Modified teleparallel gravity: Inflation without an inflaton

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The Born-Infeld strategy to smooth theories having divergent solutions is applied to the teleparallel equivalent of general relativity. Differing from other theories of modified gravity, modified teleparallelism leads to second order equations, since the teleparallel Lagrangian only contains first derivatives of the vierbein. We show that the Born-Infeld-modified teleparallelism solves the particle horizon problem in a spatially flat Friedmann-Robertson-Walker (FRW) universe by providing an initial exponential expansion without resorting to an inflaton field.

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I. MODIFIED GRAVITY: A BORN-INFELD APPROACH

In 1934 Born and Infeld (BI) [1] proposed the following scheme for modifying a field theory governed by a Lagrangian density $\mathcal{L} = \sqrt{-g}L$:

$$\mathcal{L} \rightarrow \mathcal{L}_{\rm BI} = \sqrt{-g}\lambda\left[\sqrt{1+\frac{2L}{\lambda}}-1\right]. \qquad (1)$$

The basic idea was to introduce a new scale λ with the aim of smoothing singularities. The scheme (1) is essentially the way for going from the classical free particle Lagrangian to the relativistic one; in such case, the scale is $\lambda = -mc^2$, which smooths the particle velocity by preventing its unlimited growth. Besides, Born and Infeld subtracted the "rest energy" to get that $\mathcal{L}_{\rm BI}$ vanishes when $\mathcal{L}$ is zero. We can then expect that Born-Infeld dynamics will differ from the original dynamics for those configurations where L is large. In fact, Born and Infeld looked for a reformulation of Maxwell's electrodynamics in order to smooth the divergence of the pointlike charge electric field, and they have succeeded in obtaining a finite self-energy for this configuration. On the other hand, the original Lagrangian is recovered if $L \ll \lambda$; hence the solutions of both theories do not appreciably differ in these regions. Nowadays Born-Infeld Lagrangians have reappeared in developments of string theories at low energies [2–7]; they have also been used in quintessence theories for modeling matter fluids able to drive both inflation and the present accelerated expansion [8]. However, although the subject has received some attention [9–11], no gravitational BI analogue leading to second order equations was yet proposed in four dimensions.

A wide variety of modified gravity theories have been considered in the last decades. For instance, the Lovelock Lagrangian is a polynomial in Riemann curvature which leads to second order equations for the metric tensor [12]. Nevertheless, the Lovelock Lagrangian only differs from the Einstein-Hilbert (EH) Lagrangian, $\mathcal{L}_{\mathbf{EH}}[g_{\mu\nu}(x)] = -(16\pi G)^{-1}\sqrt{-g}\mathcal{R}$ ($\mathcal{R}$ being the scalar curvature), for a dimension larger than 4. On the other hand "$f(\mathcal{R})$" theories are being currently studied, mostly connected with the attempts to explain the cosmic acceleration without resorting to quintessence models [13,14]. For instance, a $f(\mathcal{R})$ theory could be obtained by using the Born-Infeld scheme:

$$\mathcal{L} = -\frac{1}{16\pi G}\sqrt{-g}\lambda\left[\sqrt{1+\frac{2\mathcal{R}}{\lambda}}-1\right]. \qquad (2)$$

However we find this strategy unsatisfactory because: (1) fourth order dynamical equations will result, since $\mathcal{R}$ contains second derivatives of the metric (a feature that is common to $f(\mathcal{R})$ theories); (2) this strategy is unable to smooth black holes, since they have $\mathcal{R} = 0$ (then the scale λ could not play any role).

Concerning the first objection, it is well known that the second derivatives of the metric in the Einstein-Hilbert Lagrangian do not lead to fourth order equations because they only give rise to surface terms in the action. This characteristic only remains valid in Lovelock Lagrangians but is lost in $f(\mathcal{R})$ theories.

II. TELEPARALLEL EQUIVALENT OF GENERAL RELATIVITY

In order to build a modified gravity leading to second order equations in four dimensions, we will start not from the Einstein-Hilbert Lagragian but from the teleparallel equivalent of general relativity (TEGR). While general relativity uses the Levi-Civita connection (curvature but no torsion), teleparallelism uses the Weitzenböck connection [15] (torsion but no curvature). In this sense teleparallelism [16] is a sector of Einstein-Cartan theories [17,18], which describe gravity by means of a connection having both torsion and curvature. In teleparallelism the dynami-

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cal object is the vierbein field $\{\mathbf{h}_i(x^\mu)\}$, $i = 0, 1, 2, 3$. Each vector $\mathbf{h}_i$ is described by its components h_i^μ, $\mu = 0, 1, 2, 3$, in a coordinate basis. The matrix (h_i^μ) is inversible; i.e. there exists a matrix (h^i_μ) fulfilling

$$h_i^\mu h^j_\mu = \delta_i^j, \qquad h_i^\mu h^i_\nu = \delta^\mu_\nu. \tag{3}$$

The Weitzenböck connection,

$$\overset{\mathrm{w}}{\Gamma}{}^\lambda_{\mu\nu} = -h^i_\mu \partial_\nu h_i^\lambda = h_i^\lambda \partial_\nu h^i_\mu, \tag{4}$$

is such that the Weitzenböck covariant derivative of a vector $\mathbf{V} = V^i \mathbf{h}_i = V^i h_i^\mu \partial_\mu$ becomes

$$\overset{\mathrm{w}}{\nabla}_\nu V^\mu = \partial_\nu V^\mu + \overset{\mathrm{w}}{\Gamma}{}^\mu_{\lambda\nu} V^\lambda = \partial_\nu (V^i h_i^\mu) + \overset{\mathrm{w}}{\Gamma}{}^\mu_{\lambda\nu} V^i h_i^\lambda$$
$$= h_i^\mu \partial_\nu V^i. \tag{5}$$

Hence a vector $\mathbf{V}$ will be autoparallel if its components $V^i = h^i_\mu V^\mu$ are constant.

The Weitzenböck connection has zero Riemann curvature and non-null torsion:

$$T^\lambda{}_{\mu\nu} = \overset{\mathrm{w}}{\Gamma}{}^\lambda_{\nu\mu} - \overset{\mathrm{w}}{\Gamma}{}^\lambda_{\mu\nu} = h_i^\lambda (\partial_\mu h^i_\nu - \partial_\nu h^i_\mu), \tag{6}$$

i.e., $h^i_\lambda T^\lambda{}_{\mu\nu}$ are the components of the 2-form $d\mathbf{h}^i$, where $\{\mathbf{h}^i\}$ is the dual basis (whose elements have components h^i_μ). The TEGR Lagrangian is [19,20]

$$\mathcal{L}_{\mathbf{T}}[h^i_\mu(x)] = \frac{1}{16\pi G} h S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}, \tag{7}$$

where $h \equiv \det(h^i_\mu)$ and $S_\rho{}^{\mu\nu}$ is given by

$$S_\rho{}^{\mu\nu} = \tfrac{1}{2}[K^{\mu\nu}{}_\rho + \delta^\mu_\rho T^{\theta\nu}{}_\theta - \delta^\nu_\rho T^{\theta\mu}{}_\theta]. \tag{8}$$

In this last equation, the *contorsion* tensor is

$$K^{\mu\nu}{}_\rho = -\tfrac{1}{2}(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}). \tag{9}$$

In (8) and (9), indexes have been raised and lowered with the metric

$$g_{\mu\nu}(x) = \eta_{ij} h^i_\mu(x) h^j_\nu(x), \qquad g^{\mu\nu}(x) = \eta^{ij} h_i^\mu(x) h_j^\nu(x) \tag{10}$$

($\eta_{ij} = \mathrm{diag}(1, -1, -1, -1)$), so it is $h = (-\det g_{\mu\nu})^{1/2}$. Note that the vierbein is orthonormal in this metric:

$$g_{\mu\nu}(x) h_i^\mu(x) h_j^\nu(x) = \eta_{ij}. \tag{11}$$

Moreover, the Weitzenböck connection proves to be metric compatible. It is easy to show that the contorsion equals the difference between the Levi-Civita connection associated with the metric (10) and the Weitzenböck connection:

$$\overset{\mathrm{w}}{\Gamma}{}^\lambda_{\mu\nu} = \overset{\mathrm{L}}{\Gamma}{}^\lambda_{\mu\nu} + K^\lambda{}_{\mu\nu}. \tag{12}$$

Taking into account the Weitzenböck connection definition (4), this means that

$$\overset{\mathrm{L}}{\nabla}_\nu h^i_\mu = h^i_\lambda K^\lambda{}_{\mu\nu}. \tag{13}$$

Equation (12) also means that the Weitzenböck four-acceleration of a freely falling particle is not zero but it is

$$\frac{d^2 x^\lambda}{d\tau^2} + \overset{\mathrm{w}}{\Gamma}{}^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = K^\lambda{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}. \tag{14}$$

Thus, the contorsion can be regarded as a gravitational force which moves particles away from the Weitzenböck autoparallel lines.

Teleparallel Euler-Lagrange equations are

$$\frac{1}{h}\partial_\sigma(h S_i{}^{\sigma\rho}) - 4\pi G j_i{}^\rho = 4\pi G h_i^\sigma T^\rho{}_\sigma, \tag{15}$$

where $S_i{}^{\sigma\rho} = h_i^\lambda S_\lambda{}^{\sigma\rho}$, $T_\rho{}^\sigma = h^{-1} h^i_\rho (\delta \mathcal{L}_{\mathrm{matter}}/\delta h^i_\sigma)$ is the energy-momentum tensor of the sources and

$$j_i^\rho = -\frac{1}{h}\frac{\partial \mathcal{L}_{\mathbf{T}}}{\partial h^i_\rho} = \frac{h_i^\lambda}{4\pi G}\left(S_\eta{}^{\mu\rho} T^\eta{}_{\mu\lambda} - \frac{1}{4}\delta^\rho_\lambda S_\eta{}^{\mu\nu} T^\eta{}_{\mu\nu}\right). \tag{16}$$

Because of the antisymmetry of $S_i{}^{\sigma\rho}$, a conserved current appears:

$$\partial_\rho(h j_i^\rho + h h_i^\sigma T^\rho{}_\sigma) = 0, \tag{17}$$

so $j_i{}^\rho$ is associated with the vierbein energy-momentum.

The teleparallel Lagrangian (7) suffers from a defect: it is unable to govern the dynamics of the entire vierbein. In fact, the equivalence between the Lagrangian (7) and the Einstein-Hilbert Lagrangian tells us that the Lagrangian (7) does not govern the vierbein but just the metric (10). As a reflection of this character, the Lagrangian (7) is invariant under local Lorentz transformations of the vierbein, $h^i_\mu(x) \to \Lambda^{i'}_i(x) h^i_\mu(x)$, modulo boundary terms. In fact, the metric (10) does not change under this kind of vierbein transformation. But we are searching for a dynamical theory for the vierbein, which is a geometric object involving 16 functions h_i^μ instead of the ten metric components. Such a theory would govern not only the metric but also the torsion [21]. Then, a dynamical theory for the vierbein requires a Lagrangian differing from (7). In other words, each vierbein on a given manifold establishes an orthogonal grid of autoparallel lines which defines an absolute parallelism of vectors ("a vector $\mathbf{V}$ is autoparallel if its components $V^i = h^i_\mu V^\mu$ are constant"). This absolute parallelism is invariant under *global* Lorentz transformations of the vierbein. In fact $V^{i'} = \Lambda^{i'}_i V^i$ will be constant only if $\Lambda^{i'}_i$ is constant as well. Then, a dynamical theory for the vierbein should be provided with the same global invariance. Therefore, the Lagrangian (7) is not admissible.

Other Lagrangians quadratic in the torsion have also been proposed to build a dynamical theory for the vierbein. In Ref. [22] a general quadratic theory has been tried by combining three quadratic pieces, each of them associated with each of the three irreducible parts of the torsion:

vectorial, axial, and traceless-symmetric. The coefficients of the vectorial and traceless-symmetric pieces are strongly constrained by physics in the solar system. The axial part is dynamically linked to the antisymmetric part of the energy-momentum tensor (associated with the intrinsic spin of matter). This ingredient renders the theory invariant only under global Lorentz transformations of the vierbein. However the use of the axial term has been questioned (see Ref. [21]).

In spite of the above mentioned defect, the structure of the teleparallel Lagrangian (7) is very appealing because it resembles the structure for a gauge field: it is quadratic in the torsion $h_i^\lambda d\mathbf{h}^i$. In particular, it has only first derivatives of the vierbein field. This feature can be exploited to build a modified teleparallel gravity leading to second order dynamical equations. Remarkably, modified teleparallel gravity will be invariant only under global Lorentz transformations. Concretely, we are going to use a teleparallel Lagrangian à la Born-Infeld:

$$\mathcal{L}_{\mathbf{BI}} = \frac{\lambda}{16\pi G} h\left[\sqrt{1 + \frac{2S_\mu{}^{\nu\rho}T^\mu{}_{\nu\rho}}{\lambda}} - 1\right]. \tag{18}$$

Differing from $\mathcal{L}_{\mathbf{T}}$, $\mathcal{L}_{\mathbf{BI}}$ is not invariant under local Lorentz transformations of the vierbein. In fact, if such a transformation is applied on $\mathcal{L}_{\mathbf{T}}$ then a harmless boundary term will appear. But this boundary term emerging from $S_\rho{}^{\mu\nu}T^\rho{}_{\mu\nu}$ now remains trapped inside the square root, so rendering the Born-Infeld-like Euler-Lagrange equations sensitive to local Lorentz transformations. The Born-Infeld parameter λ in Eq. (18) tells that the metric for solutions of modified teleparallel gravity will approach the solutions of Einstein equations in regions where $S_\rho{}^{\mu\nu}T^\rho{}_{\mu\nu} \ll \lambda$.

III. THE COSMOLOGICAL SOLUTION

Our aim is to test Born-Infeld modified teleparallelism in a cosmological framework. For this, we will substitute a solution of the form

$$h^i_\mu = \mathrm{diag}(N(t), a(t), a(t), a(t)) \tag{19}$$

in the Euler-Lagrange equations emerging from the Lagrangian (18). The proposed solution implies a metric (10)

$$g_{\mu\nu} = \mathrm{diag}(N^2(t), -a(t)^2, -a(t)^2, -a(t)^2), \tag{20}$$

i.e., a spatially flat FRW cosmological model. Then we will use as the source a homogeneous and isotropic fluid; so $T^\rho{}_\sigma = \mathrm{diag}(\rho, -p, -p, -p)$ in the comoving frame. Of course, the dynamical equations get more involved than the GR-equivalent ones (15). The high symmetry of the proposed solution renders some of the 16 equations trivial. Finally only two independent equations are left: a first order equation

$$\left(1 - \frac{12H^2}{N^2\lambda}\right)^{-1/2} - 1 = \frac{16\pi G}{\lambda} N^2 \rho, \tag{21}$$

which results from varying with respect to h^0_0 ($H(t) = \dot{a}(t)/a(t)$ is the Hubble parameter), and a second order one,

$$\left(\frac{16H^2}{N^2\lambda} + \frac{4H^2}{N^2\lambda} q - 1\right)\left(1 - \frac{12H^2}{N^2\lambda}\right)^{-3/2} + 1 = \frac{16\pi G}{\lambda} p, \tag{22}$$

which results from varying with respect to h^i_σ with $i = \sigma = 1, 2$, or 3 ($q = -\ddot{a}a/\dot{a}^2$ is the deceleration parameter). Actually Eqs. (21) and (22) could also be obtained by replacing the proposed solution (19) in the Lagrangian (18) and then varying with respect to $N(t)$ and $a(t)$; this is a typical feature of high symmetry solutions. Note that $S_\mu{}^{\nu\rho}T^\mu{}_{\nu\rho} = -6H(t)^2/N(t)^2$, so λ in (18) will prevent the Hubble parameter from becoming infinite. As it was expected, Eq. (21) is not a dynamical equation for $N(t)$ but a constraint for $a(t)$ (“initial value equation”), as a consequence of the fact that $N(t)$ is not a genuine degree of freedom: $N(t)$ can be absorbed by redefining the t coordinate, so we will choose $N(t) = 1$.

By differentiating Eq. (21) with respect to t and combining it with Eq. (22), the fluid energy-momentum conservation is obtained:

$$\frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}a^3. \tag{23}$$

If the fluid is described by the state equation $p = \omega\rho$ then one obtains

$$a^{3(1+\omega)}\rho = \mathrm{constant} = a_o^{3(1+\omega)}\rho_o, \tag{24}$$

where a_o and ρ_o indicate the presentday values.

Combining Eqs. (21) and (22) it results

$$1 + q = \frac{3}{2}\frac{(1+\omega)}{(1 + \frac{16\pi G}{\lambda}\rho)(1 + \frac{8\pi G}{\lambda}\rho)}. \tag{25}$$

In general relativity ($\lambda \to \infty$) an accelerated expansion ($q < 0$) is only possible if $\omega < -1/3$ (negative pressure). Instead, in Born-Infeld modified teleparallelism an accelerated expansion can be handled without resorting to negative pressure; a large density ρ is sufficient:

$$\frac{32\pi G}{\lambda}\rho > -3 + \sqrt{13 + 12\omega}. \tag{26}$$

Actually, for $\rho \to \infty$ in (25), it is $q \to -1$ and the expansion becomes exponential.

In a context where the cosmological model had spatial curvature, Eq. (21) would define the *critical density* ρ_c making the universe spatially flat. Therefore, it is useful to measure the contributions to the density coming from different constituents as fractions $\Omega_i = \rho_i/\rho_c$. In this way, by combining Eqs. (21) and (24) we obtain

$$\left(1-\frac{12\dot{a}^2}{\lambda a^2}\right)^{-1/2}-1=\frac{16\pi G}{\lambda}\sum_i \rho_{oi}\left(\frac{a}{a_o}\right)^{-3(1+\omega_i)}, \quad (27)$$

which can be rewritten in the form

$$\dot{\mathrm{x}}^2+\mathcal{V}(\mathrm{x})=0, \qquad \mathrm{x}=\frac{a}{a_o}, \quad (28)$$

$\mathcal{V}(\mathrm{x})$ being an effective potential given by

$$\mathcal{V}(\mathrm{x})=\frac{\lambda}{12}\mathrm{x}^2\left[\left(1+\beta_o\sum_i \Omega_{oi}\mathrm{x}^{-3(1+\omega_i)}\right)^{-2}-1\right], \quad (29)$$

where $\beta_o \equiv (1-12H_o^2/\lambda)^{-1/2}-1$, is a constant. The potential is always negative and vanishes with null derivative when $a \to 0$, for any value of ω. Moreover, if $\omega > -1/3$ the potential will asymptotically approach zero when x goes to infinity. Instead if $\omega < -1/3$ then $\mathcal{V}$ will be a decreasing function. More relevant is the fact that $|\mathcal{V}|$ is proportional to x^2 when x goes to zero, so giving an exponential expansion for the early universe, as was anticipated. If $\omega > -1$ then the initial behavior is $a(t) \propto \exp[(\lambda/12)^{1/2}t]$. Therefore, the Hubble parameter is equal to the maximum value $H_{\max}=(\lambda/12)^{1/2}$ at the early stage. Equation (28) also says that

$$H(z)^2=H_{\max}^2\left[1-\left(1+\beta_o\sum_i \Omega_{oi}(1+z)^{3(1+\omega_i)}\right)^{-2}\right], \quad (30)$$

where $z=a_o/a(t)-1$ is the redshift. Equation (28) for only one constituent ($\Omega=1$) can be easily integrated to obtain the evolution in an implicit way:

$$\ln[2(1+v)+2\sqrt{v(2+v)}]-\sqrt{v^{-1}(2+v)}=\mathcal{T}, \quad (31)$$

being $v=\beta_o(a/a_o)^{-3(1+\omega)}$ and $\mathcal{T}=-3(1+\omega)H_{\max}t$.

IV. CONCLUDING COMMENTS

Figure 1 shows the dimensionless scale factor $a(t)/a_0$ as a function of H_0t for several values of $\alpha=H_{\max}/H_o$, as implied by Eq. (31) with $\omega=1/3$. The standard ($a/a_0=(2H_0t)^{1/2}$) behavior is plotted as a reference (dashed curve). Remarkably, modified teleparallelism smooths the singularity because the scale factor goes to zero asymptotically.

The main feature of the scale factor behavior is its asymptotic exponential character for any value of ω. This means that $H(z)$ becomes a constant when z goes to infinity. This feature implies that the particle horizon radius $\sigma=a_0\int_0^{a_0}(a\dot{a})^{-1}da$ diverges. Hence the whole space-time ends up being causally connected, in agreement with the isotropy of the cosmic microwave background radiation. This fact appears as an essential property of modified teleparallelism which does not require any special assumption about the sources of the gravitational field.

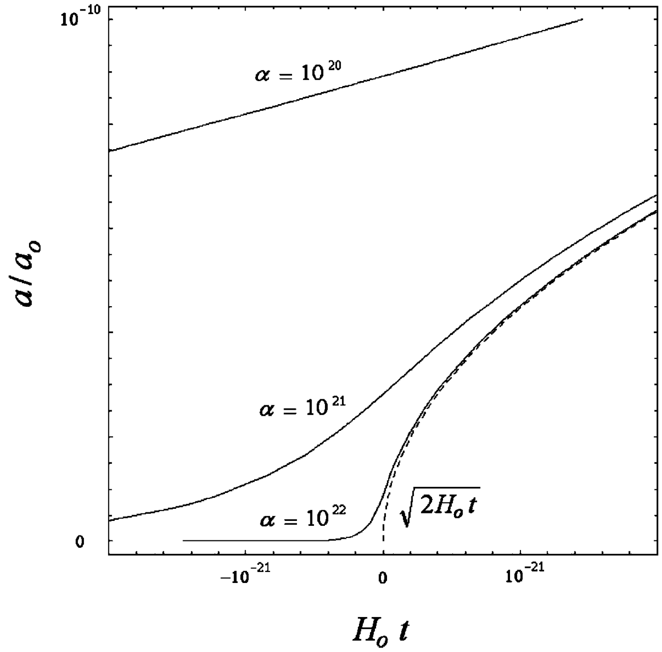

FIG. 1. Scale factor as a function of the cosmological time for $\omega=1/3$ and different values of $\alpha=H_{\max}/H_o$. The dashed line represents the GR behavior.

The standard big bang model successfully explains the relative abundances of light elements. Therefore, a modified gravity theory cannot noticeably change the standard evolution of the universe from the epoch of nucleosynthesis. This means that $H(z)$ at $z_{\mathrm{nuc}} \sim 10^9$–$10^{10}$ should not appreciably differ from its standard value. Figure 2 shows how the Hubble parameter moves away from the GR behavior, represented by the dashed line, to approach the value $H_{\max}$ as the redshift increases. The redshift z_t characterizing the transition between both behaviors can be defined as the value of z at which the asymptotic lines intersect. Since the GR behavior for only one constituent is

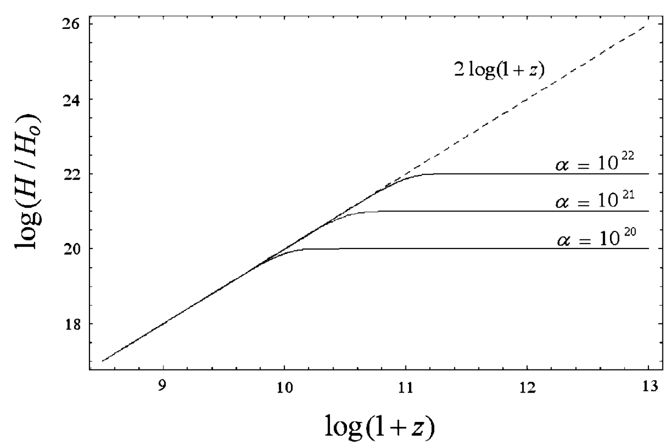

FIG. 2. Hubble parameter as a function of the redshift for $\omega=1/3$ and different values of $\alpha=H_{\max}/H_o$. The dashed line represents the GR behavior.

$\log(H/H_o) = (3/2)(1+\omega)\log(1+z)$, one obtains

$$(1+z_t)^{3(1+\omega)/2} = \frac{H_{\max}}{H_o}. \tag{32}$$

The condition $z_t \gg z_{\mathrm{nuc}}$ implies a lower bound for $H_{\max}$. For a radiation dominated universe ($\omega = 1/3$) one obtains that $H_{\max}/H_o \gg 10^{18}$.

Although inflation without inflaton was already obtained in the framework of Einstein-Cartan theories (see for instance Refs. [23,24]), those solutions rely on the existence of spinning matter (the antisymmetric part of the energy-momentum tensor does not vanish). On the contrary, an inflationary phase exists in modified teleparallel gravity for a symmetric energy-momentum tensor. In this case the inflation is ruled by the parameter λ entering the Born-Infeld Lagragian. $\lambda^{-1/2}$ has dimensions of time, and behaves as a scale governing the transition from the inflationary phase to the standard GR regime. Besides giving the value of $H_{\max} = (\lambda/12)^{1/2}$, λ controls the redshift at the transition between both regimes.

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