Gauge symmetries decrease the number of Dp-brane dimensions. II. Inclusion of the Liouville term

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(Received 10 November 2006; revised manuscript received 16 March 2007; published 18 April 2007)

The presence of the antisymmetric background field $B_{\mu\nu}$ leads to the noncommutativity of the Dp-brane manifold, while the linear dilaton field in the form $\Phi(x) = \Phi_0 + a_\mu x^\mu$ causes the appearance of the commutative Dp-brane coordinate, $x_c = a_\mu x^\mu$. In the present article we consider the case where the conformal invariance is realized by inclusion of the Liouville term. Then, the theory is conformally invariant even in the presence of the world-sheet conformal factor F, and it depends on the new parameter, the central charge c. As well as in the absence of the Liouville action, for particular relations between background fields, the local gauge symmetries appear in the theory. They turn some Neumann boundary conditions into the Dirichlet ones, and decrease the number of the Dp-brane dimensions.

DOI: 10.1103/PhysRevD.75.085011 PACS numbers: 11.10.Nx, 04.20.Fy, 11.25.−w

I. INTRODUCTION

When the ends of the open string are attached to a Dp brane with antisymmetric Kalb-Ramond field $B_{\mu\nu}$, the Dp-brane world volume becomes noncommutative [1]. The presence of the linear dilaton field Φ [2–4] turns one Dp-brane coordinate, $x_c = x^\mu \partial_\mu \Phi$, to a commutative one and the conformal part of the world-sheet metric, F, to an additional noncommutative variable [3].

The possible breaking of the conformal invariance in the open string theory by the boundary conditions has been investigated in Ref. [4]. It was shown that, besides vanishing of the β functions, in the case of the linear dilaton there are additional conditions that dilaton gradient $a_i = \partial_i \Phi$ must satisfy. It should be lightlike vector, either with respect to the closed string metric, $a^2 \equiv G^{ij} a_i a_j = 0$, or with respect to the open string (effective) metric, $\tilde{a}^2 \equiv (G_{\rm eff}^{-1})^{ij} a_i a_j = 0$. The above restrictions decrease the number of the Dp-brane dimensions, turning some Neumann to Dirichlet boundary conditions.

In the present paper we change the conditions for quantum conformal invariance. The usual requirement is vanishing of all three β functions corresponding to background fields, $\beta^G_{\mu\nu} = \beta^B_{\mu\nu} = \beta^\Phi = 0$. Here, we use the fact that the vanishing of two β functions, corresponding to the metric and antisymmetric field, $\beta^G_{\mu\nu} = \beta^B_{\mu\nu} = 0$, implies that the third one, corresponding to dilaton field, is constant, $\beta^\Phi = c$ [5]. Instead of choosing this constant to be zero, as we did in the previous paper [4], in this article we add the Liouville term in order to cancel constant contribution to the conformal anomaly. This approach is more general because the theory and, particularly, the noncommutativity parameter depend on arbitrary central charge c. The advantage is the achievement of the conformal invariance without the requirement for decoupling of the conformal factor of the world-sheet metric, F. Consequently, for $c \neq 0$ the presence of the field F in boundary conditions does not break conformal invariance as in Ref. [4].

In order to clarify notation and terminology we will distinguish two descriptions of the open string theory. We start with variable x^μ and background field $G_{\mu\nu}$, where the theory is described by equations of motion and boundary conditions. We are able to solve boundary conditions and introduce the effective theory defined only by equations of motion. This is again string theory, but in terms of effective coordinates q^μ (symmetric under transformation $\sigma \to -\sigma$) and effective background field $G^{\rm eff}_{\mu\nu}$. Following Seiberg and Witten [1], we use the names closed string metric for $G_{\mu\nu}$ and open string metric for $G^{\rm eff}_{\mu\nu}$ (the metric tensors seen by the closed and open string, respectively).

The Liouville action itself is the kinetic term for the field F. So, we are going to treat it equally with other variables. In particular, we choose the Neumann boundary condition for F. Note that, although by simple changes of variables the new field ${}^\star F$ decouples, and the term with linear dilaton disappears, the case is nontrivial because the new metric ${}^\star G_{ij}$ and the corresponding effective one ${}^\star G^{\rm eff}_{ij}$ become singular for $\alpha a^2 = 1$ and $\alpha \tilde{a}^2 = 1$, respectively [α is a useful constant defined in (2.10) proportional to the inverse central charge]. We use the mark star ($\star$) to distinguish description in terms of variables (x^i, ${}^\star F$) from that of (x^i, F).

Up to the changing of the conditions on the dilaton gradient a_i ($a^2 = 0 \to a^2 = \frac{1}{\alpha}$ and $\tilde{a}^2 = 0 \to \tilde{a}^2 = \frac{1}{\alpha}$), there is a complete analogy of the noncommutativity properties with the cases of the previous paper [4]. Note that here we have a whole one-parameter class of theories with the same properties and, in particular, the result of the previous paper has been obtained for ($\alpha \to \infty \Leftrightarrow c = 0$).

In the first case, $1 - \alpha a^2$ is a coefficient in front of the velocity $\dot{x}_0 = a_i \dot{x}^i$, so that condition $\alpha a^2 = 1$ produces the standard canonical constraint. By simple analysis we con-

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clude that it is of the first class. For $\alpha\tilde{a}^2 = 1$, some of the constraints originating from the boundary conditions turn from second class into first class constraints.

The first class constraints generate local gauge symmetries. They turn some of the initial Neumann boundary conditions into Dirichlet ones and decrease the number of the Dp-brane dimensions. String coordinates, which depend on the effective ones but also on the corresponding momenta, define the noncommutative subspace of Dp brane. The noncommutativity parameter is proportional to the antisymmetric field B_{ij}. The field ${}^\star F$ decouples from the rest. So, it plays the role of the commutative variable instead the variable $x_c = a_i x^i$ in the case without the Liouville term.

At the end of this paper, in the Concluding remarks, we summarize the results of the investigation. Also there are three appendixes. The first one is devoted to the projectors, which help us to express the results clearly. In the second appendix we introduce the redefined closed and open string star metrics, while in the third one we discussed the separation of the center of mass variables.

II. CONFORMAL INVARIANCE WITH THE HELP OF LIOUVILLE ACTION

The action

$$S_{(G+B+\Phi)} = \kappa \int_\Sigma d^2\xi \sqrt{-g}\left\{\left[\frac{1}{2}g^{\alpha\beta}G_{\mu\nu}(x) + \frac{\varepsilon^{\alpha\beta}}{\sqrt{-g}}B_{\mu\nu}(x)\right]\partial_\alpha x^\mu \partial_\beta x^\nu + \Phi(x)R^{(2)}\right\} \tag{2.1}$$

describes the evolution of the open string in the background consisting of the space-time metric $G_{\mu\nu}(x)$, Kalb-Ramond antisymmetric field $B_{\mu\nu}(x)$, and dilaton scalar field $\Phi(x)$ (for more details, see [6]). The world-sheet Σ is parametrized by $\xi^\alpha = (\tau, \sigma)$ $(\alpha = 0,\ 1)$, and the D-dimensional space-time by the coordinates x^μ $(\mu = 0, 1, 2, \ldots, D-1)$. The intrinsic world-sheet metric is $g_{\alpha\beta}$, and $R^{(2)}$ is the related scalar curvature.

There are three β functions corresponding to the space-time metric $G_{\mu\nu}$, antisymmetric field $B_{\mu\nu}$, and dilaton field Φ

$$\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{4}B_{\mu\rho\sigma}B_\nu{}^{\rho\sigma} + 2D_\mu a_\nu, \tag{2.2}$$

$$\beta^B_{\mu\nu} \equiv D_\rho B^\rho{}_{\mu\nu} - 2a_\rho B^\rho{}_{\mu\nu}, \tag{2.3}$$

$$\beta^\Phi \equiv 2\pi\kappa\frac{D-26}{6} - \frac{1}{24}B_{\mu\rho\sigma}B^{\mu\rho\sigma} - D_\mu a^\mu + 4a^2, \tag{2.4}$$

which characterize the conformal anomaly of the sigma model (2.1). The space-time Ricci tensor and covariant derivative are denoted with $R_{\mu\nu}$ and D_μ, respectively, while $B_{\mu\rho\sigma} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ is the field strength for the field $B_{\mu\nu}$ and $a_\mu = \partial_\mu\Phi$ is the gradient of the dilaton field.

It is known from Ref. [5] that vanishing of $\beta^G_{\mu\nu}$ and $\beta^B_{\mu\nu}$ implies constant value of the third β function, $\beta^\Phi = c$. We choose a particular solution of Eqs. (2.2) and (2.3)

$$G_{\mu\nu}(x) = G_{\mu\nu} = \text{const}, \qquad B_{\mu\nu}(x) = B_{\mu\nu} = \text{const}, \qquad \Phi(x) = \Phi_0 + a_\mu x^\mu \qquad (a_\mu = \text{const}). \tag{2.5}$$

Then Eq. (2.4) produces the condition

$$\beta^\Phi = 2\pi\kappa\frac{D-26}{6} + 4a^2 \equiv c, \tag{2.6}$$

under which the above solution is consistent with all equations of motion. On these conditions, the nonlinear sigma model (2.1) becomes conformal field theory. There exists a Virasoro algebra with central charge c.

The remaining anomaly, represented by the Schwinger term of the Virasoro algebra, can also be canceled by introducing a corresponding Wess-Zumino term, which in the case of the conformal anomaly takes the form of the Liouville action

$$S_L = -\frac{\beta^\Phi}{2(4\pi)^2\kappa}\int_\Sigma d^2\xi\sqrt{-g}R^{(2)}\frac{1}{\Delta}R^{(2)}, \qquad \Delta = g^{\alpha\beta}\nabla_\alpha\partial_\beta, \tag{2.7}$$

where ∇_α is the covariant derivative with respect to the intrinsic metric $g_{\alpha\beta}$. An appropriate choice of the coefficient in front of the Liouville action makes the theory fully conformally invariant and the complete action takes the form

$$S = S_{(G+B+\Phi)} + S_L. \tag{2.8}$$

We choose a particular background, decomposing the space-time coordinates $x^\mu(\xi)$ in Dp-brane coordinates denoted by $x^i(\xi)(i = 0, 1, \ldots, p)$ and the orthogonal ones $x^a(\xi)(a = p+1, p+2, \ldots, D-1)$, in such a way that $G_{\mu\nu} = 0$ $(\mu = i,\ \nu = a)$. For the other two background fields we choose $B_{\mu\nu} \to B_{ij}$, $a_\mu \to a_i$; i.e. they are nontrivial only on the Dp brane. The part of the action describing the string oscillation in x^a directions decouples from the rest.

Imposing the conformal gauge $g_{\alpha\beta} = e^{2F}\eta_{\alpha\beta}$, we obtain $R^{(2)} = -2\Delta F$ and the action (2.8) takes the form

$$S = \kappa\int_\Sigma d^2\xi\left[\left(\frac{1}{2}\eta^{\alpha\beta}G_{ij} + \epsilon^{\alpha\beta}B_{ij}\right)\partial_\alpha x^i\partial_\beta x^j + 2\eta^{\alpha\beta}a_i\partial_\alpha x^i\partial_\beta F + \frac{2}{\alpha}\eta^{\alpha\beta}\partial_\alpha F\partial_\beta F\right], \tag{2.9}$$

where we introduce the useful notation

$$\frac{1}{\alpha} = \frac{\beta^\Phi}{(4\pi\kappa)^2}. \tag{2.10}$$

The field F is a dynamical variable with the Liouville action as a kinetic term. In order to cancel the term linear in F, we change the variables, $F \to {}^\star F = F + \frac{\alpha}{2} a_i x^i$, and obtain

$$S = \kappa \int_\Sigma d^2\xi \bigg[\bigg(\frac{1}{2} \eta^{\alpha\beta}\, {}^\star G_{ij} + \epsilon^{\alpha\beta} B_{ij} \bigg) \partial_\alpha x^i \partial_\beta x^j + \frac{2}{\alpha} \eta^{\alpha\beta} \partial_\alpha {}^\star F \partial_\beta {}^\star F \bigg]. \tag{2.11}$$

This is a standard form of the action without the dilaton term and with the redefined Liouville term $F \to {}^\star F$, and redefined space-time metric ${}^\star G_{ij} = G_{ij} - \alpha a_i a_j$. The dilaton dependence is now through the metric ${}^\star G_{ij}$.

We choose Neumann boundary conditions for the redefined conformal factor of the intrinsic metric ${}^\star F$. The field ${}^\star F$ completely decouples, as well as the coordinate x^a, but because of its Neumann boundary conditions, we will treat it as a Dp-brane variable. In all cases it is a commutative variable.

All nontrivial features of the model (2.11) follow from the fact that the star metrics (${}^\star G_{ij}$ and the corresponding effective one ${}^\star G^{\rm eff}_{ij}$) are singular and consequently they produce gauge symmetries of the theory. It is easy to check that for $\alpha a^2 = 1$ and $\alpha \tilde a^2 = 1$ we have $\det {}^\star G_{ij} = 0$ and $\det {}^\star G^{\rm eff}_{ij} = 0$, respectively.

III. NONCOMMUTATIVITY FOR REGULAR STAR METRICS ${}^\star G_{ij}$ AND ${}^\star G^{\rm eff}_{ij}$ ($\alpha a^2 \neq 1$ AND $\alpha \tilde a^2 \neq 1$)

In this section we will analyze the case when both the metric ${}^\star G_{ij}$ and the corresponding effective one ${}^\star G^{\rm eff}_{ij}$ are nonsingular. Up to the field ${}^\star F$, which is decoupled from the other Dp-brane variables, there is complete formal analogy with the case without dilaton field with substitution $G_{ij} \to {}^\star G_{ij}$. For completeness we present the main steps of the procedure and add the parts corresponding to ${}^\star F$.

A. Canonical Hamiltonian in terms of currents

The momenta canonically conjugated to the fields x^i and ${}^\star F$ are

$$\pi_i = \kappa({}^\star G_{ij} \dot x^j - 2B_{ij} x'^j), \qquad \pi = \frac{4\kappa}{\alpha} {}^\star \dot F. \tag{3.1}$$

Using the definition of the canonical Hamiltonian $\mathcal{H}_c = \pi_i \dot x^i + \pi {}^\star \dot F - \mathcal{L}$, we obtain

$$H_c = \int d\sigma \mathcal{H}_c, \qquad \mathcal{H}_c = T_- - T_+,$$
$$T_\pm = \mp \frac{1}{4\kappa} \bigg[({}^\star G^{-1})^{ij}\, {}^\star j_{\pm i}\, {}^\star j_{\pm j} + \frac{\alpha}{4}\, {}^\star j_{\pm(F)}\, {}^\star j_{\pm(F)} \bigg], \tag{3.2}$$

where

$$
\begin{aligned}
{}^\star j_{\pm i} &= \pi_i + 2\kappa\, {}^\star \Pi_{\pm ij} x'^j, \\
{}^\star j_{\pm(F)} &= \pi \pm \frac{4\kappa}{\alpha} {}^\star F' \\
({}^\star \Pi_{\pm ij} &= B_{ij} \pm \tfrac{1}{2} {}^\star G_{ij})
\end{aligned}
\tag{3.3}$$

and the inverse metric $({}^\star G^{-1})^{ij}$ is introduced in Eq. (B2).

From the basic Poisson bracket algebra

$$
\begin{aligned}
\{x^i(\tau,\sigma), \pi_j(\tau,\bar\sigma)\} &= \delta^i{}_j \delta(\sigma - \bar\sigma), \\
\{{}^\star F(\tau,\sigma), \pi(\tau,\bar\sigma)\} &= \delta(\sigma - \bar\sigma),
\end{aligned}
\tag{3.4}$$

directly follows the current algebra

$$
\begin{aligned}
\{{}^\star j_{\pm i}, {}^\star j_{\pm j}\} &= \pm 2\kappa\, {}^\star G_{ij} \delta', \\
\{{}^\star j_{\pm(F)}, {}^\star j_{\pm(F)}\} &= \pm \frac{8\kappa}{\alpha} \delta', \\
\{{}^\star j_{\pm i}, {}^\star j_{\pm(F)}\} &= 0,
\end{aligned}
\tag{3.5}$$

while all opposite chirality currents commute and for simplicity we define $\delta' \equiv \partial_\sigma \delta(\sigma - \bar\sigma)$. Consequently, the Poisson brackets between canonical Hamiltonian and the currents ${}^\star j_{\pm i}$ and ${}^\star j_{\pm(F)}$ are proportional to their sigma derivatives

$$\{H_c, {}^\star j_{\pm i}\} = \mp {}^\star j'_{\pm i}, \qquad \{H_c, {}^\star j_{\pm(F)}\} = \mp {}^\star j'_{\pm(F)}. \tag{3.6}$$

B. Boundary conditions as canonical constraints

We will use Neumann boundary conditions for the fields x^i and ${}^\star F$. The boundary conditions are of the form $\gamma_i^{(0)}|_0^\pi = 0$ and $\gamma^{(0)}|_0^\pi = 0$, where

$$
\begin{aligned}
\gamma_i^{(0)} &= \frac{\partial \mathcal{L}}{\partial(\partial_\sigma x^i)} = \kappa(-{}^\star G_{ij} x'^j + 2B_{ij} \dot x^j), \\
\gamma^{(0)} &= \frac{\partial \mathcal{L}}{\partial(\partial_\sigma {}^\star F)} = -\frac{4\kappa}{\alpha} {}^\star F'.
\end{aligned}
\tag{3.7}$$

They can be rewritten in terms of the currents (3.3) as

$$
\begin{aligned}
\gamma_i^{(0)} &= ({}^\star \Pi_+ {}^\star G^{-1})_i{}^j\, {}^\star j_{-j} + ({}^\star \Pi_- {}^\star G^{-1})_i{}^j\, {}^\star j_{+j}, \\
\gamma^{(0)} &= \tfrac{1}{2}[{}^\star j_{-(F)} - {}^\star j_{+(F)}],
\end{aligned}
\tag{3.8}$$

and treated as canonical constraints. Examining the consistency of the constraints, with the help of the relations (3.6), we obtain an infinite set of constraints. Using Taylor expansion, we rewrite all the constraints at $\sigma = 0$ in a more compact σ-dependent form

$$
\begin{aligned}
\Gamma_i(\sigma) &= ({}^\star \Pi_+ {}^\star G^{-1})_i{}^j\, {}^\star j_{-j}(\sigma) + ({}^\star \Pi_- {}^\star G^{-1})_i{}^j\, {}^\star j_{+j}(-\sigma), \\
\Gamma(\sigma) &= \tfrac{1}{2}[{}^\star j_{-(F)}(\sigma) - {}^\star j_{+(F)}(-\sigma)].
\end{aligned}
\tag{3.9}$$

In the same way, we obtain similar expressions from the constraints at $\sigma = \pi$. From the fact that the differences of the corresponding constraints at $\sigma = 0$ and $\sigma = \pi$ are also constraints, we conclude that all positive chirality currents and, consequently, all variables are 2π periodic functions. Because of this periodicity the constraints at $\sigma = \pi$ can be discarded (for more details see Ref. [3]).

We complete the consistency procedure finding the Poisson brackets

$$\{H_c, \Gamma_i\} = \Gamma_i', \qquad \{H_c, \Gamma\} = \Gamma', \tag{3.10}$$

which means that there are no more constraints in the theory.

The algebra of the constraints $\chi_A = (\Gamma_i, \Gamma)$ has a simple matrix form

$$\{\chi_A(\sigma), \chi_B(\bar\sigma)\} = -\kappa M_{AB}\delta', \qquad M_{AB} = \begin{pmatrix} {}^\star G^{\rm eff}_{ij} & 0 \\ 0 & \frac{4}{\alpha} \end{pmatrix}. \tag{3.11}$$

The space-time component, which we will call the effective or open string metric, is defined in Eq. (B3). The determinant

$$\det M_{AB} = \frac{4}{\alpha}\tilde A \mathcal{A} \det G^{\rm eff}_{ij} = \frac{4}{\alpha}\frac{\tilde A^2}{A}\det G^{\rm eff}_{ij}, \tag{3.12}$$

is regular for $\tilde A \equiv 1 - \alpha\tilde a^2 \neq 0$ and $A \equiv 1 - \alpha a^2 \neq 0$, and all constraints are of the second class. The fields G_{ij} and B_{ij} are chosen in such a way that $\det G^{\rm eff}_{ij} \neq 0$.

C. Solution of the constraint equations

Let us introduce the common symbol for the coordinates and their canonically conjugated momenta, $C^A = (x^i, {}^\star F, \pi_i, \pi)$. It is useful to define the symmetric and antisymmetric parts in σ parity as

$$O^A(\sigma) = \tfrac{1}{2}[C^A(\sigma) + C^A(-\sigma)], \qquad \bar O^A(\sigma) = \tfrac{1}{2}[C^A(\sigma) - C^A(-\sigma)], \tag{3.13}$$

where to $O^A = (q^i, {}^\star f, p_i, p)$ we will refer as the effective variables. In terms of these variables, the constraints $\Gamma_i(\sigma)$ and $\Gamma(\sigma)$ have the form

$$\Gamma_i = 2(B\,{}^\star G^{-1})_i{}^j p_j - \kappa\,{}^\star G^{\rm eff}_{ij}\bar q'^j + \bar p_i, \qquad \Gamma = \bar p - \frac{4\kappa}{\alpha}{}^\star\bar f'. \tag{3.14}$$

From

$$\Gamma_i(\sigma) = 0, \qquad \Gamma(\sigma) = 0, \tag{3.15}$$

choosing integration constants $\bar q^i(\sigma = 0) = 0$ and ${}^\star\bar f(\sigma = 0) = 0$, we obtain the solution for string variables expressed in terms of the effective ones

$$x^i(\sigma) = q^i(\sigma) - 2\,{}^\star\Theta^{ij}\int_0^\sigma d\sigma_1 p_j(\sigma_1), \qquad \pi_i = p_i, \tag{3.16}$$

$${}^\star F = {}^\star f, \qquad \pi = p. \tag{3.17}$$

Note that as we explained in Sec. I, the string variables x^i and π_i describe the string dynamics before solving constraints originating from boundary conditions, while the effective ones, q^i and p_i, describe the string after solving constraints.

The parameter ${}^\star\Theta^{ij}$ is defined as

$${}^\star\Theta^{ij} = -\tfrac{1}{\kappa}({}^\star G^{-1}_{\rm eff}B\,{}^\star G^{-1})^{ij} = -\tfrac{1}{\kappa}(G^{-1}_{\rm eff}\check\Pi^0_T B G^{-1}\check\Pi^0_T)^{ij}, \tag{3.18}$$

where

$$(\check\Pi^0_T)_i{}^j = \delta_i{}^j - (1 - \tfrac{1}{A})(\Pi_0)_i{}^j, \tag{3.19}$$

and the projector $(\Pi_0)_i{}^j$ is introduced in Eq. (A4).

In terms of ${}^\star G_{ij}$, the parameter ${}^\star\Theta^{ij}$ has the same form as the parameter Θ^{ij} in terms of G_{ij} in the case without dilaton field. Note that in this approach the noncommutativity parameter ${}^\star\Theta^{ij}$ depends on central charge c.

D. Effective theory and noncommutativity relations

Let us introduce the effective currents

$${}^\star\tilde j_{\pm i} = p_i \pm \kappa\,{}^\star G^{\rm eff}_{ij}q'^j, \qquad {}^\star\tilde j_{\pm(F)} = p \pm \frac{4\kappa}{\alpha}{}^\star f'. \tag{3.20}$$

Using the solutions (3.16) and (3.17) we correlate them with currents given in Eq. (3.3)

$${}^\star j_{\pm i} = \pm 2({}^\star\Pi_\pm\,{}^\star G^{-1}_{\rm eff})_i{}^j\,{}^\star\tilde j_{\pm j}, \qquad {}^\star j_{\pm(F)} = {}^\star\tilde j_{\pm(F)}, \tag{3.21}$$

where $({}^\star G^{-1}_{\rm eff})^{ij}$ is given in Eq. (B6). Substituting these relations in the canonical Hamiltonian (3.2), we obtain

$$T_\pm = \tilde T_\pm, \qquad \mathcal{H}_c = \tilde{\mathcal{H}}_c, \tag{3.22}$$

where we introduced an effective energy-momentum tensor and Hamiltonian

$$\tilde T_\pm = \mp\frac{1}{4\kappa}\left[({}^\star G^{-1}_{\rm eff})^{ij}\,{}^\star\tilde j_{\pm i}\,{}^\star\tilde j_{\pm j} + \frac{\alpha}{4}\,{}^\star\tilde j_{\pm(F)}\,{}^\star\tilde j_{\pm(F)}\right], \qquad \tilde{\mathcal{H}}_c = \tilde T_- - \tilde T_+. \tag{3.23}$$

The effective theory is defined in the phase space spanned by the coordinates q^i and momenta p_i in the new open string background $G_{ij} \to {}^\star G^{\rm eff}_{ij}$, $B_{ij} \to 0$, and $\Phi \to 0$. The free field, which effective dynamics is described by ${}^\star f$ and p, decouples from the rest.

From the basic string variables algebra (3.4), we calculate the corresponding effective string one

$$\{q^i(\tau,\sigma), p_j(\tau,\bar\sigma)\} = \delta^i_j\delta_s(\sigma,\bar\sigma), \qquad \{{}^\star f(\tau,\sigma), p(\tau,\bar\sigma)\} = \delta_s(\sigma,\bar\sigma), \tag{3.24}$$

where $\delta_s(\sigma,\bar\sigma) = \frac{1}{2}[\delta(\sigma - \bar\sigma) + \delta(\sigma + \bar\sigma)]$.

Separating the center of mass variables according to Appendix C, we obtain

$$\{X^i(\sigma), X^j(\bar\sigma)\} = {}^\star\Theta^{ij}\Delta(\sigma+\bar\sigma), \tag{3.25}$$

$$\{X^i(\sigma), {}^\star\mathcal{F}(\bar\sigma)\} = 0, \qquad \{{}^\star\mathcal{F}(\sigma), {}^\star\mathcal{F}(\bar\sigma)\} = 0, \tag{3.26}$$

where the function $\Delta(x)$ is given in Eq. (C5), and X^i and ${}^\star\mathcal{F}$ are defined in (C3) and (C6), respectively.

The fields x^i are noncommutative variables, while the field ${}^\star F$ is a commutative one. So, the Dp brane is described by $p+1$ noncommutative and one commutative degree of freedom.

IV. NONCOMMUTATIVITY FOR SINGULAR ${}^\star G_{ij}$ ($\alpha a^2 = 1$)

In order to express the velocities in terms of the canonical momenta, the coefficients in front of the velocities must be different from zero. But the metric ${}^\star G_{ij}$ in front of $\dot x^i$ in (3.1) is singular for $\alpha a^2 = 1$ [see Eqs. (B1) and (B7)]. Consequently, a primary constraint appears in the theory [7]. For $\alpha a^2 = 1$ the projector

$$g_{ij} = (P_T^0 G)_{ij}, \tag{4.1}$$

takes the role of the metric in the subspace defined by the regular part of ${}^\star G_{ij}$.

A. Canonical Hamiltonian and gauge symmetry

Combining the coordinates x^i and their canonically conjugated momenta π_i (3.1) as

$${}^\star j \equiv a^i\pi_i + 2\kappa a^i B_{ij}x^{j\prime} = \kappa(1-\alpha a^2)a_i\dot x^i, \tag{4.2}$$

we conclude that, for $\alpha a^2 = 1$, ${}^\star j$ does not depend on velocities and consequently, it is a constraint of the theory.

The canonical Hamiltonian $\mathcal{H}_c = \pi_i\dot x^i + \pi{}^\star\dot F - \mathcal{L}$ in terms of currents has the form

$$\mathcal{H}_c = T_- - T_+,$$
$$T_\pm = \mp\frac{1}{4\kappa}\left[(g^{-1})^{ij}\,{}^\star j_{\pm i}\,{}^\star j_{\pm j} + \frac{\alpha}{4}\,{}^\star j_{\pm(F)}\,{}^\star j_{\pm(F)}\right], \tag{4.3}$$

where

$${}^\star j_{\pm i} = \pi_i + 2\kappa(B_{ij} \pm \tfrac{1}{2}g_{ij})x^{\prime j} \tag{4.4}$$

is obtained from (3.3) by imposing $\alpha a^2 = 1$, and

$$(g^{-1})^{ij} = (G^{-1}P_T^0)^{ij} \tag{4.5}$$

is the metric inverse of (4.1) in the subspace defined by the regular part of ${}^\star G_{ij}$.

The constraint ${}^\star j$ can be rewritten in terms of the current ${}^\star j_{\pm i}$ as

$${}^\star j = a^i\,{}^\star j_{\pm i}. \tag{4.6}$$

According to the Dirac theory for the constrained systems, we introduce the total Hamiltonian

$$H_T = \int d\sigma\mathcal{H}_T, \qquad \mathcal{H}_T = \mathcal{H}_c + \lambda{}^\star j, \tag{4.7}$$

where λ is a Lagrange multiplier. From the current algebra (3.5) we have

$$\{{}^\star j_{\pm i}, {}^\star j\} = 0, \qquad \{{}^\star j_{\pm(F)}, {}^\star j\} = 0 \Rightarrow \{H_T, {}^\star j\} = 0, \tag{4.8}$$

which means that ${}^\star j$ is a first class constraint. Consequently, there is a gauge symmetry in the theory.

Using expression for the gauge transformation of an arbitrary observable X, generated by symmetry generator G

$$\delta_\eta X = \{X, G\}, \qquad G \equiv \int d\sigma\eta(\sigma){}^\star j(\sigma), \tag{4.9}$$

in this particular case we obtain

$$\begin{aligned}\delta_\eta x^i &= a^i\eta, \qquad \delta_\eta{}^\star F = 0,\\ \delta_\eta\pi_i &= 2\kappa a^j B_{ji}\eta', \qquad \delta_\eta\pi = 0.\end{aligned} \tag{4.10}$$

B. Solution of constraints for particular gauge fixing

From the gauge transformations (4.10), it follows

$$\delta_\eta x_0 \equiv \delta_\eta(a_i x^i) = a^2\eta, \tag{4.11}$$

and we conclude that $x_0 = 0$ is a good gauge condition. After gauge fixing, we can treat ${}^\star j$ and x_0 as second class constraints. Implementing the conditions $x_0 = 0$ and ${}^\star j = 0$, the current ${}^\star j_{\pm i}$ changes as

$${}^\star j_{\pm i} \to j_{\pm i} = \pi_i + 2\kappa\Pi_{\pm ij}x^{\prime j} \qquad (\Pi_{\pm ij} = B_{ij} \pm \tfrac{1}{2}G_{ij}), \tag{4.12}$$

and the boundary conditions (3.8) take the form

$$\begin{aligned}\gamma_i^{(0)} &= (\Pi_+G^{-1})_i{}^j j_{-j} + (\Pi_-G^{-1})_i{}^j j_{+j},\\ \gamma^{(0)} &= \tfrac{1}{2}[{}^\star j_{-(F)} - {}^\star j_{+(F)}].\end{aligned} \tag{4.13}$$

As in the previous section, after Dirac consistency procedure, we obtain the σ-dependent form of the boundary conditions at $\sigma = 0$

$$\begin{aligned}\Gamma_i(\sigma) &= (\Pi_+G^{-1})_i{}^j j_{-j}(\sigma) + (\Pi_-G^{-1})_i{}^j j_{+j}(-\sigma),\\ \Gamma(\sigma) &= \tfrac{1}{2}[{}^\star j_{-(F)}(\sigma) - {}^\star j_{+(F)}(-\sigma)].\end{aligned} \tag{4.14}$$

Similar expressions from the constraints at $\sigma = \pi$ are solved by periodicity of all variables.

Let us mark the complete set of the constraints with $\chi_A = (\Gamma_i, \Gamma)$. The matrix form of the constraint algebra is

$$\{\chi_A(\sigma), \chi_B(\bar\sigma)\} = -\kappa M_{AB}\delta', \qquad M_{AB} = \begin{pmatrix} G^{\rm eff}_{ij} & 0\\ 0 & \frac{4}{\alpha}\end{pmatrix}, \tag{4.15}$$

where $G^{\rm eff}_{ij}$ is defined after Eq. (B3). Since we assume that

$\det G^{\rm eff}_{ij} \neq 0$, we conclude that all constraints are of the second class.

In terms of the effective variables defined in Eq. (3.13), the constraint equations $\Gamma_i(\sigma) = 0$ and $\Gamma(\sigma) = 0$ have the form

$$\bar{p}_i = 0, \qquad \bar{q}'^i = \tfrac{2}{\kappa}(G_{\rm eff}^{-1} B G^{-1})^{ij} p_j, \tag{4.16}$$

$$\bar{p} = 0, \qquad {}^\star\bar{f}' = 0. \tag{4.17}$$

Because the first class constraints and gauge fixing behave like second class constraints, from ${}^\star j = 0$ and $x_0 \equiv a_i x^i = 0$, we get additional equations

$$a^i p_i + 2\kappa (aB)_i \bar{q}'^i = 0, \qquad a^i \bar{p}_i + 2\kappa (aB)_i q'^i = 0,$$
$$q_0 \equiv a_i q^i = 0, \qquad \bar{q}_0 \equiv a_i \bar{q}^i = 0. \tag{4.18}$$

Combining the first equation in (4.16) with the second one in (4.18) we have $q'_1 \equiv (aB)_i q'^i = 0$. The fourth equation in (4.18) and the second equation in (4.16) give $p_1 \equiv (\tilde{a}B)^i p_i = 0$, while the second equation in (4.16) and the first one in (4.18) produce $p_0 \equiv \tilde{a}^i p_i = 0$. From here we conclude that the string phase space is spanned by the following coordinates and momenta:

$$(q_T)^i = ({}^\star P_T)^i{}_j q^j \equiv Q^i, \qquad (\pi_T)_i = ({}^\star P_T)_i{}^j p_j \equiv P_i, \tag{4.19}$$

where the projector ${}^\star P_T$ is defined in (A8) for $\alpha \tilde{a}^2 \neq 1$ and $\tilde{a}^2 \neq 0$. Decomposing $\bar{q}^i$ into a directions along $(aB)_i$ and a_i and the orthogonal ones

$$\bar{q}'^i = \bar{q}'^i_0 + \bar{q}'^i_1 + (\bar{q}_T)'^i, \tag{4.20}$$

from the second Eq. (4.16) we obtain

$$(\bar{q}_T)'^i = -2\, {}^\star\Theta^{ij} P_j, \qquad \bar{q}'^i_1 = \tfrac{2}{\kappa}(G_{\rm eff}^{-1}\, {}^\star P_1 B G^{-1})^{ij} P_j, \tag{4.21}$$

where the tensor

$${}^\star\Theta^{ij} = -\tfrac{1}{\kappa}(G_{\rm eff}^{-1}\, {}^\star P_T B G^{-1}\, {}^\star P_T)^{ij}, \tag{4.22}$$

is antisymmetric.

Choosing the integration constants $q_1^i(\sigma = 0) = 0$, $\bar{q}^i(\sigma = 0) = 0$, and ${}^\star\bar{f}(\sigma = 0) = 0$, the final solution of the Eqs. (4.16), (4.17), and (4.18) takes the form

$$x^i_{D_p}(\sigma) = Q^i(\sigma) - 2\, {}^\star\Theta^{ij} \int_0^\sigma d\sigma_1 P_j(\sigma_1), \qquad \pi_i^{D_p} = P_i, \tag{4.23}$$

$$x_0 = 0, \qquad \pi_0 = 0,$$
$$x_1(\sigma) = \frac{2}{\kappa}(\tilde{a}B^2 G^{-1})^i \int_0^\sigma d\sigma_1 P_i(\sigma_1), \qquad \pi_1 = 0, \tag{4.24}$$

$${}^\star F = {}^\star f, \qquad \pi = p. \tag{4.25}$$

Similar as in (4.19), we introduced the notation

$$x^i_{D_p} = ({}^\star P_T)^i{}_j x^j, \qquad \pi_i^{D_p} = ({}^\star P_T)_i{}^j \pi_j, \tag{4.26}$$

while x_0, x_1, π_0, and π_1 are defined in Eq. (A14). The solution for x_1 satisfies Dirichlet boundary conditions at $\sigma = \pi$, $x_1(\sigma = \pi) = 0$, as a consequence of the periodicity of the momenta P_i.

C. Effective theory and noncommutativity

In terms of the effective currents

$${}^\star\tilde{j}_{\pm i} = P_i \pm \kappa ({}^\star P_T G_{\rm eff})_{ij} Q'^j, \qquad {}^\star\tilde{j}_{\pm(F)} = p \pm \frac{4\kappa}{\alpha} f', \tag{4.27}$$

the currents ${}^\star j_{\pm i}$ and ${}^\star j_{\pm(F)}$ given in (3.3) and (4.4) can be expressed as

$${}^\star j_{\pm i} = \pm 2(\Pi_\pm G_{\rm eff}^{-1})_i{}^j\, {}^\star\tilde{j}_{\pm j}, \qquad {}^\star j_{\pm(F)} = {}^\star\tilde{j}_{\pm(F)}. \tag{4.28}$$

Substituting these relations in (4.3), we obtain the effective Hamiltonian

$$\tilde{\mathcal{H}}_c = \tilde{T}_- - \tilde{T}_+, \tag{4.29}$$
$$\tilde{T}_\pm = \mp \frac{1}{4\kappa}\Big[(G_{\rm eff}^{-1}\, {}^\star P_T)^{ij}\, {}^\star\tilde{j}_{\pm i}\, {}^\star\tilde{j}_{\pm j} + \frac{\alpha}{4}\, {}^\star\tilde{j}_{\pm(F)}\, {}^\star\tilde{j}_{\pm(F)}\Big].$$

The expressions for the effective current ${}^\star\tilde{j}_{\pm i}$ and the energy-momentum tensor $\tilde{T}_\pm$ show that the effective metric and its inverse are of the form

$$g^{\rm eff}_{ij} = ({}^\star P_T G^{\rm eff})_{ij}, \qquad g^{ij}_{\rm eff} = (G_{\rm eff}^{-1}\, {}^\star P_T)^{ij}. \tag{4.30}$$

Therefore, the string propagates in the subspace defined by the projector ${}^\star P_T$ in the background

$$G_{ij} \to g^{\rm eff}_{ij}, \qquad B_{ij} \to 0, \qquad \Phi \to 0. \tag{4.31}$$

The effective dynamics of the string is described by the effective variables: coordinates Q^i and momenta P_i, which satisfy the algebra

$$\{Q^i(\sigma), P_j(\bar{\sigma})\} = ({}^\star P_T)^i{}_j \delta(\sigma - \bar{\sigma}). \tag{4.32}$$

The conformal part of the effective world-sheet metric ${}^\star f$ and its momentum p are canonical variables for the scalar degree of freedom which decouples from the rest.

Using the solutions (4.23) and (4.25), and introducing the center of mass variables according to Appendix C, the noncommutativity relations take the form

$$\{X^i_{D_p}(\sigma), X^j_{D_p}(\bar{\sigma})\} = {}^\star\Theta^{ij}\Delta(\sigma + \bar{\sigma}), \tag{4.33}$$

$$\{X^i_{D_p}(\tau, \sigma), {}^\star\mathcal{F}(\tau, \bar{\sigma})\} = 0,$$
$$\{{}^\star\mathcal{F}(\tau, \sigma), {}^\star\mathcal{F}(\tau, \bar{\sigma})\} = 0, \tag{4.34}$$

where the tensor ${}^\star\Theta^{ij}$ and the function $\Delta(x)$ are defined in (4.22) and (C5).

The solutions for x_0 and x_1 satisfy the Dirichlet boundary conditions and decrease the number of the Dp-brane dimensions from $p+2$ to p. There is one commutative variable, the conformal part of the intrinsic metric ${}^\star F$, and $p-1$ noncommutative ones $x^i_{D_p}$.

V. NONCOMMUTATIVITY FOR SINGULAR ${}^\star G^{\rm eff}_{ij}$ ($\alpha\tilde{a}^2 = 1$)

For $\tilde{A} \equiv 1 - \alpha\tilde{a}^2 = 0$ and $A \equiv 1 - \alpha a^2 \neq 0$ the complete canonical analysis as well as the consistency procedure for the constraints, performed in Sec. III, can be repeated here. The difference appears in the separation of the first from the second class constraints as a consequence of the singularity of matrix M_{AB} (3.12).

A. From the second to the first class constraints

Using the expression for effective metric (B4), we obtain

$$ {}^\star G^{\rm eff}_{ij}\tilde{a}^j = \tilde{A}a_i, {}^\star G^{\rm eff}_{ij}(\tilde{a}B)^j = \frac{\tilde{A}}{A}(aB)_i, \tag{5.1}$$

so that, for $\tilde{A} = 0$ and $A \neq 0$, $\tilde{a}^i$ and $(\tilde{a}B)^i$ are singular vectors of the metric ${}^\star G^{\rm eff}_{ij}$. According to Eq. (3.11) we expect that two constraints originating from the boundary conditions turn into the first class.

In order to investigate the theory with constraints, we introduce the total Hamiltonian

$$ H_T = \int d\sigma \mathcal{H}_T, \qquad \mathcal{H}_T = \mathcal{H}_c + \lambda^i(\sigma)\Gamma_i(\sigma) + \lambda(\sigma)\Gamma(\sigma), \tag{5.2}$$

where $\mathcal{H}_c$ is defined in (3.2), Γ_i and Γ are defined in (3.9), and λ^i and λ are Lagrange multipliers. We decompose λ^i using the projectors $({}^\star\hat{P}_0)_i{}^j$, $({}^\star\hat{P}_1)_i{}^j$, and $({}^\star\hat{P}_T)_i{}^j$, defined in Appendix A

$$ \lambda^i = (\lambda_T)^i + 2\Lambda_1(\tilde{a}B)^i + \Lambda_2\tilde{a}^i, \tag{5.3}$$

where $(\lambda_T)^i = ({}^\star\hat{P}_T)^i{}_j\lambda^j$, $\Lambda_1 = -\frac{2\alpha}{1-\alpha a^2}(aB\lambda)$, and $\Lambda_2 = \alpha(a\lambda)$. The consistency conditions $\{H_T, \Gamma_i(\sigma)\} \approx 0$ and $\{H_T, \Gamma(\sigma)\} \approx 0$ enable us to calculate the coefficients

$$ \lambda' = -\frac{\alpha}{4\kappa}\Gamma', \qquad (\lambda'_T)^i = -\frac{1}{\kappa}(G^{-1}_{\rm eff}\,{}^\star\hat{P}_T)^{ij}\Gamma'_j, \tag{5.4}$$

while the coefficients Λ_1 and Λ_2 remain undetermined.

The total Hamiltonian takes the form

$$ H_T = H_c + \int_0^\pi d\sigma[(\lambda_T)^i(\Gamma_T)_i + \lambda\Gamma + \Lambda_1\Gamma_1 + \Lambda_2\Gamma_2] = H' + \int_0^\pi d\sigma(\Lambda_1\Gamma_1 + \Lambda_2\Gamma_2), \tag{5.5}$$

where

$$ \Gamma_1 = 2(\tilde{a}BG^{-1})^i\Gamma_i, \qquad \Gamma_2 = \tilde{a}^i\Gamma_i. \tag{5.6}$$

Since the constraints Γ_1 and Γ_2 are multiplied by the arbitrary coefficients Λ_1 and Λ_2, they are of the first class. On the other hand, $(\Gamma_T)_i$ and Γ, multiplied by the determined multipliers, are of the second class.

By direct calculation, from (3.11), we have

$$ \{\Gamma_1, \Gamma_i\} = 0, \qquad \{\Gamma_1, \Gamma\} = 0, \qquad \{\Gamma_2, \Gamma_i\} = 0, \qquad \{\Gamma_2, \Gamma\} = 0, \tag{5.7}$$

which is a confirmation that Γ_1 and Γ_2 are of the first class.

Calculating the algebra of the constraints $\chi_A = \{(\Gamma_T)_i, \Gamma\}$ we obtain

$$ \{\chi_A, \chi_B\} = -\kappa M_{AB}\delta', \qquad M_{AB} = \begin{pmatrix} ({}^\star\hat{P}_T G_{\rm eff})_{ij} & 0 \\ 0 & \frac{4}{\alpha} \end{pmatrix}. \tag{5.8}$$

Because the projector $({}^\star\hat{P}_T)_i{}^j$ is orthogonal to the vectors a_i and $(aB)_i$, we conclude that the rank of M_{AB} is not greater than p. Assuming that the rest of the matrix M_{AB} is regular, its rank as well as the number of the second class constraints are equal to p.

B. Gauge symmetry and solution of constraints

The gauge transformations have the form of the Eq. (4.9), with the generator

$$ G = \int_0^\pi d\sigma(\eta_1\Gamma_1 + \eta_2\Gamma_2), \tag{5.9}$$

where η_1 and η_2 are the parameters of the local transformations. The constraints

$$ \Gamma_1 = \tilde{a}^i p_i + 2(\tilde{a}BG^{-1})^i\bar{p}_i, \qquad \Gamma_2 = \tilde{a}^i\bar{p}_i + 2(\tilde{a}BG^{-1})^i p_i, \tag{5.10}$$

generate the gauge transformations of the effective variables

$$ \delta q^i = \tilde{a}^i(\eta_1)_s + 2(\tilde{a}BG^{-1})^i(\eta_2)_s, \qquad \delta\,{}^\star f = 0, \tag{5.11}$$

$$ \delta\bar{q}^i = \tilde{a}^i(\eta_2)_a + 2(\tilde{a}BG^{-1})^i(\eta_1)_a, \qquad \delta\,{}^\star\bar{f} = 0, \tag{5.12}$$

where the indices "s" and "a" denote σ symmetric and antisymmetric parts of the parameters η_1 and η_2. The particular gauge transformations

$$ \delta q_0 = \tilde{a}^2\eta_{1s}, \qquad \delta\bar{q}_0 = \tilde{a}^2\eta_{2a}, \qquad \delta q_1 = \frac{1}{2\alpha}(\alpha a^2 - 1)\eta_{2s}, \qquad \delta\bar{q}_1 = \frac{1}{2\alpha}(\alpha a^2 - 1)\eta_{1a}, \tag{5.13}$$

enable us to choose good gauge fixing

$$ q_0 = 0, \qquad \bar{q}_0 = 0, \qquad q_1 = 0, \qquad \bar{q}_1 = 0. \tag{5.14}$$

Now, the first class constraints and gauge conditions behave like second class constraints. So, the full set of expressions, Γ_i and Γ (3.14), vanishes as second class constraints.

Choosing the integration constants $\bar{q}^i(\sigma = 0) = 0$ and $\star\bar{f}(\sigma = 0) = 0$, from $\Gamma_i = 0$, $\Gamma = 0$, and gauge conditions (5.14), we get the solution

$$x^i_{D_p}(\sigma) = \hat{Q}^i(\sigma) - 2\,{}^\star\Theta^{ij}\int_0^\sigma d\sigma_1 \hat{P}_j(\sigma_1), \qquad \pi_i^{D_p} = \hat{P}_i, \tag{5.15}$$

$$\begin{aligned} x_0 &= 0, & \pi_0 &= 0, & x_1 &= 0, \\ \pi_1 &= 0, & {}^\star F &= {}^\star f, & \pi &= p. \end{aligned} \tag{5.16}$$

For $\hat{Q}^i$, $\hat{P}_i$, $x^i_{D_p}$, and $\pi_i^{D_p}$ we used the similar notation as in (4.19) and (4.26)

$$\begin{aligned} (q_T)^i &= ({}^\star\hat{P}_T)^i{}_j q^j \equiv \hat{Q}^i, \qquad (p_T)_i = ({}^\star\hat{P}_T)_i{}^j p_j \equiv \hat{P}_i, \\ x^i_{D_p} &= ({}^\star\hat{P}_T)^i{}_j x^j, \qquad \pi_i^{D_p} = ({}^\star\hat{P}_T)_i{}^j \pi_j, \end{aligned} \tag{5.17}$$

but now using the projector $({}^\star\hat{P}_T)_i{}^j$ instead of $({}^\star P_T)_i{}^j$. The vector components x_0, x_1, π_0, and π_1 are introduced in Eq. (A14), and the tensor ${}^\star\Theta^{ij}$

$${}^\star\Theta^{ij} = -\tfrac{1}{\kappa}(G_{\rm eff}^{-1}\,{}^\star\hat{P}_T B G^{-1}\,{}^\star\hat{P}_T)^{ij}, \tag{5.18}$$

is manifestly antisymmetric.

C. Effective theory

Let us introduce the effective currents

$${}^\star\tilde{j}_{\pm i} = \hat{P}_i \pm \kappa({}^\star\hat{P}_T G_{\rm eff})_{ij}\hat{Q}'^j, \qquad {}^\star\tilde{j}_{\pm(F)} = p \pm \frac{4\kappa}{\alpha}\,{}^\star f', \tag{5.19}$$

and correlate them with the currents defined in Eq. (3.3)

$$\begin{aligned} {}^\star j_{\pm i} &= \pm 2(\Pi_\pm G_{\rm eff}^{-1})_i{}^j\,{}^\star\tilde{j}_{\pm j} - 4(\Pi_\pm G_{\rm eff}^{-1}\,{}^\star\hat{P}_1 B)_i{}^j \hat{P}_j, \\ {}^\star j_{\pm(F)} &= {}^\star\tilde{j}_{\pm(F)}. \end{aligned} \tag{5.20}$$

Substituting these relations in the expression for energy-momentum tensor (3.2) we obtain

$$T_\pm = \tilde{T}_\pm, \qquad \mathcal{H}_c = \tilde{T}_- - \tilde{T}_+ \equiv \tilde{\mathcal{H}}_c, \tag{5.21}$$

where

$$\tilde{T}_\pm = \mp\frac{1}{4\kappa}\Big[(G_{\rm eff}^{-1}\,{}^\star\hat{P}_T)^{ij}\,{}^\star\tilde{j}_{\pm i}\,{}^\star\tilde{j}_{\pm j} + \frac{\alpha}{4}\,{}^\star\tilde{j}_{\pm(F)}\,{}^\star\tilde{j}_{\pm(F)}\Big]. \tag{5.22}$$

The effective theory lives in the background $G_{ij} \to g^{\rm eff}_{ij} = ({}^\star\hat{P}_T G_{\rm eff})_{ij}$, $B_{ij} \to 0$, and $\Phi \to 0$. The string dynamics is described by the effective variables $\hat{Q}^i$, $\hat{P}_j$, ${}^\star f$, and p.

D. Noncommutativity

From the algebra (3.24), we obtain the algebra of the effective variables

$$\{\hat{Q}^i(\sigma), \hat{P}_j(\bar\sigma)\} = ({}^\star\hat{P}_T)^i{}_j\delta_s(\sigma, \bar\sigma), \tag{5.23}$$

where $\delta_s(\sigma, \bar\sigma)$ is defined after Eq. (3.24).

As in the two previous cases, ${}^\star F$ is decoupled and takes the role of the commutative variable. Introducing the center of mass variables according to Appendix C, with the help of the Eqs. (5.15), we have

$$\{X^i_{D_p}(\tau, \sigma), X^j_{D_p}(\tau, \bar\sigma)\} = {}^\star\Theta^{ij}\Delta(\sigma + \bar\sigma), \tag{5.24}$$

where the antisymmetric tensor ${}^\star\Theta^{ij}$ is given in Eq. (5.18).

It follows from (5.15) and (5.16) that x_0 and x_1 are fixed and, consequently, satisfy Dirichlet boundary conditions and decrease the number of Dp-brane dimensions. All other $p - 1$ Dp-brane coordinates are noncommutative.

VI. CONCLUDING REMARKS

In this article we used the possibility to establish the conformal invariance adding the Liouville term to the action, instead to use the third space-time equation of motion, $\beta^\Phi = 0$. We showed that this change preserves main results of the previous paper [4]: (1) existence of the local gauge symmetries, which decrease the number of the Dp-brane dimensions; (2) the number of the commutative and noncommutative variables.

In fact, the Liouville action cancels the remaining constant anomaly $\beta^\Phi = c$ after imposing the first two space-time equations of motion, $\beta^G_{\mu\nu} = 0 = \beta^B_{\mu\nu}$. It also makes the conformal part of the world-sheet metric, F, dynamical variable. The theory becomes bilinear in F, with the quadratic Liouville term and linear term with the dilaton field. It is easy to change the variables, $F \to {}^\star F = F + \frac{\alpha}{2}a_i x^i$, so that term linear in F disappears. As a consequence, the quadratic term in x^i appears which changes the metric tensor, $G_{ij} \to {}^\star G_{ij} = G_{ij} - \alpha a_i a_j$. For particular values of the square of the vector a_i, with respect to the closed string metric, $a^2 = \frac{1}{\alpha}$, and to the effective one, $\tilde{a}^2 = \frac{1}{\alpha}$, the corresponding star metrics become singular [see Eqs. (B7) and (B8)].

We analyzed three cases: (1) $\alpha a^2 \neq 1$, $\alpha\tilde{a}^2 \neq 1$, (2) $\alpha a^2 = 1$, $\alpha\tilde{a}^2 \neq 1$, and (3) $\alpha\tilde{a}^2 = 1$, $\alpha a^2 \neq 1$. In all cases the field ${}^\star F$ decouples, so it is a commutative variable. The rest part of the action formally has the same form as in the dilaton free case, where the regular metric G_{ij} is substituted by the metric ${}^\star G_{ij}$, which can be singular for some choices of the background fields. The case (1) corresponds to such values of parameters that the star metric ${}^\star G_{ij}$ is regular. So, everything behaves as in the dilaton free case. In particular, all Dp-brane coordinates x^i are noncommutative.

The singularities of the star metrics have different influences to the canonical constraints. In the case (2), ${}^\star G_{ij}$ is coefficient in front of the velocity $\dot{x}^i$, so its singularity produces standard canonical constraint. In the case (3), the algebra of the constraints originating from boundary conditions, closed on ${}^\star G^{\rm eff}_{ij}$. So, the singularity of ${}^\star G^{\rm eff}_{ij}$ changes the character of the constraints, turning some of them from the second to the first class. According to Appendix B, ${}^\star G_{ij}$ has one singular direction a^i, while ${}^\star G^{\rm eff}_{ij}$ has two singular directions, $\tilde{a}^i$ and $(\tilde{a}B)^i$. Therefore, in the case (2) there is one and in the case (3) there are two first class constraints.

In both cases the first class contstraints are the symmetry generators. After the gauge fixing, gauge conditions and the first class constraints can be considered as second class constraints. Solving all second class constraints (both the original ones and the first class constraints with gauge conditions), we obtain the string coordinates in terms of the effective ones.

The solutions (3.16), (4.23), and (5.15) have the same general form

$$x^i_{D_p}(\sigma) = Q^i(\sigma) - 2{}^\star\Theta^{ij}\int_0^\sigma d\sigma_1 P_j(\sigma_1). \tag{6.1}$$

The string coordinates $x^i_{Dp} = ({}^\star P_{D_p})^i{}_j x^j$ are expressed in terms of the effective canonical variables

$$Q^i = ({}^\star P_{D_p})^i{}_j q^j, \qquad P_i = ({}^\star P_{D_p})_i{}^j p_j, \tag{6.2}$$

satisfying the algebra

$$\{Q^i(\sigma), P_j(\bar\sigma)\} = ({}^\star P_{D_p})^i{}_j \delta_s(\sigma, \bar\sigma). \tag{6.3}$$

In the second and third case, the string coordinates $x_0 \equiv (n_0)_i x^i = a_i x^i$ and $x_1 \equiv (n_1)_i x^i = (aB)_i x^i$ satisfy Dirichlet boundary conditions and decrease the number of the Dp-brane dimensions. It is known that boundary conditions are usually imposed on spacelike variables. Because the coordinates x_0 and x_1 satisfy Dirichlet boundary conditions, it is important to clarify the nature of the vectors $(n_0)_i$ and $(n_1)_i$. Let us first introduce explicit dependence on the string slope parameter $\alpha' = \frac{1}{2\pi\kappa}$, by simple redefinition of dilaton field $\Phi \to \alpha'\Phi$. Then the singularities of metrics ${}^\star G_{ij}$ and ${}^\star G^{\rm eff}_{ij}$ occur at $a^2 = \frac{1}{\alpha\alpha'^2}$ and $\tilde{a}^2 = \frac{1}{\alpha\alpha'^2}$, respectively, and from (2.6) and (2.10) we obtain

$$\frac{1}{\alpha\alpha'^2} = \frac{\beta^\Phi}{4} = \frac{D-26}{24\alpha'} + a^2. \tag{6.4}$$

From the first singularity condition the a^2 dependence disappears and we obtain that string must be critical, $D = 26$. Because there are no conditions on $(n_0)_i$ and $(n_1)_i$, we can choose them to be spacelike variables, $n_0^2 = a^2 > 0$ and $n_1^2 = -(aB^2a) > 0$. From the second singularity condition, with the help of the relation $\tilde{a}^2 - a^2 = 4\tilde{a}B^2a = 4aB^2a + 16\tilde{a}B^4a$, we obtain the conditions for the vectors $(n_0)_i$ and $(n_1)_i$ to be spacelike

$$n_0^2 = \tilde{a}^2 - \frac{D-26}{24\alpha'} > 0, \qquad n_1^2 = 4\tilde{a}B^4a - \frac{D-26}{96\alpha'} > 0. \tag{6.5}$$

For $D \leq 26$, in order to satisfy these conditions, it is enough to choose $\tilde{a}^2 \geq 0$ and $\tilde{a}B^4a \geq 0$.

The string components $x^i_{D_p}$ are noncommutative degrees of freedom, because they depend on the effective coordinates and momenta. The noncommutativity relation between the Dp-brane coordinates has the same form in all three cases

$$\{X^i_{D_p}(\tau,\sigma), X^j_{D_p}(\tau,\bar\sigma)\} = {}^\star\Theta^{ij}\Delta(\sigma+\bar\sigma), \qquad [X^i_{D_p}(\sigma) = x^i_{D_p}(\sigma) - (x^i_{D_p})_{cm}]. \tag{6.6}$$

The interior of the string is commutative and noncommutativity occurs on the string end points. The noncommutativity parameter ${}^\star\Theta^{ij}$ in the first case is given in (3.18), while in the other two cases it can be expressed in terms of the projectors ${}^\star P_{D_p}$

$${}^\star\Theta^{ij} = -\tfrac{1}{\kappa}(G^{-1}_{\rm eff}\,{}^\star P_{D_p} B G^{-1}\,{}^\star P_{D_p})^{ij}. \tag{6.7}$$

All important results of this analysis are presented in Table I where D_{Dp} is the number of the Dp-brane dimensions, the symbol V_{Dbc} is related to variables with Dirichlet boundary condition, and the effective metrics are denoted by $g^{\rm eff}_{ij}$. All projectors are defined in Appendix A.

Let us stress that the solution of the boundary conditions, the number of the Dp-brane dimensions, the number of the commutative and noncommutative coordinates as well as the form of the noncommutativity parameter, in the approach with the Liouville action are the same as in the approach presented in Ref. [4]. There are two formal differences. When we deal with the Liouville action, the gauge symmetries appear for $\alpha a^2 = 1$ and $\alpha\tilde{a}^2 = 1$ when star metrics, ${}^\star G_{ij}$ and ${}^\star G^{\rm eff}_{ij}$, are singular instead for $a^2 = 0$ and $\tilde{a}^2 = 0$ in the absence of the Liouville term. Also some

TABLE I. Dp-brane features dependence on background fields.

Case	D_{Dp}	$({}^\star P_{D_p})_i{}^j$	V_{Dbc}	$x^i_{D_p}$	$g^{\rm eff}_{ij}$
$\alpha\tilde{a}^2 \neq 1$, $\alpha a^2 \neq 1$	$p+2$	$\delta_i{}^j$	...	x^i	${}^\star G^{\rm eff}_{ij}$
$\alpha a^2 = 1$, $\alpha\tilde{a}^2 \neq 1$	p	$({}^\star P_T)_i{}^j$	x_0, x_1	$({}^\star P_T x)^i$	$({}^\star P_T G_{\rm eff})_{ij}$
$\alpha\tilde{a}^2 = 1$, $\alpha a^2 \neq 1$	p	$({}^\star\hat{P}_T)_i{}^j$	x_0, x_1	$({}^\star\hat{P}_T x)^i$	$({}^\star\hat{P}_T G_{\rm eff})_{ij}$

commutative and noncommutative variables switched the roles, $x_0 \to {}^\star F$ and $F \to x_0$.

The inclusion of the Liouville term produces few advantages. First, there are only two space-time equations of motion (originated from $\beta^G_{\mu\nu} = 0$ and $\beta^B_{\mu\nu} = 0$) instead of three ones without Liouville. Second, the presence of F does not break the closed string conformal invariance. Consequently, there is no possibility that F-dependent open string boundary conditions break this invariance and there is no need for additional restrictions on background fields, as in the absence of the Liouville term. Finally, the complete solution including noncommutative parameter and effective variables depends on an additional parameter, the central charge c.

It is interesting to mention that the effect of boundary conditions reduces the dimension of Dp brane by 2, from p to $p-2$, as well as double T duality. In fact any T duality relates Dp brane wrapped around compact direction with radius R to the D$(p-1)$-brane with dual compact radius $\tilde{R}$. So, two T dualities along $x_0 = a_i x^i$ and $x_1 = (aB)_i x^i$ with compactification radii R_0 and R_1, could transform Dp brane to D$(p-2)$ brane with compactified radii $\tilde{R}_0$ and $\tilde{R}_1$. Possible deeper understanding of our result in terms of T dualities is under investigation.

ACKNOWLEDGMENTS

This work was supported in part by the Serbian Ministry of Science and Environmental Protection, under Contract No. 141036.

APPENDIX A: PROJECTORS

In this appendix we introduce projector operators in order to separate noncommutative and nonphysical variables on the Dp brane as well as to express the noncommutativity parameter.

The projectors on the direction n_i and on the subspace orthogonal to vector n_i are

$$(\Pi)_i{}^j = \frac{n_i n^j}{n^2}, \qquad (\Pi_T)_i{}^j = \delta_i{}^j - (\Pi)_i{}^j, \tag{A1}$$

where $n^i = g^{ij} n_j$ and $n^2 = n^i n_i$. The transposed operator is

$$\Pi^i{}_j = g^{ik} \Pi_k{}^l g_{lj}. \tag{A2}$$

1. Case $n_i = a_i$ and $g_{ij} = G_{ij}$

For $n_i \to (n_0)_i = a_i$ and $g_{ij} \to G_{ij}$ we obtain

$$(\Pi)_i{}^j \to (P_0)_i{}^j = \frac{a_i a^j}{a^2}, \qquad (\Pi_T)_i{}^j \to (P^0_T)_i{}^j = \delta_i{}^j - (P_0)_i{}^j. \tag{A3}$$

2. Case $(n_0)_i = a_i$ and $(n_1)_i = (aB)_i$ and $g_{ij} = G^{\text{eff}}_{ij}$

Let us construct the projector orthogonal to the vectors $(n_0)_i = a_i$ and $(n_1)_i = (aB)_i$ with respect to the effective metric G^{eff}_{ij}. These two vectors are mutually orthogonal and it is enough to use the projectors on the direction a_i

$$(\Pi_0)_i{}^j = \frac{a_i \tilde{a}^j}{\tilde{a}^2}, \tag{A4}$$

and on the direction $(aB)_i$

$$(\Pi_1)_i{}^j = \frac{4}{\tilde{a}^2 - a^2} (Ba)_i (\tilde{a}B)^j, \tag{A5}$$

to construct the projector orthogonal on them

$$(\Pi_T)_i{}^j = \delta_i{}^j - (\Pi_0)_i{}^j - (\Pi_1)_i{}^j. \tag{A6}$$

In the case when $\alpha a^2 = 1$ we have

$$({}^\star P_1)_i{}^j = (\Pi_1)_i{}^j|_{\alpha a^2=1} = \frac{4\alpha}{\alpha\tilde{a}^2 - 1} (Ba)_i (\tilde{a}B)^j, \tag{A7}$$

$$({}^\star P_T)_i{}^j = (\Pi_T)_i{}^j|_{\alpha a^2=1} = \delta_i{}^j - ({}^\star P_0)_i{}^j - ({}^\star P_1)_i{}^j, \tag{A8}$$

where by definition we put

$$({}^\star P_0)_i{}^j = (\Pi_0)_i{}^j = \frac{a_i \tilde{a}^j}{\tilde{a}^2}. \tag{A9}$$

Similarly for $\alpha\tilde{a}^2 = 1$ we get

$$({}^\star \hat{P}_0)_i{}^j = (\Pi_0)_i{}^j|_{\alpha\tilde{a}^2=1} = \alpha a_i \tilde{a}^j, \qquad ({}^\star \hat{P}_1)_i{}^j = (\Pi_1)_i{}^j|_{\alpha\tilde{a}^2=1} = \frac{4\alpha}{1 - \alpha a^2} (Ba)_i (\tilde{a}B)^j, \tag{A10}$$

$$({}^\star \hat{P}_T)_i{}^j = (\Pi_T)_i{}^j|_{\alpha\tilde{a}^2=1} = \delta_i{}^j - ({}^\star \hat{P}_0)_i{}^j - ({}^\star \hat{P}_1)_i{}^j. \tag{A11}$$

An arbitrary contravariant vector x^i decomposes as

$$x^i = (x_0)^i + (x_1)^i + (x_T)^i, \qquad (x_0)^i = (\Pi_0)^i{}_j x^j, \qquad (x_1)^i = (\Pi_1)^i{}_j x^j, \qquad (x_T)^i = (\Pi_T)^i{}_j x^j, \tag{A12}$$

as well as an arbitrary covariant vector π_i

$$\pi_i = (\pi_0)_i + (\pi_1)_i + (\pi_T)_i, \qquad (\pi_0)_i = (\Pi_0)_i{}^j \pi_j, \qquad (\pi_1)_i = (\Pi_1)_i{}^j \pi_j, \qquad (\pi_T)_i = (\Pi_T)_i{}^j \pi_j. \tag{A13}$$

It is useful to introduce the following notation for the projections of vectors x^i and π_i:

$$x_0 = (n_0)_i x^i = a_i x^i, \qquad x_1 = (n_1)_i x^i = (aB)_i x^i, \qquad \pi_0 = \tilde{n}^i_0 \pi_i = \tilde{a}^i \pi_i, \qquad \pi_1 = \tilde{n}^i_1 \pi_i = (\tilde{a}B)^i \pi_i. \tag{A14}$$

APPENDIX B: THE STAR METRICS ${}^\star G_{ij}$ AND ${}^\star G^{\text{eff}}_{ij}$

Here we are going to introduce the expressions for the redefined metrics in the presence of the Liouville action, ${}^\star G_{ij}$ and ${}^\star G^{\text{eff}}_{ij}$. The metric ${}^\star G_{ij}$ is defined as

$$\begin{aligned} {}^\star G_{ij} &= G_{ij} - \alpha a_i a_j = (P_T^0 + AP_0)_i{}^k G_{kj}, \\ A &= 1 - \alpha a^2, \end{aligned} \tag{B1}$$

while, for $A \neq 0$, its inverse is

$$({}^\star G^{-1})^{ij} = G^{ij} + \frac{\alpha}{1-\alpha a^2} a^i a^j = G^{ik}\left(P_T^0 + \frac{1}{A}P_0\right)_k{}^j. \tag{B2}$$

The effective metric ${}^\star G_{ij}^{\text{eff}}$ has the same form as in the dilaton free case up to the substitution $G_{ij} \to {}^\star G_{ij}$

$$\begin{aligned} {}^\star G_{ij}^{\text{eff}} &= {}^\star G_{ij} - 4(B\,{}^\star G^{-1}B)_{ij} \\ &= G_{ij}^{\text{eff}} - \alpha a_i a_j - \frac{4\alpha}{1-\alpha a^2}(Ba)_i(aB)_j, \end{aligned} \tag{B3}$$

where $G_{ij}^{\text{eff}} = G_{ij} - 4B_{ik}G^{kl}B_{lj}$. In terms of the projectors, we have

$${}^\star G_{ij}^{\text{eff}} = (\Pi_T + \tilde{A}\Pi_0 + \mathcal{A}\Pi_1)_i{}^k G_{kj}^{\text{eff}}, \tag{B4}$$

where

$$\tilde{A} = 1 - \alpha\tilde{a}^2, \qquad \mathcal{A} = \frac{\tilde{A}}{A} = \frac{1-\alpha\tilde{a}^2}{1-\alpha a^2}. \tag{B5}$$

With the help of (B4), for $\tilde{A} \neq 0$ and $A \neq 0$, we obtain

$$\begin{aligned} ({}^\star G_{\text{eff}}^{-1})^{ij} &= (G_{\text{eff}}^{-1})^{ik}\left(\Pi_T + \frac{1}{\tilde{A}}\Pi_0 + \frac{1}{\mathcal{A}}\Pi_1\right)_k{}^j \\ &= (G_{\text{eff}}^{-1})^{ij} + \frac{\alpha}{1-\alpha\tilde{a}^2}[\tilde{a}^i\tilde{a}^j + 4(B\tilde{a})^i(\tilde{a}B)^j]. \end{aligned} \tag{B6}$$

According to Eq. (B1) the determinant of ${}^\star G_{ij}$ is of the form

$$\det{}^\star G_{ij} = A \det G_{ij}, \tag{B7}$$

while the determinant of the effective metric ${}^\star G_{ij}^{\text{eff}}$ (B4) is

$$\det{}^\star G_{ij}^{\text{eff}} = \tilde{A}\mathcal{A}\det G_{ij}^{\text{eff}} = \frac{\tilde{A}^2}{A}\det G_{ij}^{\text{eff}}. \tag{B8}$$

For $A = 0$, we have $\det{}^\star G_{ij} = 0$ and the vector a^i is singular for the metric ${}^\star G_{ij}$, what is obvious from

$${}^\star G_{ij}a^j = Aa_i. \tag{B9}$$

For $\tilde{A} = 0$ and $A \neq 0$ the effective metric ${}^\star G_{ij}^{\text{eff}}$ is singular. From the relations

$${}^\star G_{ij}^{\text{eff}}\tilde{a}^j = \tilde{A}a_i, \qquad {}^\star G_{ij}^{\text{eff}}(\tilde{a}B)^j = \frac{\tilde{A}}{A}(aB)_i, \tag{B10}$$

follows that $\tilde{a}^i$ and $(\tilde{a}B)^i$ are singular vectors of the star effective metric.

APPENDIX C: SEPARATION THE CENTER OF MASS VARIABLE

We will explain separation the center of mass variable and define the corresponding functions $\theta(x)$ and $\Delta(x)$. Let variable x^i satisfies the Poisson bracket

$$\{x^i(\tau,\sigma), x^j(\tau,\bar\sigma)\} = 2\,{}^\star\Theta^{ij}\theta(\sigma+\bar\sigma), \tag{C1}$$

where the function $\theta(x)$ is defined as

$$\theta(x) = \begin{cases} 0 & \text{if } x = 0, \\ 1/2 & \text{if } 0 < x < 2\pi, \\ 1 & \text{if } x = 2\pi. \end{cases} \tag{C2}$$

Separating the center of mass variable

$$x_{cm}^i = \frac{1}{\pi}\int_0^\pi d\sigma x^i(\sigma), \qquad x^i(\sigma) = x_{cm}^i + X^i(\sigma), \tag{C3}$$

we obtain

$$\{X^i(\tau,\sigma), X^j(\tau,\bar\sigma)\} = {}^\star\Theta^{ij}\Delta(\sigma+\bar\sigma), \tag{C4}$$

where the function $\Delta(x)$

$$\Delta(x) = 2\theta(x) - 1 = \begin{cases} -1 & \text{if } x = 0, \\ 0 & \text{if } 0 < x < 2\pi, \\ 1 & \text{if } x = 2\pi, \end{cases} \tag{C5}$$

is different from zero only on the string end points.

The same procedure can be applied to variable ${}^\star F(\sigma)$ with notation

$${}^\star F(\sigma) = {}^\star F_{cm} + {}^\star\mathcal{F}(\sigma). \tag{C6}$$

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