^{\rm DBI} &= T_4 \int d^4x d\tau e^{-\phi}\sqrt{-\det(g_{MN} + 2\pi\alpha' \mathcal{F}_{MN})} \\ &\sim T_4 \frac{1}{4}(2\pi\alpha')^2 \frac{\delta\tau}{g_s}\int d^4x 2\,{\rm tr}\mathcal{F}_{\mu\nu}^2 + O(\mathcal{F}^4), \\ &\quad (M, N = 0\text{–}4,\ \mu, \nu = 0\text{–}3) \end{aligned} \tag{8}$$

where $\alpha' \equiv l_s^2$ is the Regge slope parameter and $T_p \equiv \frac{1}{(2\pi)^p l_s^{p+1}}$ is the surface tension of the Dp brane. The gravitational energy is abbreviated in Eq. (8) for simplicity. The fifth component of gauge fields $\mathcal{A}_\tau$ is also neglected in Eq. (8) for its large mass due to the complete SUSY-breaking on the D4 brane, discussed in the previous section. In the rescaled Yang-Mills theory with $g_{\rm YM}\mathcal{A}_\mu \to \mathcal{A}_\mu$, the coupling constant appears in the action as $\frac{1}{2g_{\rm YM}^2}{\rm tr}\mathcal{F}_{\mu\nu}^2$. Therefore, by comparing with the action (8), $g_{\rm YM}$ can be written by the string parameters as

$$g_{\rm YM}^2 = \frac{g_s}{T_4(2\pi\alpha')^2\delta\tau} = (2\pi)^2 g_s l_s/\delta\tau. \tag{9}$$

By inverting the relations (7) and (9) and also using Eq. (5), the string parameters R, $U_{\rm KK}$, and g_s can be written by those in the Yang-Mills theory side as follows:

$$R^3 = \frac{1}{2}\frac{g_{\rm YM}^2 N_c l_s^2}{M_{\rm KK}}, \qquad U_{\rm KK} = \frac{2}{9}g_{\rm YM}^2 N_c M_{\rm KK} l_s^2, \qquad g_s = \frac{1}{2\pi}\frac{g_{\rm YM}^2}{M_{\rm KK} l_s}. \tag{10}$$

Now, we see the conditions for the reliability of the classical supergravity description. First, it is required that the curvature everywhere is sufficiently small relative to the fundamental string tension, so that higher-derivative string corrections can be neglected. Second, local string coupling e^ϕ should be small to suppress the string loop effects. These conditions give the constraints for the string parameters and, using the relations (10), they can be expressed in terms of the Yang-Mills theory, expected to be dual with classical supergravity solution, as follows [10]:

$$g_{\rm YM}^4 \ll \frac{1}{g_{\rm YM}^2 N_c} \ll 1, \tag{11}$$

which is achieved by $g_{\rm YM} \to 0$, $N_c \to \infty$, and $\lambda \equiv g_{\rm YM}^2 N_c$ fixed and large. λ is the 'tHooft coupling, appearing as the effective coupling constant in the large-N_c gauge theory. Therefore, massless large N_c QCD in the strong coupling regime can be dual with classical supergravity description of D4 branes with D8-$\overline{\rm D8}$ probes. In other words, nonperturbative property of QCD can be successfully described by the tree-level string theory.

Here, we comment about the realization of chiral-symmetry breaking and color confinement in the holographic model. With the supergravity background of the D4 brane (4), probe D8 and $\overline{\rm D8}$ branes are smoothly interpolated with each other in this curved space-time as in Fig. 2 [see D8 brane configuration (15) in Sec. II C], and $U(N_f)_{\rm D8} \times U(N_f)_{\overline{\rm D8}}$ gauge symmetry breaks into the single $U(N_f)$ gauge symmetry, which is understood as a holographic manifestation of chiral symmetry breaking in QCD. Actually global chiral symmetry still remains as the element of $U(N_f)$ local gauge symmetry discussed in later sections, and the interpolation of D8-$\overline{\rm D8}$ branes induces its spontaneous breaking in a vacuum state. Chiral symmetry breaking is thus realized by the geometrical connection of D8 and $\overline{\rm D8}$ branes.

Furthermore, color confinement is realized in the holographic QCD as follows. Since color quantum number is carried only by N_c-folded D4 branes, colored objects appear as the fluctuation modes of open strings with at least one end located on N_c-folded D4 branes, e.g., gluons from 4-4 strings and flavor quarks from 4-8 strings. Thus these colored modes are localized around the origin of the extra five dimensions: $x_{\alpha=5\text{–}9} = 0$, where the D4 brane exists. Therefore, in the supergravity background of the D4 brane, it can be interpreted that these colored objects would locate

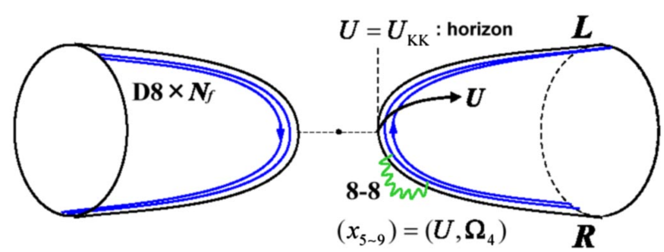

FIG. 2 (color online). Probe D8 branes with D4 supergravity background (4). Flat four-dimensional space-time $x_{0\text{–}3}$ are not plotted. The radial coordinate U in the extra five dimensions $x_{5\text{–}9}$ is bounded from below by a horizon $U_{\rm KK}$ as $U \geq U_{\rm KK}$. Color-singlet mesons appear as the fluctuation modes of 8-8 string shown by the waving line. The two "corns" shown above are continuously connected along the angle directions Ω_4 in $x_{5\text{–}9}$. For the argument of SO(5) singlet modes around Ω_4, only single "corn" is to be considered, as shown in Fig. 3.

behind the horizon $U_{\rm KK}$ and become invisible from outside, which can be understood as the color-confinement phenomena at the low-energy scale. In fact, in the holographic QCD, color confinement is realized as ''loss of direct color information'' at the low-energy scale, due to the appearance of ''nontrivial large spacial curvature'' in the extra fifth-direction as shown in Fig. 2, while the informations from color degrees of freedom are hidden at large distances due to extremely strong correlations in usual four-dimensional QCD. The appearance of the horizon in the classical supergravity solution, which is reliable for large N_c and large 't Hooft coupling λ, seems consistent with the low-energy property of large-N_c Yang-Mills theory, inducing self-coupling of gauge fields and the confinement.

These considerations may suggest that chiral symmetry breaking and color confinement occur simultaneously; two independent chirality spaces are connected by the ''worm hole'' into which colored objects are absorbed as shown in Fig. 2.

The close relations between chiral symmetry breaking and color confinement have been observed in lattice QCD calculations at finite temperatures [13]. In lattice QCD at finite temperatures, chiral symmetry breaking is expressed by the quark condensate $\langle\bar{q}q\rangle = {\rm tr}G_q[\mathcal{A}_\mu]$ as its order parameter with the light quark propagator $G_q[\mathcal{A}_\mu] = \frac{1}{\not{\partial}+g\not{\mathcal{A}}+m_q}$ in the Euclidean metric. The quark confinement property can also be expressed by the vacuum expectation value of the Polyakov-loop $\langle P\rangle$ with $P \equiv {\rm tr}Pe^{ig\int_0^{1/T}d\tau\mathcal{A}_4(\mathbf{x},\tau)}$, which is the order parameter of the center symmetry $\mathbf{Z}^{N_c} \in {\rm SU}(N_c)$ and is physically related to the single static-quark energy E_q as $\langle P\rangle = e^{-E_q/T}$. In fact, the chiral-symmetry restoration and the deconfinement phase transition are found to occur simultaneously at the critical temperature T_c in both quenched and full lattice QCD with $N_c = 2$, 3 and $N_f = 0$, 1, 2, 3, 4: both of the order parameters, i.e., the Polyakov loop $\langle P\rangle$ and the quark condensate $\langle\bar{q}q\rangle$, drastically change around T_c near the chiral limit.

In spite of the lattice QCD evidence, the relations are still unclear between chiral symmetry breaking and color confinement, especially from the analytical frameworks. Furthermore, at finite baryon-number density, which is also an important axis in the QCD phase diagram, lattice QCD calculations do not work well due to the ''sign problem.'' Therefore there is still missing the sufficient consensus about the relations of chiral symmetry breaking and color confinement, particularly in the finite-density QCD. From these viewpoints, it would be desired to perform the well-defined generalization of the holographic models to the finite-temperature and finite-density QCD in future.

After the supergravity description of background D4 branes, there only appear colorless objects as the fluctuation modes of residual probe D8 branes: mesons and also baryons. By the analysis of large-N_c gauge theory [6], large-N_c QCD is found to be equivalent with the weak-coupling system of mesons (and glueballs), and baryons do not directly appear as the dynamical degrees of freedom. Therefore, in this large-N_c holographic QCD, baryons are expected to appear not directly but as some solitonlike topological objects, which is discussed in later sections.

C. Non-Abelian DBI action of probe D8 brane

In this section, we treat the multiflavor system with $N_f = 2$, by using the non-Abelian Dirac-Born-Infeld (DBI) action of the D8 brane as

$$S_{\rm D8}^{\rm DBI} = T_8\int d^9x e^{-\phi}\sqrt{-\det(g_{MN}+2\pi\alpha' F_{MN})} \tag{12}$$

$$= T_8\int d^9x e^{-\phi}\sqrt{-\det g_{MN}} \times \{1 + \tfrac{1}{4}(2\pi\alpha')^2 g^{JK}g^{PQ}2\,{\rm tr}(F_{JP}F_{KQ})\} + O(F^4), \tag{13}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M + i[A_M, A_N]$ is the field strength tensor in nine-dimensional space-time of the D8 brane, and $A_{\mu=0\text{–}3}$, A_z, and $A_{\alpha=5\text{–}8}$ ($\alpha = 5$–8 are coordinate indices on S^4) are the Hermite U(N_f) gauge fields as the fluctuation modes of 8-8 string. Factor 2 appears in the second term of (13) according to the normalization of generators: ${\rm tr}T^aT^b = \frac{1}{2}\delta^{ab}$. In fact, gauge field A_M is color-singlet and obeys the adjoint representation of U(N_f) flavor space, eventually producing the meson degrees of freedom in the holographic QCD.

In $A_M \to 0$ limit, the action (13) becomes

$$S_{\rm D8}^{\rm DBI}|_{A_M\to 0} = T_8\int d^9x e^{-\phi}\sqrt{-\det g_{MN}}, \tag{14}$$

which gives the gravitational energy of the D8 brane in general coordinates. By minimizing the action (14) with D4 supergravity background (4), the configuration of probe D8 brane in ten-dimensional space-time is determined [1] to be

$$\tau(U) = \delta\tau/4, \tag{15}$$

which is shown in Fig. 2, and coordinates on the D4 supergravity solution in detail with the stabilized D8 brane configuration (15) are shown in Fig. 3.

Now, we discuss the gauge fields as the fluctuation modes of D8 brane configuration (15). SO(5) rotational symmetry around S^4 does not exist in QCD with four-dimensional space-time. Therefore, in the D4/D8/$\overline{\rm D8}$ holographic model, only SO(5) singlet modes are considered and gauge fields $A_{\alpha=5\text{–}8}$ are neglected as SO(5) vectors. A_μ and A_z are also assumed to be independent of these extra coordinates on S^4. Then the integral along the S^4 direction in the action (13) can be performed as

$$S_{\rm D8}^{\rm DBI} - S_{\rm D8}^{\rm DBI}|_{A_M\to 0} = \tilde{T}_8(2\pi\alpha')^2 \int d^4xdz 2\,{\rm tr}\left\{\frac{R^3}{4U_z(z)} g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + \frac{9}{8}\frac{U_z^3(z)}{U_{\rm KK}} g^{\mu\nu}F_{\mu z}F_{\nu z}\right\} + O(F^4), \quad (16)$$

where $\tilde{T}_8 \equiv \frac{2}{3}R^{3/2}U_{\rm KK}^{1/2}T_8V_4g_s^{-1}$ and the fifth coordinate z on the D8 brane configuration (15) and the z-dependent function $U_z(z)$ are defined as

$$\frac{z^2}{U_{\rm KK}^2} \equiv \frac{U^3}{U_{\rm KK}^3} - 1, \quad (17)$$

$$U_z(z) \equiv (U_{\rm KK}^3 + U_{\rm KK}z^2)^{1/3}. \quad (18)$$

In the action (16), the gravitational energy (14) is subtracted as a vacuum energy relative to the gauge sectors. The curvature in the fifth dimension is shown by the induced measures $\frac{R^3}{4U_z}$ and $\frac{9}{8}\frac{U_z^3}{U_{\rm KK}}$ in Eq. (16); $F_{\mu z}$ includes a field strength along the z direction, so that the measure for $F_{\mu z}$ differs from that for $F_{\mu\nu}$. The five-dimensional Yang-Mills theory (16) is regarded as the unified theory of mesons, where mode expansion of the gauge fields gives the meson degrees of freedom, discussed in later sections.

Now one can easily show that, using the relations (10), DBI action of the D8 brane (16) with D4 supergravity background has no explicit dependence on the string length l_s. This is because the effective action with classical supergravity background of the D4 brane is constructed with the condition $U_{\rm KK}^{1/2}R^{3/2} \gg l_s^2$ ($U_{\rm KK}^{1/2}R^{3/2}$ is the typical curvature in D4 classical supergravity solution [10]), i.e., the reliability of the local approximation for the fundamental strings, which gives the large 'tHooft coupling condition $\lambda \gg 1$ in the gauge theory side discussed in Eq. (11). Therefore, without loss of generality, we can set the string tension as sufficiently small value $l_s^2 = \frac{9}{2}\lambda^{-1}M_{\rm KK}^{-2}$ to be consistent with the reliability of the local approximation for the fundamental strings. Then the relations between the dimensional quantities R, $U_{\rm KK}$, and $M_{\rm KK}$ in Eqs. (10) can be rewritten as

$$R^3 = \tfrac{9}{4}M_{\rm KK}^{-3}, \qquad U_{\rm KK} = M_{\rm KK}^{-1}. \quad (19)$$

In this setup, all the dimensional quantities are simply scaled with the Kaluza-Klein mass $M_{\rm KK}$, and therefore we take the convenient unit $M_{\rm KK} = 1$ for the derivation of the four-dimensional effective theory. At the end of calculations, the dimensional quantities can be recovered by multiplying the proper power of $M_{\rm KK}$.

The final form of the DBI action in $M_{\rm KK} = 1$ unit is written as the five-dimensional Yang-Mills theory with flat four-dimensional Euclidean space-time x, i.e., x_{0-3} and other fifth dimension z with curved measure as

$$S_{\rm D8}^{\rm DBI} - S_{\rm D8}^{\rm DBI}|_{A_M\to 0} = \kappa \int d^4xdz\,{\rm tr}\{\tfrac{1}{2}K(z)^{-1/3}F_{\mu\nu}F_{\mu\nu} + K(z)F_{\mu z}F_{\mu z}\} + O(F^4), \quad (20)$$

where the overall factor κ is defined in the $M_{\rm KK} = 1$ unit as

$$\kappa \equiv \tilde{T}_8(2\pi\alpha')^2R^3 = \frac{\lambda N_c}{108\pi^3}, \quad (21)$$

and $K(z) \equiv 1 + z^2$ expresses the nontrivial curvature in the fifth direction z induced by the supergravity background of the D4 brane.

In the $M_{\rm KK} = 1$ unit, the factor $\alpha'(= l_s^2)$ appearing in front of the field strength tensor in the DBI action (12) can be written as $\alpha' = \frac{9}{2}\lambda^{-1}$. Thus it becomes clear that the expansion of the DBI action with respect to the field strength tensor corresponds to the expansion about the "inverse" of the large 'tHooft coupling λ with condition (11), where the nonperturbative aspects of QCD with large 'tHooft coupling λ can be manifestly linked with the tree-level Yang-Mills theory induced on the probe D8 brane with D4 supergravity background. This is similar to the nonlinear sigma model [14] as the meson effective theory having soft-momentum expansion, which gives a workable perturbative treatment in the strong-coupling regime of QCD. We now treat nontrivial leading order $O(F^2)$ of the DBI action (20), corresponding to the leading of $1/N_c$ and $1/\lambda$ expansions in the holographic QCD, for the argument of the nonperturbative (strong coupling) properties of QCD.

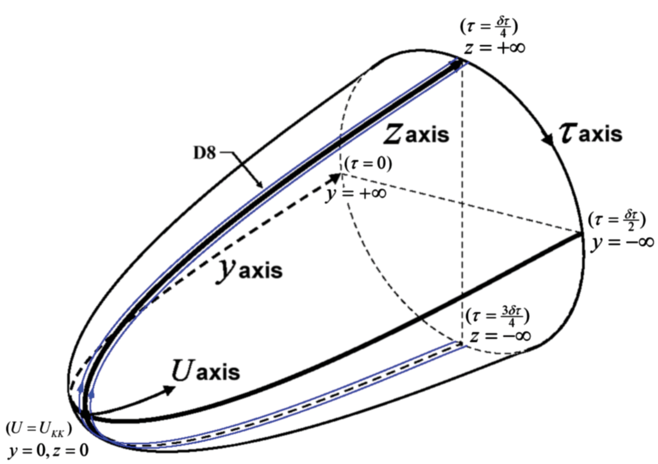

FIG. 3 (color online). Coordinates (U, τ) and (y, z) on the D4 supergravity solution (4); coordinates (y, z) are introduced as $\frac{y}{U_{\rm KK}} \equiv (\frac{U^3}{U_{\rm KK}^3} - 1)^{1/2}\cos(\frac{2\pi}{\delta\tau}\tau)$ and $\frac{z}{U_{\rm KK}} \equiv (\frac{U^3}{U_{\rm KK}^3} - 1)^{1/2}\sin(\frac{2\pi}{\delta\tau}\tau)$. Stabilized D8 brane configuration (15) with the curved space-time background is also shown.

D. Mode expansion of gauge field

In this section, to obtain the four-dimensional effective theory reflecting definite parity and G-parity of QCD, we perform the mode expansion of the gauge field with respect to the four-dimensional parity eigenstates by using proper orthogonal basis in the z direction.

The gauge field $A_M(x_N)$ ($M, N = 0$–$3, z$) transforms with U(N_f) local gauge symmetry of D8 brane as

$$A_M(x_N) \to A_M^g(x_N) \equiv g(x_N)A_M(x_N)g^{-1}(x_N) + \frac{1}{i}g(x_N)\partial_M g^{-1}(x_N). \tag{22}$$

In order that the action (20) is to be finite as the tree level, the field strength tensor F_{MN} should vanish at $z \to \pm\infty$. This can be achieved by the gauge field A_μ which goes to the pure-gauge form in $z \to \pm\infty$ as $A_M \to (1/i)U^\dagger \partial_M U$ [$U \in U(N_f)$]. Now, by applying the gauge-transformation (22), we begin with the gauge fixing satisfying

$$A_M(x_N) \to 0 \quad (\text{as } z \to \pm\infty). \tag{23}$$

In this gauge fixing, the gauge degrees of freedom is fixed only at the 4-dimensional "surfaces" $z = \pm\infty$ in five-dimensional space-time, and there still remains large gauge-degrees of freedom with gauge function $g^{\text{res}}(x_M)$ as

$$\partial_M g^{\text{res}}(x_N) \to 0, \quad (\text{as } z \to \pm\infty) \tag{24}$$

which means that $g^{\text{res}}(x_M)$ goes to constant matrix $g_\pm \in U(N_f)$ as $g_\pm \equiv \lim_{z\to\pm\infty} g^{\text{res}}(x_M)$. In the holographic model, (g_+, g_-) are regarded as the elements of chiral symmetry $\text{U}(N_f)_L \times \text{U}(N_f)_R$ in QCD; the global chiral symmetry $(U(N_f)_L, U(N_f)_R)$ appears as the elements of the local $U(N_f)$ gauge symmetry of D8 brane at the surfaces $z = \pm\infty$ in the five-dimensional space-time. Thus, in the five-dimensional description, $U(N_f)_L$ and $U(N_f)_R$ appear and act at spatially different surfaces $z = \pm\infty$, by holding local U(N_f) gauge symmetry between two of them.

Now, pion field is introduced in Refs. [1,15] as

$$U(x^\mu) = P\exp\left\{-i\int_{-\infty}^{\infty} dz' A_z(x_\mu, z')\right\} \in U(N_f), \tag{25}$$

where P denotes the path-ordered product along the z coordinate with the upper-edge z variable arranged in left-hand side. Since $g^{\text{res}}(x_M)$ is x-independent at $z = \pm\infty$, $U(x^\mu)$ transforms by the gauge function $g^{\text{res}}(x_M)$ as the global transformation as

$$U(x^\mu) \to g^{\text{res}}(x_\mu, z = +\infty)U(x^\mu)g^{\text{res}}(x_\mu, z = -\infty)^{-1} = g_+ U(x^\mu) g_-^{-1}. \tag{26}$$

This transformation property is the same as that for the pion field in the chiral Lagrangian of a nonlinear sigma model with respect to global chiral transformation [1,14,15].

Here, we also introduce the variables $\xi_\pm(x_\mu)$ as

$$\xi_\pm^{-1}(x^\mu) = P\exp\left\{-i\int_{z_0(x_\mu)}^{\pm\infty} dz' A_z(x_\mu, z')\right\} \in U(N_f), \tag{27}$$

where $z_0(x_\mu)$ is a single-valued arbitrary function of x_μ. Then the chiral field (25) can be written as

$$U(x^\mu) = \xi_+^{-1}(x_\mu)\xi_-(x_\mu). \tag{28}$$

$\xi_\pm(x_\mu)$ transform with the gauge function $g^{\text{res}}(x_M)$ as

$$\xi_\pm(x_\mu) \to h_{z_0}(x_\mu)\xi_\pm(x_\mu)g_\pm^{-1}, \tag{29}$$

where $h_{z_0}(x_\mu) \equiv g^{\text{res}}(x_\mu, z = z_0(x_\mu))$ is the element of U(N_f) local gauge symmetry on the four-dimensional surface $z_0(x_\mu)$. This local gauge symmetry of $h_{z_0}(x_\mu)$ corresponds to the hidden local symmetry for vector mesons in Ref. [3], which is later discussed.

Now, we change to the $A_z = 0$ gauge, which is a popular gauge in the holographic models. The $A_z = 0$ gauge is appropriate for the construction of the four-dimensional effective theory because A_z behaves as a scalar field in the flat four-dimensional space-time and it is to be absorbed by the four-dimensional gauge field A_μ (see Sec. 3.4 of the former reference in [1]). In fact, in the $A_z = 0$ gauge, the breaking of U(N_f) local gauge symmetry becomes manifest like the unitary gauge in the non-Abelian Higgs theory, which eventually induces the finite mass for gauge fields, especially for the (axial) vector mesons appearing as a part of A_μ. One can take the $A_z = 0$ gauge by applying gauge function

$$g^{-1}(x_\mu, z) = P\exp\left\{-i\int_{z_0(x_\mu)}^{z} dz' A_z(x_\mu, z')\right\}, \tag{30}$$

which changes the boundary behavior of A_μ as

$$A_\mu(x_N) \to \frac{1}{i}\xi_\pm(x_\nu)\partial_\mu\xi_\pm^{-1}(x_\nu) \quad (\text{as } z \to \pm\infty). \tag{31}$$

Now, we perform the mode expansion of A_μ with the boundary condition (31) by using proper orthogonal basis $\psi_\pm(z)$ and $\psi_n(z)$ ($n = 1, 2, \cdots$) as follows:

$$A_\mu(x_N) = l_\mu(x_\nu)\psi_+(z) + r_\mu(x_\nu)\psi_-(z) + \sum_{n\geq 1} B_\mu^{(n)}(x_\nu)\psi_n(z), \tag{32}$$

$$l_\mu(x_\nu) \equiv \frac{1}{i}\xi_+(x_\nu)\partial_\mu\xi_+^{-1}(x_\nu), \tag{33}$$

$$r_\mu(x_\nu) \equiv \frac{1}{i}\xi_-(x_\nu)\partial_\mu\xi_-^{-1}(x_\nu). \tag{34}$$

The basis $\psi_\pm$ are chosen to support all parts of the gauge field A_μ at $z \to \pm\infty$ as $\psi_\pm(z \to \pm\infty) = 1$ and $\psi_\pm(z \to \mp\infty) = 0$, which are here defined as

$$\psi_{\pm}(z) \equiv \tfrac{1}{2} \pm \hat{\psi}_0(z), \tag{35}$$

$$\hat{\psi}_0(z) \equiv \frac{1}{\pi} \arctan z. \tag{36}$$

In order to diagonalize the DBI action (20) with the curved measures $K(z)^{-1/3}$ and $K(z)$ induced in the fifth direction, the basis ψ_n ($n = 1, 2, \cdots$) are taken to be the normalizable eigenfunction satisfying

$$-K(z)^{1/3} \frac{d}{dz}\left\{K(z)\frac{d\psi_n}{dz}\right\} = \lambda_n \psi_n, \qquad (\lambda_1 < \lambda_2 < \cdots) \tag{37}$$

with normalization condition as

$$\kappa \int dz K(z)^{-1/3} \psi_m \psi_n = \delta_{nm}. \tag{38}$$

It should be noted that $\psi_{\pm}$, i.e., $\hat{\psi}_0$ in Eqs. (35) and (36) can be regarded as the zero-modes of (37) with eigenvalue $\lambda_0 \equiv 0$ as

$$-K(z)^{1/3} \frac{d}{dz}\left\{K(z)\frac{d\psi_{\pm}}{dz}\right\} = 0, \tag{39}$$

although they are not normalizable.

Equations (35)–(38) also give other conditions for the basis with z-derivatives as

$$\kappa \int dz K(z) \partial_z \psi_n \partial_z \psi_m = \lambda_n \delta_{nm}, \tag{40}$$

$$\kappa \int dz K(z) \partial_z \hat{\psi}_0 \partial_z \psi_m = \frac{\kappa}{\pi} \int dz \partial_z \psi_m = 0. \tag{41}$$

Conditions (38)–(41) for orthogonal basis are used to give the well-defined mode decoupling of the action, which is later discussed in Sec. II E.

In the holographic model, the fields $B_\mu^{(n=1,2,\cdots)}$ in Eq. (32) are identified as the (axial) vector mesons, which belong to the adjoint representation of the $U(N_f)$ gauge group as $B_\mu = B_\mu^a T_a$. After diagonalizing the DBI action with infinite number of (axial) vector mesons $B_\mu^{(n=1,2,\cdots)}$, their mass terms appear with $m_n^2 \equiv \lambda_n$ [1]. Note here that, in the mode expansion of the gauge field (32), $A_M(x_N)$ is a five-dimensional vector field obeying $A_M(x_N) \to -A_M(x_N)$ under the five-dimensional parity transformation, and we can choose ψ_n as the parity eigenstates of (37) as $\psi_n(-z) = (-)^{n-1}\psi_n(z)$ using the translational invariance along z-direction. Then, $B_\mu^{(n=1,2,\cdots)}$ transform for the four-dimensional parity transformation as $B_\mu^{(n)}(x_\nu) \to (-)^n B_\mu^{(n)}(x_\nu)$. These considerations show that vector and axial vector mesons appear alternately in the excitation spectra [1].

Even with $A_z = 0$ gauge fixing, there still remains residual gauge symmetry appearing in (29) as

$$A_\mu(x_N) \to h_{z_0}(x_\nu) A_\mu(x_N) h_{z_0}^{-1}(x_\nu) + \frac{1}{i} h_{z_0}(x_\nu) \partial_\mu h_{z_0}^{-1}(x_\nu), \tag{42}$$

$$B_\mu^{(n)}(x_\nu) \to h_{z_0}(x_\nu) B_\mu^{(n)}(x_\nu) h_{z_0}^{-1}(x_\nu). \tag{43}$$

Note that there also exist the symmetry of deforming the surface $z_0(x_\mu)$ to the z-direction, apart from the gauge symmetry as in Eqs. (42) and (43). This means that there exist infinite number of the hidden local symmetries for infinite number of (axial) vector mesons with respect to the surface degrees of freedom. This is similar to the "open moose model" discussed by Son *et al.* [15], which is the phenomenological five-dimensional meson theory constructed by embedding infinite number of hidden local symmetries in the chiral field defined on the flat four-dimensional space-time.

Equation (32) can also be written by using useful variables α_μ and β_μ as

$$A_\mu(x_N) = \alpha_\mu(x_\nu)\hat{\psi}_0(z) + \beta_\mu(x_\nu) + \sum_{n\geq 1} B_\mu^{(n)}(x_\nu)\psi_n(z), \tag{44}$$

where α_μ and β_μ are defined as

$$\alpha_\mu(x_\nu) \equiv l_\mu(x_\nu) - r_\mu(x_\nu), \tag{45}$$

$$\beta_\mu(x_\nu) \equiv \tfrac{1}{2}\{l_\mu(x_\nu) + r_\mu(x_\nu)\}. \tag{46}$$

α_μ and β_μ correspond to "axial vector" and "vector" current, respectively. With the residual gauge symmetry with the gauge function $h_{z_0}(x_\mu)$ in Eqs. (42) and (43), α_μ and β_μ transform as

$$\alpha_\mu \to h_{z_0} \alpha_\mu h_{z_0}^{-1}, \qquad \beta_\mu \to h_{z_0} \beta_\mu h_{z_0}^{-1} + \frac{1}{i} h_{z_0} \partial_\mu h_{z_0}^{-1}. \tag{47}$$

Now we take $\xi_+^{-1}(x_\mu) = \xi_-(x_\mu)$ gauge for the residual gauge symmetry in Eqs. (42) and (43). This type of gauge fixing is used in the hidden local symmetry approach [3], which gives the breaking of hidden local symmetry $SU(N_f)_V$ and generates the mass of vector mesons as the Higgs mechanism. The chiral field $U(x_\mu)$ is written in analogy with that of the chiral Lagrangian as $U(x_\mu) = e^{2i\pi(x_\mu)/f_\pi} \in U(N_f)$ with pion field $\pi(x_\mu)$. Then, in the $\xi_+^{-1}(x_\mu) = \xi_-(x_\mu)$ gauge, $\xi_\pm$ can be written from Eq. (28) as

$$\xi_+^{-1}(x_\mu) = \xi_-(x_\mu) \equiv \xi(x_\mu) = e^{i\pi(x_\mu)/f_\pi}. \tag{48}$$

With this gauge fixing (48), $\xi_\pm$ become parity and G-parity partners with each other in four-dimensional space-time, and α_μ and β_μ transform as axial vector and vector fields. Therefore, if the action in the $\xi_+^{-1}(x_\mu) = \xi_-(x_\mu)$ gauge is parity or G-parity transformed in four-dimensional space-time, then the gauge-fixing condition shifts into $\xi_-^{-1}(x_\mu) = \xi_+(x_\mu)$, which is the same gauge fixing as the previous one. In this sense, the effective action in the $\xi_+^{-1}(x_\mu) = \xi_-(x_\mu)$ gauge is realized as a scalar (i.e., invariant) for the four-dimensional parity and G-parity transformation. In fact, other gauge, e.g., $\xi_-(x_\mu) = 1$ gauge gives ρ-3π couplings like $(\partial_\mu \boldsymbol{\pi})^2(\partial_\nu \boldsymbol{\pi} \cdot \boldsymbol{\rho}_\nu)$ and $(\partial_\mu \boldsymbol{\pi} \cdot \boldsymbol{\rho}_\nu) \times (\partial_\mu \boldsymbol{\pi} \cdot \partial_\nu \boldsymbol{\pi})$ in the four-dimensional effective action, which violate both parity and G-parity. In terms of the parity and G-parity classification in the strong interaction, it is essential to take the $\xi_+^{-1}(x_\mu) = \xi_-(x_\mu)$ gauge to construct four-dimensional effective theory of QCD.

E. Effective action of pion and ρ meson

In this paper, we treat pions together with ρ mesons as the lightest vector mesons. We neglect higher mass excitation modes of the vector (axial vector) mesons in discussing the low-energy properties of QCD. Especially as for the profiles of chiral solitons, the effects of heavier (axial) vector mesons are expected to be small with the small coupling constants between pions and heavier (axial) vector mesons, which is discussed in the end of this subsection. Now, the ρ meson field is introduced as $\rho_\mu \equiv B_\mu^{(1)}$, which couples with the lowest mode $\psi_1(z)$ in the fifth direction. Then the gauge field can be written as

$$A_\mu(x_\nu, z) = l_\mu(x_\nu)\psi_+(z) + r_\mu(x_\nu)\psi_-(z) + \rho_\mu(x_\nu)\psi_1(z), \tag{49}$$

and the five-dimensional field strength tensor can be also written with the $A_z = 0$ gauge as

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \\ &= (\partial_\mu l_\nu - \partial_\nu l_\mu)\psi_+ + (\partial_\mu r_\nu - \partial_\nu r_\mu)\psi_- + (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\psi_1 + i\{[l_\mu, l_\nu]\psi_+^2 + [r_\mu, r_\nu]\psi_-^2 + [\rho_\mu, \rho_\nu]\psi_1^2\} \\ &\quad + i\{([l_\mu, r_\nu] + [r_\mu, l_\nu])\psi_+\psi_- + ([l_\mu, \rho_\nu] + [\rho_\mu, l_\nu])\psi_+\psi_1 + ([r_\mu, \rho_\nu] + [\rho_\mu, r_\nu])\psi_-\psi_1\} \\ &= -i[\alpha_\mu, \alpha_\nu]\psi_+\psi_- + (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\psi_1 + i[\rho_\mu, \rho_\nu]\psi_1^2 + i\{([\alpha_\mu, \rho_\nu] + [\rho_\mu, \alpha_\nu])\hat{\psi}_0\psi_1 \\ &\quad + ([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])\psi_1\}, \end{aligned} \tag{50}$$

$$F_{z\mu} = \partial_z A_\mu = \alpha_\mu \partial_z \hat{\psi}_0 + \rho_\mu \partial_z \psi_1. \tag{51}$$

In the derivation of Eqs. (50) and (51), we have used the Maurer-Cartan equations, $\partial_\mu l_\nu - \partial_\nu l_\mu + i[l_\mu, l_\nu] = 0$ and $\partial_\mu r_\nu - \partial_\nu r_\mu + i[r_\mu, r_\nu] = 0$, and definitions of α_μ and β_μ in Eqs. (45) and (46).

The second term of the DBI action (20) can be diagonalized by using Eqs. (40) and (41) as

$$\begin{aligned} &\kappa \int d^4x dz \,\mathrm{tr}\{K(z) F_{\mu z} F_{\mu z}\} \\ &= \kappa \int d^4x dz K(z) \,\mathrm{tr}(\alpha_\mu \partial_z \hat{\psi}_0 + \rho_\mu \partial_z \psi_1)^2 \\ &= \int d^4x \left\{\frac{f_\pi^2}{4} \mathrm{tr}(L_\mu L_\mu) + m_\rho^2 \,\mathrm{tr}(\rho_\mu \rho_\mu)\right\}, \end{aligned} \tag{52}$$

$$L_\mu \equiv \frac{1}{i} U^\dagger \partial_\mu U, \tag{53}$$

where the relation $\alpha_\mu = \xi_- L_\mu \xi_-^{-1}$ is used. In Eq. (52), the pion decay constant f_π is introduced, comparing to the chiral Lagrangian, as

$$\frac{f_\pi^2}{4} \equiv \kappa \int dz K(z)(\partial_z \hat{\psi}_0)^2 = \frac{\kappa}{\pi}, \tag{54}$$

and the mass of ρ meson field $\rho_\mu \equiv B_\mu^{(1)}$ is presented from the orthogonal condition (40) as

$$m_\rho^2 \equiv m_1^2 = \lambda_1. \tag{55}$$

The first term of the DBI action (20) can also be written as

$$\begin{aligned} &\frac{\kappa}{2} \int d^4x dz \,\mathrm{tr}\{K(z)^{-1/3} F_{\mu\nu} F_{\mu\nu}\} \\ &= \frac{\kappa}{2} \int d^4x dz K(z)^{-1/3} \,\mathrm{tr}[-i[\alpha_\mu, \alpha_\nu]\psi_+\psi_- \\ &\quad + (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\psi_1 + i[\rho_\mu, \rho_\nu]\psi_1^2 \\ &\quad + i\{([\alpha_\mu, \rho_\nu] + [\rho_\mu, \alpha_\nu])\hat{\psi}_0\psi_1 \\ &\quad + ([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])\psi_1\}]^2. \end{aligned} \tag{56}$$

After diagonalizing (56) by using Eqs. (37) and (38), we can finally get the effective action of pion and ρ meson in flat four-dimensional Euclidean space-time from Eqs. (52) and (56) as follows:

$$S_{\rm D8}^{\rm DBI} - S_{\rm D8}^{\rm DBI}|_{A_M \to 0} = \kappa \int d^4x dz \,\mathrm{tr}\left\{\frac{1}{2} K(z)^{-1/3} F_{\mu\nu} F_{\mu\nu} + K(z) F_{\mu z} F_{\mu z}\right\} \tag{57}$$

$$= \frac{f_\pi^2}{4} \int d^4x \,\mathrm{tr}(L_\mu L_\mu) \qquad \text{(chiral term)} \tag{58}$$

$$+ m_\rho^2 \int d^4x \, \mathrm{tr}(\rho_\mu \rho_\mu) \qquad (\rho\text{-mass term}) \tag{59}$$

$$+ \frac{1}{2} \frac{1}{16e^2} [(-i)^2] \int d^4x \, \mathrm{tr}[L_\mu, L_\nu]^2 \qquad (\text{Skyrme term}) \tag{60}$$

$$+ \frac{1}{2} \int d^4x \, \mathrm{tr}(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)^2 \qquad (\rho\text{-kinetic term}) \tag{61}$$

$$+ \frac{1}{2} 2g_{3\rho}[i] \int d^4x \, \mathrm{tr}\{(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)[\rho_\mu, \rho_\nu]\} \quad (3\rho \text{ coupling}) \tag{62}$$

$$+ \frac{1}{2} g_{4\rho}[i^2] \int d^4x \, \mathrm{tr}[\rho_\mu, \rho_\nu]^2 \qquad (4\rho \text{ coupling}) \tag{63}$$

$$- ig_1 \int d^4x \, \mathrm{tr}\{[\alpha_\mu, \alpha_\nu](\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)\} \quad (\partial\rho\text{-}2\alpha \text{ coupling}) \tag{64}$$

$$+ g_2 \int d^4x \, \mathrm{tr}\{[\alpha_\mu, \alpha_\nu][\rho_\mu, \rho_\nu]\} \qquad (2\rho\text{-}2\alpha \text{ coupling}) \tag{65}$$

$$+ g_3 \int d^4x \, \mathrm{tr}\{[\alpha_\mu, \alpha_\nu]([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])\} \quad (\rho\text{-}2\alpha\text{-}\beta \text{ coupling}) \tag{66}$$

$$+ ig_4 \int d^4x \, \mathrm{tr}\{(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])\} \quad (\rho\text{-}\partial\rho\text{-}\beta \text{ coupling}) \tag{67}$$

$$- g_5 \int d^4x \, \mathrm{tr}\{[\rho_\mu, \rho_\nu]([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])\} \quad (3\rho\text{-}\beta \text{ coupling}) \tag{68}$$

$$- \frac{1}{2} g_6 \int d^4x \, \mathrm{tr}([\alpha_\mu, \rho_\nu] + [\rho_\mu, \alpha_\nu])^2 \qquad (2\rho\text{-}2\alpha \text{ coupling}) \tag{69}$$

$$- \frac{1}{2} g_7 \int d^4x \, \mathrm{tr}([\beta_\mu, \rho_\nu] + [\rho_\mu, \beta_\nu])^2 \qquad (2\rho\text{-}2\beta \text{ coupling}). \tag{70}$$

In the term (60), the Skyrme parameter e is introduced, comparing to the standard Skyrme model [8], as

$$\frac{1}{16e^2} \equiv \kappa \int dz K(z)^{-1/3} \psi_+^2 (1 - \psi_+)^2. \tag{71}$$

It should be noted that the chiral term and the Skyrme term naturally appear in terms (58) and (60) from the holographic approach. The appearance of the chiral term is reasonable and less surprising, because only the chiral term is allowed as the two-derivative term of pion fields due to chiral symmetry and Lorentz invariance. On the other hand, the unique appearance of the Skyrme term $\sim\mathrm{tr}[L_\mu, L_\nu]^2$ as the four-derivative term of pion fields in the holographic approach is fairly remarkable. Actually, at the leading order of $1/N_c$ and $1/\lambda$ expansions, the holographic D4/D8/$\overline{\text{D8}}$ QCD never induces other four-derivative terms like $\mathrm{tr}\{L_\mu, L_\nu\}^2$ and $\mathrm{tr}(\partial_\mu L_\mu)^2$ and higher-derivative terms allowed by the chiral symmetry and Lorentz invariance, where these terms occasionally cause an instability of Skyrme solitons [16,17]. The reason of the unique appearance of the Skyrme term can be naturally understood from the starting-point of the DBI action (20), because this action of $O(F^2)$ includes only "two time-derivatives" at most, which gives a severe restriction on the possible terms in the effective meson theory. Indeed, other two candidates, $\mathrm{tr}\{L_\mu, L_\nu\}^2$ and $\mathrm{tr}(\partial_\mu L_\mu)^2$ include four time-derivatives and are consequently forbidden from the holographic point of view. Thus, the term with four derivatives of pion fields naturally and inevitably appears as the Skyrme term with two time-derivatives in the holographic framework as the leading order of $1/N_c$ and $1/\lambda$ expansions.

In the term (61), owing to Eq. (38), $\psi_1(z)$ is canonically normalized as

$$\kappa \int dz K(z)^{-1/3} \psi_1^2 = 1, \tag{72}$$

to give the proper kinetic term of the ρ meson field. The self-couplings of ρ-mesons, $g_{3\rho}$ and $g_{4\rho}$, are expressed with the basis ψ_1 as

$$g_{3\rho} \equiv \kappa \int dz K(z)^{-1/3} \psi_1^3, \tag{73}$$

$$g_{4\rho} \equiv \kappa \int dz K(z)^{-1/3} \psi_1^4. \tag{74}$$

Because of the κ-dependence of the basis $\psi_1 \propto \frac{1}{\sqrt{\kappa}}$ with the normalization condition (72), the ratio $g_{3\rho}^2/g_{4\rho}$ is independent of κ and uniquely determined in the holographic QCD as $g_{3\rho}^2/g_{4\rho} \simeq 0.90$, i.e., $g_{3\rho}^2 \neq g_{4\rho}$. This means that the ρ-meson part in the four-dimensional effective theory obtained from the holographic QCD slightly differs from the massive Yang-Mills theory, while most phenomenological approaches adopt the massive Yang-Mills-type interaction ($g_{3\rho}^2 = g_{4\rho}$) for the ρ-meson sector.

Other coupling constants g_{1-7} in terms (64)–(70) are defined with the basis $\psi_\pm$ (or $\hat{\psi}_0$) and ψ_1 as follows:

$$g_1 \equiv \kappa \int dz K(z)^{-1/3} \psi_1 \psi_+ \psi_-, \tag{75}$$

$$g_2 \equiv \kappa \int dz K(z)^{-1/3} \psi_1^2 (\tfrac{1}{4} - \hat{\psi}_0^2), \tag{76}$$

$$g_3 \equiv \kappa \int dz K(z)^{-1/3} \psi_1 \psi_+ \psi_- = g_1, \tag{77}$$

$$g_4 \equiv \kappa \int dz K(z)^{-1/3} \psi_1^2 = 1, \tag{78}$$

$$g_5 \equiv \kappa \int dz K(z)^{-1/3} \psi_1^3 = g_{3\rho}, \tag{79}$$

$$g_6 \equiv \kappa \int dz K(z)^{-1/3} \psi_1^2 \hat{\psi}_0^2 = \tfrac{1}{4} - g_2, \tag{80}$$

$$g_7 \equiv \kappa \int dz K(z)^{-1/3} \psi_1^2 = 1. \tag{81}$$

The holographic model has two parameters $\kappa = \frac{\lambda N_c}{108\pi^3}$ and Kaluza-Klein mass $M_{\rm KK}$ as the ultraviolet cutoff scale of the theory. κ appears in front of the effective action (20) because the effective action of D8 brane with D4 supergravity background expanded up to $O(F^2)$ corresponds to the leading order of $1/N_c$ and $1/\lambda$ expansions (see the end of Sec. II C). Therefore, by fixing two parameters, e.g., experimental inputs for f_π and m_ρ, then all the masses and the coupling constants are uniquely determined, which is a remarkable consequence of the holographic approach. In particular, the dimensionless coupling constants (73)–(81) are determined only by the dimensionless parameter κ. For instance, with the typical value of $\kappa \simeq 7.460 \times 10^{-3}$, which reproduces the experimental ratio of f_π and m_ρ [1], we numerically find $g_{3\rho} = g_5 \simeq 5.17$, $g_{4\rho} \simeq 29.7$, $g_1 = g_3 \simeq 0.0341$, and $g_2 = 1/4 - g_6 \simeq 0.180$, as well as the trivial relations $g_4 = g_7 = 1$.

Functions $\psi_\pm(z)$ and $\psi_n(z)$ in Eq. (32) correspond to the "wave functions" of pions and (axial) vector mesons in the curved fifth dimension. When the "oscillation" of the wave function in the fifth dimension increases, it can be realized as the larger mass of the (axial) vector mesons in the four-dimensional space-time, which is indicated by the relation $m_n^2 = \lambda_n$; the origin of the mass is the oscillation in the extra fifth direction. As for the pions, they also include slightly oscillating component $\psi_\pm$ in their wave functions. However the fifth coordinate is curved by D4 supergravity background, and $\psi_\pm$ actually appear as the zero-modes in (37) with the curved measure. In this sense, pions are the "geodesic" of the curved five-dimensional space-time and they can be realized as the massless objects.

Concerning the interaction terms with more than three bodies of pions and (axial) vector mesons, the wave functions of the heavier (axial) vector mesons have smaller overlap with those of pions in the fifth direction because of its large oscillation. Such smaller overlapping of wave functions in the extra fifth direction between pions and "largely oscillating" heavier (axial) vector mesons predicts the smaller coupling constants after the projection of the action into the flat four-dimensional space-time [15], which is numerically checked by the z-integral in the coupling constants. Actually, some recent experiments indicate that the heavier (axial) vector mesons tend to have smaller width for the decay into pions [18], which is consistent with the prediction of holographic QCD as their smaller coupling constants mentioned above. Therefore, for the study of chiral solitons as the large-amplitude pion fields, we can expect smaller effects from heavier (axial) vector mesons, so that we consider only pions and ρ mesons as the lowest massive mode ($\rho_\mu \equiv B_\mu^{(1)}$) along the later discussions of chiral solitons in the holographic QCD.

III. BRANE-INDUCED SKYRMIONS

According to the large-N_c gauge theory discussed by 'tHooft [6], baryons do not directly appear as the dynamical degrees of freedom in the large-N_c limit. In this viewpoint, there should also occur the problem of how to describe baryons in the large-N_c holographic models of QCD. In this section we describe a baryon as a solitonlike topological object in the four-dimensional meson effective action (58)–(70) induced by the holographic QCD, which we now call the brane-induced Skyrmion, comparing with the standard Skyrmion [8] based on the nonlinear sigma model as its meson effective action. Several properties of the brane-induced Skyrmions like meson field configurations, total energy, energy density, and also the size in the coordinate space are discussed by showing the numerical results.

A. Standard Skyrmions and brane-induced Skyrmions

In 1961, Skyrme proposed the idea to describe nucleons as mesonic solitons in a nonlinear meson field theory. In this picture, a baryon is identified as a "chiral soliton" of the Nambu-Goldstone field, which was called "Skyrmion," corresponding to the nontrivial mapping from the three-dimensional coordinate space to the group manifold $[\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R]/\mathrm{SU}(2)_V \simeq \mathrm{SU}(2)_A$, in accordance with spontaneous chiral-symmetry breaking. This concept of chiral soliton provides an interesting topological picture for baryons, describing many properties of baryons with large phenomenological successes [19].

In terms of QCD, which was proposed as the fundamental theory of the strong interaction by Nambu [20] in 1966 and Gross, Wilczek [21], and Politzer [22] in 1973, the Skyrme model had several problems; justification for regarding the winding number of Skyrmion as the physical baryon number, the ambiguity for the effective action of mesons, and the unclear connection of it with QCD because the Skyrme model has actually no quarks and gluons.

In the 1970's, the analysis for large-N_c QCD gives the revival of the Skyrme model. In 1974, 't Hooft prevailed that, assuming confinement, large-N_c gauge theory becomes equivalent to the weak coupling system of mesons (and glueballs) with the coupling strength $g \sim 1/N_c$ [6]. However, this survey gives the problem of how baryons should appear in large-N_c gauge theory. In fact, in 1979, Witten showed that in large-N_c limit the mass of a baryon gets increase and proportional to N_c [7], which explains the reason why baryons do not directly appear as the dynamical degrees of freedom in large-N_c limit. Witten also remarked that, in large-N_c limit, the mass of a baryon becomes proportional to the inverse of coupling strength of mesons; $M_{\rm baryon} \propto 1/g$ [7]. As for most of the topological solitons in scalar field theories, their classical mass is proportional to the inverse of the coupling strength of the scalar fields in the action [23], which means a soliton is a nonperturbative phenomenon on scalar fields. These considerations about the large-N_c gauge theory and general property of soliton waves revived Skyrme's identification of baryons as mesonic solitons, and the Skyrme model was expected to be a nonperturbative (low-energy) effective theory of QCD.

The action of the Skyrme model is originally based on the concept of nonlinear sigma model, which is the low-energy effective theory of mesons reflecting spontaneous chiral-symmetry breaking $SU(2)_L \times SU(2)_R \to SU(2)_V$ in QCD. In this model, higher mass excitations (ρ, ω, a_1, etc.) are neglected and only three physical pions, which are light pseudoscalar Nambu-Goldstone bosons with respect to the chiral symmetry breaking, are included as the three independent local parameters of the coset space $[SU(2)_L \times SU(2)_R]/SU(2)_V$. If these pion fields appear without derivatives in the effective action, it gives the explicit breaking of the global chiral symmetry, because constant pion fields correspond to the parameters of global chiral transformation. Therefore, pions as the local parameters of the coset space should appear accompanying with derivative in the action, giving low-energy expansion (or derivative expansion) for soft pions as the nonlinear realization [14]. The leading term respecting chiral symmetry is uniquely defined containing two derivatives of pion field $U(x_\mu)$ as

$$\mathcal{L}_1 = \frac{f_\pi^2}{4}\,\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U), \qquad (U(x_\mu) \equiv e^{2i\pi(x_\mu)/f_\pi}) \tag{82}$$

which is called the chiral term.

According to Derrick's theorem, stable soliton solutions cannot exist only with the chiral term (82); other terms with four or more derivatives of the pion fields are at least needed for the existence of stable soliton solutions in the scalar field theory in the three-dimensional coordinate space [23]. Skyrme added a term with four derivatives as

$$\mathcal{L}_2 = \frac{1}{32e^2}\,\mathrm{tr}[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2, \tag{83}$$

which is called the Skyrme term. Skyrme recognized the appearance of this term as the kinetic term of 1-form of chiral field; $L_\mu = \frac{1}{i}U^\dagger\partial_\mu U$, which can be seen by using the Maurer-Cartan equation $\partial_\mu L_\nu - \partial_\nu L_\mu + i[L_\mu, L_\nu] = 0$ for the Skyrme term (83) as

$$\mathcal{L}_2 \propto \tfrac{1}{2}\mathrm{tr}(\partial_\mu L_\nu - \partial_\nu L_\mu)^2. \tag{84}$$

In this viewpoint, the chiral term (82) can be regarded as the mass term of L_μ [19]. Since there exist two other candidates for four-derivative terms [17], the effective action up to four derivatives is not uniquely determined.

Derrick's theorem also suggests that stable soliton solutions in the scalar field theory may exist by including other degrees of freedom like vector and/or spinor fields. Furthermore, large N_c QCD reduces to the weak coupling system including not only pions but also other mesons like ρ, ω, and a_1. With these considerations, the relations of stable Skyrmions with vector and axial vector mesons are discussed in some context [24,25]. As for the effective action between pions and (axial) vector mesons, a hidden local symmetry approach was suggested as a phenomenological treatment for the low-energy meson dynamics [3]. In this approach, the existence of $SU(2)_V$ local symmetry is assumed in the chiral Lagrangian, and ρ mesons are introduced as the gauge boson for this hidden local symmetry. The kinetic term of ρ mesons is also assumed to be dynamically generated by some quantum effects in QCD as the dynamical pole generations. To discuss the effect of vector mesons for the Skyrme solitons, Igarashi *et al.* [24] included ρ mesons as the dynamical gauge boson of hidden local symmetry, without adding four-derivative terms like the Skyrme term, and showed that the kinetic term of ρ mesons becomes the Skyrme term in some parameter limit. However there are large varieties for the hidden local symmetric terms, and action is not still uniquely determined in that framework.

Furthermore, there actually exist infinite excitation modes of vector and axial vector mesons. The consistent framework with pions and these large varieties of (axial) vector mesons was not sufficiently developed, and inclusive discussions about the effects of these mesons for the Skyrme soliton properties were not achieved at all at that time.

Now, following the D4/D8/$\overline{\mathrm{D8}}$ holographic QCD discussed in the previous section, we can uniquely get the four-dimensional meson effective action with the coupling terms of pions and (axial) vector mesons starting from QCD. In this framework, massless QCD is constructed from the multi D-brane configurations of D4/D8/$\overline{\mathrm{D8}}$ system, and classical supergravity description of the D4 brane with D8/$\overline{\mathrm{D8}}$ probes eventually gives meson effective action as the low-energy effective theory of QCD. In this effective action, all the coupling constants are uniquely determined just by two experimental parameters like f_π and m_ρ, and the four-derivative term of the chiral field $U(x_\mu)$ naturally appear as Skyrme's type [8].

As for the (axial) vector mesons in the holographic QCD, the hidden local symmetry, which is embedded in the framework of Bando *et al.* [3], naturally appears as a part of the $\mathrm{U}(N_f)$ local gauge symmetry on the D8 brane with D4 supergravity background, also including the kinetic term of the gauge field. In analogy with the "open moose model" discussed by Son *et al.* [15], there appear infinite excitation modes of massive (axial) vector mesons in the holographic QCD, through the mode expansion of the gauge field with respect to the parity eigenstates. Because of this mode expansion of the gauge field, (axial) vector mesons do not directly correspond to the dynamical gauge boson of the hidden local symmetry, but they appear as some homogeneous component of it like Eq. (43).

In this paper we discuss the effect of ρ mesons for the Skyrme soliton properties by using the effective action (58)–(70), and the contributions from the higher mass excitation modes of (axial) vector mesons are neglected. The reliability of this strategy is supported by the smaller coupling constant between pions and heavier (axial) vector mesons, given by the oscillation of the meson wave functions in the extra fifth direction (see Sec. II E); if we assume that the main part of the Skyrme soliton is still constructed by the large-amplitude pion fields, these smaller couplings may suggest the smaller effects of heavier (axial) vector mesons for the Skyrme soliton profiles. Although the discussion is restricted for pions and ρ mesons, the holographic QCD intrinsically includes infinite excitation modes of (axial) vector mesons by the existence of the continuous fifth dimension. Therefore, ρ meson does not, in a strict sense, correspond to the dynamical gauge boson of hidden local symmetry on the D8 brane as discussed above.

B. Hedgehog Ansatz for pion and ρ meson configurations

The meson effective action (58)–(70) induced by the holographic QCD has $\mathbf{R}^3$ physical coordinate space and also the group manifold $\mathrm{SU}(2)_A$ as its internal isospin space. In order that the action (58)–(70) is to be finite, chiral field $U(\mathbf{x})$ and ρ meson field $\rho_\mu(\mathbf{x})$ in the action should satisfy the boundary conditions as

$$U(\mathbf{x}) \to U_0, \qquad \rho_\mu(\mathbf{x}) \to 0, \qquad (\text{for } |\mathbf{x}| \to \infty) \tag{85}$$

where we now conventionally take the constant matrix U_0 as a unit matrix; $U_0 = \mathbf{1}$. The action is equal to the energy in the Euclidean metric, and so a configuration $U(\mathbf{x}) = \mathbf{1}$ and $\rho_\mu(\mathbf{x}) = 0$ corresponds to a classical vacuum with zero energy. Therefore the boundary condition (85) means that pion and ρ meson fields have the same classical vacuum for any direction at $|\mathbf{x}| \to \infty$; directional variables in the coordinate space $\mathbf{R}^3$ become redundant at infinity. This means that $\mathbf{R}^3$ can be compactified at infinity into the closed manifold S^3 with infinite radius. Now, according to the homotopical classification $\pi_3(\mathrm{SU}(2)_A) = \mathbf{Z}$ [26], the action (58)–(70) has the Skyrme soliton solution as nontrivial mapping from the compactified physical space S^3 to internal $\mathrm{SU}(2)_A$ manifold, which is the same mechanism to the standard Skyrme model [8].

Now we take the hedgehog Ansatz [8] for the chiral field in the Skyrmions as

$$U^\star(\mathbf{x}) = e^{i\tau_a \hat{x}_a F(r)}, \qquad \left(\hat{x}_a \equiv \frac{x_a}{r},\, r \equiv |\mathbf{x}|\right) \tag{86}$$

where τ_a is Pauli matrix, and $F(r)$ is a dimensionless profile function with boundary conditions $F(0) = \pi$ and $F(\infty) = 0$, giving topological charge equal to unity. Ansatz (86) means $2\pi_a(\mathbf{x}) = \hat{x}_a F(r)$ for the pion field. This configuration is called "maximally symmetric" in the sense that it is invariant under a combination of space and isospace rotations [19].

Here we also take the hedgehog configuration Ansatz for ρ meson field as

$$\rho_0^\star(\mathbf{x}) = 0, \qquad \rho_i^\star(\mathbf{x}) = \rho_{ia}^\star(\mathbf{x})\frac{\tau_a}{2} = \{\varepsilon_{iab}\hat{x}_b \tilde{G}(r)\}\tau_a, \qquad (\tilde{G}(r) \equiv G(r)/r) \tag{87}$$

where $G(r)$ is a dimensionless profile function. This Ansatz for ρ meson field is also called "Wu-Yang-'tHooft-Polyakov Ansatz" [24], and the same configuration Ansatz can be seen in the context of the gauge field in the 'tHooft-Polyakov monopole [23].

ρ meson field is sometimes described by two pion configurations as $\rho_{ia} \sim [\boldsymbol{\pi} \times \partial_i \boldsymbol{\pi}]_a$, for example, in the hidden local symmetry approach [3]. If we apply the hedgehog Ansatz (86) for this two pion configurations, the ρ meson field can be written as

$$\rho_{ia} \sim [\boldsymbol{\pi} \times \partial_i \boldsymbol{\pi}]_a = \varepsilon_{abc}\pi_b \partial_i \pi_c = \varepsilon_{iab}\hat{x}_b (F^2/r), \tag{88}$$

which should be compared with Eq. (87). This comparison means that the Ansatz for pion and ρ meson fields in Eqs. (86) and (87) are consistent at least for small amplitude, from the viewpoint of hidden local symmetry approach. These solution Ansatz as in Eqs. (86) and (87) are often taken in the discussion of soliton waves [23]; pion and ρ meson degrees of freedom are now represented by the radial-dependent functions $F(r)$ and $\tilde{G}(r)$ respectively, and the effective action is reduced into the one-dimensional space and becomes solvable.

With the hedgehog Ansatz (86) for pion field, L_μ, α_μ and β_μ appearing in the action (58)–(70) can be written by using $\hat{\delta}_{ia} \equiv \delta_{ia} - \hat{x}_i\hat{x}_a$ and $F'(r) \equiv dF/dr$ as follows:

$$L_0^\star(\mathbf{x}) = 0, \tag{89}$$

$$L_i^\star(\mathbf{x}) = \left(\hat{x}_i\hat{x}_a F' + \hat{\delta}_{ia}\frac{\sin F\cdot\cos F}{r} + \varepsilon_{iab}\hat{x}_b\frac{\sin^2 F}{r}\right)\tau_a, \tag{90}$$

$$\alpha_0^\star(\mathbf{x}) = 0, \qquad \alpha_i^\star(\mathbf{x}) = \left(\hat{x}_i\hat{x}_a F' + \hat{\delta}_{ia}\frac{\sin F}{r}\right)\tau_a, \tag{91}$$

$$\beta_0^\star(\mathbf{x}) = 0, \qquad \beta_i^\star(\mathbf{x}) = \frac{1}{2}\varepsilon_{iab}\hat{x}_b\frac{1-\cos F}{r}\tau_a. \tag{92}$$

C. Energy and Euler-Lagrange equation for hedgehog soliton

Now, by substituting Eqs. (86), (87), and (89)–(92) into the action (58)–(70), we can get the energy of the brane-induced Skyrmion with the hedgehog configuration Ansatz as follows:

$$E[F(r),\tilde{G}(r)] \equiv [S_{\rm D8}^{\rm DBI} - S_{\rm D8}^{\rm DBI}|_{A_M\to 0}]_{\rm hedgehog} \equiv \int_0^\infty 4\pi dr r^2\varepsilon[F(r),\tilde{G}(r)]. \tag{93}$$

For the soliton solution, the energy E is to be minimized with the boundary conditions, $F(0)=\pi$ and $F(\infty)=0$. The energy density multiplied by the metric r^2 reads

$$\begin{aligned} r^2\varepsilon[F(r),\tilde{G}(r)] = {}& \frac{f_\pi^2}{4}[2(r^2F'^2 + 2\sin^2 F)] \qquad (\text{chiral term}) \\ &+ m_\rho^2[4r^2\tilde{G}^2] \qquad (\rho\text{-mass term}) \\ &+ \frac{1}{32e^2}\left[16\sin^2 F\left(2F'^2 + \frac{\sin^2 F}{r^2}\right)\right] \qquad (\text{Skyrme term}) \\ &+ \frac{1}{2}[8\{3\tilde{G}^2 + 2r\tilde{G}(\tilde{G}') + r^2\tilde{G}'^2\}] \qquad (\rho\text{-kinetic term}) \\ &- g_{3\rho}[16r\tilde{G}^3] \qquad (3\rho \text{ coupling}) \\ &+ \frac{1}{2}g_{4\rho}[16r^2\tilde{G}^4] \qquad (4\rho \text{ coupling}) \\ &+ g_1[16\{F'\sin F\cdot(\tilde{G} + r\tilde{G}') + \sin^2 F\cdot\tilde{G}/r\}] \qquad (\partial\rho\text{-}2\alpha \text{ coupling}) \\ &- g_2[16\sin^2 F\cdot\tilde{G}^2] \qquad (2\rho\text{-}2\alpha \text{ coupling}) \\ &- g_3[16\sin^2 F\cdot(1-\cos F)\tilde{G}/r] \qquad (\rho\text{-}2\alpha\text{-}\beta \text{ coupling}) \\ &- g_4[16(1-\cos F)\tilde{G}^2] \qquad (\rho\text{-}\partial\rho\text{-}\beta \text{ coupling}) \\ &+ g_5[16r(1-\cos F)\tilde{G}^3] \qquad (3\rho\text{-}\beta \text{ coupling}) \\ &+ g_6[16r^2F'^2\tilde{G}^2] \qquad (2\rho\text{-}2\alpha \text{ coupling}) \\ &+ g_7[8(1-\cos F)^2\tilde{G}^2] \qquad (2\rho\text{-}2\beta \text{ coupling}). \end{aligned} \tag{94}$$

Then, following the same procedure as that of Adkins, Nappi, and Witten [27], we introduce the energy unit $E_{\rm ANW} \equiv \frac{f_\pi}{2e}$ and length unit $r_{\rm ANW} \equiv \frac{1}{ef_\pi}$, and rewrite variables in this "Adkins-Nappi-Witten (ANW) unit" as $\bar{E} \equiv \frac{1}{E_{\rm ANW}}E = \frac{2e}{f_\pi}E$ and $\bar{r} \equiv \frac{1}{r_{\rm ANW}}r = ef_\pi r$. With this scaled unit, the energy density can be rewritten as follows (overlines of $\bar{E}$ and $\bar{r}$ below are abbreviated for simplicity);

$$
\begin{aligned}
r^2\varepsilon[F(r),\tilde{G}(r)] = {} & (r^2F'^2 + 2\sin^2 F) \qquad \text{(chiral term)} \\
& + 2\left(\frac{m_\rho}{f_\pi}\right)^2[4r^2\tilde{G}^2] \qquad (\rho\text{-mass term}) \\
& + \sin^2 F\left(2F'^2 + \frac{\sin^2 F}{r^2}\right) \qquad \text{(Skyrme term)} \\
& + (2e^2)\frac{1}{2}[8\{3\tilde{G}^2 + 2r\tilde{G}(\tilde{G}') + r^2\tilde{G}'^2\}] \qquad (\rho\text{-kinetic term}) \\
& - (2e^2)g_{3\rho}[16r\tilde{G}^3] \qquad (3\rho \text{ coupling}) \\
& + (2e^2)\frac{1}{2}g_{4\rho}[16r^2\tilde{G}^4] \qquad (4\rho \text{ coupling}) \\
& + (2e^2)g_1[16\{F'\sin F\cdot(\tilde{G} + r\tilde{G}') + \sin^2 F\cdot\tilde{G}/r\}] \qquad (\partial\rho\text{-}2\alpha \text{ coupling}) \\
& - (2e^2)g_2[16\sin^2 F\cdot\tilde{G}^2] \qquad (2\rho\text{-}2\alpha \text{ coupling}) \\
& - (2e^2)g_3[16\sin^2 F\cdot(1-\cos F)\tilde{G}/r] \qquad (\rho\text{-}2\alpha\text{-}\beta \text{ coupling}) \\
& - (2e^2)g_4[16(1-\cos F)\tilde{G}^2] \qquad (\rho\text{-}\partial\rho\text{-}\beta \text{ coupling}) \\
& + (2e^2)g_5[16r(1-\cos F)\tilde{G}^3] \qquad (3\rho\text{-}\beta \text{ coupling}) \\
& + (2e^2)g_6[16r^2F'^2\tilde{G}^2] \qquad (2\rho\text{-}2\alpha \text{ coupling}) \\
& + (2e^2)g_7[8(1-\cos F)^2\tilde{G}^2] \qquad (2\rho\text{-}2\beta \text{ coupling}). \qquad (95)
\end{aligned}
$$

Here we comment about the scaling property of brane-induced Skyrmions. The holographic QCD with D4/D8/$\overline{\text{D8}}$ system has just two parameters like κ and M_{KK}. Therefore, using Eqs. (54), (55), and (71), all the parameters like the pion decay constant f_π, the ρ meson mass m_ρ and the Skyrme parameter e can be written by the two parameters κ and M_{KK} in the holographic framework as follows:

$$f_\pi = 2\sqrt{\frac{\kappa}{\pi}}M_{\text{KK}}, \qquad (96)$$

$$m_\rho = \sqrt{\lambda_1}M_{\text{KK}} \simeq \sqrt{0.67}M_{\text{KK}}, \qquad (97)$$

$$e = \frac{1}{4}\left[\kappa\int dz K^{-1/3}\psi_+^2(1-\psi_+)^2\right]^{-1/2} \simeq \frac{1}{4\sqrt{0.157}}\frac{1}{\sqrt{\kappa}}, \qquad (98)$$

where the energy unit M_{KK} is recovered. With these relations (96)–(98), ANW units E_{ANW} and r_{ANW} can be written in the holographic framework by the two parameters κ and M_{KK} as

$$E_{\text{ANW}} = \frac{f_\pi}{2e} = \text{const}\cdot\kappa M_{\text{KK}}, \qquad (99)$$

$$r_{\text{ANW}} = \frac{1}{ef_\pi} = \text{const}\cdot\frac{1}{M_{\text{KK}}}. \qquad (100)$$

The parameter $\kappa = \frac{\lambda N_c}{108\pi^3}$ originally appears only in front of the effective action (20) as an overall factor of the theory because the effective action corresponds to the leading order term of the expansion about $1/N_c$ and $1/\lambda$ (see the end of Sec. II C). M_{KK} is also the sole energy scale as the ultraviolet cutoff scale of this holographic model. Therefore, by taking the energy unit E_{ANW} ($\propto \kappa M_{\text{KK}}$) as in Eq. (99), the total energy appears as a scale invariant variable. In fact, by introducing the rescaled ρ meson field $\hat{G}(r)$ as

$$\hat{G}(r) \equiv \frac{1}{\sqrt{\kappa}}\tilde{G}(r), \qquad (101)$$

and taking into account the κ-dependence of the basis ψ_1 as $\psi_1 \propto \frac{1}{\sqrt{\kappa}}$ indicated by the normalization condition (38), one can show that every energy density in each term of Eq. (95) and meson field configurations $F(r)$ and $\hat{G}(r)$ are scale invariant variables, being independent of the holographic two parameters, κ and M_{KK}.

Such scaling property of the brane-induced Skyrmion can hold even by introducing other (axial) vector meson degrees of freedom in this holographic framework like $a_1, \rho', a_1', \cdots$ with masses $m_{a_1}, m_{\rho'}, m_{a_1'}, \cdots$, appearing in the mode expansion of the gauge field (32), because the holographic QCD is still expressed just by two parameters, κ and M_{KK}. Therefore, all the effects of physical parameters like $e, f_\pi, m_\rho, m_{a_1}, m_{\rho'}, m_{a_1'}, \cdots$ (or, κ and M_{KK}) can be fully extracted in the units E_{ANW} and r_{ANW}, which is a remarkable consequence of the holographic approach. Such a scaling property can also be realized in the standard Skyrme model composed only by pion fields. However this scaling property cannot be seen in the traditional phenomenological treatment of Skyrmions with (axial) vector mesons introduced as the external degrees of freedom.

Euler-Lagrange equations for the pion field $F(r)$ and ρ meson field $\tilde{G}(r)$ can also be derived from the functional derivative of the energy E with the energy density (95) as follows (note that, in the ANW unit, E and r are dimensionless):

$$\frac{1}{4\pi}\left\{\frac{\delta E}{\delta F(r)} - \frac{d}{dr}\left(\frac{\delta E}{\delta F'(r)}\right)\right\} = (-4rF' - 2r^2F'' + 4\sin F\cdot\cos F) + (-4\sin F\cdot\cos F\cdot F'^2 - 4\sin^2F\cdot F'' + 4\sin^3F\cdot\cos F\cdot 1/r^2) + (2e^2)g_1[16\{2\sin F\cdot\cos F\cdot\tilde{G}/r - \sin F\cdot(2\tilde{G}' + r\tilde{G}'')\}] - (2e^2)g_2[16(2\sin F\cdot\cos F\cdot\tilde{G}^2)] - (2e^2)g_3[16(\sin F + 2\sin F\cdot\cos F - 3\sin F\cdot\cos^2F)\tilde{G}/r] - (2e^2)g_4[16(\sin F\cdot\tilde{G}^2)] + (2e^2)g_5[16(r\sin F\cdot\tilde{G}^3)] + (2e^2)g_6[16(-4rF'\tilde{G}^2 - 2r^2F''\tilde{G}^2 - 4r^2F'\tilde{G}\tilde{G}')] + (2e^2)g_7[8\{2(1-\cos F)\sin F\cdot\tilde{G}^2\}] = 0, \tag{102}$$

$$\frac{1}{4\pi}\left\{\frac{\delta E}{\delta\tilde{G}(r)} - \frac{d}{dr}\left(\frac{\delta E}{\delta\tilde{G}'(r)}\right)\right\} = 2\left(\frac{m_\rho}{f_\pi}\right)^2[4(2r^2\tilde{G})] + (2e^2)\frac{1}{2}[8(4\tilde{G} - 4r\tilde{G}' - 2r^2\tilde{G}'')] - (2e^2)g_{3\rho}[16(3r\tilde{G}^2)] + (2e^2)\frac{1}{2}g_{4\rho}[16(4r^2\tilde{G}^3)] + (2e^2)g_1[16(\sin^2F/r - \cos F\cdot F'^2r - \sin F\cdot F''r)] - (2e^2)g_2[16(2\sin^2F\cdot\tilde{G})] - (2e^2)g_3[16\sin^2F(1-\cos F)/r] - (2e^2)g_4[16\{2(1-\cos F)\tilde{G}\}] + (2e^2)g_5[16\{3r(1-\cos F)\tilde{G}^2\}] + (2e^2)g_6[16\{2r^2F'^2\tilde{G}\}] + (2e^2)g_7[8\{2(1-\cos F)^2\tilde{G}\}] = 0. \tag{103}$$

In $\tilde{G}(r)\to 0$ limit, Euler-Lagrange equation (102) with the variation of the pion field coincides with that of the standard Skyrme model without ρ mesons:

$$\frac{1}{4\pi}\left\{\frac{\delta E}{\delta F(r)} - \frac{d}{dr}\left(\frac{\delta E}{\delta F'(r)}\right)\right\}_{\tilde{G}(r)\to 0} = 2(-2rF' - r^2F'' + 2\sin F\cdot\cos F) - 4\left(\sin F\cdot\cos F\cdot F'^2 + \sin^2F\cdot F'' - \frac{\sin^3F\cdot\cos F}{r^2}\right) = 0. \tag{104}$$

However even in the $\tilde{G}(r)\to 0$ limit, the Euler-Lagrange equation (103) with the variation of the ρ meson field still gives a constraint for pion field $F(r)$ as follows:

$$\frac{1}{4\pi}\left\{\frac{\delta E}{\delta\tilde{G}(r)} - \frac{d}{dr}\left(\frac{\delta E}{\delta\tilde{G}'(r)}\right)\right\}_{\tilde{G}(r)\to 0} = (2e^2)g_1[16(\sin^2F/r - \cos F\cdot F'^2r - \sin F\cdot F''r)] - (2e^2)g_3[16\sin^2F(1-\cos F)/r] = 0, \tag{105}$$

which is not included in (104); Eqs. (104) and (105) give over-conditions for pion field configurations. This means that standard Skyrme configuration without ρ mesons does not correspond to the (local maximum, local minimum, or saddle-point) solution of Euler-Lagrange equation (102) and (103) of the brane-induced Skyrme model. In this sense, the brane-induced Skyrme model has explicit instability into the nontrivial configuration of ρ meson field ($\rho \neq 0$). This instability is given by the correlation processes including one ρ meson, i.e., $\partial\rho$-2α and ρ-2α-β coupling terms as in Eq. (105), affecting the dynamics of pions even at the small amplitude limit: $\tilde{G}(r)\to 0$.

D. Numerical results

Now we show the numerical results on the profiles of brane-induced Skyrmions appearing in the meson effective action (95) induced by the holographic QCD. First we present the numerical results in ANW unit, where meson field configurations $F(r)$ and $\hat{G}(r)$, total energy, energy densities, and root-mean-square radius of brane-induced Skyrmions are found as the scale invariant variables, being independent of physical parameters e, f_π, and m_ρ (or, κ and $M_{\rm KK}$); all the effects of the physical parameters are fully extracted in the units $E_{\rm ANW}$ and $r_{\rm ANW}$ as previously discussed. The recovering of the physical unit is discussed in the last part of this subsection.

By numerically solving the Euler-Lagrange equations (102) and (103), we find that the stable soliton solution exists as brane-induced Skyrmion with the chiral profile $F(r)$ and rescaled ρ-meson profile $\hat{G}(r)$ as in Fig. 4, which should be a correspondent of a baryon in the large-N_c holographic QCD. To see the effect of ρ meson fields for a baryon, the standard Skyrme configuration

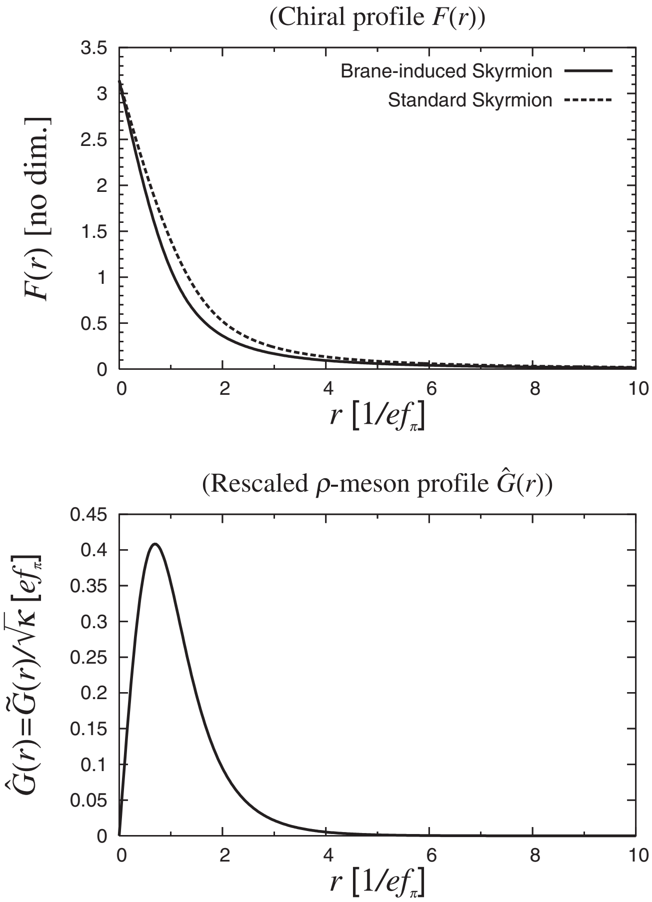

FIG. 4. Chiral profile $F(r)$ and rescaled ρ-meson profile $\hat{G}(r)$ of the brane-induced Skyrmion as the hedgehog soliton solution in the holographic QCD. The dashed profile in the upper figure shows the chiral profile of the standard Skyrmion without ρ mesons.

without ρ mesons is also shown for comparison in Fig. 4. Through the interactions between pions and ρ mesons in the holographic QCD, pion contribution seems to be slightly replaced by ρ meson degrees of freedom, which results in the shrinkage of $F(r)$ from the standard Skyrme configuration as in Fig. 4.

The energy contributions of all the terms in the action (95) of brane-induced Skyrmion are presented in Fig. 5. Those of chiral and Skyrme terms in the standard Skyrme model are also shown for comparison in this figure. The energy contribution of the chiral term decreases with the shrinkage of pion field $F(r)$ because a trivial solution $F(r) = 0$ is most energetically favored only with the chiral term from the Derrick's theorem [23]. On the other hand, the energy contribution of the Skyrme term with four derivatives increases with the shrinkage of $F(r)$, and gives the slight energy enhancement for the sum of chiral and Skyrme terms. This is natural because the Skyrme configuration denoted by the dashed profile in Fig. 4 is most stabilized in the standard Skyrme model without ρ mesons [8]. In the brane-induced Skyrme model, there exist the interactions between pion and ρ mesons, giving the total stability of brane-induced Skyrmion with nontrivial $\rho \neq 0$ configuration.

Now by solving the Euler-Lagrange equations (102) and (103), we find the energy of brane-induced Skyrmion in ANW unit as

$$E \simeq 1.115 \times 12\pi^2\left[\frac{f_\pi}{2e}\right], \qquad (106)$$

which is compared with that of the standard Skyrmion without ρ mesons; $E \simeq 1.231 \times 12\pi^2[\frac{f_\pi}{2e}]$, given by Adkins *et al.* [27]. The total energy of Skyrmion in ANW unit is often written as in Eq. (106) by the ratio relative to $12\pi^2$, which is called the Bogomol'nyi-Prasad-

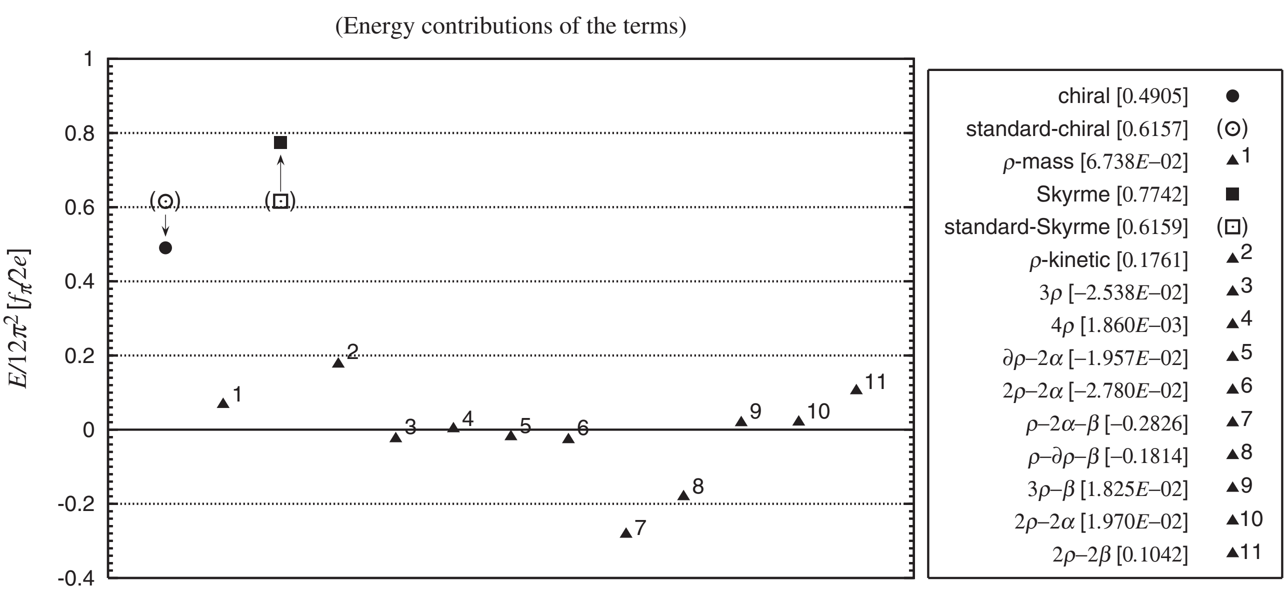

FIG. 5. Energy contributions of all the terms in the action (95) of brane-induced Skyrmion. Those of the standard Skyrme model with the chiral and Skyrme terms are also shown for comparison.

Sommerfield (BPS) saturation energy originated from the unit topological charge of a Skyrmion (see the appendix). The result (106) suggests that the total energy of brane-induced Skyrmion is reduced by ~10% relative to the standard Skyrmion because of the interactions between pions and ρ mesons in the meson effective action (58)–(70) induced by the holographic QCD.

The total energy of the standard Skyrmion is known to be slightly enhanced by ~23% relative to the BPS saturation energy ($12\pi^2$) [27]. In this sense, the standard Skyrmion is called to have no BPS saturation. The origin of this energy enhancement and non-BPS nature of the Skyrmion was discussed from the viewpoint of nonlinear elasticity theory by Manton *et al.* [28]. In his survey, the energy of a Skyrmion is regarded as a geometrical distortion energy of the physical manifold $S^3_{(\infty)}$ with infinite radius wrapped on the other internal manifold $SU(2)_A \simeq S^3_{(\langle\sigma\rangle=1)}$ with unit radius, comparing to the "strain energy" of one deformed material winded on the other material. The two closed manifolds, $S^3_{(\infty)}$ and $SU(2)_A \simeq S^3_{(\langle\sigma\rangle=1)}$ of a Skyrmion have different radius, i.e., different curvatures with each other. Thus the difference of the curvatures on these two manifolds is thought to give the enhancement of the total energy of a Skyrmion relative to only winding energy, i.e., BPS saturation energy (see the appendix). From these viewpoints, the result (106) may suggest that the metrical distortion energy of a Skyrmion is slightly relieved by the interactions with other degrees of freedom as ρ mesons induced by the holographic QCD.

It should also be noted that, even in the $m_\rho \to \infty$ limit, the energy of the brane-induced Skyrmion in Eq. (106) is unchanged, i.e., it does not coincide with that of the standard Skyrmion without ρ mesons. This m_ρ-independence of the total energy of the brane-induced Skyrmion in ANW unit is originated from its scaling property discussed in Sec. III C. In fact, ρ-mesons are naturally introduced as the internal degrees of freedom from the fluctuation modes of open strings in the holographic framework. Therefore only the ρ-meson contributions cannot be suppressed even by taking heavy mass limit: $m_\rho \to \infty$, in comparison with the other phenomenological treatment, where ρ mesons are included as the external degrees of freedom.

The total energy density and energy density of each term in the action (95) of brane-induced Skyrmion are shown in Figs. 6 and 7. The total energy density of standard Skyrmion is also shown in Fig. 6 for comparison. We calculate the root-mean-square radius of the Skyrme configuration by using the normalized energy density $\bar{\varepsilon}(r) \equiv \varepsilon(r)/E$ ($\varepsilon(r)$ is total energy density and E is total energy of a Skyrmion) as

$$\sqrt{\langle r^2\rangle} \equiv \left[\int_0^\infty 4\pi r^2 dr \bar{\varepsilon}(r) r^2\right]^{1/2}, \quad (107)$$

which gives a rough estimation of baryon size. We numerically find the root-mean-square radius of brane-induced Skyrmion in ANW unit as

$$\sqrt{\langle r^2\rangle} \simeq 1.268\left[\frac{1}{ef_\pi}\right], \quad (108)$$

which is compared with that of the standard Skyrmion: $\sqrt{\langle r^2\rangle} \simeq 1.422[\frac{1}{ef_\pi}]$. In the brane-induced Skyrmion, some part of total mass is carried by the heavy vector mesons in the soliton core, which should give the shrinkage of the total size by ~10% relative to the standard Skyrmion as in Fig. 6.

The comparison between total energy density and ρ meson contributions in the brane-induced Skyrmion is shown in Fig. 8. The effects of ρ meson degrees of freedom for the total energy and root-mean-square radius of a Skyrmion are found to be relatively small with about 10% modifications discussed above. However, Fig. 8 shows that ρ-meson components are rather active in the core region of baryons (Skyrmions) through various interaction terms in the four-dimensional effective action. Particularly the ρ-2α-β coupling term is found to give about 25% negative (attractive) contribution for the total baryon mass as shown in Fig. 5. This active ρ-meson component inside baryons may be a new striking picture for baryons suggested from the holographic QCD. Recently the projects with high-energy meson-baryon scattering are proposed in the J-PARC experiments. There it may be possible to see the effect of active ρ-meson components in the deeper interior of baryons by using high-energy resolutions.

In this paper we do not perform the semiclassical quantization for the brane-induced Skyrmions. Nucleon N and Δ isobar can be treated by isospin rotating the brane-induced Skyrmion as the excited hedgehog solitons. The proper procedure for such semiclassical quantization with ρ meson fields becomes nontrivial and should be discussed

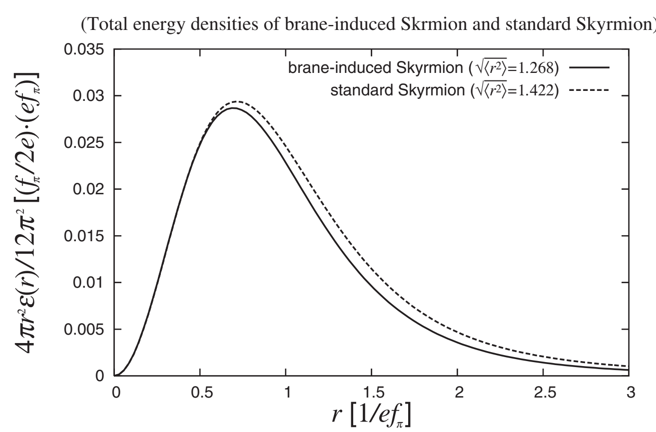

FIG. 6. Total energy densities $4\pi r^2\varepsilon(r)$ per BPS value of $12\pi^2$ for the brane-induced Skyrmion (solid curve) and the standard Skyrmion (dashed curve).

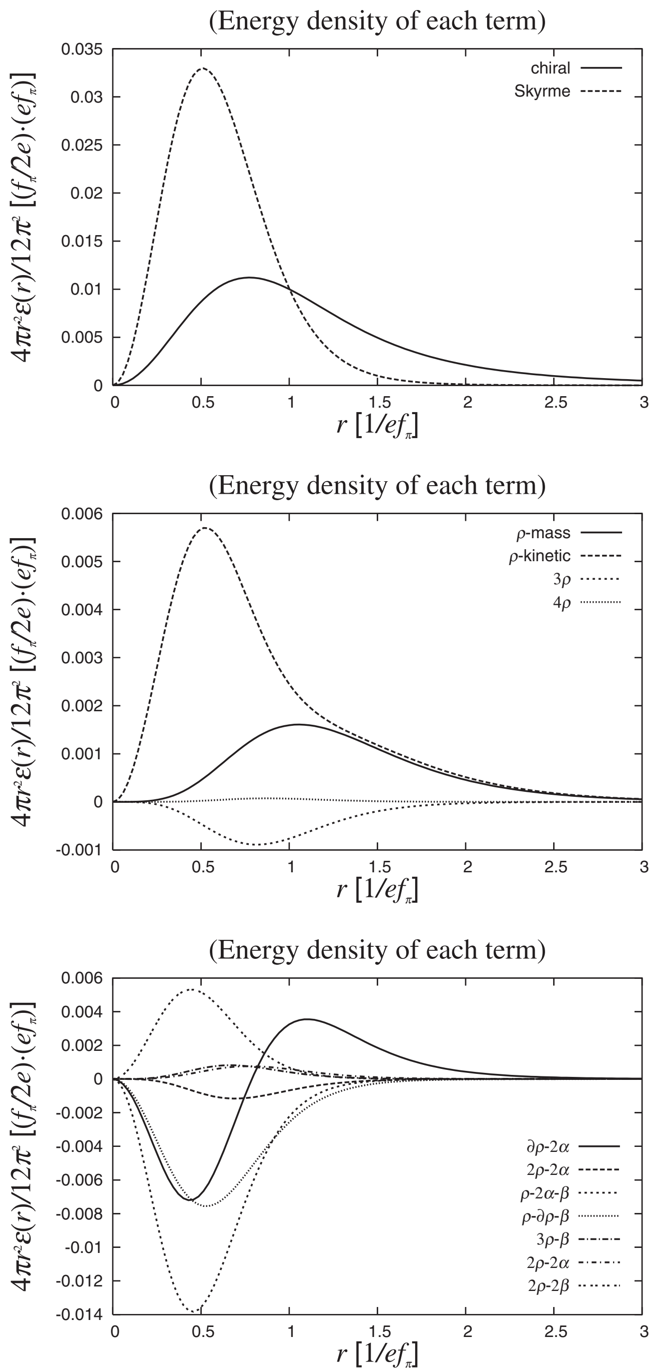

FIG. 7. Contributions of all the terms in the effective action (95) to the energy density $4\pi r^2 \varepsilon(r)/12\pi^2$ of the brane-induced Skyrmion.

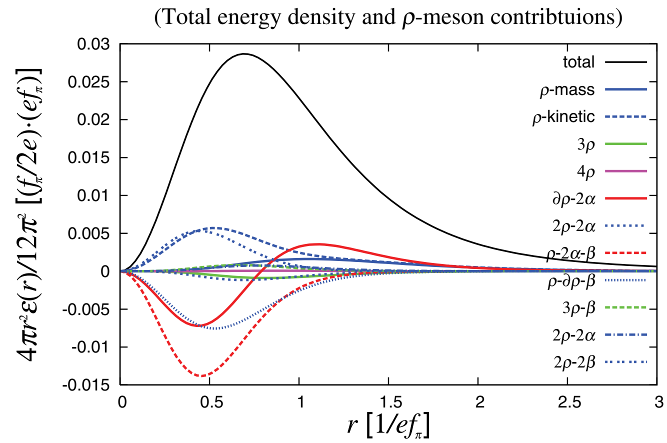

FIG. 8 (color online). Contributions of ρ-meson interaction terms in the effective action (95) to the energy density $4\pi r^2 \varepsilon(r)/12\pi^2$ of the brane-induced Skyrmion with the total energy density.

in future to get the physical interpretations of interaction channels in the action (58)–(70).

Finally we discuss the recovering of the physical unit for the mass and root-mean-square radius of the brane-induced Skyrmion. The holographic QCD has two parameters, and now we take f_π and m_ρ as experimental values;

$$f_\pi = 92.4 \text{ MeV}, \qquad m_\rho = 776.0 \text{ MeV}. \tag{109}$$

With these two experimental inputs, all the variables in the holographic QCD like κ, $M_{\rm KK}$, and e are uniquely determined by Eqs. (96)–(98) as follows:

$$\kappa \simeq 7.460 \times 10^{-3}, \qquad M_{\rm KK} \simeq 948.0 \text{ MeV}, \qquad e \simeq 7.315. \tag{110}$$

With these variables, the static soliton energy (the baryon mass) and the root-mean-square radius of the brane-induced Skyrmion with hedgehog configuration are found to be

$$E \simeq 1.115 \times 12\pi^2 \left[\frac{f_\pi}{2e}\right] \simeq 834.0 \text{ MeV}, \tag{111}$$

$$\sqrt{\langle r^2 \rangle} \simeq 1.268 \left[\frac{1}{ef_\pi}\right] \simeq 0.37 \text{ fm}. \tag{112}$$

With these results, the brane-induced Skyrmion in Eq. (111) seems to give a reasonable baryon mass. (The hedgehog soliton mass is slightly smaller than the nucleon mass [19].) However, the size in Eq. (112) seems to be smaller in comparison with experimental data for the proton charge radius (~ 0.8 fm) [29] and also with the size of baryon core without dynamical pion cloud (~ 0.6 fm) suggested by the quark model calculations [30]. By rotating the static hedgehog configurations of the brane-induced Skyrmion as a semiclassical quantization procedure and also taking into account the dynamical pion cloud, some enhancement of the total size of the brane-induced Skyrmion is still expected for the argument of baryons experimentally observed like nucleons.

In the framework of the standard Skyrme model without ρ mesons, the masses of nucleon N and Δ isobar are classically estimated by rotating the Skyrme configuration with moment of inertia $\mathcal{J}$ [27] as

$$M_I = M_{\rm HH} + \frac{I(I+1)}{2\mathcal{J}}, \tag{113}$$

where $M_{\rm HH}$ denotes the mass of a Skyrmion as a static hedgehog soliton, and isospin I denotes $I = \frac{1}{2}$ for N and $I = \frac{3}{2}$ for Δ. By taking the ANW unit, Eq. (113) can be written as

$$M_I\left[\frac{f_\pi}{2e}\right] = M_{\rm HH}\left[\frac{f_\pi}{2e}\right] + (4e^4)\frac{I(I+1)}{2\mathcal{J}}\left[\frac{f_\pi}{2e}\right], \tag{114}$$

where e^4 explicitly appears in the second term and, in this sense, N-Δ splitting has a scale dependence even in the ANW unit. In order to fit the N-Δ splitting experimentally observed, Adkins *et al.* [27] took a relatively small Skyrme parameter $e \simeq 5.44$, and, to fit the experimental data for nucleon mass, the pion decay constant is taken as a smaller value $f_\pi \simeq 64.5$ MeV. This also gives the root-mean-square radius of a baryon as $\sqrt{\langle r^2\rangle} \simeq 1.422[\frac{1}{ef_\pi}] \simeq 0.80$ fm.

Actually, however, N-Δ splitting corresponds to the next-to-next-leading order of $1/N_c$ expansion. Therefore the Skyrme model as the leading order $O(N_c)$ of $1/N_c$ expansion might be difficult to reproduce the experimental values belonging to the higher-order contributions like N-Δ splitting. This N_c-counting mismatch may result in the deviation of pion decay constant $f_\pi(\simeq 64.5$ MeV) from the experimental value by fitting the N-Δ splitting in the standard Skyrme model.

The comparison between the profiles of brane-induced Skyrmion and standard Skyrmion are summarized in Table II with some experimental data for baryons. In the holographic QCD, the consistency on the pion decay constant f_π and the ρ meson mass m_ρ gives the reasonable hedgehog baryon mass and the smaller baryon size as shown in this table.

In the holographic QCD with D4/D8/$\overline{\rm D8}$ system, other meson degrees of freedom like ω and η' with $U_A(1)$ anomaly are discussed by introducing the Chern-Simons (CS) term in the effective action of the D8 brane with D4 supergravity background. In fact, the DBI part and CS part correspond to the same order $O(N_c)$ of $1/N_c$ expansion because the effective action of the D8 brane is composed on the D4 supergravity background as the large-N_c effective theory, and also CS part is actually the next-order contribution relative to the DBI part with respect to the expansion about 'tHooft coupling $1/\lambda$ [$\simeq 0.120$ from Eqs. (21) and (110)]. Furthermore, most of the CS part includes one time-derivative of pion fields and does not affect the static properties of hedgehog solitons. Nevertheless, the CS term still has some time-independent part including ω_0 [1], and it may affects soliton properties to some extent, although the CS part itself corresponds to the higher order of $1/\lambda$ expansion as mentioned above. Anyway, the possibility of some improvements in the consistency of N-Δ splitting and root-mean-square radius of baryons with experimental data should be delicately discussed in future, even with the CS term in the holographic approach.

IV. SUMMARY AND DISCUSSION

We have studied baryons as brane-induced Skyrmions and numerically obtained the hedgehog soliton solution in the four-dimensional meson effective action induced by the holographic QCD with D4/D8/$\overline{\rm D8}$ system.

We have reviewed the D4/D8/$\overline{\rm D8}$ holographic QCD and its low-energy effective meson theory from the viewpoints of recent hadron physics and QCD phenomenologies. Infinite number of hidden local symmetries are shown to appear in the holographic model as the counterparts for the infinite tower of (axial) vector mesons, which is consistent with the construction process of a phenomenological five-dimensional Yang-Mills theory from the open moose model by piling up infinite number of hidden local symmetries embedded in the chiral field [15]. Physical interpretations for the gauge fixing conditions in the $U(N_f)$ local gauge symmetry of the D8 brane are also discussed; $A_z = 0$ gauge manifestly shows the mass generation of gauge fields to give massive (axial) vector mesons similar to the unitary gauge in non-Abelian Higgs theories, and, as

TABLE II. Profiles of the brane-induced Skyrmion and standard Skyrmion with experimental data for baryons.

	Brane-induced Skyrmion	Standard Skyrmion	Experiment
f_π	92.4 MeV (input)	64.5 MeV	92.4 MeV
m_ρ	776.0 MeV (input)	—	776.0 MeV
e	7.32	5.44	—
$E_{\rm ANW} \equiv \frac{f_\pi}{2e}$	6.32 MeV	5.93 MeV	—
$r_{\rm ANW} \equiv \frac{1}{ef_\pi}$	0.29 fm	0.56 fm	—
$M_{\rm HH}$	834.0 MeV	864.3 MeV	—
$\sqrt{\langle r^2\rangle}$	0.37 fm	0.80 fm	0.60–0.80 fm
M_N	—	938.9 MeV (input)	938.9 MeV
M_Δ-M_N	—	293.1 MeV (input)	293.1 MeV

for the residual hidden local symmetries, $\xi_+^{-1} = \xi_-$ gauge gives definite parity and G-parity for the four-dimensional meson effective action induced by the holographic QCD.

With the mode expansion of the gauge fields, four-dimensional meson effective action is uniquely derived from the holographic QCD, without small amplitude expansion of meson fields. We have discussed the origin of the mass of (axial) vector mesons as the oscillations of meson wave functions in the extra fifth dimension. The smaller coupling constants between pions and heavier (axial) vector mesons can also be suggested by the smaller overlapping of meson wave functions in the fifth dimension because of the large oscillation of heavier (axial) vector meson wave functions. This is consistent with the smaller decay width of heavier (axial) vector mesons into pions observed in particle data group experiments. Therefore, by assuming that the main part of the chiral soliton is constructed by the large amplitude pion fields, the effects of higher mass excitation modes of (axial) vector mesons for chiral solitons are expected to be small, and so only pions and ρ mesons are treated along the discussion of chiral solitons.

The energy functional and the Euler-Lagrange equation of brane-induced Skyrmion are derived with the hedgehog configuration Ansatz for pion and ρ meson fields in the four-dimensional meson effective action induced by the holographic QCD. The scaling property of brane-induced Skyrmion is also discussed by taking the ANW unit, where all the effects of the physical parameters are shown to be fully extracted into the units of quantities. This scaling property can hold even by including other (axial) vector meson degrees of freedom because the holographic QCD has intrinsically two parameters κ and Kaluza-Klein mass $M_{\rm KK}$ as a ultraviolet cutoff scale of the theory. This scaling property cannot be seen in the traditional discussion of the Skyrme model with (axial) vector mesons included as the external degrees of freedom.

By numerically solving the Euler-Lagrange equation, we have found that stable Skyrme soliton solution exits, which should be a correspondent of a baryon in the large-N_c holographic QCD. Numerical results about the meson wave functions, total energy, energy density, and root-mean-square radius of the brane-induced Skyrmion are presented in the ANW unit as the scale invariant properties of the brane-induced Skyrmion. Total energy of the Skyrmion tends to decrease by about 10% (in ANW unit) through the interactions of pions and ρ mesons. Total size also shrinks by about 10% (in ANW unit) with the existence of ρ mesons because some part of total baryon mass is carried by the heavy ρ mesons in the core of the baryon. By comparing the total energy density with ρ-meson contributions in the brane-induced Skyrme action, ρ mesons are found to be rather active in the deeper interior of a baryon through various interaction terms in the effective action induced by the holographic QCD.

Finally we have recovered the physical unit for the dimensionless quantities by taking experimental inputs for the pion decay constant f_π and ρ meson mass m_ρ as $f_\pi = 92.4$ MeV and $m_\rho = 776.0$ MeV. Then the mass and root-mean-square radius of the brane-induced Skyrmion are found to be $E \simeq 1.115 \times 12\pi^2[\frac{f_\pi}{2e}] \simeq 834.0$ MeV and $\sqrt{\langle r^2 \rangle} \simeq 1.268[\frac{1}{ef_\pi}] \simeq 0.37$ fm. Thus, the brane-induced Skyrmion has a reasonable baryon mass of about 1 GeV. However, the size seems to be smaller in comparison with the value experimentally observed and also the size of the baryon core without dynamical pion cloud suggested by quark model calculations [30]. By rotating the static hedgehog configuration as the semiclassical quantization procedure, and taking into account the dynamical pion cloud, the enhancement in the total size of the brane-induced Skyrmion is still expected.

Recently there an idea has been suggested that a baryon can be described as an instanton in the five-dimensional Yang-Mills action in the context of the D4/D8/$\overline{\rm D8}$ holographic model [1]. According to the concept of Witten's baryon vertex [31], N_c fundamental strings are induced from the newly wrapped N_c folded D4 branes around the S^4 (S^4 is a four-sphere around the extra coordinates x_{5-9}), which is regarded as a bound state of N_c quarks as a baryon. In the D4/D8/$\overline{\rm D8}$ holographic model, the existence of these newly wrapped D4 branes are represented by an instanton on the D8 brane through the duality of "brane within brane." The complemental relations between these two kinds of topological nontrivialities as an instanton in the five-dimensional space-time and Skyrme soliton in the four-dimensional space-time derived in our framework should be discussed in future, if both describe the same object as a baryon.

As for the further complicated aspects of nonperturbative QCD with including nonzero current quark masses, it would also be desired to construct more simplified effective theories from the viewpoint of the holographic QCD. To this end, a pioneering phenomenological construction of the five-dimensional meson theory [15,32] may give a guide line of the framework. It is also valuable to compare the present holographic QCD with the analyses of QCD and SUSY-QCD using the other type of effective theories inspired by the AdS/CFT correspondence [33,34]. In any case, the holographic QCD is one of the new powerful approaches for the nonperturbative region of QCD, and is expected to continuously present new striking ideas and concepts to the hadron physics for the future.

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APPENDIX: TOPOLOGICAL NATURES OF SKYRMIONS

In this appendix, we summarize the topological natures of Skyrmions from the viewpoint of manifold theories.

The total energy of standard Skyrmions is known to have a topological lower bound, which can be shown as the Cauchy-Schwaltz inequality relation for the chiral and Skyrme terms in ANW unit as follows:

$$\begin{aligned}\frac{1}{2}\int d^3x\,\mathrm{tr}(L_jL_j)-\frac{1}{16}\int d^3x\,\mathrm{tr}[L_j,L_k]^2 &= -\frac{1}{2}\int d^3x\,\mathrm{tr}(iL_jiL_j)-\frac{1}{16}\int d^3x\,\mathrm{tr}[iL_j,iL_k]^2\\ &= -\int d^3x\frac{1}{2}\,\mathrm{tr}\left\{iL_j\pm\frac{1}{4}\varepsilon_{jkl}[iL_k,iL_l]\right\}^2\\ &\quad\pm 12\pi^2\int d^3x\frac{1}{2\pi^2}\frac{1}{2\cdot 3!}\varepsilon_{jkl}\,\mathrm{tr}(iL_jiL_kiL_l). \qquad \text{(A1)}\end{aligned}$$

$L_j=\frac{1}{i}U^\dagger\partial_jU$ is the Hermite 1-form of the chiral field $U\in \mathrm{SU}(2)_A$, which can be expanded by the Pauli matrices as $L_j\equiv L_j^a\tau^a$ ($L_j^a\in\mathbf{R}$). Then the first term of Eq. (A1), which is called a nontopological part, is shown to be non-negative as

$$\begin{aligned}(-1)^2\int d^3x\tfrac{1}{2}\mathrm{tr}\{(L_j^a\mp\tfrac{1}{4}\varepsilon_{jkl}\varepsilon^{abc}L_k^bL_l^c)\tau^a\}^2 &= \int d^3x(L_j^a\mp\tfrac{1}{4}\varepsilon_{jkl}\varepsilon^{abc}L_k^bL_l^c)^2\geq 0. \qquad \text{(A2)}\end{aligned}$$

The physical meaning of the second term of Eq. (A1) is discussed as follows. The Skyrmion is a nontrivial mapping from the flat three-dimensional coordinate space $\mathbf{R}^3$ compactified at infinity into 3-sphere $S^3_{(\infty)}$ with infinite radius, to the group manifold $\mathrm{SU}(2)_A\simeq S^3_{(\langle\sigma\rangle=1)}$ with a radius equal to the chiral condensate $\langle\sigma\rangle$, which is conventionally taken to be a unit radius. Regarding the second term of Eq. (A1), which is called a topological part, $\frac{1}{2\cdot 3!}\varepsilon_{jkl}\,\mathrm{tr}(iL_jiL_kiL_l)$ corresponds to the volume form on $S^3_{(\langle\sigma\rangle=1)}$ and $2\pi^2$ is the total volume of $S^3_{(\langle\sigma\rangle=1)}$ with unit radius. The $\mathbf{x}$ integral in this topological part runs all over the physical coordinate space $S^3_{(\infty)}$, and therefore topological part corresponds to $12\pi^2$ times the degree of mapping of Skyrmions from physical space to the internal space. Actually the Skyrmion is the mapping between the two closed manifolds $S^3_{(\infty)}$ and $S^3_{(\langle\sigma\rangle=1)}$, and this mapping is homotopically equivalent with the integer group as $\pi_3(\mathrm{SU}(2)_A)=\mathbf{Z}$ [26]. Therefore the degree of mapping of Skyrmions must be integer and, using Eq. (A2), the energy from chiral and Skyrme terms is found to have a topological lower bound originated from the degree of mapping as follows:

$$\tfrac{1}{2}\int d^3x\,\mathrm{tr}(L_jL_j)-\tfrac{1}{16}\int d^3x\,\mathrm{tr}[L_j,L_k]^2\geq 12\pi^2|B|, \qquad \text{(A3)}$$

$$B\equiv\int d^3x\frac{1}{2\pi^2}\frac{1}{2\cdot 3!}\varepsilon_{jkl}\,\mathrm{tr}(iL_jiL_kiL_l). \qquad \text{(A4)}$$

B is called the topological charge and is conserved for the continuous distortion of waves with finite energy, because wave should overcome an infinite energy barrier to cross into the different homotopical sectors. B is identified as the conserved baryon number in the Skyrme model [8], which prevents a proton from decaying into pions [28]. Therefore a standard Skyrmion with a unit degree, which corresponds to one baryon with $B=1$, has topological lower bound for its energy as $E\geq 12\pi^2$.

Regarding several solitons (Skyrmions, monopoles, instantons, etc.), they are realized as some kinds of topological nontriviality, and their energies tend to have a lower bound originated from its topological charge. If the soliton energy is exactly equal to this topological lower bound, then this soliton is called to have Bogomol'nyi-Prasad-Sommerfield (BPS) saturation [23]. BPS saturation is known to occur in some cases of solitons with the gauge theories like 'tHooft-Polyakov monopole in the SU(2) Higgs model, and also the instantons with the Euclidean metric.

As for the case of Skyrmions, Adkins *et al.* [27] numerically showed that the standard Skyrmion has no BPS saturation, and there exists slight enhancement of total energy by ~23% relative to the BPS saturation energy; $E\simeq 1.231\times 12\pi^2[\frac{f_\pi}{2e}]$. The origin for this enhancement of total energy and non-BPS nature of Skyrmions is discussed from the viewpoint of nonlinear elasticity theory by Manton *et al.* [28]. In this survey, the energy of the Skyrmion is regarded as a measure of the geometrical distortion induced by the mapping between two manifolds $S^3_{(\infty)}$ and $S^3_{(\langle\sigma\rangle=1)}$, which is compared to the strain energy of one deformed material winded on the other material. If the two manifolds are not isometric with each other, the distortion energy of one manifold winded around the other one is enhanced by the difference of curvatures, relative to the only winding energy, i.e., BPS saturation energy. Standard Skyrmion is a mapping between two nonisometric manifolds; $S^3_{(\infty)}$ with infinite radius and $S^3_{(\langle\sigma\rangle=1)}$ with unit radius, which then gives the enhancement of Skyrmion energy relative to the BPS saturation energy.

By solving the Euler-Lagrange equations (102) and (103) induced by the holographic model, we find the energy of brane-induced Skyrmion as

$$E \simeq 1.115 \times 12\pi^2 \left[\frac{f_\pi}{2e}\right], \tag{A5}$$

which is lower by $\sim$10% relative to that of standard Skyrmion. This may be understood that the metrical distortion energy is slightly relieved by the interactions with other degrees of freedom as ρ mesons, which is discussed in Eq. (106).

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