New Born-Infeld and Dp-brane actions under 2-metric and 3-metric prescriptions

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The parent action method is utilized to the Born-Infeld and Dp-brane theories. Various new forms of Born-Infeld and Dp-brane actions are derived by using this systematic approach, in which both the already known 2-metric and newly proposed 3-metric prescriptions are considered. An auxiliary worldvolume tensor field, denoted by $\omega_{\mu\nu}$, is introduced and treated probably as an additional worldvolume metric because it plays a similar role to that of the auxiliary worldvolume (also called *intrinsic*) metric $\gamma_{\mu\nu}$. Some properties, such as duality, permutation and Weyl invariance as a local worldvolume symmetry of the new forms are analyzed. In particular, a new symmetry, i.e. the double Weyl invariance is discovered in 3-metric forms.

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I. INTRODUCTION

The remarkable progress of string theory [1] is the discovery of Dirichlet p branes [2], i.e. Dp branes. Geometrically, the Dp branes are $(p+1)$-dimensional hypersurfaces that are embedded in a higher dimensional spacetime. Dynamically, they are solitonic solutions to string equations that are "branes" on which open strings attach with Dirichlet boundary conditions. The dynamics of Dp branes is induced by the open strings and governed in general by the action of Born-Infeld type [3]. Recently, many different actions for generalizations of the Born-Infeld action have been proposed to describe the effective worldvolume theories of Dp branes. For instance, see Refs. [4–17] where quite interesting are the action form [14] that is quadratic in Abelian field strengths[1] and its conformal invariant development [15]. The merit of this formalism, as promised and expected by Refs. [14,15], is on simplifying quantization and dualization of the gauge fields, which originates from the fact that the string action [18] with an auxiliary worldsheet metric and conformal invariance greatly simplifies the analysis of string theory and allows a covariant quantization [19].

The idea of the parent action method was introduced [20] in order to establish, at the level of Lagrangians instead of equations of motion, the equivalence or so-called duality between the Abelian self-dual and Maxwell-Chern-Simons models in $(2+1)$-dimensional spacetime. Recently the method has been developed and applied to a quite wide region. For instance, one direct development [21] is the building of the duality between the non-Abelian self-dual and Yang-Mills-Chern-Simons models. One interesting application related closely to the present paper focuses on chiral bosons [22] and bosonic p branes [23]. For chiral bosons and their extensions to chiral p forms, the self-duality of various chiral boson actions has been built [24] with the modification [25] of the method with one more auxiliary field to preserve the manifest Lorentz invariance of the actions. For bosonic p branes, some new actions of bosonic p branes have been worked out and their classification to dual sets fixed [26] as well, and furthermore the canonical Hamiltonian analyses of the new actions have therefore been performed [27]. In the present work, the term duality is used to refer to different actions for the same Dp brane, rather than the duality of the Dp branes themselves.

The main idea of the parent action method [28] originates from the Legendre transformation and contains the meaning of the two aspects: (a) to introduce auxiliary fields and then construct a parent or master action by adding a Lagrange multiplier term to a known action, and (b) to make variation of the parent action with respect to each auxiliary field, solve one auxiliary field in terms of other fields and then substitute the solution into the parent action. Through making variations with respect to different auxiliary fields, we can obtain different forms of the actions. The actions are, of course, equivalent classically, and the relation between them is usually referred to as duality. If the resulting actions are the same, the relation is called self-duality.

The content and arrangement of this paper are as follows. In the next section, we begin to construct 2-metric forms in terms of the parent action method, and then build, as a by-product of the method, duality structures of the action forms. In the application of the method, we consider two proposals of writing parent actions one of which, introduced firstly by the present author [26], has given rise to interesting bosonic p brane actions with highly nonlinear terms of induced metrics [27]. Because of some symmetric formalism of the 1-metric source action (see Sec. II for details) we start with, we obtain a variety of

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[1]Different quadratic formulations with nonsymmetric auxiliary worldvolume metrics are also given by Refs. [13,16]. However, they do not have such a permutation symmetry between $g_{\mu\nu}$ and $\mathcal{G}_{\mu\nu}$ [cf. Eq. (3)] that is dealt with as our starting point and are therefore not discussed in this paper.

new actions of Dp branes (especially for 3-metric forms, see Sec. III for details). Although some forms do not provide useful formulations for simplifying the analysis of the Dp-brane theory, such as S'^{II} [cf. Eqs. (18)–(23)] that are not quadratic in field strengths, they supply the base for us to go further beyond the actions with 2 metrics. This is just the task of the following section. In Sec. III, we propose the 3-metric prescription whose key is the introduction of an additional worldvolume metric. Simply speaking, the motivation[2] is to look for such a Dp-brane action that is, besides quadratic in field strengths, a rational functional of the induced metric. It is obvious that a rational formulation is more convenient to be analyzed than a rooted one. Using the parent action method two successive times to the 1-metric source form, together with the consideration of the two proposals just mentioned, we get a large amount of new Dp-brane actions with 3-metrics and also fix their duality structures. Quite surprising is the richness of the 3-metric forms in permutation and conformal symmetry. We therefore devote Sec. IV to analyze the properties in detail. We give an interesting one-to-one correspondence of the 3-metric actions under permutation transformations of the two auxiliary worldvolume metrics, and, in particular, discover a new symmetry, i.e. the double conformal invariance in some 3-metric action forms. Finally, a conclusion is made in Sec. V.

In this paper we name various forms of actions at first by using series number that is just the number of metrics involved in, and then classify them within series II and series III (series I includes only one form) in terms of some important properties they have, such as duality, permutation and Weyl symmetries.

The notation we use throughout this paper is as follows. Some Greek lowercase letters, for example, μ, ν, λ, σ, running over $0, 1, \cdots, p$, are used as indices in the worldvolume that is spanned by $p+1$ arbitrary parameters ξ^{μ}. Incidentally, spacetime indices are suppressed because our discussions only involve in the worldvolume. The Dp-brane kinetic term takes the form [2]

$$S = -T_p \int d^{p+1}\xi e^{-\phi}\sqrt{-\det(g_{\mu\nu} + \mathcal{F}_{\mu\nu})}, \tag{1}$$

where

$$\mathcal{F}_{\mu\nu} \equiv F_{\mu\nu} - B_{\mu\nu}, \tag{2}$$

ϕ, $g_{\mu\nu}$, and $B_{\mu\nu}$ are pullbacks to the worldvolume of the background dilaton, metric and NS antisymmetric two-form fields, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, with $A_\mu(\xi)$ the $U(1)$ worldvolume gauge field. T_p is the Dp-brane tension.

[2]This idea is necessary for the investigation of Dp branes in general though the typical 2-metric action [14], when it describes the low-energy effective worldvolume theory for an open type I string, can be reduced to its static gauge form which is quadratic in the derivatives of both gauge fields and spacetime coordinates.

Equation (1) can be rewritten as [14]

$$S^{\text{I}} = -T_p \int d^{p+1}\xi e^{-\phi}(-g)^{1/4}(-\mathcal{G})^{1/4}, \tag{3}$$

where

$$\mathcal{G}_{\mu\nu} = g_{\mu\nu} - g^{\lambda\sigma}\mathcal{F}_{\mu\lambda}\mathcal{F}_{\sigma\nu}, \tag{4}$$

and $g \equiv \det(g_{\mu\nu})$, $\mathcal{G} \equiv \det(\mathcal{G}_{\mu\nu})$; $g^{\mu\nu}$ is the inverse of $g_{\mu\nu}$. This is a convenient form for our following discussions, and treated as the source form of Dp-brane actions we start with in this paper. It is named as 1-metric series with the superscript I, i.e. series I, which contains solely the induced metric $g_{\mu\nu}$.

II. 2-METRIC FORM AND DUALITY

Starting from the 1-metric form of Dp-brane actions, we now derive new forms with 2-metrics, called 2-metric series or series II, in terms of the parent action method which has been shown [26] powerful to discovering new actions and fixing their dualities for bosonic p branes. We point out that this method is also applicable to Born-Infeld and Dp-brane theories and that new forms of actions will be obtained in a systematic way. This means that we obtain not only the forms proposed already [14,15] but also some new forms unknown before. As the method has been utilized in detail to bosonic p branes in our previous work [26], here we just follow the main procedure of the method and write down results.

According to the parent action method [20,28], we introduce two auxiliary worldvolume second-rank tensor fields $\Lambda^{\mu\nu}$ and $\gamma_{\mu\nu}$, where $\Lambda^{\mu\nu}$ is dealt with as a Lagrange multiplier, and write down one parent action of the 1-metric form

$$S^{\text{I}}_{\text{P1}} = -T_p \int d^{p+1}\xi e^{-\phi}[(-g)^{1/4}(-\gamma)^{1/4} + \Lambda^{\mu\nu}(\gamma_{\mu\nu} - \mathcal{G}_{\mu\nu})], \tag{5}$$

where $\gamma \equiv \det(\gamma_{\mu\nu})$. Note that the Lagrange multiplier term in the above equation is composed of the contravariant $\Lambda^{\mu\nu}$ multiplied by the covariant $\gamma_{\mu\nu} - \mathcal{G}_{\mu\nu}$. This is the first proposal we suggest for construction of parent actions while the second, just opposite to the first, will be given later in this section. We will find that $\gamma_{\mu\nu}$ is just the auxiliary worldvolume (*intrinsic*) metric, but at present it is treated as an independent auxiliary field.

Now varying Eq. (5) with respect to $\Lambda^{\mu\nu}$ gives the relation $\gamma_{\mu\nu} = \mathcal{G}_{\mu\nu}$, together with which Eq. (5) turns back to the 1-metric form Eq. (3). This shows the classical equivalence between the parent and 1-metric actions. However, varying Eq. (5) with respect to $\gamma_{\mu\nu}$ leads to the expression of $\Lambda^{\mu\nu}$ in terms of $\gamma_{\mu\nu}$:

$$\Lambda^{\mu\nu} = -\tfrac{1}{4}(-g)^{1/4}(-\gamma)^{1/4}\gamma^{\mu\nu}, \tag{6}$$

where $\gamma^{\mu\nu}$ is the inverse of $\gamma_{\mu\nu}$. Substituting Eq. (6) into Eq. (5), we obtain one dual version of the 1-metric action

$$S_1^{\rm II} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-g)^{1/4}(-\gamma)^{1/4} \times [\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)]. \quad (7)$$

This is the 2-metric Dp-brane action with the auxiliary field $\gamma_{\mu\nu}$ that now plays the role of the auxiliary worldvolume metric. It was obtained but its duality to the 1-metric form was not uncovered in Ref. [14]. The superscript II of the symbol in Eq. (7) means that the action belongs to series II, i.e. the forms with 2-metrics and the subscript i (here $i = 1$) corresponds to the forms that are derived in terms of the first proposal of writing parent actions mentioned above.

As $S_1^{\rm II}$ has no Weyl invariance for the general case of $p \neq 3$, we then adopt the approach [16,26] utilized in bosonic p branes to derive other 2-metric Dp-brane actions that possess such an invariance. To this end, by introducing an auxiliary scalar field $\Phi(\xi)$, and rescaling the worldvolume metric $\gamma_{\mu\nu} \to \Phi\gamma_{\mu\nu}$ in the parent action of the 1-metric form, i.e. in Eq. (5), we write down the second parent action[3]

$$S_{\rm P2}^{\rm I} = -T_p\int d^{p+1}\xi e^{-\phi}[\Phi^{(p+1)/4}(-g)^{1/4}(-\gamma)^{1/4} + \Lambda^{\mu\nu}(\Phi\gamma_{\mu\nu} - \mathcal{G}_{\mu\nu})], \quad (8)$$

where $\Phi(\xi)$ should be a scalar in both the spacetime and worldvolume in order to keep Eq. (8) invariant under the Lorentz transformation and reparametrization.

Varying Eq. (8) with respect to $\Lambda^{\mu\nu}$ brings about $\gamma_{\mu\nu} = \Phi^{-1}\mathcal{G}_{\mu\nu}$, which leads to nothing new but the classical equivalence between the 1-metric and second parent actions. However, varying the equation with respect to $\gamma_{\mu\nu}$, we solve $\Lambda^{\mu\nu}$ as follows:

$$\Lambda^{\mu\nu} = -\tfrac{1}{4}\Phi^{(p-3)/4}(-g)^{1/4}(-\gamma)^{1/4}\gamma^{\mu\nu}. \quad (9)$$

Substituting Eq. (9) back to Eq. (8), we derive one more dual action of the 1-metric form

$$S_2^{\rm II} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-g)^{1/4}(-\gamma)^{1/4} \times [\Phi^{(p-3)/4}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)\Phi^{(p+1)/4}], \quad (10)$$

which was obtained too but whose duality to $S^{\rm I}$ was not uncovered either in Ref. [15]. This action is interesting because it has Weyl invariance that will be analyzed in detail in Sec. IV.

Moreover, we are able to deduce from $S_2^{\rm II}$ another Weyl invariant form without the auxiliary scalar field $\Phi(\xi)$. To this end, let us vary Eq. (10) with respect to $\Phi(\xi)$, which gives rise to the relation for the general case of $p \neq 3$:

[3] Parent actions are not unique.

$$\Phi(\xi) = \frac{1}{p+1}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}. \quad (11)$$

Substituting Eq. (11) into Eq. (10), we therefore have the Weyl invariant 2-metric action that does not involve in $\Phi(\xi)$ but contains higher (than two) order terms of field strengths for Dp branes of $p > 3$

$$S_3^{\rm II} = -T_p\int d^{p+1}\xi e^{-\phi}(-g)^{1/4}(-\gamma)^{1/4} \times \left(\frac{1}{p+1}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}\right)^{(p+1)/4}. \quad (12)$$

As to the special case of $p = 3$, $\Phi(\xi)$ disappears automatically from $S_2^{\rm II}$. Furthermore, we find that $S_i^{\rm II}$ ($i = 1, 2, 3$) coincide with each other in this case. Equation (12) is therefore suitable for describing $D3$ branes. Incidentally, a similar situation also happened to bosonic strings (1 branes) [23].

Besides the above two parent actions, we are able to write down according to the first proposal other different forms of parent actions that associate with the 1-metric source action if we follow the way of dealing with bosonic p branes. As the procedure is quite straightforward to Dp branes, we here omit it and simply conclude that no more actions are derived but more dualities among $S^{\rm I}$ and $S_i^{\rm II}$ ($i = 1, 2, 3$) that consist of a closed set of dual actions are exposed. The dualities are shown in detail by the schematic representation of Fig. 1.

At the present stage, although we uncover the dualities that exist in the 1-metric and three 2-metric forms, we merely reproduce through the parent action method the 2-metric Dp-brane actions, $S_i^{\rm II}$ ($i = 1, 2, 3$), that have been proposed [14,15] with the consideration different from ours. As we pointed out before that the parent action method is powerful to finding new action forms and building their dualities, our next goal is to derive other 2-metric forms. To this end, let us introduce the second proposal for writing parent actions which has been exploited to bosonic p branes [26]. Contrary to it in Eq. (5), now the Lagrange multiplier term is composed of the covariant $\Lambda_{\mu\nu}$ multiplied by the contravariant $\gamma^{\mu\nu} - \mathcal{G}^{\mu\nu}$, where $\Lambda_{\mu\nu}$ and $\gamma^{\mu\nu}$ are two auxiliary worldvolume second-rank tensor fields we introduce, and $\mathcal{G}^{\mu\nu}$ is the inverse of $\mathcal{G}_{\mu\nu}$. Note that $\mathcal{G}^{\mu\nu}$ contain of course highly nonlinear terms of field strengths. As a result, we construct one parent action that associates with the second proposal:

$$S_{\rm P\bar{1}}^{\rm I} = -T_p\int d^{p+1}\xi e^{-\phi}[(-g)^{1/4}(-\gamma)^{1/4} + \Lambda_{\mu\nu}(\gamma^{\mu\nu} - \mathcal{G}^{\mu\nu})], \quad (13)$$

where subscript $\bar{i}$ (here $\bar{i} = \bar{1}$) corresponds to our second proposal. At first sight $S_{\rm P\bar{1}}^{\rm I}$ looks like $S_{\rm P1}^{\rm I}$ in Eq. (5), just with the exchange of superscripts and subscripts in the second term. We note that if $\mathcal{G}^{\mu\nu}$ were understood as

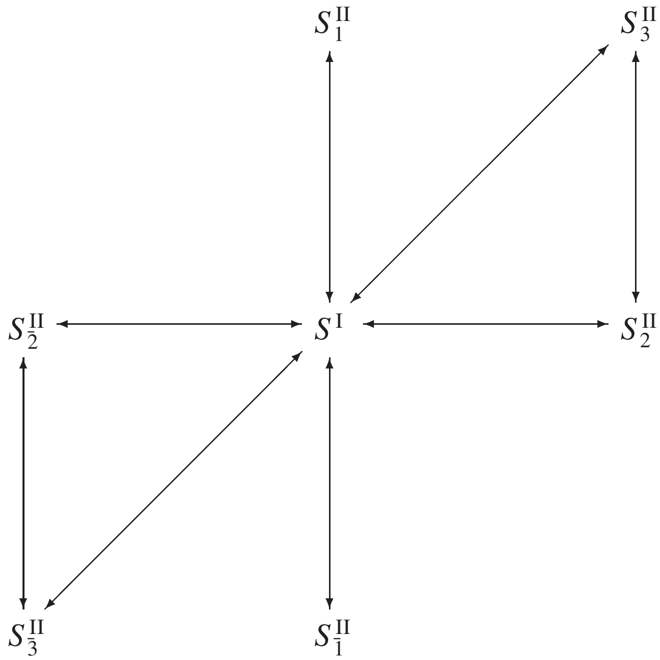

FIG. 1. Dualities in the two sets of dual actions, ($S^{\rm I}$, $S^{\rm II}_1$, $S^{\rm II}_2$, $S^{\rm II}_3$) and ($S^{\rm I}$, $S^{\rm II}_{\bar 1}$, $S^{\rm II}_{\bar 2}$, $S^{\rm II}_{\bar 3}$). The line with two arrows connects two actions that are dual to each other. As a source action, the 1-metric form, $S^{\rm I}$, appears in both sets. It is quite noticeable that in each set the Weyl noninvariant form, $S^{\rm II}_1$ ($S^{\rm II}_{\bar 1}$), has no *direct* dualities to its corresponding Weyl invariant forms, $S^{\rm II}_2$ and $S^{\rm II}_3$ ($S^{\rm II}_{\bar 2}$ and $S^{\rm II}_{\bar 3}$).

$\gamma^{\mu\lambda}\gamma^{\nu\sigma}\mathcal{G}_{\lambda\sigma}$, $S^{\rm I}_{{\rm P}\bar 1}$ was exactly the same as $S^{\rm I}_{\rm P1}$ and nothing new could be deduced from Eq. (13) but the Dp-brane action Eq. (7). Actually $\mathcal{G}^{\mu\nu}$ in Eq. (13) is defined as the inverse of $\mathcal{G}_{\mu\nu}$ and is thus independent of $\gamma_{\mu\nu}$. With this in mind, we follow the usual procedure described above and derive a new 2-metric Dp-brane action that is dual and equivalent to $S^{\rm I}$:

$$S^{\rm II}_{\bar 1} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-g)^{1/4}(-\gamma)^{1/4} \times[-\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)]. \tag{14}$$

The new Dp-brane action is Lorentz invariant and reparametrization invariant as well. Under the reparametrization, the factor $d^{p+1}\xi e^{-\phi}(-g)^{1/4}(-\gamma)^{1/4}$ remains unchanged and $\gamma_{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$ transform as the worldvolume covariant and contravariant tensors, respectively, which keeps $\gamma_{\mu\nu}\mathcal{G}^{\mu\nu}$ invariant.

$S^{\rm II}_{\bar 1}$, different from $S^{\rm II}_1$ of Eq. (7), is Weyl noninvariant in any dimension of worldvolume because of the nonzero term of $p+5$ (for further details, see Sec. IV). However, this invariance can be recovered by the above procedure done for $S^{\rm II}_1$. That is, introducing an auxiliary scalar field $\Phi(\xi)$, and rescaling the worldvolume metric as $\Phi\gamma_{\mu\nu}$ in $S^{\rm I}_{{\rm P}\bar 1}$ of Eq. (13), we write down one more parent action associated with the second proposal of construction of parent actions

$$S^{\rm I}_{{\rm P}\bar 2} = -T_p\int d^{p+1}\xi e^{-\phi}[\Phi^{(p+1)/4}(-g)^{1/4}(-\gamma)^{1/4} + \Lambda_{\mu\nu}(\Phi^{-1}\gamma^{\mu\nu} - \mathcal{G}^{\mu\nu})]. \tag{15}$$

Simply following the procedure under Eq. (8), i.e. $S^{\rm I}_{\rm P2}$, we thus deduce another new action with 2 metrics

$$S^{\rm II}_{\bar 2} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-g)^{1/4}(-\gamma)^{1/4} \times[-\Phi^{(p+5)/4}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)\Phi^{(p+1)/4}], \tag{16}$$

which restores the Weyl invariance. Furthermore, the other Weyl invariant form can be derived by eliminating the auxiliary scalar field $\Phi(\xi)$ from $S^{\rm II}_{\bar 2}$ as the way that $S^{\rm II}_3$ has been deduced from $S^{\rm II}_2$

$$S^{\rm II}_{\bar 3} = -T_p\int d^{p+1}\xi e^{-\phi}(-g)^{1/4}(-\gamma)^{1/4} \times\left(\frac{1}{p+1}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu}\right)^{-(p+1)/4}. \tag{17}$$

Obviously $S^{\rm II}_{\bar i}$ ($\bar i = \bar 1, \bar 2, \bar 3$) contain highly nonlinear terms of field strengths because of our special definition of $\mathcal{G}^{\mu\nu}$.

Similar to the situation of $S^{\rm II}_i$ ($i = 1, 2, 3$), we can also write down other parent actions besides $S^{\rm I}_{{\rm P}\bar 1}$ and $S^{\rm I}_{{\rm P}\bar 2}$, however, we obtain no more new forms of actions with 2-metrics under our second proposal but more dualities among the three new forms $S^{\rm II}_{\bar i}$ ($\bar i = \bar 1, \bar 2, \bar 3$) that, together with the 1-metric source action, also constitute a closed set of dual actions. The schematic representation of dual sets, combined with that of $S^{\rm II}_i$ ($i = 1, 2, 3$), is shown in Fig. 1.

As a consequence, if we apply the parent action method to the 1-metric form with respect to $\mathcal{G}_{\mu\nu}$, i.e., moving $\mathcal{G}_{\mu\nu}$ out of one-fourth root while keeping $g_{\mu\nu}$ unchanged, we obtain the six 2-metric actions: $S^{\rm II}_i$ and $S^{\rm II}_{\bar i}$, where $i = 1, 2, 3$ and $\bar i = \bar 1, \bar 2, \bar 3$. The first three forms with subscript i are derived in terms of the parent action method by adding a Lagrange multiplier term that associates with $\Lambda^{\mu\nu}$ (a contravariant Lagrange multiplier tensor field) and $\mathcal{G}_{\mu\nu}$, and have been proposed [14,15] by quite different consideration from ours; while the other three with subscript $\bar i$ are deduced by adding a Lagrange multiplier term that involves in $\Lambda_{\mu\nu}$ (a covariant Lagrange multiplier tensor field) and the inverse of $\mathcal{G}_{\mu\nu}$, i.e. $\mathcal{G}^{\mu\nu}$, and are new actions that contain highly nonlinear terms of field strengths. The two proposals of construction of parent actions divide the six 2-metric forms into two parts, each of which, together with the 1-metric form, consists of a closed set. Here we mention in advance that this notation will be utilized again in series III, that is, subscript i corresponds to the first proposal of writing parent actions, while subscript $\bar i$ corresponds to the second.

Schematic representation of dual sets composed of the above six actions in series II together with series I as the source, that is, (S^{I}, S_1^{II}, S_2^{II}, S_3^{II}) and (S^{I}, $S_{\bar{1}}^{\mathrm{II}}$, $S_{\bar{2}}^{\mathrm{II}}$, $S_{\bar{3}}^{\mathrm{II}}$), is shown by Fig. 1.

The permutation symmetry of series I, i.e. S^{I} between $\mathcal{G}_{\mu\nu}$ and $g_{\mu\nu}$ makes us consider a further application of the parent action method with respect to $g_{\mu\nu}$. This idea is quite natural after our fulfilment to $\mathcal{G}_{\mu\nu}$. That is, if we apply the parent action method to the 1-metric action with respect to the induced metric $g_{\mu\nu}$ instead, or in other words, moving $g_{\mu\nu}$ out of one-fourth root while keeping $\mathcal{G}_{\mu\nu}$ unchanged, we then derive six more 2-metric forms of Dp-brane actions, denoted by S'^{II} (subscripts suppressed), by completely following the above procedure in this section

$$S_1^{\prime\mathrm{II}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\mathcal{G})^{1/4}(-\gamma)^{1/4} \times [\gamma^{\mu\nu}g_{\mu\nu} - (p-3)], \quad (18)$$

$$S_2^{\prime\mathrm{II}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\mathcal{G})^{1/4}(-\gamma)^{1/4} \times [\Phi^{(p-3)/4}\gamma^{\mu\nu}g_{\mu\nu} - (p-3)\Phi^{(p+1)/4}], \quad (19)$$

$$S_3^{\prime\mathrm{II}} = -T_p\int d^{p+1}\xi e^{-\phi}(-\mathcal{G})^{1/4}(-\gamma)^{1/4} \times \left(\frac{1}{p+1}\gamma^{\mu\nu}g_{\mu\nu}\right)^{(p+1)/4}, \quad (20)$$

$$S_{\bar{1}}^{\prime\mathrm{II}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\mathcal{G})^{1/4}(-\gamma)^{1/4} \times [-\gamma_{\mu\nu}g^{\mu\nu} + (p+5)], \quad (21)$$

$$S_{\bar{2}}^{\prime\mathrm{II}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\mathcal{G})^{1/4}(-\gamma)^{1/4} \times [-\Phi^{(p+5)/4}\gamma_{\mu\nu}g^{\mu\nu} + (p+5)\Phi^{(p+1)/4}], \quad (22)$$

$$S_{\bar{3}}^{\prime\mathrm{II}} = -T_p\int d^{p+1}\xi e^{-\phi}(-\mathcal{G})^{1/4}(-\gamma)^{1/4} \times \left(\frac{1}{p+1}\gamma_{\mu\nu}g^{\mu\nu}\right)^{-(p+1)/4}, \quad (23)$$

which appear, to our knowledge, for the first time. They are classically equivalent and, together with the 1-metric form, constitute the two dual sets of actions, (S^{I}, $S_1^{\prime\mathrm{II}}$, $S_2^{\prime\mathrm{II}}$, $S_3^{\prime\mathrm{II}}$) and (S^{I}, $S_{\bar{1}}^{\prime\mathrm{II}}$, $S_{\bar{2}}^{\prime\mathrm{II}}$, $S_{\bar{3}}^{\prime\mathrm{II}}$). Of course, this kind of separation or classification depends on the two different ways of construction of parent actions as mentioned above. We note that S'^{II} can be obtained from S^{II} simply under the permutation between $\mathcal{G}_{\mu\nu}$ and $g_{\mu\nu}$ (for further details on permutation, see Sec. IV). Although they do not supply a simpler formulation related to field strengths, S'^{II} provide us the possibility that richer contexts of new forms beyond series II should be considered. That is the content of our next section. For the schematic representation of dualities of S'^{II}, see Fig. 2, which has the same structure as that of Fig. 1 only with the replacement of S^{II} by S'^{II}.

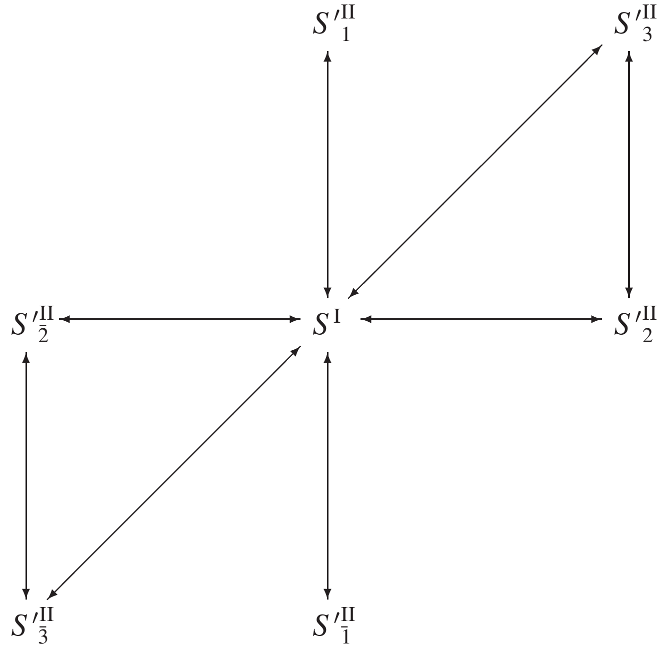

FIG. 2. Dualities in the two sets of dual actions, (S^{I}, $S_1^{\prime\mathrm{II}}$, $S_2^{\prime\mathrm{II}}$, $S_3^{\prime\mathrm{II}}$) and (S^{I}, $S_{\bar{1}}^{\prime\mathrm{II}}$, $S_{\bar{2}}^{\prime\mathrm{II}}$, $S_{\bar{3}}^{\prime\mathrm{II}}$). The line with two arrows connects two actions that are dual to each other. As a source action, the 1-metric form, S^{I}, appears in both sets. It is quite noticeable that in each set the Weyl noninvariant form, $S_1^{\prime\mathrm{II}}$ ($S_{\bar{1}}^{\prime\mathrm{II}}$), has no *direct* dualities to its corresponding Weyl invariant forms, $S_2^{\prime\mathrm{II}}$ and $S_3^{\prime\mathrm{II}}$ ($S_{\bar{2}}^{\prime\mathrm{II}}$ and $S_{\bar{3}}^{\prime\mathrm{II}}$).

As a summary, in series II, i.e., 2-metric series, which contains one induced metric $g_{\mu\nu}$ and one auxiliary worldvolume (sometimes called *intrinsic*) metric $\gamma_{\mu\nu}$ (both of them are symmetric), we obtain 12 different forms of actions that are equivalent at classical level. As of derivation of series II, six of them, S_i^{II} and $S_{\bar{i}}^{\mathrm{II}}$, where $i = 1, 2, 3$ and $\bar{i} = \bar{1}, \bar{2}, \bar{3}$, are related to the application of the parent action method to series I with respect to $\mathcal{G}_{\mu\nu}$, while the other six, $S_i^{\prime\mathrm{II}}$ and $S_{\bar{i}}^{\prime\mathrm{II}}$, however, with respect to $g_{\mu\nu}$. Moreover, in both of S^{II} and S'^{II}, the two proposals for construction of parent actions are considered. The clear advantage of this method exists, we may say, in the two aspects, one is its systematicness and the other its natural connection with duality as we have seen in the above discussions. The result shows a richer context of new actions than that of bosonic p branes in which only appears the induced metric $g_{\mu\nu}$ in the Nambu-Goto form as a source action [26].

III. 3-METRIC FORM AND DUALITY

The symmetric status of $\mathcal{G}_{\mu\nu}$ and $g_{\mu\nu}$ in series I, i.e. S^{I} as the original form provides us the possibility to go further beyond the region of series II. That is, we can consider a new kind of action forms that are *not only* quadratic in the Abelian field strength *but also* "formally quadratic" in the coordinate of spacetime. In other words, the new actions are *not only* linear in $\mathcal{G}_{\mu\nu}$ *but also* seemingly linear in $g_{\mu\nu}$. Because the definition of $\mathcal{G}_{\mu\nu}$ contains the inverse of the induced metric $g_{\mu\nu}$, the new forms are not able to be quadratic in the coordinate of spacetime but *gain an advantage over one-fourth roots* that are nonrational. If we try to move both $\mathcal{G}_{\mu\nu}$ and $g_{\mu\nu}$ out of one-fourth roots by following the same procedure as that from series I to series II, one direct way is to introduce one more auxiliary worldvolume tensor field $\omega_{\mu\nu}$ that will play a similar role to that of the intrinsic metric $\gamma_{\mu\nu}$ introduced in series II. For a deeper understanding of the relation between $\gamma_{\mu\nu}$ and $\omega_{\mu\nu}$, see the next section. If so, there will appear three metrics in an action all of which are symmetric, that is, one induced, one intrinsic and one newly introduced intrinsic-like metrics. According to the result in Sec. II, six 3-metric forms will be deduced from each 2-metric source action in terms of the parent action method together with the two proposals of constructing parent actions. We thus have $6 \times 6 = 36$ new forms of 3-metric actions, denoted by S^{III} (subscripts suppressed here and explained in the following paragraphs), by treating the six 2-metric forms, S_i^{II} and $S_{\bar{i}}^{\mathrm{II}}$, as source actions; and moreover, we still have the other 36 new forms, S'^{III} (subscripts suppressed here and explained in the following paragraphs), by using the remaining six 2-metric source forms, $S_i'^{\mathrm{II}}$ and $S_{\bar{i}}'^{\mathrm{II}}$, where $i = 1, 2, 3$ and $\bar{i} = \bar{1}, \bar{2}, \bar{3}$. Consequently, we will acquire $36 + 36 = 72$ newly proposed 3-metric forms in total.

In series III, i.e., 3-metric series, which includes an additional worldvolume metric $\omega_{\mu\nu}$ besides the induced and intrinsic ones, 72 new forms of Dp-brane actions can be derived as we stated above. Alternatively, they can also be obtained by treating the 1-metric action S^{I} as the source and doing two successive times of application of the parent action method to it. If we apply the parent action method to the 1-metric action with respect firstly to $\mathcal{G}_{\mu\nu}$, and secondly to the induced metric $g_{\mu\nu}$, called order $(\mathcal{G}_{\mu\nu}, g_{\mu\nu})$, that is, moving $\mathcal{G}_{\mu\nu}$ out of its one-fourth root at first and then moving $g_{\mu\nu}$, we therefore gain the 36 forms S^{III}. As the derivation of the new 3-metric forms is, though tedious, straightforward by following one of the two schemes (one is to apply the parent action method just once by treating the 2-metric forms as source actions, and the other is to apply the parent action method twice but starting from the 1-metric form as the source), we omit the procedure but just simply list the forms as a whole in Appendix A. However, we write down, as an example, one of typical actions with 3-metrics as follows:

$$S_{11}^{\mathrm{III}} = -\frac{T_p}{16} \int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)][\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)], \quad (24)$$

which is obtained by following the order $(\mathcal{G}_{\mu\nu}, g_{\mu\nu})$ with the consideration of the first proposal of constructing parent actions for both $\mathcal{G}_{\mu\nu}$ and $g_{\mu\nu}$. This is a rational form in the induced metric, which gains an advantage over one-fourth roots with the price paid by introducing one more auxiliary worldvolume metric $\omega_{\mu\nu}$ as was mentioned in the above paragraph. When describing $D3$ branes, i.e. Eq. (33), this action form turns to possess the double Weyl invariance (see the next section for details). Moreover, if we do in the reverse order, i.e., order $(g_{\mu\nu}, \mathcal{G}_{\mu\nu})$, we get the other 36 forms S'^{III} that are listed in Appendix B. Of course, the 72 forms are equivalent at classical level, that is, all of them give rise to the same equations of motion. Note that in the notation of series III two subscripts are needed in which subscript i corresponds to the first proposal of writing parent actions, while subscript $\bar{i}$ corresponds to the second because of twice of application of the parent action method. This is different from the case of series II in which the parent action method is applied only once. For example, the symbol $S_{i\bar{j}}^{\mathrm{III}}$ denotes (a) S^{III} corresponds to order $(\mathcal{G}_{\mu\nu}, g_{\mu\nu})$, and (b) subscript i corresponds to firstly moving $\mathcal{G}_{\mu\nu}$ in terms of the first proposal while subscript $\bar{j}$ to secondly moving $g_{\mu\nu}$ in terms of the second proposal. The other symbols, such as S_{ij}^{III}, $S_{\bar{i}j}^{\mathrm{III}}$, and $S_{\bar{i}\bar{j}}^{\mathrm{III}}$, depend on the other modes of permutations and combinations of twice

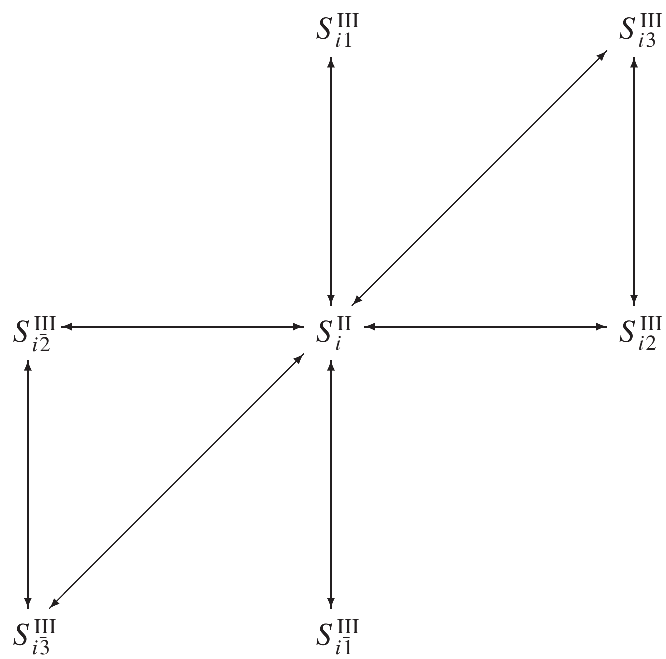

FIG. 3. Dualities in the six sets of dual actions, (S_i^{II}, S_{i1}^{III}, S_{i2}^{III}, S_{i3}^{III}) and (S_i^{II}, $S_{i\bar{1}}^{\mathrm{III}}$, $S_{i\bar{2}}^{\mathrm{III}}$, $S_{i\bar{3}}^{\mathrm{III}}$), where $i = 1, 2, 3$. The line with two arrows connects two actions that are dual to each other. As a source action, each 2-metric form, S_i^{II}, appears in its corresponding two sets. It is quite noticeable that in each set the Weyl noninvariant form, S_{i1}^{III} ($S_{i\bar{1}}^{\mathrm{III}}$), has no *direct* dualities to its corresponding Weyl invariant forms, S_{i2}^{III} and S_{i3}^{III} ($S_{i\bar{2}}^{\mathrm{III}}$ and $S_{i\bar{3}}^{\mathrm{III}}$).

use of the first and second proposals. In addition, we emphasize that one more auxiliary scalar field $\Psi(\xi)$ is necessary to be introduced for restoration of single Weyl invariance or for further improvement to double Weyl invariance besides $\Phi(\xi)$ for series III because of the existence of two intrinsic metrics, where $\Phi(\xi)$ and $\Psi(\xi)$, functions of worldvolume parameters, are scalars in both the spacetime and worldvolume, and correspond to $\gamma_{\mu\nu}$ and $\omega_{\mu\nu}$, respectively. For details of the single and double Weyl invariance, see the next section.

It is much more complicated to classify series III into dual sets than to classify series II because it depends on two successive times of use of the parent action method with respect to $\mathcal{G}_{\mu\nu}$ and $g_{\mu\nu}$, respectively, and on the consideration of two proposals of construction of parent actions. There are different kinds of classification for different choices of order of $\mathcal{G}_{\mu\nu}$ and $g_{\mu\nu}$ and of order of the two proposals. However, one direct and convenient approach is to adopt the classification for series II, which results, of course, in more number of dual sets in series III. In this way, 36 Dp-brane actions listed in Appendix A, i.e. $S^{\rm III}$ can be classified into 12 dual sets. Schematic representations of the 12 dual sets between each 2-metric source action and its corresponding six 3-metric forms are shown in Figs. 3 and 4. Similarly, $S'^{\rm III}$ constitute their own 12 dual sets of actions if we adopt the same classification as that used for $S^{\rm III}$. We ignore the schematic representations of dual sets for $S'^{\rm III}$ but just mention that they have the completely same structure as that of Figs. 3 and 4 with only the replacement of all S (including both series II and series III) by S'.

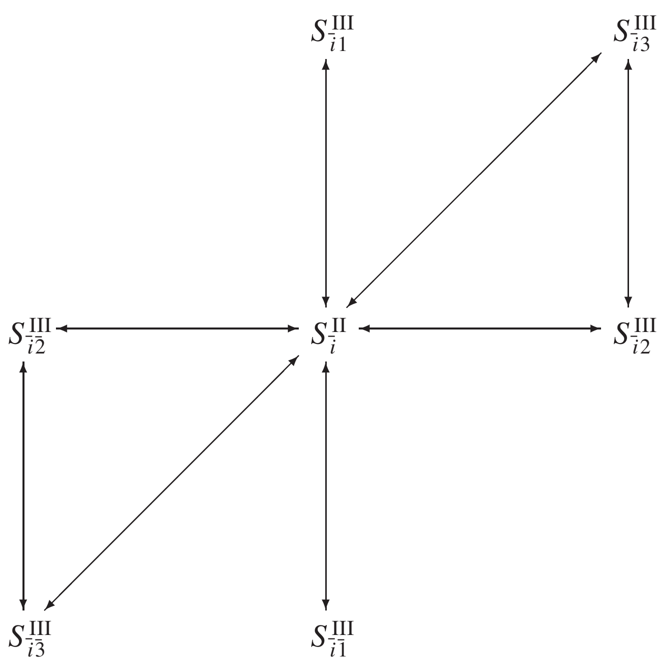

FIG. 4. Dualities in the six sets of dual actions, ($S^{\rm II}_{\bar{i}}$, $S^{\rm III}_{\bar{i}1}$, $S^{\rm III}_{\bar{i}2}$, $S^{\rm III}_{\bar{i}3}$) and ($S^{\rm II}_{\bar{i}}$, $S^{\rm III}_{\bar{i}\bar{1}}$, $S^{\rm III}_{\bar{i}\bar{2}}$, $S^{\rm III}_{\bar{i}\bar{3}}$), where $\bar{i} = \bar{1}, \bar{2}, \bar{3}$. The line with two arrows connects two actions that are dual to each other. As a source action, each 2-metric form, $S^{\rm II}_{\bar{i}}$, appears in its corresponding two sets. It is quite noticeable that in each set the Weyl noninvariant form, $S^{\rm III}_{\bar{i}1}$ ($S^{\rm III}_{\bar{i}\bar{1}}$), has no *direct* dualities to its corresponding Weyl invariant forms, $S^{\rm III}_{\bar{i}2}$ and $S^{\rm III}_{\bar{i}3}$ ($S^{\rm III}_{\bar{i}\bar{2}}$ and $S^{\rm III}_{\bar{i}\bar{3}}$).

IV. PERMUTATION AND WEYL SYMMETRIES

Nonetheless, the 72 forms of series III have some relations among themselves that might be interesting. If we divide them by the kinds of orders ($\mathcal{G}_{\mu\nu}$, $g_{\mu\nu}$) and ($g_{\mu\nu}$, $\mathcal{G}_{\mu\nu}$) that are related to the ways of following the parent action method and of proposing possible parent actions, we then obtain two parts or groups, $S^{\rm III}$ and $S'^{\rm III}$, each of which contains 36 actions as listed in Appendix A and B. We find that $S^{\rm III}$ changes to $S'^{\rm III}$ and *vice versa* under the permutation transformation between the intrinsic metric $\gamma_{\mu\nu}$ and the intrinsiclike metric $\omega_{\mu\nu}$, and simultaneously between their corresponding auxiliary scalar fields $\Phi(\xi)$ and $\Psi(\xi)$, that is,

$$\gamma_{\mu\nu} \rightleftharpoons \omega_{\mu\nu}, \qquad \Phi(\xi) \rightleftharpoons \Psi(\xi). \tag{25}$$

The one-to-one correspondence between the two groups exists as follows:

$$\begin{aligned} S^{\rm III}_{ij} &\rightleftharpoons S'^{\rm III}_{ji}, \qquad & S^{\rm III}_{i\bar{j}} &\rightleftharpoons S'^{\rm III}_{\bar{j}i}, \\ S^{\rm III}_{\bar{i}j} &\rightleftharpoons S'^{\rm III}_{j\bar{i}}, \qquad & S^{\rm III}_{\bar{i}\bar{j}} &\rightleftharpoons S'^{\rm III}_{\bar{j}\bar{i}}, \end{aligned} \tag{26}$$

where $i, j = 1, 2, 3$ and $\bar{i}, \bar{j} = \bar{1}, \bar{2}, \bar{3}$. Such a correspondence shows that $\gamma_{\mu\nu}$ and $\omega_{\mu\nu}$ have an equivalent status in series III. It is not surprising to have the conclusion because both of them are auxiliary worldvolume tensor fields. This is the reason that we call $\omega_{\mu\nu}$ an intrinsiclike metric. As to series II, although a similar one-to-one correspondence, $S^{\rm II}_{i} \rightleftharpoons S'^{\rm II}_{i}$ and $S^{\rm II}_{\bar{i}} \rightleftharpoons S'^{\rm II}_{\bar{i}}$, appears under the permutation between $\mathcal{G}_{\mu\nu}$ and $g_{\mu\nu}$, this just reflects our application of the parent action method to series I with respect to $\mathcal{G}_{\mu\nu}$ or $g_{\mu\nu}$ but nothing else.

Let us turn to investigate Weyl invariance of Dp-brane actions in series II and series III. We just consider $S^{\rm II}$ and $S^{\rm III}$. As to $S'^{\rm II}$ and $S'^{\rm III}$, similar results can be acquired through the permutation between S and S' (series II and series III, respectively).

In the six forms of $S^{\rm II}$, $S^{\rm II}_{1}$ is Weyl noninvariant for the general case of $p \neq 3$, and $S^{\rm II}_{\bar{1}}$ that contains highly nonlinear terms of field strengths is Weyl noninvariant in any dimension of worldvolume. The Weyl noninvariance of $S^{\rm II}_{\bar{1}}$ is *inevitable* because of the definition of the inverse formulation $\mathcal{G}^{\mu\nu}$ which brings about the nonzero term of $p + 5$, and a similar phenomenon happened in one of new forms for string theory [26]. However, the remaining four forms

$$S^{\rm II}_{2}, \qquad S^{\rm II}_{3}; \qquad S^{\rm II}_{\bar{2}}, \qquad S^{\rm II}_{\bar{3}}, \tag{27}$$

as they have been proposed, keep Weyl invariant for the general case under the conformal transformation:

$$\gamma_{\mu\nu}(\xi) \rightarrow \exp(\eta(\xi))\gamma_{\mu\nu}(\xi), \quad \Phi(\xi) \rightarrow \exp(-\eta(\xi))\Phi(\xi), \tag{28}$$

where $\eta(\xi)$ is an arbitrary real function of worldvolume parameters. For the special case of $p = 3$, S_1^{II} of $D3$ branes restores the Weyl invariance, which is similar to the string case of bosonic p branes [23]. Moreover, S_2^{II}, S_3^{II}, $S_{\bar{2}}^{\mathrm{II}}$, and $S_{\bar{3}}^{\mathrm{II}}$ retain their Weyl invariance, but both S_2^{II} and S_3^{II} degenerate into S_1^{II} in this special case. We note that this kind of degeneracy also appears in series III but, however, gives rise to richer contents, see below for details. Therefore, there are only four independent 2-metric forms in S^{II}, i.e., S_1^{II} and $S_{\bar{i}}^{\mathrm{II}}$ ($\bar{i} = \bar{1}, \bar{2}, \bar{3}$), that describe $D3$ branes.

It seems to be more complicated but is in fact more interesting to analyze the Weyl invariance for series III as two intrinsic metrics ($\gamma_{\mu\nu}$ and $\omega_{\mu\nu}$) appear in every form of actions. Here we point out in advance that a new phenomenon that does not exist in series II is that some actions possess a so-called bi-Weyl or double Weyl invariance. It is the appearance of two intrinsic metrics that the new phenomenon occurs. This shows the richness of series III *not only* in number of various forms as mentioned before *but also* in the aspect of Weyl invariance if compared with series II.

We now analyze Weyl invariance for the 36 forms of S^{III} in the general case of $p \neq 3$. Four of them

$$S_{11}^{\mathrm{III}}, \qquad S_{1\bar{1}}^{\mathrm{III}}, \qquad S_{\bar{1}1}^{\mathrm{III}}, \qquad S_{\bar{1}\bar{1}}^{\mathrm{III}}, \tag{29}$$

do not have such an invariance. In particular, $S_{\bar{1}\bar{1}}^{\mathrm{III}}$ is Weyl noninvariant in any dimension of worldvolume, which is *inevitable* because of the nonzero term related to $p + 5$ as happened to $S_{\bar{1}}^{\mathrm{II}}$ in series II. However, 16 forms of series III

$$\begin{cases} S_{i1}^{\mathrm{III}}, & S_{i\bar{1}}^{\mathrm{III}}, & S_{\bar{i}1}^{\mathrm{III}}, & S_{\bar{i}\bar{1}}^{\mathrm{III}}; \\ S_{1i}^{\mathrm{III}}, & S_{\bar{1}i}^{\mathrm{III}}, & S_{1\bar{i}}^{\mathrm{III}}, & S_{\bar{1}\bar{i}}^{\mathrm{III}}, \end{cases} \tag{30}$$

where $i = 2, 3$ and $\bar{i} = \bar{2}, \bar{3}$, possess the usual, or specifically, *single* Weyl invariance. To emphasize *single* is just to distinguish it from the so-called *double* Weyl invariance that will be seen soon. Eight forms on the first line of Eq. (30) correspond to the transformation Eq. (28), while the other eight on the second line of Eq. (30) maintain Weyl invariance under the transformation

$$\omega_{\mu\nu}(\xi) \rightarrow \exp(\rho(\xi))\omega_{\mu\nu}(\xi), \quad \Psi(\xi) \rightarrow \exp(-\rho(\xi))\Psi(\xi), \tag{31}$$

where $\rho(\xi)$ is another arbitrary real function of worldvolume parameters that is, in general, different from $\eta(\xi)$. At last, the remaining 16 forms of Dp-brane actions with 3-metrics

$$S_{ij}^{\mathrm{III}}, \qquad S_{i\bar{j}}^{\mathrm{III}}, \qquad S_{\bar{j}i}^{\mathrm{III}}, \qquad S_{\bar{i}\bar{j}}^{\mathrm{III}}, \tag{32}$$

where i, $j = 2$, 3 and $\bar{i}$, $\bar{j} = \bar{2}$, $\bar{3}$, possess *double* Weyl invariance, that is, they are invariant under both conformal transformations Eqs. (28) and (31). Because there are two intrinsic metrics in series III (the corresponding scalar fields $\Phi(\xi)$ and $\Psi(\xi)$ appear only in some of 3-metric forms), two Weyl transformations occur naturally. This is the new symmetry we have discovered to Dp branes.

For the special case of $p = 3$, the investigation of Weyl invariance for series III is of particular interests. To the first three forms of Eq. (29), S_{11}^{III}, reduced to be

$$S_{11}^{\mathrm{III}}(p = 3) = -\frac{T_3}{16}\int d^4\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4}(\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}) \times (\omega^{\lambda\sigma}g_{\lambda\sigma}), \tag{33}$$

where μ, ν, λ, $\sigma = 0$, 1, 2, 3, now possesses the double Weyl invariance; $S_{1\bar{1}}^{\mathrm{III}}$ and $S_{\bar{1}1}^{\mathrm{III}}$ restore the single Weyl invariance related to the transformations Eq. (28) and (31), respectively. In Eq. (30) the eight forms with subscript 1

$$S_{i1}^{\mathrm{III}}, \quad S_{\bar{i}1}^{\mathrm{III}}, \quad S_{1i}^{\mathrm{III}}, \quad S_{1\bar{i}}^{\mathrm{III}}, \quad (i = 2, 3; \bar{i} = \bar{2}, \bar{3}), \tag{34}$$

now extend their Weyl invariance from *single* to *double*, while the other eight with subscript $\bar{1}$,

$$S_{i\bar{1}}^{\mathrm{III}}, \quad S_{\bar{i}\bar{1}}^{\mathrm{III}}, \quad S_{\bar{1}i}^{\mathrm{III}}, \quad S_{\bar{1}\bar{i}}^{\mathrm{III}}, \quad (i = 2, 3; \bar{i} = \bar{2}, \bar{3}), \tag{35}$$

keep their original single Weyl invariance unchanged. Finally, the 16 forms in Eq. (32) still maintain the double Weyl invariance in the special case, which is obvious as they have such a new invariance in any dimension of worldvolume.

In this special case it seems that more actions restore single or double Weyl invariance, however, it is, in fact, that some forms of actions coincide with each other and thus the number of independent forms of S^{III} decreases from 36 to 16. Note that a similar but simpler case occurred to S^{II} as mentioned above. We classify the 16 independent forms into three sets in accordance with their Weyl invariance. The first set with *no* Weyl invariance contains only one action $S_{\bar{1}\bar{1}}^{\mathrm{III}}$ that is Weyl noninvariant in any dimension of worldvolume as we pointed out before. The second set with *single* Weyl invariance includes six different forms of actions

$$(S_{i\bar{1}}^{\mathrm{III}}), \qquad (S_{\bar{1}i}^{\mathrm{III}}), \qquad S_{\bar{i}\bar{1}}^{\mathrm{III}}, \qquad S_{\bar{1}\bar{i}}^{\mathrm{III}}, \tag{36}$$

where $i = 1$, 2, 3 and $\bar{i} = \bar{2}$, $\bar{3}$, and a bracket means that action forms inside coincide with each other. That is, the independent number of actions inside a bracket is one. The last set with *double* Weyl invariance takes the following nine forms of actions

$$\begin{matrix} (S^{\rm III}_{ij}), & (S^{\rm III}_{i\bar{2}}), & (S^{\rm III}_{i\bar{3}}), \\ (S^{\rm III}_{\bar{2}i}), & (S^{\rm III}_{\bar{3}i}), & S^{\rm III}_{\bar{i}\bar{j}}, \end{matrix} \tag{37}$$

where $i, j = 1, 2, 3$ and $\bar{i}, \bar{j} = \bar{2}, \bar{3}$, and the bracket has the same meaning as above. To the notations of Eq. (36) and (37), we give a brief explanation. The first term of Eq. (37), for instance, means that $S^{\rm III}_{i1}$ and $S^{\rm III}_{1i}$ of Eq. (34) and $S^{\rm III}_{ij}$ of Eq. (32) with double Weyl invariance, eight forms in total, are reduced to $S^{\rm III}_{11}$ of Eq. (33). We may utilize "degree of degeneracy" to describe this situation happened in the case of $p = 3$. If so, we may say the degree of degeneracy for $S^{\rm III}_{11}$ is 9. Other terms with brackets have a similar meaning but different degrees of degeneracy. The terms without brackets have the degree of degeneracy 1. In a sense, $D3$ branes are singular to the other Dp branes whose dimension of worldvolume is not equal to four. In fact, the $D3$ branes have played a central role in recent studies of Dp-brane dynamics and string theory especially for the anti-de Sitter/conformal field theory correspondence [29]. Among the 16 3-metric forms of $D3$ branes, $S^{\rm III}_{11}$ of Eq. (33) with the largest degree of degeneracy is the simplest and, in particular, double Weyl invariant. It is known [18,19] that the Weyl invariance of string theory has greatly simplified the theory's analysis and allowed its covariant quantization. The *double* Weyl invariance of $D3$ branes (Dp branes in general) may shed some light in this direction.

V. CONCLUSION

In fact, this paper focuses on a natural extension of our previous work [26] from bosonic p branes to Dp branes, that is, deriving various forms of Dp-brane actions and establishing their dualities in terms of the parent action method. In Ref. [26], we mentioned that the parent action method could be applied to Dp branes and thought naively that parallel, or precisely speaking, trivial conclusions would be made. However, our discussions above are quite nontrivial, which shows much richer contents on new forms, dualities and conformal symmetries of Dp branes than that of p branes. The richness and/or nontriviality presents, in particular, in the two aspects: (a) both 2-metric and 3-metric prescriptions are considered, and (b) an interesting new symmetry, i.e. the double conformal invariance is discovered for the first time in some action forms with 3-metrics.

Under the 2-metric prescription, we obtain 12 different forms of Dp-brane actions which are classically equivalent, that is, which give rise to the same equations of motion. The forms can be classified into the four groups or sets: $S^{\rm II}_i$, $S^{\rm II}_{\bar{i}}$, and $S'^{\rm II}_i$, $S'^{\rm II}_{\bar{i}}$, where $i = 1, 2, 3$ and $\bar{i} = \bar{1}, \bar{2}, \bar{3}$. Each set, together with the 1-metric source action $S^{\rm I}$, consists of a closed set of dual actions, and is shown visually by Figs. 1 and 2. For the conformal symmetry of $S^{\rm II}$, $S^{\rm II}_1$, and $S^{\rm II}_{\bar{1}}$ are Weyl noninvariant, while the rest (four actions) Weyl invariant in the general case of $p \neq 3$; however, $S^{\rm II}_i$ ($i = 1, 2, 3$) coincide with each other (called in this paper *degeneracy* which appears in series III with varieties) and possess the Weyl invariance, and the other three $S^{\rm II}_{\bar{i}}$ ($\bar{i} = \bar{1}, \bar{2}, \bar{3}$) keep their Weyl noninvariance or invariance unchanged in the special case of $p = 3$. As to $S'^{\rm II}$, similar results appear. The advantage of the parent action method is the systematicness, which has been pointed out in the study of bosonic p branes [26]. That is, we obtain a series of results that cover, on the one hand, the known actions (cf. $S^{\rm II}_i$) proposed by others [14,15] and provide, especially on the other hand, new actions and interesting duality structures combining series I with series II. We note that the systematicness has brought about more new actions and richer dualities in series III. Moreover, we emphasize that our two proposals for writing parent actions play an important role in the procedure of deriving new actions and building their dualities. The importance exists *not only* in the 2-metric prescription *but also* in the 3-metric prescription that is going to be summarized next.

The motivation to introduce the additional auxiliary worldvolume metric $\omega_{\mu\nu}$, or in other words, to propose the 3-metric prescription, lies in the construction of such an action form that does not involve in the one-fourth root of the induced metric $g_{\mu\nu}$. That is, our aim is to acquire such a form that is a rational functional of the induced metric as the rational formalism would probably be useful for us to analyze and/or covariantly quantize it according to the experience from the string theory [18,19]. Under the 3-metric prescription, we work out 72 equivalent Dp-brane actions, $S^{\rm III}$ and $S'^{\rm III}$, each of which contains 36 different forms. Although the classification of series III into dual sets is more complicated than that of series II, and, in particular, is not unique but depends on different procedures followed by for deriving the actions, it becomes easier and the resulting duality structure is simpler if we adopt the approach utilized in series II. For $S^{\rm III}$, 36 forms are classified into 12 dual sets as shown in detail by Figs. 3 and 4, which is the most convenient classification. As to $S'^{\rm III}$, the same duality structure exists. A quite interesting relation between $S^{\rm III}$ and $S'^{\rm III}$ is the permutation under the transformation Eq. (25), and the concrete one-to-one correspondence of actions is given by Eq. (26). It is the relation that we conclude the equivalent status of the two auxiliary worldvolume metrics $\gamma_{\mu\nu}$ and $\omega_{\mu\nu}$ in series III. We note that all the three metrics, $g_{\mu\nu}$, $\gamma_{\mu\nu}$ and $\omega_{\mu\nu}$ appear in a symmetric way in each action of series III. In addition, the Weyl noninvariance and invariance of all action forms of series III have been analyzed completely for the general case of $p \neq 3$ and for the special case of $p = 3$ as well in Sec. IV. Here what we want to emphasize for 3-metric actions is the double Weyl invariance that the 16 forms in Eq. (32) possess. This symmetry is associated closely with the appearance of the two worldvolume metrics and pointed out in Dp branes, to our knowledge, for the first time.

The supersymmetric generalization of the action of the Born-Infeld type Eq. (1) or of series I is straightforward [9–12]. This seems to be maintained in some forms of series II [13,16], which presumably depends on the conjecture that the supersymmetrization of spacetime might be unentangled with the introduction of the auxiliary worlvolume metric, i.e. with the 2-metric prescription. However, it was argued [10] that this would create considerable algebraic complications[4] if one attempted to introduce the auxiliary worldvolume metric field in the Dp-brane action of the Born-Infeld type. We may say, it remains somehow ambiguous and needs further studies to construct the supersymmetric 3-metric actions of Dp branes.

As to possible quantum theories of the actions with 3-metrics, we do not know how an exact role the double Weyl invariance plays in the procedure of quantization, but do know that it should not be useless because the Weyl invariance of the string action with an auxiliary worldsheet metric has played a crucial role in the covariant quantization. However, the Dp-brane case is much more complicated than that of p branes (strings when $p = 1$). In fact, we are trying to make canonical Hamiltonian analyses for the simplest formulation that possesses the double Weyl invariance, i.e. the $D3$-brane action Eq. (33) as our first example on dealing with this problem, and will, if available, give the result in a separate work. Other questions related to the 3-metric actions, such as duality with respect to field strengths and, on the other hand, construction of new conformal couplings in any worldvolume dimension to the auxiliary scalar fields $\Phi(\xi)$ and $\Psi(\xi)$ promoted as dynamical variables [30], are also under consideration.

Finally, if we set the dilaton field and NS antisymmetric two-form field be zero and deal with metrics, gauge fields, and field strengths back to spacetime instead of worldvolume in all series of Dp-brane actions derived above, we therefore obtain the results for the Born-Infeld theory.

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[4]No such difficulties occur for super p branes, see, for instance, Ref. [15].

APPENDIX A: 36 FORMS OF SERIES III WITH ORDER $(\mathcal{G}_{\mu\nu}, g_{\mu\nu})$

$$S^{\rm III}_{11} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)][\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)], \tag{A1}$$

$$S^{\rm III}_{12} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)] \times[\Psi^{(p-3)/4}\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{A2}$$

$$S^{\rm III}_{13} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)]\left(\frac{1}{p+1}\omega^{\lambda\sigma}g_{\lambda\sigma}\right)^{(p+1)/4}, \tag{A3}$$

$$S^{\rm III}_{1\bar{1}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)][-\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)], \tag{A4}$$

$$S^{\rm III}_{1\bar{2}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)] \times[-\Psi^{(p+5)/4}\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{A5}$$

$$S^{\rm III}_{1\bar{3}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)]\left(\frac{1}{p+1}\omega_{\lambda\sigma}g^{\lambda\sigma}\right)^{-(p+1)/4}, \tag{A6}$$

$$S^{\rm III}_{21} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\Phi^{(p-3)/4}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times[\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)], \tag{A7}$$

$$S^{\rm III}_{22} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\Phi^{(p-3)/4}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times[\Psi^{(p-3)/4}\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{A8}$$

$$S^{\rm III}_{23} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\Phi^{(p-3)/4}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times\left(\frac{1}{p+1}\omega^{\lambda\sigma}g_{\lambda\sigma}\right)^{(p+1)/4}, \tag{A9}$$

$$S^{\rm III}_{2\bar{1}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\Phi^{(p-3)/4}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times[-\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)], \tag{A10}$$

$$S^{\rm III}_{2\bar{2}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\Phi^{(p-3)/4}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times[-\Psi^{(p+5)/4}\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{A11}$$

$$S^{\rm III}_{2\bar{3}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[\Phi^{(p-3)/4}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times\left(\frac{1}{p+1}\omega_{\lambda\sigma}g^{\lambda\sigma}\right)^{-(p+1)/4}, \tag{A12}$$

$$S^{\rm III}_{31} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}\right)^{(p+1)/4}[\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)], \tag{A13}$$

$$S^{\rm III}_{32} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}\right)^{(p+1)/4} \times[\Psi^{(p-3)/4}\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{A14}$$

$$S^{\rm III}_{33} = -T_p\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}\right)^{(p+1)/4}\left(\frac{1}{p+1}\omega^{\lambda\sigma}g_{\lambda\sigma}\right)^{(p+1)/4}, \tag{A15}$$

$$S^{\rm III}_{3\bar{1}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}\right)^{(p+1)/4}[-\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)], \tag{A16}$$

$$S^{\rm III}_{3\bar{2}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}\right)^{(p+1)/4} \times[-\Psi^{(p+5)/4}\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{A17}$$

$$S^{\rm III}_{3\bar{3}} = -T_p\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma^{\mu\nu}\mathcal{G}_{\mu\nu}\right)^{(p+1)/4} \times\left(\frac{1}{p+1}\omega_{\lambda\sigma}g^{\lambda\sigma}\right)^{-(p+1)/4}, \tag{A18}$$

$$S^{\rm III}_{\bar{1}1} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)][\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)], \tag{A19}$$

$$S^{\rm III}_{\bar{1}2} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)] \times[\Psi^{(p-3)/4}\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{A20}$$

$$S^{\rm III}_{\bar{1}3} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)]\left(\frac{1}{p+1}\omega^{\lambda\sigma}g_{\lambda\sigma}\right)^{(p+1)/4}, \tag{A21}$$

$$S^{\rm III}_{\bar{1}\bar{1}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)] \times[-\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)], \tag{A22}$$

$$S^{\rm III}_{\bar{1}\bar{2}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)] \times[-\Psi^{(p+5)/4}\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{A23}$$

$$S^{\rm III}_{\bar{1}\bar{3}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)]\left(\frac{1}{p+1}\omega_{\lambda\sigma}g^{\lambda\sigma}\right)^{-(p+1)/4}, \tag{A24}$$

$$S^{\mathrm{III}}_{\bar{2}1} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [-\Phi^{(p+5)/4}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times [\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)], \tag{A25}$$

$$S^{\mathrm{III}}_{\bar{2}2} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [-\Phi^{(p+5)/4}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times [\Psi^{(p-3)/4}\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{A26}$$

$$S^{\mathrm{III}}_{\bar{2}3} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [-\Phi^{(p+5)/4}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times \left(\frac{1}{p+1}\omega^{\lambda\sigma}g_{\lambda\sigma}\right)^{(p+1)/4}, \tag{A27}$$

$$S^{\mathrm{III}}_{\bar{2}\bar{1}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [-\Phi^{(p+5)/4}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times [-\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)], \tag{A28}$$

$$S^{\mathrm{III}}_{\bar{2}\bar{2}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [-\Phi^{(p+5)/4}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times [-\Psi^{(p+5)/4}\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{A29}$$

$$S^{\mathrm{III}}_{\bar{2}\bar{3}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [-\Phi^{(p+5)/4}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times \left(\frac{1}{p+1}\omega_{\lambda\sigma}g^{\lambda\sigma}\right)^{-(p+1)/4}, \tag{A30}$$

$$S^{\mathrm{III}}_{\bar{3}1} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu}\right)^{-(p+1)/4}[\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)], \tag{A31}$$

$$S^{\mathrm{III}}_{\bar{3}2} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu}\right)^{-(p+1)/4} \times [\Psi^{(p-3)/4}\omega^{\lambda\sigma}g_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{A32}$$

$$S^{\mathrm{III}}_{\bar{3}3} = -T_p\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu}\right)^{-(p+1)/4} \times \left(\frac{1}{p+1}\omega^{\lambda\sigma}g_{\lambda\sigma}\right)^{(p+1)/4}, \tag{A33}$$

$$S^{\mathrm{III}}_{\bar{3}\bar{1}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu}\right)^{-(p+1)/4}[-\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)], \tag{A34}$$

$$S^{\mathrm{III}}_{\bar{3}\bar{2}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu}\right)^{-(p+1)/4} \times [-\Psi^{(p+5)/4}\omega_{\lambda\sigma}g^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{A35}$$

$$S^{\mathrm{III}}_{\bar{3}\bar{3}} = -T_p\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma_{\mu\nu}\mathcal{G}^{\mu\nu}\right)^{-(p+1)/4} \times \left(\frac{1}{p+1}\omega_{\lambda\sigma}g^{\lambda\sigma}\right)^{-(p+1)/4}. \tag{A36}$$

APPENDIX B: 36 FORMS OF SERIES III WITH ORDER ($g_{\mu\nu}$, $\mathcal{G}_{\mu\nu}$)

$$S'^{\mathrm{III}}_{11} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\gamma^{\mu\nu}g_{\mu\nu} - (p-3)][\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)], \tag{B1}$$

$$S'^{\mathrm{III}}_{12} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\gamma^{\mu\nu}g_{\mu\nu} - (p-3)] \times [\Psi^{(p-3)/4}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{B2}$$

$$S'^{\mathrm{III}}_{13} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\gamma^{\mu\nu}g_{\mu\nu} - (p-3)]\left(\frac{1}{p+1}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma}\right)^{(p+1)/4}, \tag{B3}$$

$$S'^{\rm III}_{1\bar{1}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\gamma^{\mu\nu}g_{\mu\nu} - (p-3)][-\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)], \tag{B4}$$

$$S'^{\rm III}_{1\bar{2}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\gamma^{\mu\nu}g_{\mu\nu} - (p-3)] \times [-\Psi^{(p+5)/4}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{B5}$$

$$S'^{\rm III}_{1\bar{3}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\gamma^{\mu\nu}g_{\mu\nu} - (p-3)]\left(\frac{1}{p+1}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma}\right)^{-(p+1)/4}, \tag{B6}$$

$$S'^{\rm III}_{21} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\Phi^{(p-3)/4}\gamma^{\mu\nu}g_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times [\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)], \tag{B7}$$

$$S'^{\rm III}_{22} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\Phi^{(p-3)/4}\gamma^{\mu\nu}g_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times [\Psi^{(p-3)/4}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{B8}$$

$$S'^{\rm III}_{23} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\Phi^{(p-3)/4}\gamma^{\mu\nu}g_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times \left(\frac{1}{p+1}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma}\right)^{(p+1)/4}, \tag{B9}$$

$$S'^{\rm III}_{2\bar{1}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\Phi^{(p-3)/4}\gamma^{\mu\nu}g_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times [-\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)], \tag{B10}$$

$$S'^{\rm III}_{2\bar{2}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\Phi^{(p-3)/4}\gamma^{\mu\nu}g_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times [-\Psi^{(p+5)/4}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{B11}$$

$$S'^{\rm III}_{2\bar{3}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [\Phi^{(p-3)/4}\gamma^{\mu\nu}g_{\mu\nu} - (p-3)\Phi^{(p+1)/4}] \times \left(\frac{1}{p+1}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma}\right)^{-(p+1)/4}, \tag{B12}$$

$$S'^{\rm III}_{31} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma^{\mu\nu}g_{\mu\nu}\right)^{(p+1)/4}[\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)], \tag{B13}$$

$$S'^{\rm III}_{32} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma^{\mu\nu}g_{\mu\nu}\right)^{(p+1)/4} \times [\Psi^{(p-3)/4}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \tag{B14}$$

$$S'^{\rm III}_{33} = -T_p\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma^{\mu\nu}g_{\mu\nu}\right)^{(p+1)/4}\left(\frac{1}{p+1}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma}\right)^{(p+1)/4}, \tag{B15}$$

$$S'^{\rm III}_{3\bar{1}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma^{\mu\nu}g_{\mu\nu}\right)^{(p+1)/4}[-\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)], \tag{B16}$$

$$S'^{\rm III}_{3\bar{2}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma^{\mu\nu}g_{\mu\nu}\right)^{(p+1)/4} \times [-\Psi^{(p+5)/4}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \tag{B17}$$

$$S'^{\rm III}_{3\bar{3}} = -T_p\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times \left(\frac{1}{p+1}\gamma^{\mu\nu}g_{\mu\nu}\right)^{(p+1)/4} \times \left(\frac{1}{p+1}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma}\right)^{-(p+1)/4}, \tag{B18}$$

$$S'^{\rm III}_{\bar{1}1} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times [-\gamma_{\mu\nu}g^{\mu\nu} + (p+5)][\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)], \tag{B19}$$

$$S'^{\rm III}_{\bar{1}2} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}g^{\mu\nu} + (p+5)] \times[\Psi^{(p-3)/4}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \quad (B20)$$

$$S'^{\rm III}_{\bar{1}3} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}g^{\mu\nu} + (p+5)]\left(\frac{1}{p+1}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma}\right)^{(p+1)/4}, \quad (B21)$$

$$S'^{\rm III}_{\bar{1}\bar{1}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}g^{\mu\nu} + (p+5)] \times[-\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)], \quad (B22)$$

$$S'^{\rm III}_{\bar{1}\bar{2}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}g^{\mu\nu} + (p+5)] \times[-\Psi^{(p+5)/4}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \quad (B23)$$

$$S'^{\rm III}_{\bar{1}\bar{3}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\gamma_{\mu\nu}g^{\mu\nu} + (p+5)]\left(\frac{1}{p+1}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma}\right)^{-(p+1)/4}, \quad (B24)$$

$$S'^{\rm III}_{\bar{2}1} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\Phi^{(p+5)/4}\gamma_{\mu\nu}g^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times[\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)], \quad (B25)$$

$$S'^{\rm III}_{\bar{2}2} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\Phi^{(p+5)/4}\gamma_{\mu\nu}g^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times[\Psi^{(p-3)/4}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \quad (B26)$$

$$S'^{\rm III}_{\bar{2}3} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\Phi^{(p+5)/4}\gamma_{\mu\nu}g^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times\left(\frac{1}{p+1}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma}\right)^{(p+1)/4}, \quad (B27)$$

$$S'^{\rm III}_{\bar{2}\bar{1}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\Phi^{(p+5)/4}\gamma_{\mu\nu}g^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times[-\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)], \quad (B28)$$

$$S'^{\rm III}_{\bar{2}\bar{2}} = -\frac{T_p}{16}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\Phi^{(p+5)/4}\gamma_{\mu\nu}g^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times[-\Psi^{(p+5)/4}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \quad (B29)$$

$$S'^{\rm III}_{\bar{2}\bar{3}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times[-\Phi^{(p+5)/4}\gamma_{\mu\nu}g^{\mu\nu} + (p+5)\Phi^{(p+1)/4}] \times\left(\frac{1}{p+1}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma}\right)^{-(p+1)/4}, \quad (B30)$$

$$S'^{\rm III}_{\bar{3}1} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma_{\mu\nu}g^{\mu\nu}\right)^{-(p+1)/4}[\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)], \quad (B31)$$

$$S'^{\rm III}_{\bar{3}2} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma_{\mu\nu}g^{\mu\nu}\right)^{-(p+1)/4} \times[\Psi^{(p-3)/4}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma} - (p-3)\Psi^{(p+1)/4}], \quad (B32)$$

$$S'^{\rm III}_{\bar{3}3} = -T_p\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma_{\mu\nu}g^{\mu\nu}\right)^{-(p+1)/4} \times\left(\frac{1}{p+1}\omega^{\lambda\sigma}\mathcal{G}_{\lambda\sigma}\right)^{(p+1)/4}, \quad (B33)$$

$$S'^{\rm III}_{\bar{3}\bar{1}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma_{\mu\nu}g^{\mu\nu}\right)^{-(p+1)/4}[-\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)], \quad (B34)$$

$$S'^{\mathrm{III}}_{\bar{3}\bar{2}} = -\frac{T_p}{4}\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma_{\mu\nu}g^{\mu\nu}\right)^{-(p+1)/4} \times[-\Psi^{(p+5)/4}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma} + (p+5)\Psi^{(p+1)/4}], \quad \text{(B35)}$$

$$S'^{\mathrm{III}}_{\bar{3}\bar{3}} = -T_p\int d^{p+1}\xi e^{-\phi}(-\gamma)^{1/4}(-\omega)^{1/4} \times\left(\frac{1}{p+1}\gamma_{\mu\nu}g^{\mu\nu}\right)^{-(p+1)/4} \times\left(\frac{1}{p+1}\omega_{\lambda\sigma}\mathcal{G}^{\lambda\sigma}\right)^{-(p+1)/4}. \quad \text{(B36)}$$

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